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An Investigation of the Instructional Norms of Mathematical Communication when Students Present Geometry Proofs at the Board

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AN INVESTIGATION OF THE INSTRUCTIONAL NORMS OF MATHEMATICAL COMMUNICATION WHEN STUDENTS PRESENT GEOMETRY PROOFS AT THE BOARD

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An Abstract of the Thesis Presented
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Expert mathematical communication is a synthesis of speaking, writing, diagramming, and gesturing. What opportunities are there for secondary mathematics students to learn the discipline-specific ways in which these modalities can be combined to communicate proofs? In an initial effort to investigate this question, Dimmel and Herbst (2020) conducted a multimedia survey experiment to test a hypothesis about how secondary mathematics teachers expect students to communicate when presenting proofs at the board in secondary geometry classrooms. Their hypothesis, based on an analysis of episodes of instruction from a small sample of secondary geometry classrooms, was that teachers expected student presentations of proofs to default to mark-for-mark reproductions of previously completed proofs. Dimmel and Herbst (2020) referred to this practice as *proof transcription*. A key aspect of this practice is that proofs were not expected to be evaluated as mathematical arguments until the transcription was completed. They tested their hypothesis with a survey experiment that used storyboard representations of episodes of geometry instruction to gauge teacher's reactions to instructional

practices that endorsed or challenged proof transcription. They found participants reacted more positively to instructional practices that endorsed proof transcription. While this study provided insight into secondary classroom mathematical communication practices, the result of the study is representative of only a regional population (Midwestern) of secondary mathematics teachers (N=60) all of whom were not geometry teachers. Therefore, I extended this research work by investigating whether the communication practices of secondary school students observed by Dimmel & Herbst (2020) are generally recognized by a larger, nationally representative sample of specifically secondary geometry teachers. My research work is both descriptive—investigating the normative ways teachers expect students to present their proofs—and expansive—investigating teachers' reaction to alternative communication practices thereby drawing attention to ways in which teachers could create opportunities for students to develop multimodal communication skills during their proof presentations.

I analyzed data from a multimedia survey experiment with a nationally representative sample (N = 405) of secondary geometry teachers to investigate instructional communication practices during proof presentation. Participants were shown storyboard depictions of instructional episodes and asked to rate the appropriateness of the (hypothetical) teacher's actions using a Likert-like response format. I analyzed participants' responses using ANOVA. The purpose of the experiment was to investigate how secondary geometry teachers expect students to communicate when presenting proofs during class. My study (1) replicated findings from an investigation of what teachers expect when students present proofs Dimmel & Herbst (2020), (2) investigated how geometry teachers reacted to instructional practices that attempted to steer student presentations of proofs toward disciplinary communication practices, and (3) investigated how geometry teachers justify or criticize hypothetical instructional scenarios where proof transcription were either allowed or disrupted. The findings from this study support findings from existing literature that teachers expect students' presentation of proofs to default to proof transcription. Similarly, it provides evidence that teachers can create opportunities for students to develop and practice disciplinary approximate communication practices during their proof presentations. The warrant for this claim is evidenced by teachers' generally positive reactions to episodes that steered students toward engaging in multimodal and disciplinary-approximate presentations of their proofs.

DEDICATION

I would like to dedicate this work to my family and friends. I appreciate your support.

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CHAPTER ONE

INTRODUCTION

1.1 Background to the Study

According to the National Council of Teachers of Mathematics, “Communication is a fundamental aspect of mathematics and mathematics education” (NCTM, 2000, p.60). The communication standard of NCTM emphasizes how crucial it is for young children to coherently express their mathematical reasoning to their peers and teachers using mathematical language. Mathematical communication enhances students' cognitive functioning by helping them articulate their ideas and strategy (Jordak & Abu Zein, 1998; Kosko & Wilkins, 2010). Engaging in discussions, an integral aspect of communication, allows students to reflect on topics through social interactions with others participating in the same activity. Students also get familiar with specific ways of explaining mathematics while they are doing mathematics (Kosko & Wilkins, 2010; Lee, 2006).

Proof plays an essential role in creating, developing, and communicating mathematical knowledge and is considered an important aspect of students’ mathematical learning experiences ((Ball & Bass, 2003; Herbst & Brach, 2006; Stylianides, 2019). Giaquinto (2005) proposed three general categories of actions for engaging in any piece of mathematics: creating, presenting, and evaluating it. In the context of proving, these three processes may be regarded as constructing a new argument, presenting a constructed argument, and reading an existing argument (Mejía-Ramos, 2008; Mejía-Ramos & Inglis, 2009). Many research studies have focused on secondary students' constructions of mathematical proofs using research instruments that analyzed students written proofs (e.g., Healy & Hoyles, 2000; Kuchemann & Hoyles, 2001; Knuth et al., 2009). Findings from these studies indicate that many secondary students have difficulties with producing written arguments that meet the standard of mathematical proof.

Compared to the literature on students' written proofs, relatively little has been written about how students present proofs (Dimmel& Herbst, 2020; Kokushkin et al., 2022; Stylianides, 2019). Various modes of representation can be used when presenting proofs, including written and oral presentations, gestures and diagrams, and figure representations (Stylianides, 2019). Several studies (e.g., Lai & Weber, 2014; Weber, 2004; Artemeva & Fox, 2011) have demonstrated how mathematicians employ various combinations of these presentation techniques when presenting proofs in front of the class. For example, Weber (2004) examined the case of a professor presenting proofs in an introductory real analysis course. The findings from the study indicated that the professor verbalized the proof's steps as they were written, offered commentary on the proof's development, and gave an overview of the proof before generating it.

Considerably fewer studies have researched students' presentation of proofs at the board. At the undergraduate mathematics research level, Stylianides (2019) compared the oral arguments that secondary students made in front of the class with the written arguments they created during small group work for the same assertions. The results of the study indicated that the verbal expression of proofs might relieve students from the need to compress their inner speech and provide an opportunity for students to communicate the intended meaning behind implicit arguments, which might otherwise be impossible because of the structure of written arguments. While this study explored how students communicate their proofs when an expert requires them to do so in disciplinary-specific ways, it neither provided any insight into the broader communication practices typical in a mathematics classroom nor investigated teachers' expectations during such presentations.

To describe these broader communication practices, Dimmel & Herbst (2020) conducted a two-part, mixed methods study. First, they analyzed video records of geometry classrooms when students were asked to present proofs. They found that, in contrast to the multimodal practices of mathematicians, students engaged in mark-for-mark reproductions of previously completed proofs—an action described by the authors as *proof transcription*. A key aspect of this practice is that proofs were not expected to be evaluated as mathematical arguments until the transcription was completed. Second, they conducted a multimedia survey experiment in which hypothetical episodes of geometry instruction, represented by cartoon storyboards, were shown to a sample ($n = 60$) of midwestern secondary mathematics teachers. The storyboards showed students transcribing proofs and depicted different actions a teacher might take in response. One version of the storyboard depicted the teacher interfering with the students' proof transcription by insisting that the students' proof be mathematically coherent as it is being produced—e.g., adding diagram labels before using such labels in their written statements. The other storyboard depicted the teacher allowing the students to simply transcribe their proofs. Participants in the study reacted more positively to episodes of instruction that depicted teachers allowing students to transcribe their proofs than to episodes of instruction that depicted teachers interrupting those transcriptions. Their study suggests that secondary mathematics teachers recognized proof transcription as an acceptable means for students to present proofs (Dimmel & Herbst, 2020). That teachers recognized proof transcription as routine is significant because it suggests that there are limited opportunities for students to develop fluency with the multimodal communication practices for presenting proofs.

This present study builds on this prior work in three ways. One, this study replicated the statistical tests described by Dimmel & Herbst (2018, 2020) with a larger, nationally representative sample of secondary geometry teachers. Dimmel & Herbst (2018, 2020) were limited to a regional sample of midwestern teachers, not all of whom were geometry teachers at the time the study was conducted. The replication part of the study is significant because it provides an opportunity for further empirical testing of a phenomenon that has been previously described and, considering the ongoing replication crises in human subject research, such an investigation is non-trivial (Shrout & Rodgers, 2018). The replication part of the study investigated whether this representative sample of geometry teachers recognized proof transcription as a routine instructional practice during proof presentation.

Two, building on the foundation that students' proof presentations are expected to default to a mark-for-mark reproduction of written proofs, this study used two additional storyboards created to depict instructional actions that attempted to steer students toward developing multimodal communication skills during their proof presentations in geometry classrooms. While the findings from the study conducted by Dimmel & Herbst (2020) offer some insight into secondary mathematics teachers' expectations regarding how their students' present proofs, the design of the storyboards leaves open the possibility that teachers were merely responding or attending to teaching actions that represented a departure from other classroom instructional norms—e.g., an expectation of how students should be treated when they are in socially vulnerable situations such as when sharing their written work at the board, rather than attending to the communication practice depicted in the instructional scenario. Therefore, this present study provided a different way of scaffolding expert communication practices by designing storyboards targeting alternative communication practices designed to steer students away from simply transcribing their written proofs toward a multimodal presentation of their proofs. These two storyboards were designed to investigate how teachers would react to instructional episodes that require that students' modes of communication exemplify the disciplinary practices of mathematical communication rather than an unreflective transcription of proofs. I probed teachers' reactions to such alternative communication practices (e.g., giving a verbal overview of the plan for the proof before writing it on the board) to investigate (1) whether secondary geometry teachers find such disciplinary approximate communication skills to be disciplinary and pedagogically valuable, and also to enumerate (2) the potential challenges with enacting instructional changes in relation to mathematical communication in the secondary mathematics classroom. This is significant because it reveals what opportunities might be available for secondary students to hone their mathematical communication skills during mathematics instruction.

Three, I investigated the *practical rationality* (Herbst & Chazan, 2012) behind the enacted norms by analyzing how teachers justified or criticized compliance or breach of the practice of proof transcription using the framework of professional obligations (Herbst & Chazan, 2012). This is significant because the social and institutional environment of instruction impacts teachers' instructional practices just as much as their individual characteristics. Hence analyzing teachers' perception of their obligations and how that influences their judgment of and reaction to different instructional situations provided an in-depth insight into the practice of mathematics teaching that could help bridge the gap between practice and research. This study thus contributes fine-grained, subject-specific knowledge to our understanding of instructional norms that relate to mathematical communication practices, as well as practitioner-centric accounts of the challenges of implementing instructional changes aimed at enhancing students' mathematical communication abilities.

1.2 Research Question

I operationalized my overarching question about how students learn discipline-specific mathematical communication practices into three research questions:

1. Do secondary geometry teachers recognize proof transcription as a routine means for students to present proofs on the board?
2. How do secondary geometry teachers react to instructional actions that ask students to engage in approximations to disciplinary communication practices?
3. How do secondary geometry teachers justify or criticize hypothetical instructional actions of geometry teachers as they pertain to student presentations of proofs?

CHAPTER TWO

REVIEW OF RELEVANT LITERATURE

2.1 Mathematical Communication

Mathematical cognition and competence are developed through communication (Sfard,2008). Mathematical communication skills include the ability to present mathematical ideas using various modalities such as writing, speaking, drawing diagrams, gesturing, and other representations. A key component of mathematical communication is the ability to express and justify arguments (NCTM, 2000). Students' knowledge of mathematics can be improved through oral and written mathematical communication. (Triana & Zubainur, 2019). Mathematical communication can also facilitate the exchange of ideas among students (Chung et al., 2016).

Bill Barton (2008), in his study "The evidence from Language", explored the intersection between language and mathematics. He emphasized that "Mathematics is created by communicating, that is, mathematics is created in the act of communication. Learning mathematics, and doing mathematics, involves talking mathematics: the more we talk mathematics, the better we will learn it and do it" (Barton, 2008, p. 173- 174). Even though communication plays a significant role in the teaching and learning of mathematics, it has been noted that mathematical communication skills are rarely explicitly taught in mathematics curricula (Wood, 2012). How then can students learn fluency with mathematical communication practices used by experts? One classic viewpoint is the practice of apprenticeship, in which "novices tacitly develop discipline-specific communication skills as they apprentice into a field" (Dimmel & Herbst, 2017, p.151). This viewpoint stresses the fact that students can only learn to communicate in discipline-approximate ways as they become experts, Nevertheless, studies on mathematical discourse have shown that discipline-specific communication practices can both be investigated and explicitly taught. (Fang, 2012; O'Halloran, 2010; Wood, 2012; Yore, Pimm, & Tuan, 2007).

But what characterizes expert mathematical communication? Weber (2012), in a study on mathematicians' perspectives on their pedagogical practices with respect to proof, noted that certain mathematicians, when asked how they would present specific proofs to their students, described their steps as follows " Accompany the proof with an example, draw and use a diagram, emphasize new features of a proof that were not present in the previous proofs, recall relevant facts prior to presenting the proof, use dialog format so students are involved with the proof construction"(Weber, 2012, p.11). The study thus emphasized the notion that mathematicians employ multiple modalities of communication when they present their mathematical knowledge to an audience. In a different study, Forman (1996) identified specific argumentation styles, such as clarity, conciseness, and logical coherence, as characteristics of academic mathematical communication practices. Other highly regarded techniques in mathematical communication are abstraction and generalization (Moschkovich, 2007). One way that we can further explore experts' modes of communication during mathematical discourse is to explore their modes of argumentation or proving.

Proofs are a crucial component of mathematical practice and an essential element of mathematics education. Mathematicians and mathematics educators have similar goals for presenting proofs to students. Such goals include presenting proofs for explanations, creating conviction, and modeling the mathematical culture of proving to students (Bell, 1976, 2003; Bieda & Lepak, 2014; Flores, 2006; Harel & Sowder, 1998; Weber, 2012). Thus, proofs and proving activities provide a framework for investigating mathematical communication practices. What are the opportunities available for students to hone their communication skills during proof presentations? In the rest of this review, I report existing educational research on proofs and proof presentation—particularly, experts and students' modalities of communication when presenting their proofs.

2.2 Proofs and Proof Presentation in Mathematics Education Research

In the literature, three primary proof-related tasks have been identified: proof construction, proof presentation, and proof reading (Mejia-Ramos, 2008). According to Mejia-Ramos and Inglis (2009), proof reading activities can be further subdivided into comprehension- and validation-based reading. Existing literature on students' written proofs in mathematics education has primarily focused on proof construction (e.g., Basturk, 2010; Healy & Hoyles, 2000). Such studies have found that students struggle to create formally presented deductive arguments and to distinguish between deductive proof and empirical evidence (Chazan, 1993; Harel & Sowder, 1998; Healy & Hoyles, 2000; Schoenfeld, 1989; Harel & Sowder, 2007), have insufficient specific content-area knowledge (Azrou & Khelladi, 2019; Moore, 1994), and have difficulty interpreting the logical structure of the statement to be proved (Zandieh et al., 2011).

Mejía-Ramos & Inglis (2009) suggested that proof reading (comprehension and validation) and proof presentation are two crucial actions involved in the evaluation of students' proving skills. Still, neither has received a significant amount of scholarly attention. I review existing literature on these key proving activities.

2.2.1 Proof Reading

At the undergraduate level, students are typically introduced to new mathematical concepts through reading proofs in class, lecture notes, and textbooks (Rav,1999; Selden & Selden,2003;

Weber, 2004). However, existing research studies have shown that students struggle with reading proofs that have been given to them (Inglis & Alcock, 2012; Lin & Yang, 2007) and evaluating the validity of proofs (Hoyles & Healy, 2007). Selden & Selden (2003) found that students concentrate majorly on the surface characteristics of given proofs. Using eye tracking, Inglis and Alcock (2012) affirmed that undergraduates focused more attention on the algebraic portions of proofs and paid less attention to the surrounding proof text where logical claims are often made explicit. They also found that mathematicians made much more transitions from one line of proof to the next in an attempt to infer implicit between-line warrants, whereas students read more linearly when attempting to comprehend written proofs.

Mejia-Ramos et al. (2012) developed a theoretical model for assessing proof comprehension to address the limited research on proof reading. This model comprises seven dimensions which address (2012) both a “proof's local comprehension and holistic comprehension” (Selden & Selden, 2017). Local comprehension was defined as knowing the definitions of important terms, the logical standing of the claims in the argument, understanding the structure of the evidence, and understanding how and why each statement proceeded from the ones before it. While holistic comprehension is defined as having the ability to instantiate the proof using examples, recognize the subproofs and how they relate to the proof's structure, and transfer the ideas of the proof to other proving tasks (Selden & Selden, 2017).

2.2.2 Proof Presentation

Proof presentation involves the act of presenting an argument to demonstrate one’s understanding of it (Mejía-Ramos & Inglis, 2009). Hence, it represents the process of engaging in mathematical talk and sharing one’s mathematical knowledge with an audience, usually a mathematical audience or classroom community. Having students present their proofs at the board offers many benefits including providing students with the opportunity of demonstrating their knowledge of mathematical proofs and proving, hence proof presentation has gained significant attention from mathematics education researchers interested in investigating students' conception of proofs.

Various modes of representation can be used when presenting proofs, including written and oral presentations, gestures and diagrams, and figure representations (Stylianides, 2019). Few research studies have focused on how mathematicians, mathematics educators, or students employ various combinations of these presentation techniques when presenting proofs in front of the class. For example, Pantaleon et al. (2018) investigated the communication process of a prospective mathematics teacher in a proving task of geometry and algebra. The authors found that when presenting their proofs, the pre-service mathematics teachers' sequence of actions includes an explanation of what is understood, an expression of the idea in the form of a drawing or symbol, an explanation of the idea or argument, a presentation of the proving steps, and reinforcement of the results obtained. Similarly, other studies have shown that mathematics professors when presenting their proofs give an overview of the proof before generating it, explain the proof's steps as they are written, and provide commentary on the proof's development. (e.g., Weber, 2004)

Within the undergraduate proof literature, Soto-Johnson (2012) assessed undergraduate students' oral and written proofs in abstract algebra. The findings from this study indicated that whereas students struggled with writing their abstract ideas coherently, they were able to generate sound oral arguments. In a different research, Fukawa-Connelly (2012) described a case study of a college-level abstract algebra class where students were encouraged to present and discuss proofs. The author, through observation and analysis of video recordings of class meetings, discusses the various norms enacted by the class communities during and after the presentation of their proofs. For example, student presenters were expected to explain and defend their work while the class decided on the validity of the presented proof. In another study, Malmstrom and Eriksson (2018) created an intervention for a multivariable calculus course for undergraduates in a Scandinavian country. In this intervention, students were required to present a summary of the information learned over the previous class week on the chalkboard. “The intervention's goal was to give students a chance to participate in chalk talk, the discipline-specific communication techniques used by mathematicians to regenerate arguments in real-time at the chalkboard” (Dimmel & Herbst, 2020, p.74).

Fewer studies have investigated students’ communication of proofs at the secondary school level. Winer & Battista (2022) analyzed students' oral proof explanations and their written proofs in geometry. They found that while most students used sound reasoning in their spoken explanations, they had trouble expressing that reasoning in their written two-column proofs. They argued that students’ difficulties with written proofs are a consequence of formalizations rather than a lack of deductive reasoning abilities. They emphasized that students are not fully aware of or do not see the necessity for all the details required to write formal proofs with the proper logical and axiomatic structuring. The findings from this study are consistent with similar studies conducted by Stylianides (2019), who found that students provided a sound oral argument compared to their written expressions of the same arguments.

The studies described above have primarily investigated novices' or experts’ presentation of proofs with the goal of understanding their proof conception. While this is a valuable research strand, proof presentation can also extend our understanding of experts' and novices' communication practices. This is an equally important research strand that has gathered very little attention. The research work conducted by Dimmel & Herbst (2020) as described in chapter 1 of this thesis, attempted to bridge this research gap by investigating teachers' expectations of students' communication practices during proof presentations in geometry classrooms. Similarly, this present study extends our understanding of secondary classroom communication practices and the potential of implementing instructional changes that would improve students' mathematical communication.

2.3 Theoretical Framework

2.3.1 Instructional situations and Background Expectations

Mathematics classrooms can be modeled as didactical systems that consist of interactions among teachers, students, and mathematical content (Chevallard, 1980; Cohen et al., 2003; Herbst & Chazan, 2012). To analyze such systems, Brousseau (1997) proposed the existence of a didactical contract - a set of implicit rules that binds the teacher, students, and the mathematical content to each other and the environment of instruction or the instructional milieu. A simple statement of the general didactical contract is the teacher is expected to teach content that students are expected to learn (Herbst & Chazan, 2012). In the specific setting of geometry classrooms, the role of a geometry teacher ties the teacher contractually to teach the content "geometry". On the other hand, the students are expected to engage in the learning activities devised by the teacher. Both the teacher and the students are expected to satisfy the environmental conditions of the instructional system e.g., the school, which requires the teacher to teach concepts relevant to the mathematical domain of geometry within a given timeframe and expects the students to satisfy institutional prerequisites before taking the class. The existence of the didactical contract allows for a conceptualization of teaching as a set of socially situated practices that occur in instructional situations (Herbst, 2006)—i.e., "identifiable segments of instruction that are organized around specific kinds of mathematical work, such as doing proofs in geometry or solving equations in algebra" (Dimmel & Herbst, 2020, p.7).

Norms are familiar ways of observing, believing, evaluating, and behaving in an environment (Goodnough, 1971). They are defined in terms of the features of a social situation that not only regularly occur but that participants expect to occur (Garfinkel, 1963; Herbst & Chazan, 2003). These norms are generally unstated yet transparent to those within the social situation. Although unstated, norms have been shown to have a considerable influence on human behavior (Boileau, 2021; Legros & Cislaghi, 2020). This means that "norms may carry weight without needing to be formally institutionalized and that they affect behavior simply by someone perceiving that a particular rule ought to be followed"(Hora & Anderson, 2012).

For instance, "how groups of strangers organize themselves to wait in lines when boarding an airplane, or how passengers stand when riding in an elevator are examples of the unwritten rules people are expected to follow"(Dimmel & Herbst, 2020, p.6). Similarly, within the instructional setting, there are unspoken norms that guide the actions of teachers and students for example, many (unspoken) didactical contracts for the teaching and learning of algebra contain as a norm that the teacher expects students to provide traceable written records of the methods by which they solved a set of problems—i.e., students are expected to show their work (Herbst & Chazan, 2012). Students could breach this norm by including only a solution, or perhaps even by using a different solution method than the method that the teacher expected for a particular problem. In such instances, the teacher could sanction the students by deducting their grade points, which, from the perspective of the students, might appear to be a breach of the didactical contract—e.g., *I got the correct answer, why did I lose points?* Thus, identifying the norms of mathematics instruction is a means to describe the current state of teaching and learning mathematics in schools.

Instructional situations provide a way of framing the routine instructional activities of teachers and students in those situations, thereby investigating implicit teaching actions without mediating or making them explicit. Researchers have identified and developed ways of investigating routine instructional practices of teachers and students within mathematics instruction (Dimmel & Herbst, 2018; Herbst & Chazan, 2015; Herbst, 2006). Thus, normative instructional activities, though invisible, can be investigated by observing how teachers react to scenarios that represent a departure from situational norms (Herbst & Chazan, 2015). The present study used the breaching experiment approach (Garfinkel, 1963) within a specific instructional situation—proof presentation in geometry (Herbst, 2006)—to investigate: (1) the normative ways secondary mathematics teachers expect students to present proofs at the board; and (2) secondary mathematics teachers' reactions to alternative mathematical communication practices. Specifically, I analyzed teachers' reactions to instructional scenarios that represented departures from the norm of proof transcription.

2.3.2 Practical rationality

Herbst & Chazan (2012) proposed the framework of practical rationality, which, among other features, outlines the sources of justification teachers can use when accounting for their instructional decisions or choices. The practical rationality framework conceptualizes teaching as "a system of activities involving positions, roles, and relationships, where individual choices can be made but at a cost” (Herbst & Chazan, 2012, p.). Teaching is therefore explained as a set of socially situated practices regulated by norms and expectations.

The above professional resource complements studies of beliefs (e.g., Clark & Peterson, 1986; Fang, 1996; Kagan, 1992; Thompson, 1992) and knowledge (e.g., Hill, Rowan & Ball, 2005; Hill et al., 2008) by taking into account that instruction takes place within larger educational systems with a variety of stakeholders and thus the rationality of teaching can be explained not only by the individual choices of the teachers but in terms of the norms of the instructional activity and the professional obligations of the mathematics teacher (Herbst & Chazan, 2012).

Professional Obligations

Herbst and Chazan (2012) proposed four stakeholders within the environment of instruction to which every mathematics teacher as a professional must respond: the knowledge represented by the discipline of mathematics, the society, the students, and the organization. These four stakeholders correspond to four categories of professional obligations: disciplinary obligation, interpersonal obligation, individual obligation, and institutional obligation (see Table 1 for definitions).

Table 1: Professional Obligations, Definitions Adapted from Chazan et al. (2016)

Disciplinary Obligation: The expectation that mathematics educators should represent mathematical knowledge in a way authentic to mathematics as a discipline.
Individual Obligation: The expectation that teachers acknowledge students as individuals and attend to their individual needs including their social, emotional, and cognitive needs.
Interpersonal Obligation: The expectation that the teacher shares and stewards their medium of interaction with others in the classroom.
Institutional Obligation: The expectation that the teacher follows institutional policies such as behavioral policies, school curriculum, equipment, and facilities.

In accordance with the discipline of mathematics, mathematics teachers have the responsibility to provide an accurate representation of mathematical knowledge and practices (disciplinary obligation). With regards to the organization, the mathematics teacher has institutional obligations to the school which include the department (e.g., textbook selection, curriculum coverage), school schedules (e.g., calendar, bell schedules), school district (e.g., assessment goals), and professional associations and unions (e.g., workday length). In response to the students, the mathematics teacher has the obligation to treat each student as an individual with specific needs, abilities, and skills. The obligation to the students also extends to the social environment of learning which includes the shared classroom resources of knowledge and space. The mathematics teacher has the obligation of managing the community of learners in the classroom and ensuring equitable access to classroom resources.

Instructional Norms

Normative or routine teaching actions can be described as recognizable sequences of instructional actions or events in the mathematics classroom (Erickson et al., 2021). One example of an instructional norm of doing geometry proofs is that “students are expected to justify a statement in a proof with a reason before moving on to the next statement” (Herbst & Chazan 2011, p.412). While mathematics instruction can be described as abiding by specific sets of norms, these norms are not absolute, and thus they can be broken. For instance, one could find a geometry teacher who had let a student make a new claim without having justified the previous claim.

Although a teacher may not be required to justify teaching actions that are deemed to be routine or normative due to the general acceptance of such actions, a breaching action may require “special maneuvers and the explicit support of justifications that appeal to one of four professional obligations: to the individual student, to the classroom community of learners, to the discipline of mathematics, and the institution of schooling” (Erickson et al., 2021, p.4). Thus, the professional obligations of mathematics teachers might provide a source of justification for more ambitious teaching activities. For example, a geometry teacher might appeal to the disciplinary obligation as justification for asking students to declare diagrammatic givens in a specific proof activity, such as asking them to provide statements and justifications for the existence of a point of intersection between two angle bisectors of a parallelogram, a detail that Dimmel & Herbst (2018) noted is not typically required from student proofs. Such a teacher might defend this instructional strategy on the grounds of the need to uphold disciplinary standards by giving students the opportunity to comprehend the relationships between mathematical axioms and theorems. The individual obligations may also support a teacher's decision to forgo requiring such specifics in a student's proof because doing so could result in students being overburdened with information.

In this study, I investigated how teachers draw on their professional obligations to justify or criticize teachers' actions in the hypothesized instructional scenarios represented in both the replication study and the alternative communication practices study.

For the replication part of the study, I was interested in analyzing how participants in the treatment storyboard might justify departures from the routine practice of proof transcription and how participants in the control storyboards might criticize the routine practice of proof transcription. The purpose of this analysis is descriptive, as it could be the case that participants do not reference any obligation in criticizing or justifying their reactions to the storyboards.

For the alternative communication study, I analyzed how teachers might use the professional obligations to support more ambitious but discipline-approximate mathematical communication practices, such as those depicted in the alternative communication storyboards, or how these obligations might prevent teachers from enacting these multimodal communication practices in their instructions. One possibility in that regard could be that teachers simply may not think that the teaching actions depicted in the instructional episodes merit the instructional time allotted to it and that there are more important aspects of instruction to concentrate on. Thus, such a teacher might cite their institutional obligation as a justification for criticizing the instructional action depicted in the episodes. An investigation of teachers' justifications of instructional practices that could be viewed as a step toward reforming mathematics instruction may provide useful information that would support upcoming reforms aimed at enhancing students' mathematical communication or improving mathematics instruction in general.

CHAPTER THREE

METHODS AND ANALYSIS

3.1 Research Design

This chapter includes details of the methods of the present research. It contains a description of the data collection instruments and a description of the participants. I also enumerate the data analysis approaches for the quantitative and qualitative aspects of the study.

3.1.1 VIRTUAL BREACHING EXPERIMENTS

Expanding on the research conducted by Dimmel and Herbst (2018, 2020), the present study used a virtual breaching experiment and a planned comparison between individuals who were randomly assigned to control or treatment conditions to investigate teachers' recognition of a hypothesized instructional norm. The breaching experiment was virtual in the sense that participants were shown storyboard depictions of instruction rather than being asked to take part in those circumstances firsthand (Dimmel & Herbst, 2020; Herbst & Chazan, 2015). To allow participants to project their own experiences onto the episodes of teaching that were created, storyboard representations of classroom instructional scenarios similar to those used by Dimmel and Herbst (2020) were employed. Storyboard representations were made to create an alternative instructional scenario different from routine classroom instruction (Dimmel & Herbst, 2020; Dimmel & Herbst, 2018; Herbst et al., 2011). The alternative scenarios were used to assess secondary mathematics teachers' responses to episodes of teaching that, according to the literature, deviated from what instructors normally anticipated. The study's first goal was to determine whether participants' reactions to the episodes differed depending on the instructional actions that were represented in the different scenarios. The study's second goal was to investigate how teachers in the breach and routine conditions justify or criticize departures from the normative practice of proof transcription. Particularly, we were interested in investigating whether or not teachers recognize the departure from the practice of proof transcription as an action that is justifiable within the disciplinary standards of mathematical communication.

3.1.2 Sequence Norm

The idea that it is normative for students to act as transcribers when presenting their proofs and that a transcription-based proof is not reviewable until the transcription is completed was described as the *sequence norm hypothesis* by Dimmel and Herbst (2020). The sequence norm was developed by Dimmel & Herbst (2020) through a study of video recordings from geometry classes, which showed that students' presentations of proofs consisted of unremarkable replication of written proofs that were not presented in a way that would have made them mathematically coherent. For example, students may complete the transcription of their proofs without labeling the diagram that supports the proof, thereby making the written arguments unclear. They may also write the statements and reasons before drawing the diagram, writing what was given, or expressing the propositions to be proved.

3.1.3 Storyboard Design

In this section, I describe the design of the two storyboards used for the replication study and then the two storyboards targeting alternative communication practices.

Storyboards Targeting Proof Transcription

Storyboards consisted of 12 – 20 frames of classroom activity that were represented using simple cartoon graphics. They were designed in breach/routine pairs: Up until a 3-5 frame segment of storyboard (i.e., the *segment of interest*), the storyboards in a breach/routine pair were identical.

During the segments of interest, students were depicted transcribing proofs. In the routine storyboards, the teacher allowed the student to transcribe the proof without comment or interference. In the breach storyboards, by contrast, the teachers attempted to make the presentation of the proof mathematically accountable as it was being transcribed. Each storyboard had a distracter segment where a teacher performed another distinct teaching activity in addition to the segment of interest; these activities were considered routine instructional activities, e.g., a teacher asking students to check their work, and they were the same for both the breach and routine versions of a storyboard.

The first and second storyboards were targeted at how students employed labels and reasons, respectively, when presenting proofs. The breach versions of these two storyboards show a teacher interfering with the student's presentation in places where the student's presented work could be described as lacking mathematical coherence from a disciplinary standpoint—e.g., referring to labeled angles in the written statements of a proof before adding those labels in the diagram (Figure 1). The control version shows a teacher allowing the students to transcribe their proof without interference. Versions of these storyboards were used in the original study (Dimmel & Herbst, 2020).

3.1.4Storyboard 1 – Label Storyboard

In the label storyboard, a student uses labels for the points in the given figure but labels the angles within the statements of the proof before writing these labels in the diagram (Figure 1). In the breach version of the storyboard, the teacher asks the student to erase the steps, label the angles in the diagram, and then try again. In the routine version, the teacher lets the student add the angle labels after the reasons were written.

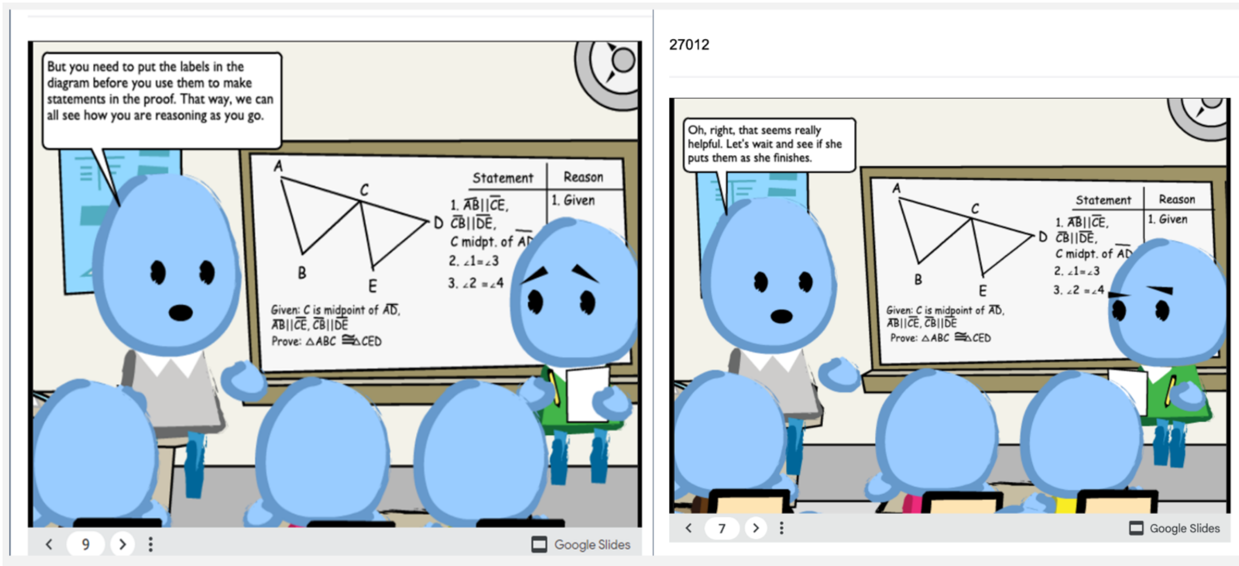

Figure1: Frames from the label storyboard (Left frame – Breach storyboard; Right frame – Routine Storyboard)

3.1.5 Storyboard 2 - Reason Storyboard

In the reasons storyboard, a student copies the proof and omits to provide a justification for one of its claims (Fig. 2). The teacher asks the student to recreate the proof starting from the point where the justification was missed in the breach version of the storyboard. In the routine version, the student just adds the missing justification to the proof without changing any of the other parts.

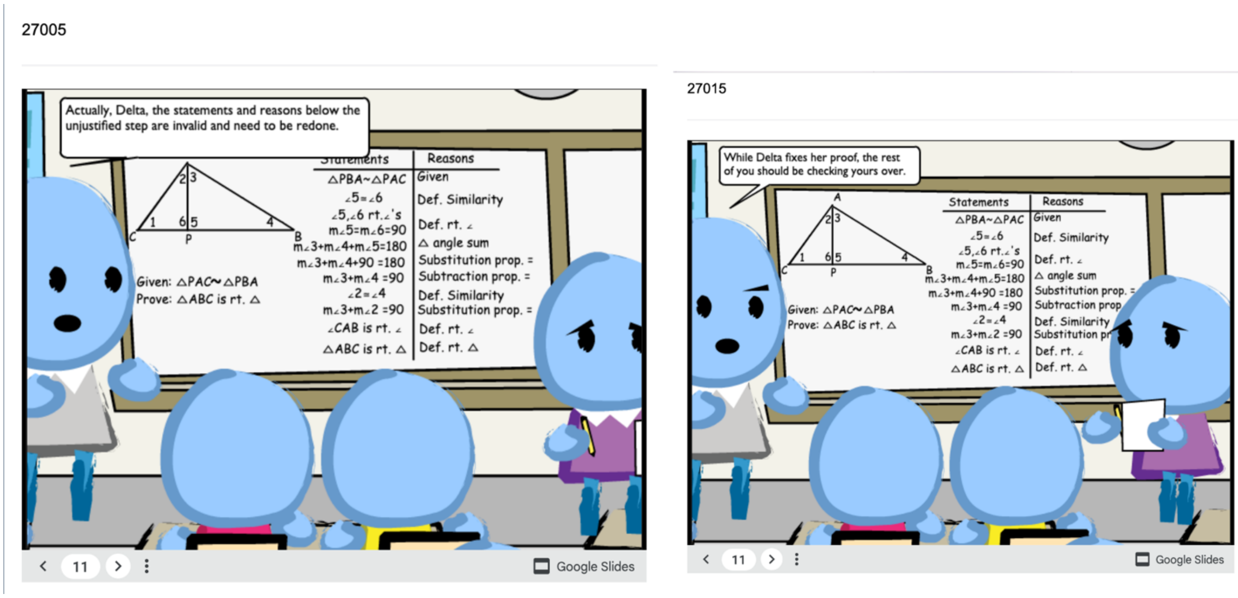

Figure 2: Frames from the Reason storyboard (Left frame – Breach storyboard; Right frame – Routine Storyboard)

3.1.6 Storyboards Targeting Alternative Communication Practices

In addition to the replication of the original results, two additional storyboard pairs were designed to investigate how secondary geometry teachers would react to proof presentations that approximated disciplinary communication practices.

Storyboard A

Storyboard A (Figure 2) depicts an instructional scenario where two students presented a proof at the board. In the treatment version, one student writes the proof and the other verbalizes the proof as it is being written, while in the control version, the students took turns transcribing the proof (Figure 3). When experts present a mathematical argument, they frequently use a variety of communication techniques, such as gesturing, diagramming, and verbal explanations of the proof's processes. What would instruction in a secondary geometry class that aimed to model this style of communication look like? The above question was a motivation behind the design of the instructional scenario in Storyboard A. While it might be unrealistic to expect secondary students' proof presentations to be a replica of an expert's, sharing the responsibility of the proof presentation—having one student write the proof statement and another explains the proof verbally—can be accepted as a realistic approximation of multimodal proof communication in a secondary geometry classroom. Storyboard A was thus designed to illustrate such an instructional scenario.

Storyboard B

Storyboard B compared two possible interventions that a teacher could make to steer a student's presentation of proof toward the multimodal practices that are typical in the discipline. In treatment 1, (Figure 4), the teacher asked the student to provide an overview of the proof strategy and use a conceptual register (Herbst, Kosko, & Dimmel, 2013) to describe the steps in the proof, rather than simply read the statements as they were written on the board. For instance, Dimmel (2014) contrasted novice and experts proof communication using the example:

> while a student could point to "segment AB" and then "segment CD" in a diagram and declare, "segment AB is congruent to segment CD," verbally reading the written statement,…an expert, e.g. a teacher, might verbalize the same step using conceptual speech (e.g., "the opposite sides are congruent") that is co-expressed with non-pointing gestures (e.g., a metaphorical gesture that indicates an equivalence of opposite things)" (Dimmel n.d.(2014), p.298).

Hence comments on proof strategy and register switching are touchstones of mathematicians' proof presentations (Weber, 2004). In treatment 2, the teacher asks the student to include appropriate congruence markings and to point at the diagram. Thus, in storyboard B, both treatments 1 and 2 were created to illustrate further aspects of the disciplinary communication techniques used by experts and how proof instructions may be designed to explicitly teach students these techniques. Although I am aware that the instructional scenarios may be viewed as ambitious for secondary students, I was interested in finding out whether teachers recognized such communication techniques as disciplinary and as having pedagogical value.

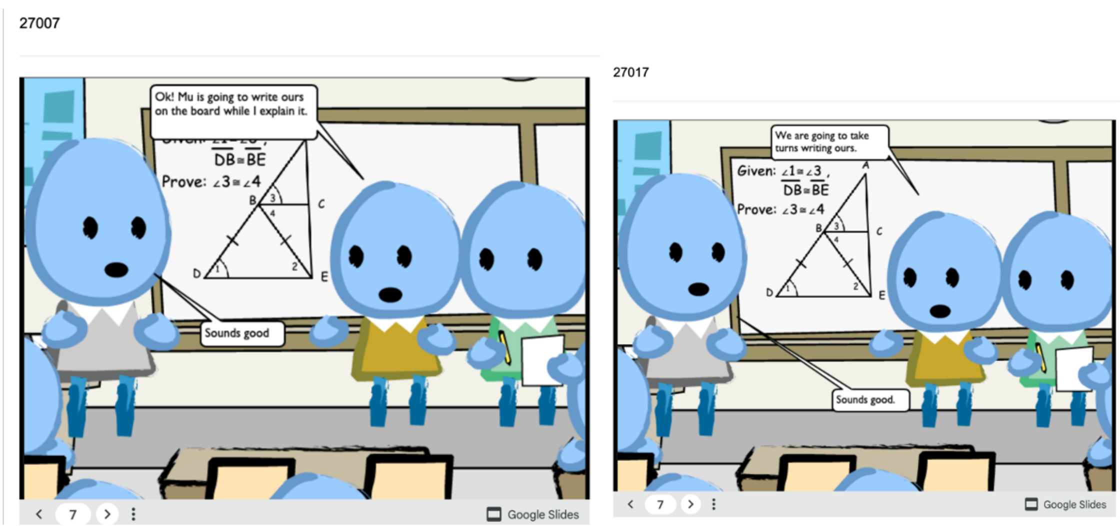

Figure 3: Frames from Alternative Communication Storyboard A (Left frame – Breach storyboard; Right frame – Routine Storyboard)

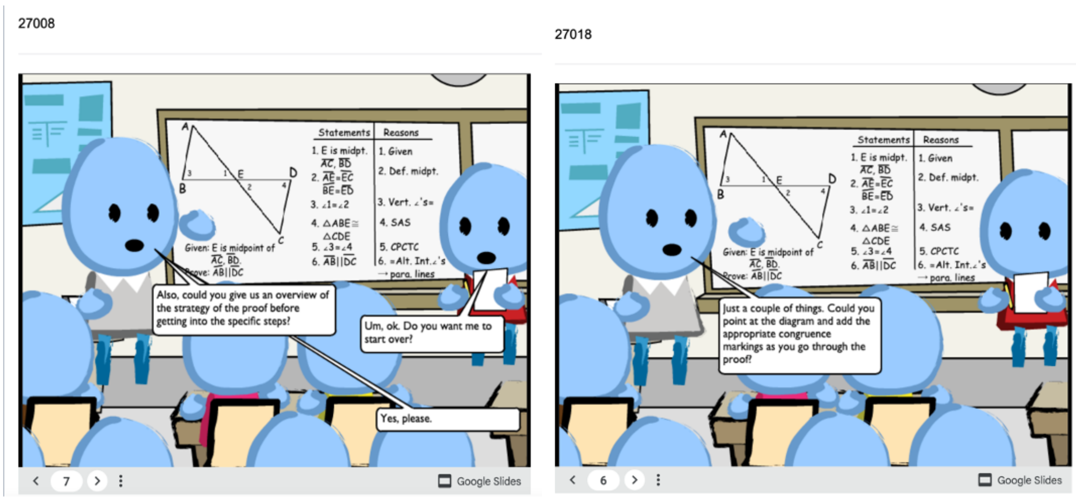

Figure 4: Frames from the Alternative Communication Storyboard B (Left frame – Breach storyboard; Right frame – Routine Storyboard)

3.2 PARTICIPANTS

The participants for this study were 405 secondary mathematics teachers from 46 states within the United States. All participants have experienced geometry teachers with an average of 7 years of experience teaching geometry. 84.6% of them identified as White, 6.31% as Black, 2.78% as Asian, 2.02% as Hispanic, and 0.76% as other. 60.1% of them were female and 39.9% were male.

3.3 DATA COLLECTION

Using the storyboards described above as probes, data was collected using a multimedia survey assigned to designated experiment groups. The survey was administered remotely from 2015 – 2016 and was one of a suite of research instruments that were deployed in a large-scale study of the instructional practices of secondary mathematics teachers (Herbst et al., 2015). The experiment groups were designed such that each participant in the group viewed either a routine or a breach version of the storyboard in each routine/breach storyboard pair. Four open-ended and three closed-ended questions were presented to participants after seeing each storyboard. The first open-ended question the participants were asked was, “What did you see happening in this scenario?" next, participants were asked to use a 6-valued Likert-like answer scale to rate how appropriate they thought the teacher's actions were. The first close-ended rating question was “How appropriate was the teacher’s review of the proof?” This targeted the teacher’s actions throughout the entire instructional episode, The second and third close-ended rating questions “How appropriate were the teacher’s actions in this segment of the story?” was targeted at getting participants to rate the teacher’s actions in the segment of interest and the distracter segments of the storyboard. Each of the three rating questions was followed by a request for participants to "Please explain your rating". The rating choices for the closed-ended rating questions were: 1 (very inappropriate), 2 (inappropriate), 3 (somewhat inappropriate), 4 (somewhat appropriate), 5 (appropriate), and 6 (very appropriate).

3.4 Analysis of Close-Ended Responses

I analyzed participant responses to the closed-ended questions by creating planned comparisons within and between experimental groups. To analyze whether there was a difference between the mean ratings of participants across experimental groups, I generated two sets of hypotheses for the planned comparison of the two storyboards targeting proof transcription. I hypothesized that participants would rate the teacher's work more negatively in episodes that showed a departure from proof transcription than in episodes that showed students transcribing their proofs. I hypothesized that the mean ratings of *episode appropriateness* (EA) and *segment-of-interest appropriateness* (SIA) would be lower in episodes that depicted a departure from proof transcription compared to the episodes that showed transcription of proofs. Similarly, I hypothesized that there would be no difference in the mean ratings of the *distracter segment* *appropriateness* (DSA) across conditions. The second hypothesis regards a comparison within conditions. I compared the segment-of-interest and distracter ratings for both the breach and routine conditions, I hypothesized that the mean ratings of SIA would be lower than that of DSA in the breach condition and there would be no difference between the ratings in the routine condition.

For the two storyboards targeting alternative communication practices, I hypothesized that participants would rate the teacher's work more negatively in episodes that showed disciplinary-approximate practices of proof communication e.g., requiring that students provide an overview of proof strategies before presenting the proofs than in episodes that showed compliance with routine presentation practices. I hypothesized that the mean ratings of *episode appropriateness* (EA) and *segment-of-interest appropriateness* (SIA) would be lower in episodes that depicted a departure from routine communication practices compared to the episodes that showed compliance with routine practices of proof presentation. Similarly, I hypothesized that there would be no difference in the mean ratings of the *distracter segment appropriateness* (DSA) across conditions. I conducted a repeated measures ANOVA with planned comparison to test our hypotheses.

Table 2: Summary of hypothesis generated for planned comparison across and within experimental groups.

ACROSS-GROUP COMPARISON	WITHIN-GROUP COMPARISON
H0: EA Treatment = EA Control H1: EA Treatment < EA Control H0: SIA Treatment = SIA Control H1: SIA Treatment < SIA Control H0: DSA Treatment = DSA Control H1: DSA Treatment ≠DSA Control	H0: SIA Treatment = DSA Treatment H1: SIA Treatment < DSA Treatment H0: SIA Control = DSA Control H1: SIA Control ≠ DSA Control

3.5 Analysis of Open-Ended Responses

3.5.1 Justification Coding Scheme

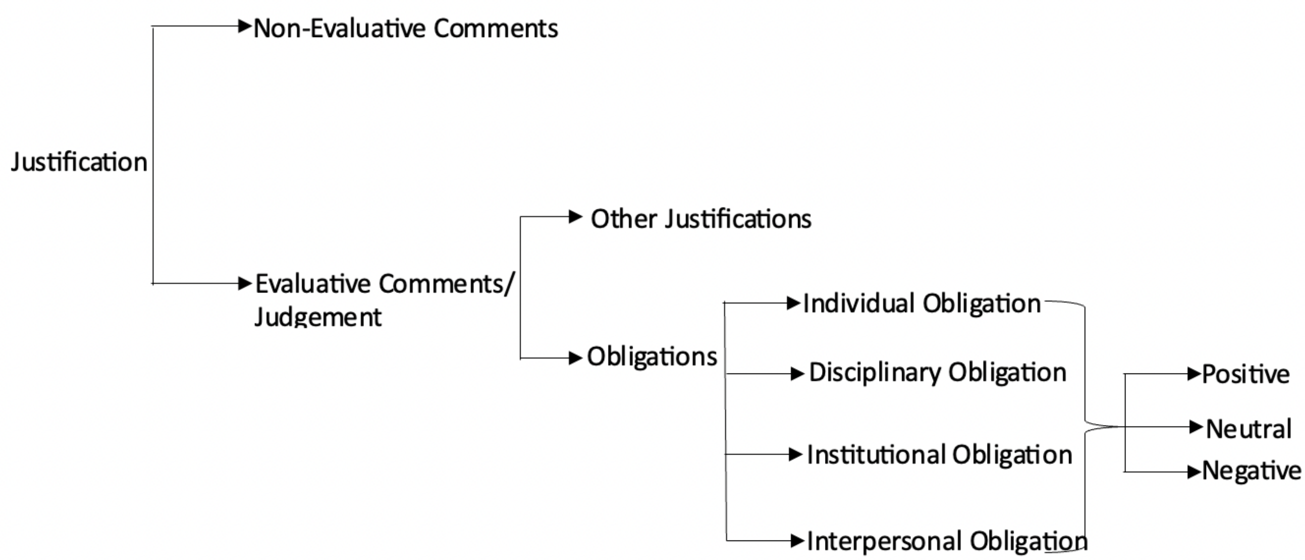

Figure 5: A network diagram showing the justification coding scheme for coding open response data.

To investigate how participants justified or criticized the hypothetical instructional actions of geometry teachers, I developed a coding scheme for coding participants' open-ended responses for justifications as represented in the network diagram shown in figure 5. By justification, I mean the rationale for teachers' decision to either rate the storyboards positively or negatively. How might teachers justify or criticize the instructional actions depicted in the proof transcription and alternative communication storyboards? According to Herbst & Chazan (2012), teachers rarely need to justify their actions or decisions, "however, teacher actions could be rationally justified by constructing narratives that from the researcher's perspectives capture the givens and possibilities available to the teachers as attested, among other things, by what they perceive and value in narratives about their action" (Herbst & Chazan, 2012, p. 602). Therefore, my goal was to examine how teachers perceive and or value the communication practices as depicted by the hypothetical instructional scenarios in the storyboard.

To investigate teachers' justification of instructional actions depicted by the storyboards, first, I blinded participants' responses in terms of the breach/routine conditions. Next, I distinguished between participants' responses that represent a justification and those that do not by using a dichotomous code 0/1. I defined responses that represent a justification as evaluative comments that reference teaching actions within the instructional episodes and pass judgment on (a) why the teaching actions are right or wrong; (b) refute the validity of the teaching action; or (c) give a defense of the teaching actions. Martin & White (2005) in *The Language of Evaluation* classified judgment as a class of attitude through which people express positive or negative feelings about other people or actions. The authors further defined judgment as an institutionalized affect based on shared societal norms, ethical principles, or other standards of proper behavior. (Martin & White, 2005). For example, "She is a brilliant student" provides a positive judgment of an individual who takes on the role of a student within the limits of some systemic sets of standards (school standards).

Hence, I choose to code for teachers' justification of the instructional episodes by analyzing how teachers evaluate or pass judgment on the instructional episodes based on their perceived value of the actions depicted in such a scenario. For example, one participant said:

> *Wow. I don't like this teacher. Putting it bluntly she is a jerk. Or maybe I am just too sensitive to students' feelings. After she did the proof, I wouldn't have asked her to fix it because obviously she couldn't, no need to make a fool of her or give her anxiety. She shouldn't have started to address students until Delta sat down. At that point, I would ask*

what would fix her proof and I would allow the "complements" student to put up his simultaneously while I worked with the class to fix delta's." (Storyboard R_ST; participant 4998).

I did not code non-evaluative comments i.e., comments that simply describe the teaching actions in the storyboard because by describing the episodes only, they do not provide any evidence of their perception, appraisal, or value of the teaching actions in the episodes and so I cannot categorize these responses as either a justification or criticism of the instructional scenarios. For example, one participant wrote: "Student sharing their proof solution on the board. Others suggesting an alternate method." (Storyboard L_SC; participant 4226).

Next, I coded participants' responses for justification by looking at how their comments appeal to any of the four professional obligations that impact decisions within the instructional system as a result of its hold on the mathematics teacher (Herbst & Chazan, 2016). While there are many frameworks for analyzing the rationale for teachers' instructional decisions including teachers' mathematical knowledge, teachers' beliefs, etc., the professional obligation framework emphasizes the impact of the environment of instruction on the position of the mathematics teacher (P. Herbst & Chazan, 2012) and thus proposes that some of the rationality behind teachers' instructional decisions might be outside of the control of the individual teacher. Since the research focus is investigating instructional norms as a social construct that is independent of any individual teacher the choice of professional obligation as a justification framework is deemed reasonable.

The professional obligations coding corresponded to the four obligations previously defined (disciplinary, individual, interpersonal, institutional). Participants' responses were coded as referencing the individual obligation if they contained evaluative comments regarding how teaching actions in the storyboard affected the individual student in the classroom e.g., *"The teacher should not be telling the student that everything she did after the missing statement was invalid and needs to be re-done. she is embarrassing her in front of the class*." (Storyboard R_ST; participant 5097). Responses were coded as referencing the interpersonal obligation if they contained evaluative comments regarding how teaching actions in the storyboard affected the classroom community of mathematics students, including the interactions among class members and the use of the shared classroom resources of time, knowledge, or physical space. For example, "*There is too much time spent on the students writing on the board. The rest of the students are probably not really paying attention since that one student is doing all the work."* (Storyboard R_SC; participant 4430). Similarly, responses were coded as referencing the institutional obligation if they contained evaluative comments about how teaching actions explicitly or implicitly inhibit or support the teacher's abilities to fulfill institutional responsibilities within or outside the scope of that classroom instruction. This included comments that reference responsibility towards the school department or district e.g., recommended textbooks, assessment goals, curriculum coverage, school schedule- calendar/ bell schedule, etc. e.g., "*Too much downtime for a majority of the class*." (Storyboard R_ST; participant 4462). This comment was coded as a reference to the institutional obligation because it references time which is regarded as an institutional resource (P. Herbst & Chazan, 2012).

Finally, I coded responses as referencing the disciplinary obligation if they contained references to how the teaching actions were/were not valid representations of the mathematical knowledge, values, and practices authentic to the discipline of mathematics teaching. E.g., "*The teacher does not correct reason, adjacent congruent angles are not always right angles"* (Storyboard R_SC;

participant 4247). There was an obligations coding scheme that was independent of evaluation; then, each instance of an obligation code could be positive or negative or ambiguous/neutral. An example of a positive obligation reference is: "*I really liked this scenario. The teacher was letting the students teach each other and there was a lot of good interaction with the class. They seemed to be engaged and understanding what they were doing."* (Storyboard M_SC; participant 4430), this response was coded as a positive interpersonal obligation. Likewise, an example of a negative obligation reference is: "*Too much class time wasted. Also, not really any praise for the student who had put a lot of work into the proof at this point.*" (Storyboard R_SC; participant 5397), this response was coded as a negative institutional and individual obligation. An example of a neutral obligation reference is: "The teacher asks the student to put up some work and then the student doesn't really explain the work. The teacher just wants students to look at it and check it." (Storyboard R_SC; participant 5453), this response was coded as a neutral individual obligation.

I created a frequency table from the dichotomously coded responses to indicate how participants referenced their professional obligations when justifying their positions regarding breaching or compliance with the sequence norm. By using a chi-square statistical test, I looked at the association between the experimental conditions and a possible positive or negative reference to the professional obligations. Finally, by coding the open-ended responses for participants' responses to comparable storyboard frames under the routine and breached conditions, I experimentally investigated the existence of social norms. I hypothesized that participants would respond more positively to the identical teaching actions in the routine condition and more negatively to the same actions in the breached condition.

3.5.2 Reliability of the Justification coding scheme

The justification coding scheme was tested for reliability by comparing the coded responses of two independent coders using Kappa statistical analysis. A sample of 60 responses was randomly selected from the blinded data set and the coding scheme was applied. The kappa statistics justification and obligation coding were 0.84 and 0.81 respectively. The kappa statistics for the reference to the individual, interpersonal, disciplinary, and Institutional obligations are 0.77, 0.86, 0.92, and 0.92 respectively. The kappa scores indicate high (0.77) and very high (0.81, 0.84, 0.86, 0.92) agreements between the two independent coders.

3.5.3 Data Selection for Open response coding

To investigate the third research question: *How do secondary geometry teachers justify or criticized hypothetical instructional actions of geometry teachers as they pertained to student presentations of proofs?* I analyzed participants' open-ended responses for references to their professional obligations. Data from three open responses were used for this analysis: participants' descriptions of what is happening in the storyboard scenario, participants' open-ended responses to the episode appropriateness ratings, and participants' open-ended responses to the segment of interest ratings. The three data points were chosen because, in the first place, they show participants' initial reactions to the storyboard frames without any prompting from the researchers. Additionally, it shows participants' reactions to both specific storyboard segments where the sequence norm was either breached or followed as well as their reactions to the entire storyboard as one instructional scenario.

For this analysis, two storyboards were chosen, the first is the reason storyboard from the replication study and the second is Storyboard A from the study of alternative communication practices. In the reason storyboard, the breach condition shows a teacher interrupting the students' presentation by insisting that the student recreate the proof due to an omitted justification. The routine condition does not include an interruption since the teacher simply asks the student to insert the missing justification. In storyboard A, two students presented proof at the board. In the treatment version, one student writes the proof and the other verbalizes the proof as it is being written, while in the control version, the students took turns transcribing the proof. The choice of these two storyboards is to distinguish between participants' reactions to the practice of proof transcription and participants' reactions to instructional practices that attempted to steer student presentations of proofs toward a more disciplinary communication practice.

Due to the size of the study, it was thought fair to limit the number of open-ended responses to code by only coding the extreme scores. Participants with a higher rating of (5-appropriate and 6-very appropriate) and lower ratings of (1- very inappropriate and 2- inappropriate) on the close-ended responses to the segment of interest ratings (SIE) and episode ratings (EA) were selected. The selection of the extreme cases was also deemed reasonable because the study hypothesized that participants would rate teaching actions represented in the breached condition more negatively, so a positive rating of the breached actions represents a deviation from what is expected and thus presents an interesting case. Similarly, I predicted that participants would rate teaching actions more positively in the routine condition, and thus a negative rating of the routine condition represents an interesting case for analysis. Table 3 shows the number of open responses coded by treatment and control condition for the reason storyboard and storyboard A

Table 3: Number of participants selected for open-ended response coding by storyboards and storyboard conditions.

Storyboards	Treatment (N)			Control (N)		
	Very High Ratings (5-6)	Very Low Ratings (1-2)	Total	Very High Ratings (5-6)	Very Low Ratings (1-2)	Total
Reason Storyboard	6	53	59	44	15	60
Storyboard A	135	1	136	67	3	70

CHAPTER FOUR

RESULTS

4.1 Summary of descriptive statistics

Closed-response rating questions were incorporated into the test to determine how participants responded to storyboards in which the teacher was seen as departing from the hypothesized norm. The closed response-rating questions were made to elicit responses at two different levels: participants' reaction to the entire storyboard (EA - Episode Appropriateness Rating) and participants' reaction to specific segments of the storyboard (SIA - Segment of Interest Appropriateness Rating, and DSA - Distracter Segment Appropriateness Rating). The segment of interest and distracter segments represents where breaching actions and routine instructional actions occur respectively.

Below are figures that display the box-and-whiskers plots for the storyboards in the routine and breach conditions. Each of the storyboards was seen by 191 participants in the routine condition, and 214 participants in the breach condition all of whom responded to questions regarding them. Figure 6 displays box-and-whisker plots for the episode appropriateness ratings for both storyboards targeting proof transcription and those targeting alternative communication practices respectively. The routine and breach conditions for the proof transcription storyboards are represented by the first two box-and-whisker plots closest to the left, and the routine and breach conditions for the alternative communication storyboards are represented by the second two box-and-whisker plots on the right. EA1 and EA2 each represent the episode appropriateness ratings for the reason and label storyboards, the two storyboards targeting proof transcription, and EA3 and EA4 each represent the episode appropriateness ratings for Storyboard B and Storyboard A, the two storyboards targeting alternative communication storyboards.

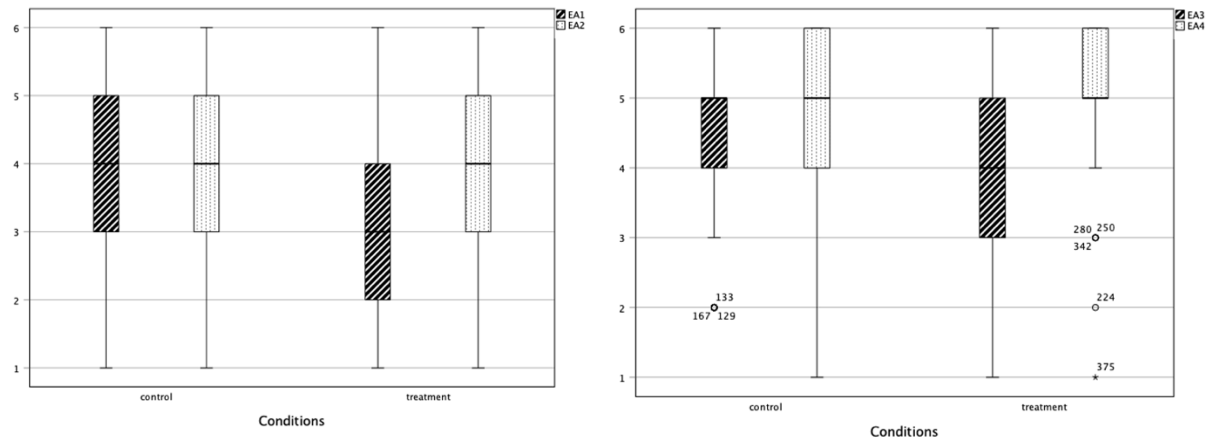

Figure 6: Box-and-whiskers plots for responses to Episode Appropriateness (EA) Ratings. Left plot– Proof transcription storyboards; Right plot – Alternative Communication Storyboard.

The plots of the response to the first rating question indicate that the responses to the breach storyboards had median, lower-quartile, and upper-quartile scores that were less than or equal to the median, lower-quartile, and upper-quartile scores of the control storyboards for both the proof transcription storyboards and the alternative communication storyboards. Open circles are used to identify mild outliers, as shown above. In the alternative communication storyboards, three outliers exist in total for the control storyboard and four in the treatment storyboard. These outliers, which in both cases deviate from the predicted response pattern, were included in the data set because doing so analyzes the closed-response data more cautiously. The means and standard deviations for the episode appropriateness ratings are reported in Table 4a and Table 4b respectively.

Table4a: Mean and Standard Deviation of Episode Appropriateness Ratings for Storyboards Targeting Proof Transcription.

		Group	Mean	Standard Deviation	N
Label Storyboard	EA2	Control	3.88	1.241	189
		Treatment	3.05	1.263	212
Reason Storyboard	EA1	Control	4.23	1.193	189
		Treatment	3.68	1.295	212

Table4b: Mean and standard deviation of Episode Appropriateness Ratings for Storyboards Targeting Alternative Communication Practices.

		Group	Mean	Standard Deviation	N
Storyboard A	EA4	Control	4.92	0.948	190
		Treatment	5.32	0.779	212
Storyboard B	EA3	Treatment 1	4.58	1.127	190
		Treatment 2	3.90	1.233	212

Figure 7 displays box-and-whisker plots for the segment of interest appropriateness ratings SIA. The routine and breach conditions for the proof transcription storyboards are represented by the first two box-and-whisker plots closest to the left, and the routine and breach conditions for the alternative communication storyboards are represented by the second two box-and-whisker plots on the right. SIA1 and SIA2 each represent the segment of interest appropriateness ratings for the reason and label storyboards, the two storyboards targeting proof transcription and SIA3 and SIA4 each represent the segment of interest appropriateness ratings for Storyboard B and Storyboard A, the two storyboards targeting alternative communication storyboards.

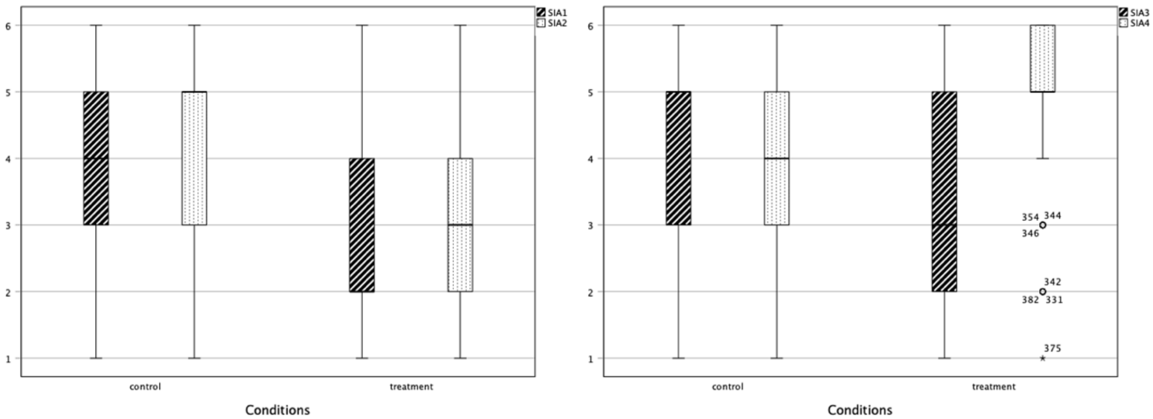

Figure 7: Box-and-whiskers plots for responses to Segment of Interest Appropriateness (SIA) Ratings. Left plot– Proof transcription storyboards; Right plot – Alternative Communication Storyboard.

The plots of the response to this rating question indicate that for the storyboards targeting proof transcription, the responses to the breach storyboards had median, lower-quartile, and upper-quartile scores that were less than or equal to the median, lower-quartile, and upper-quartile scores of the control storyboards. For the storyboards targeting alternative communication practices, storyboard B had responses to the breach condition with median, lower-quartile, and upper-quartile scores that were less than or equal to the median, lower-quartile, and upper-quartile scores of the control storyboard while storyboard A had responses to the breach condition with median, lower-quartile, and upper-quartile scores that were greater than the median, lower-quartile, and upper-quartile scores of the control condition. six outliers exist in the treatment storyboard. The means and standard deviations for the segment of interest appropriateness ratings are reported in Table 5a and Table 5b respectively.

Table5a: Mean and Standard Deviation of Segment of Interest Appropriateness Ratings for Storyboards Targeting Proof Transcription.

		Group	Mean	Standard Deviation	N
Label Storyboard	SIA2	Control	3.98	1.235	189
		Treatment	2.72	1.256	212
Reason Storyboard	SIA1	Control	4.31	1.404	189
		Treatment	2.92	1.457	212

Table5b: Mean and Standard Deviation of Segment of Interest Appropriateness Ratings for Storyboards Targeting Alternative Communication Practices.

		Group	Mean	Standard Deviation	N
Storyboard A	SIA4	Control	4.12	1.312	190
		Treatment	4.96	1.002	212
Storyboard B	SIA3	Treatment 1	4.28	1.368	190
		Treatment 2	3.47	1.497	212

Figure 8 displays box-and-whisker plots for the distracter segment appropriateness ratings DSA. The routine and breach conditions for the storyboards targeting proof transcription are represented by the first two box-and-whisker plots closest to the left, and the routine and breach conditions for the alternative communication storyboards are represented by the second two box-and-whisker plots on the right. DSA1 and DSA2 each represent the distracter segment appropriateness ratings for the reason and label storyboards, the two storyboards targeting proof transcription and DSA3 and DSA4 each represent the distracter segment appropriateness ratings for Storyboard B and Storyboard A, the two storyboards targeting alternative communication storyboards.

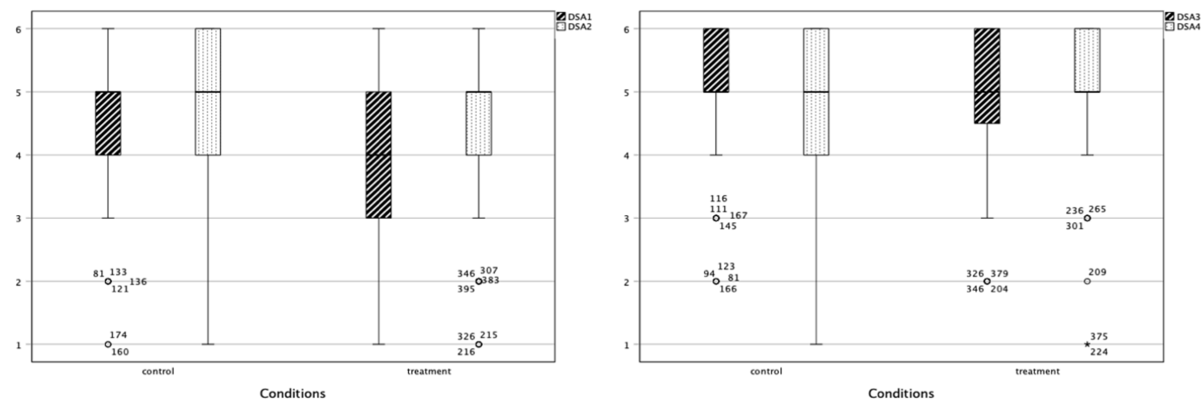

Figure 8: Box-and-whiskers plots for responses to Distracter Segment Appropriateness (DSA) Ratings. Left plot– Proof transcription storyboards; Right plot – Alternative Communication Storyboard.

The plots of the response to this rating question indicate that for the storyboards targeting proof transcription, the responses to the breach storyboards had median, lower-quartile, and upper-quartile scores that were less than or equal to the median, lower-quartile, and upper-quartile scores of the control storyboards. For the storyboards targeting alternative communication practices, storyboard B had responses to the breach condition with median, lower-quartile, and upper-quartile scores that were less than or equal to the median, lower-quartile, and upper-quartile scores of the control storyboard while storyboard A had responses to the breach condition with median, lower-quartile, and upper-quartile scores that were greater than the median, lower-quartile, and upper-quartile scores of the control condition. Open circles are used to identify mild outliers, as shown above. The means and standard deviations for the segment of interest appropriateness ratings are reported in Table 6a and Table 6b respectively.

Table6a: Mean and Standard Deviation of Distracter Segment Appropriateness Ratings for Storyboards Targeting Proof Transcription.

		Group	Mean	Standard Deviation	N
Label Storyboard	DSA2	Control	4.74	1.112	189
		Treatment	4.67	1.161	212
Reason Storyboard	DSA1	Control	4.42	1.072	189
		Treatment	4.09	1.219	212

Table6b: Mean and Standard Deviation of Distracter Segment Appropriateness Ratings for Storyboards Targeting Alternative Communication Practices.

		Group	Mean	Standard Deviation	N
Storyboard A	DSA4	Control	4.93	0.945	190
		Treatment	5.21	0.886	212
Storyboard B	DSA3	Treatment 1	5.13	0.880	190
		Treatment 2	4.91	0.952	212

4.2 Results of Inferential Test

4.2.1 Across and Within Group Comparison

The general hypothesis that guided the instrument's design was that participants would react negatively to a teacher deviating from proof transcription. I developed five specific hypotheses (See Table 2) to examine participant responses across and within experimental conditions to address our first research question: Do secondary geometry teachers recognize the hypothesized norm of proof transcription when students present proofs at the board?

The three across-group hypotheses involve comparisons of participants who saw storyboard iterations as treatments or controls. For these comparisons, the mean appropriateness ratings to questions following storyboards where proof transcription was breached (treatment storyboard) were compared to mean appropriateness ratings to questions following storyboards where proof transcription was allowed (control storyboard). The two within-group hypotheses look for differences in how participants rated segments of the storyboards where the teacher was seen breaching the practice of proof transcription versus segments of the storyboard where the teacher allowed students to transcribe their proof within each of the experimental conditions (treatment and control). I predicted that the treatment storyboards would have lower mean ratings than the control storyboards and thus the alternate hypotheses (I), (II), and (IV) are pointed in that direction. With hypothesis (III) and (V), I predicted that there won't be any significant differences in mean ratings for DSA. All the hypotheses were tested for the two storyboards targeting proof transcription. To test these hypotheses, a one-way repeated measure ANOVA was conducted. All participants in each condition were asked to rate the teacher's actions in the storyboards on three separate occasions, and all participants provided rating responses to comparable questions on two different storyboards, so the statistical test chosen was reasonable. The hypothesis testing used two defined within-subject factors. The first factor is described as "storyboards," and there are two levels, both of which represent the reason and label storyboards. The second factor is described as "Ratings" with three levels, each level denoting ratings for episode appropriateness (EA), segment of interest (SIA), and distracter segment (DSA). A between-subject factor was also defined with the label “conditions” which represents the control and breached conditions. A two-tailed distribution was utilized to assess the significance of the hypotheses.

4.2.2 Results of Repeated Measure ANOVA

A. Test of Sphericity

Table 7: Mauchly's Test of Sphericity.

	Measure: Measure1				Epsilon
Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig	Greenhouse-Geisser
Ratings	.747	116.001	2	<.001	.798
Ratings*Storyboards	.752	113.099	9	<.001	.879

Mauchly's test of sphericity indicated that the assumption of sphericity had been violated, $\chi2(2)$ = 116.001, $p < .001$ for ratings and $\chi2(2)$ = 113.099, $p < .001$ for ratings versus conditions. Epsilon (ε) was 0.798 and 0.879 respectively, as calculated according to Greenhouse and Geisser (1959) and this was used to correct the one-way repeated measures ANOVA.

B. Test of within Subject Effect

The results from Table 8 show that there is a statistical difference between the mean appropriateness ratings of the storyboard and the experimental conditions, F (1.596) = 80.148, $p < .001$. Similarly, there is a statistical difference within the appropriateness ratings, F (1.596,636.959) = 419.749, $p < .001$. More details on the variations in mean ratings across and within experimental groups are provided by the pairwise comparison post hoc test, which is detailed in the replication results below. The results of the pairwise comparison display not only the level of significance but also the direction of detected differences in mean.

Table 8: Test of within-subject effect.

Source		Type III Sum of Squares	df	Mean Square	F	**Sig.**
Ratings	Greenhouse-Geisser	706.740	1.596	442.71	419.749	**<.001**
Ratings* Conditions	Greenhouse-Geisser	134.948	1.596	84.533	80.148	**<.001**
Error (Ratings)	Greenhouse-Geisser	671.805	636.959	1.055		

4.2.3 Replication Results

For the across-conditions comparison, participant ratings of the treatment and control storyboards were as predicted for the episode and segment of interest for the two storyboards that replicated the original study. In comparison to the routine storyboards, the breach storyboards had significantly lower means on the episode appropriateness rating questions (Table 9 and 10, row 1) and the segment of the interest rating questions (Table 9 and 10, row 2). The distracter segment of the reason storyboard showed a significant difference in mean ratings (Table 10, row 3). I attributed the significant difference in the mean of the distracter segment to 1) A carry-over effect of the generally negative reaction to the teacher's breach of the practice of proof transcription in the treatment storyboards. This claim is deemed reasonable because routine actions are salient as to become invisible, hence individuals within a social situation where a normative action is breached might be unable to pinpoint what is different about such a situation, this could result in a more critical appraisal of the entire situation and as a result a generally negative reaction to the overall situation. 2) the large sample size, due to the large sample size

(N=405) it is more likely that we find a statistically significant difference in the mean between the experimental groups and so we carried out an effect size analysis to investigate the size of the effect.

The effect-size statistical analysis reveals that while the distracter segments of this storyboard had statistically significant differences in mean ratings, the size of the effect was small, with d =.278. These results thus support the existing literature that when participants view instructional episodes in which teachers allowed students to transcribe their proofs, the work of the teacher was rated higher than in those episodes in which the teacher interfered with the transcriptions.

Table 9: Across-condition pairwise comparison for Label Storyboard

Appropriateness Measure	R (n=189)	B (n=212)	μ1-μ2	Std. Error	Sig..[b]	95% Confidence Interval for Difference: Lower Bound	95% Confidence Interval for Difference: Upper Bound
Episode	4.233	3.684	0.549	.125	<.001	.303	.794
Segment of Interest	4.312	2.920	1.392	.143	<.001	1.111	1.674
Distracter Segment	4.741	4.675	0.066	.114	.561	-.158	.290

Based on estimated marginal means
* The mean difference is significant at the .05 level
[b] Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustment)

Table 10: Across-condition pairwise comparison for Reason Storyboard

Appropriateness Measure	R (n=189)	B (n=212)	μ1-μ2	Std. Error	Sig..[b]	95% Confidence Interval for Difference: Lower Bound	95% Confidence Interval for Difference: Upper Bound
Episode	3.884	3.052	0.832	.125	<.001	.585	1.078
Segment of Interest	3.984	2.717	1.267	.125	<.001	1.022	1.512
Distracter Segment	4.418	4.090	0.328	.115	.005	.102	.555

Based on estimated marginal means
* The mean difference is significant at the .05 level
[b] Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustment)

For the within-condition comparison, the findings shown in Table 11 and Table 12 provide evidence that within the experimental condition, there is a significant difference between the means of the distracter segment and the segment of interest. The negative sign implies that, in both the breach and control circumstances, the mean ratings of the segment of interest (SIA) were lower than the mean ratings of the distracter segments. Although this result supports my hypothesis for the treatment group, it contradicts it for the control group. The routine storyboard depicts teaching actions that what was considered to be routine or normative for both the segment of interest and the distracter segments, hence I anticipated that there would be no discernible difference between the mean ratings. I investigated whether the significant difference in these mean ratings is a result of the large sample size, so I conducted an effect size analysis to investigate the size of the effect.

According to the findings of the effect size analysis, both the reason and label storyboards (d = -0.340 and -0.380, respectively) only slightly differ in the mean ratings of SIA and DSA. This indicates that the difference in the mean is negligible even if it is statistically significant.

Table 11: Within condition pairwise comparison for Label Storyboard

Condition	SIA	DSA	$\mu1$-$\mu2$	Std. Error	Sig..[b]	95% Confidence Interval for Difference: Lower Bound	Upper Bound	Cohen's D
Routine	4.312	4.741	-.429	.105	<.001	-.635	-.223	-.340
Breach	2.920	4.675	-1.755	.099	<.001	-1.949	-1.560	-1.328

Based on estimated marginal means
* The mean difference is significant at the .05 level
[b] Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustment)

Table 12: Within condition pairwise comparison for Reason Storyboard

Condition	SIA	DSA	μ1-μ2	Std. Error	Sig..^{b}	95% Confidence Interval for Difference Lower Bound	Upper Bound	Cohen's D
Routine	3.984	4.418	-.434	.085	<.001	-.601	-.267	-.380
Breach	2.717	4.090	-1.373	.080	<.001	-1.530	-1.215	-1.106

Based on estimated marginal means
* The mean difference is significant at the .05 level
[b] Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustment)

4.2.4 Results Alternative Communication Practices

In storyboard A, in comparison to the routine storyboards, the breach storyboards had significantly higher means on the episode appropriateness ratings (Table 13, row 1) and the segment of the interest ratings (Table 13, row 2). These results were different from what we hypothesized. The distracter segment showed a significant difference in mean ratings (Table 13, row 3) however the effect-size statistical analysis reveals that while the distracter segments of this storyboard had statistically significant differences in mean ratings, the size of the effect was small, with d =-0.25. The result thus suggests that in episodes where students were required to engage in more authentic disciplinary and multimodal communication practices, such as explaining their proof as it is being created, participants rated the work of the teacher more positively than those where the teacher allowed the students to simply transcribe their proofs.

Table 13: Across condition pairwise comparison for Communication Storyboard A

Appropriateness Measure	R (n=189)	B (n=212)	μ1-μ2	Std. Error	Sig..[b]	95% Confidence Interval for Difference Lower Bound	Upper Bound
Episode	4.921	5.321	-.400	.086	<.001	-.570	-.230
Segment of Interest	4.111	4.962	-.851	.116	<.001	-1.079	-.623
Distracter Segment	4.979	5.212	-.233	.091	.011	.054	.413

Based on estimated marginal means
* The mean difference is significant at the .05 level
[b] Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustment)

In storyboard B, treatment 1 had significantly lower means on the episode appropriateness ratings (Table 14, row 1) and the segment of interest ratings (Table 14, row 2) in comparison to treatment 2. The distracter segment showed a significant difference in mean ratings (Table 14, row 3) however the effect-size statistical analysis reveals that while the distracter segments of this storyboard had statistically significant differences in mean ratings, the size of the effect was small, with d =.234. The more negative reaction to the alternative communication practices in treatment 2 could be explained as a consequence of the more ambitious communication practice depicted in this storyboard. However, the fact that the mean ratings of this storyboard (Table 14, column 3) are generally greater than the mean ratings of the treatment versions of both the label and reason storyboard (Table 9 and 10, column 3) where proof transcription was allowed suggests that participants considered instructions where the teacher steers the students towards a more disciplinary approximate communication practice to be a justifiable instructional practice.

Table 14: Across condition pairwise comparison for Communication Storyboard B

Appropriateness Measure	T1 (n=189)	T2 (n=212)	μ1-μ2	Std. Error	Sig	95% Confidence Interval for Difference: Lower Bound	Upper Bound
Episode	4.587	3.901	0.686	.119	<.001	.453	.920
Segment of Interest	4.275	3.472	0.803	.144	<.001	.521	1.086
Distracter Segment	5.132	4.910	0.222	.092	.016	.041	.403

Based on estimated marginal means
* The mean difference is significant at the .05 level
[b] Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustment)

4.3 Result of Open-Ended Coding

4.3.1 Results of Justification Coding Targeting Proof Transcription

After coding the open-ended responses using the justification coding scheme, the data was unblinded in relation to the breach/control condition of the storyboard. The results of the coding were then contrasted with the breach/control conditions from the storyboards. The comparison's objective is to investigate how participants justify or criticize the hypothetical actions of the geometry teacher in the breach and routine versions of the storyboard. Using the chi-square statistical test where it is feasible and the fisher exact test when it is not, we examine the association between storyboard conditions and a positive or negative reference to the professional obligations. Table 15 shows the count of references to the professional obligations across experimental conditions for the reason storyboard.

Table 15: Counts of responses that contained references to professional obligations by condition for reason storyboard.

Professional Obligations	Condition C = 59 T = 60	Positive	Negative	Ambiguous	Total
Individual Obligation	Control	34	8	0	42
	Treatment	3	55	0	58
Interpersonal Obligation	Control	9	24	1	34
	Treatment	1	25	0	26
Disciplinary Obligation	Control	4	4	0	8
	Treatment	6	20	1	27
Institutional Obligation	Control	3	6	0	9
	Treatment	0	31	0	31

Note:

According to Table 15, a large percentage of participants in the control version of the reason storyboard referenced their individual and interpersonal obligations when justifying their ratings, with 61% and 49% respectively, citing these obligations. The disciplinary and Institutional obligations were the least referenced obligations with only 11% and 12% references respectively. One possible reason why there were more references to the other obligations than the disciplinary obligation could be because, in the control version of this storyboard, the proof was presented by students in a way that was intended to reflect the transcribing practices that are thought to be commonplace in geometry classrooms when students write proofs on the board. And thus, participants were more likely to focus on other classroom teaching strategies within the storyboard frames while the disciplinary practice of communication portrayed during the proof presentation naturally recedes into the background.

In the treatment version of the storyboard, there were more references to individual obligations while there was a relatively even distribution of references to the other professional obligations. Since the treatment version of this storyboard presents teaching actions depicted as breaching the hypothesized sequence norm, the disciplinary practice of the proof presentation was more explicit, and thus participants provide more commentary on the disciplinary obligations of proving and proof presentation.

Comparing the references to the professional obligations across the breach and routine conditions, we observed that 94.8% of the references to the individual obligation are negative in the breach version of the storyboard while 81% of the references are positive in the routine version of the storyboard. A chi-square test of association indicates there is a significant relationship between the breach/control conditions and positive or negative reference to the individual obligation ($\chi 2$ (2) = 120.026, $p < .001$). One possible reason for the observed differences in the positive or negative reference to the individual obligations in the reason storyboard could be a result of the presence or absence of an interruption of students' presentation in the treatment and control versions of this storyboard.

In addition to reacting to the breach of the norm, participants' negative reaction to the teacher's direction for the students to recreate their proof for having omitted a reason in the treatment version of this storyboard could stem from the obligation of the teacher towards the needs of the student at the board. One participant wrote: *"I am all for telling students when they make mistakes, but it tone of the statement is almost derogatory. There is no support and encouragement afterward. I am sure the student is embarrassed. I weaker student might just give up."* (27005-5360).

Another participant commented "*Just because there is a missing "reason", the statement is not necessarily false, and the rest of the proof could very well be correct. The reason adds justification to the proof—and the absence of the reason does not necessarily nullify the proof. Requiring the student to "redo" the subsequent steps is unnecessary."* (27005- participant 4999).

While the two comments were negative, the second participant's comment was geared towards the disciplinary validity of asking the student to recreate the proof while the first participant was responding to the emotional needs of the student presenting the proof.

There were more negative than positive references to the interpersonal obligation in both the control and treatment version of this storyboard. The chi-square test of association could not be used to statistically test for a significant relationship between the reference type (positive/negative) and the conditions (treatment/control) because one of the values of the reference cells is less than 5. However, using Fisher's exact test yields a p-value (0.032) which indicates that the association is significant. To test for association between the treatment/control conditions and positive or negative references to the disciplinary and Institutional obligation, we used the Fishers exact text which results in a p-value (0.195) for the disciplinary obligation and a p-value (0.009) for the institutional obligation. These results indicate that there is no significant association between the experimental conditions and a positive or negative reference to the disciplinary obligation but there is a significant association between the experimental conditions and a positive or negative reference to the institutional obligation. The bar graphs in figure 9 and figure 10 give a visual representation of the references to the professional obligations for the treatment and control versions of the storyboards respectively.

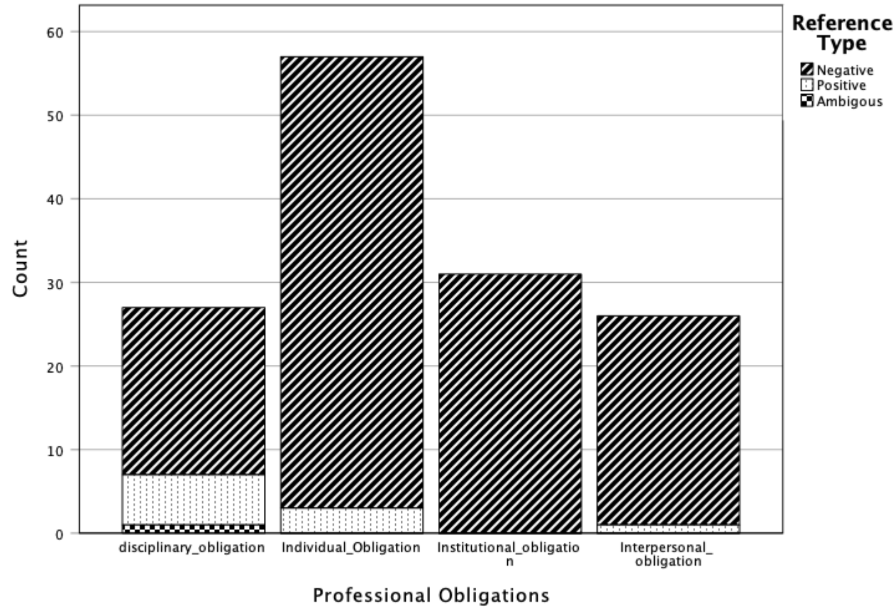

Figure 9: Bar graph Showing References to Professional Obligation for Treatment Storyboard

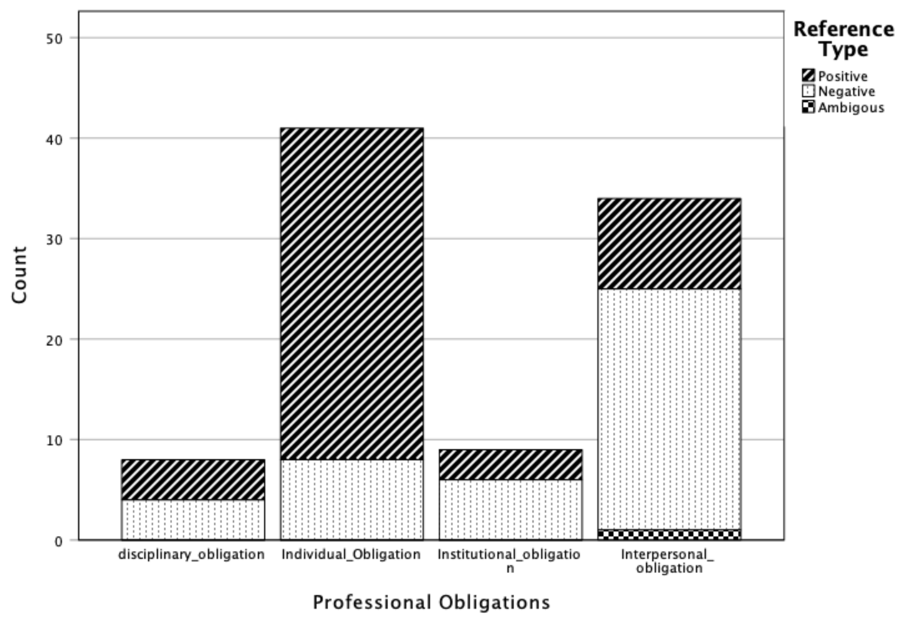

Figure 10: Bar graph Showing References to Professional Obligation for Control Storyboard

4.3.2 Results of Justification Coding Targeting Alternative Communication Practices

The open-ended responses for storyboard A were coded using the justification coding scheme. The result in table 16 shows that participants in the routine condition mostly referenced the individual and interpersonal and institutional obligations when justifying or criticizing the instructional actions of the hypothetical geometry teacher in the storyboard scenario. Again, the fact that the disciplinary obligation was sparingly referenced by participants can be explained by the invisibility of routine activities. Since teaching actions in this scenario depicted instructional practices that were considered to be normative for teachers, students taking turns transcribing their proofs on the board, such actions naturally fade to the background causing teachers to focus on other classroom instructional actions of the hypothetical geometry teacher e.g., how the hypothetical teacher interacted with the students presenting their proofs at the board: " *The teacher allows the pair of students the freedom to decide how they break up the task of writing up their proof."* (27017,4637). This participant justified the teacher's action by referencing how the teacher gave the students choice/agency in their presentation (Positive reference to the Individual Obligation). It is possible that teachers simply did not notice that the students were taking turns transcribing their proofs and this could account for why participants did not comment on the non-disciplinary nature of the presentation. But this seems unlikely since we have participants that explicitly required that students' presentations utilize a multimodal presentation technique. For example, one participant said "*Great, but I would have one student explain while the other was writing so the other students were not just sitting there."* (27017, 4277). Another participant said "*One student could have put the proof on the board while the other pointed out the reasons as progress is made through the proof. Or one student could have written the proof on the board while someone from another pair could have shown their work* *also."* (27017, 4995). While participants' reaction to the instructional episode was still mostly positive, these comments suggest that participants recognize that the student's proof presentation could be steered towards a more multimodal exchange where students provide commentary on proof steps as the proof is being generated.

Table 16: Counts of responses that contained references to professional obligations by condition for Storyboard A.

Professional Obligations	Condition C = 70 T = 136	Positive	Negative	Total
Individual Obligation	Control	49	2	51
	Treatment	108	1	109
Interpersonal Obligation	Control	26	6	32
	Treatment	34	1	35
Disciplinary Obligation	Control	3	4	7
	Treatment	16	8	24
Institutional Obligation	Control	11	4	15
	Treatment	16	1	17

Note:

In the treatment condition, the result from table 16 indicated that similar to the control condition, the individual obligation was the most referenced by participants as they justified the instructional actions of the hypothetical geometry teacher in the episodes. However, a good number of participants explicitly referenced the multimodal presentation technique utilized by the students in presenting their work to the class. E.g., one participant wrote "*The teacher really didn't have any actions other than allow students to present. If this were at the teacher's directions, then kudos having one student write and the other talk."* (27007, 5049). Some participants in this episode also showed evidence of requiring students' presentations to approximate higher communication standards such as students using appropriate segment names as they comment on the written proof. For instance, one participant wrote:

> *Perhaps the teacher could have asked the student to be specific when naming the congruent angles and parallel segments. But, to me, it seems as if the student is gesturing at the angles and segments under consideration. I guess, technically, it would have been nice for the student to name them, but since it is written in the proof anyway, I'm not sure that I would be too critical about that.* (27007, 4505).

Using chi-square or fishers exact test as appropriate, I investigated the association between the storyboard conditions and positive or negative references to the professional obligations. For the individual obligation, the chi-square test of association could not be used to statistically test for a significant relationship between the reference type (positive/negative) and the conditions (treatment/control) because two of the values of the reference cells are less than 5. However, using Fisher's exact test yields a p-value (.239) which indicates that the association is not significant. For the Interpersonal obligation, the Fisher exact test with a p-value of .048 indicates that there is a significant association between the storyboard conditions and a positive or negative reference to this obligation. Similarly, for the disciplinary and Institutional obligations, the Fishers exact test yields a p-value of .384 and .121 respectively indicating that the association is not significant.

These findings from the association testing seem reasonable because, under the routine condition, it is expected that the teacher's action would be evaluated more positively since it shows compliance with the norm of proof transcription. In the same vein, because the findings from the quantitative part of this study indicated that teachers reacted positively to instructional actions that steer students towards a more disciplinary approximate communication practice, it seems reasonable that there would be more positive than negative references to professional obligations in the breach condition as well. This therefore could account for the mostly nonsignificant association between the experimental conditions and the references to professional obligations.

In conclusion, findings from this analysis support the results of the statistical testing and show that teachers find instructional actions that attempt to steer students towards a disciplinary approximate communication practice to be pedagogically valuable. The evidence for this claim stems from the fact that teachers generally rated higher instructional scenarios depicting alternative communication practices than those showing proof transcriptions. These findings imply that teachers can create value for multimodal communication practices during proof presentations. This is significant because it provides research evidence that even though the classroom interaction patterns that experienced teachers have developed over years of practice are strong, there are possibilities for implementing initiatives that would shape secondary mathematics classroom instructions, particularly students' communication practices towards a more disciplinary approximate standard.

CHAPTER FIVE

DISCUSSIONS AND CONCLUSIONS

5.1 Discussion of Results

In this study, I investigated three research questions. The first research question: Do secondary geometry teachers recognize the hypothesized norm of proof transcription when students present proofs at the board? replicates a study investigating teachers' expectations during students' proof presentations. The replication study offered a robust empirical test of the hypothesis that secondary geometry teachers expect student presentations of proofs to default to proof transcriptions (Dimmel & Herbst, 2020). The findings from this empirical test corroborate existing findings in the literature that teachers expect students' presentations of proofs to default to proof transcription. This claim is evidenced by the general negative ratings of instructional episodes where teachers insist that students' proof be accountable e.g., labeling the diagrams before using such labels in their written statements, and generally positive ratings of instructional episodes where teaching actions allow for the practice of proof transcription. In terms of overall and segment-specific appropriateness ratings, the responses to the control storyboards (where proof transcriptions were allowed) were generally higher than those of the treatment storyboards (where proof transcription was not permitted). It was also noteworthy that the treatment storyboards received lower ratings than the control storyboards, even in the distracter segments of the storyboards where teaching actions were regarded as routine activities, such as asking students to check over their work.

The results of the replication study show that the control storyboards accurately represented typical or routine mathematical communication during geometry instruction while the treatment storyboard represented a deviation from the norm. This is evidenced by the higher versus lower ratings of the control and treatment storyboards respectively. The storyboards' matched breach/control design also lends evidence to the claim that this response pattern was brought on by the breach of the routine communication practice of proof transcription rather than some other features of the storyboards. The findings from the replication part of this study are significant because they provided additional evidence to support an observed social phenomenon. Given the ongoing replication crises in human subject research, such a result is non-trivial (Shrout & Rodgers, 2018). Additionally, the findings from these large-scale studies of mathematics teachers' instructional expectations and routine practices offer empirical evidence in support of theoretical studies that characterize teaching as a socially embedded activity influenced by background expectations and norms. This assertion is supported by the observation that a nationally representative sample of geometry teachers appears to recognize the practice of proof transcription.

The iteration of the study that investigated teachers' reactions to alternative communication practices addresses the second research question: How do secondary geometry teachers react to instructional actions that ask students to engage in approximations to disciplinary communication practices? The findings from this part of the study are significant because they provided evidence that teachers might be willing to deviate from the conventional practice of proof transcription and give students chances to develop their multimodal communication skills. In fact, across all four storyboard pairs, the highest mean ratings for the episode and segment of interest were linked to the storyboard that depicted two students working together to achieve a multimodal presentation of proof. Also of note, each of the alternatives that were tested in the comparison for Storyboard B had higher mean ratings than the breaches that were depicted in the original study. Rating these breached conditions more positively suggests that secondary geometry teachers might recognize that giving students the chance to develop and use multimodal presentation modalities, such as writing, explaining, gesturing, and diagramming, is a disciplinary and pedagogically justifiable action. The findings from the alternative communication practices study do not suggest that the communication practices exemplified in the storyboard scenarios are typical in secondary geometry classrooms; rather, it provides evidence that teachers appear to recognize that the task of presenting proof in a geometry classroom offers a chance for students to practice discipline-specific communication techniques.

With regards to the third research question: how do secondary geometry teachers justify or criticize hypothetical instructional actions of geometry teachers as they pertain to student presentations of proofs? I analyzed how secondary geometry teachers justify or criticize the hypothesized norm for both the storyboard targeting proof transcription and the storyboard targeting alternative communication practices.

For the storyboard targeting proof transcription, I analyzed how participants justify or criticize the hypothetical teacher's action as shown in the reason storyboard. The reason storyboard as described in chapter 3 depicted a scenario where a student copies the proof and omits justifying one of its claims. In the breach condition, the teacher asks the student to recreate the proof starting from the point where the justification was missed while in the routine version, the student just adds the missing justification to the proof without changing any of the other parts. In investigating teachers' rationality for justifying or criticizing the instructional actions portrayed in this storyboard, I found that participants mostly appealed to the individual and interpersonal obligation for justifying or criticizing the routine storyboard. with very few references to disciplinary or institutional obligations. The participants who cited the individual obligation mostly justified the teaching actions depicted in the instructional episodes (as evidenced by a higher number of positive references) while those who criticized the episodes cited the interpersonal obligation (as evidenced by a higher number of negative references). Overall, the reactions to the routine storyboard were relatively muted, with about the same number of positive and negative references focusing primarily on the obligations of the teacher to the students and the community of mathematics learners in the classroom, and little to no reference to the practice of proof transcription that was portrayed in the episode.

In contrast, the breached condition had participants citing all four obligations as reasons for criticizing the instructional actions depicted by the storyboard. This is evidenced by the higher number of negative references to all four obligations. The breach of proof transcription received a largely negative response, with many participants citing their obligations to the students or the institution's limited time supply as justifications. However, it is noteworthy that some participants found the departure from proof transcription to be disciplinarily reasonable even though they had competing obligations and hence rated the episodes negatively. For example, one participant in the treatment storyboard said:

> “The goal is for her to fix the problem by adding the necessary justification. Although technically the teacher is correct about the rest of the statements, it does not help learning to redo so much work. The teacher needs to provide experiences with more students working all at the same time.” (Storyboard 27005, Participant 5083).

Although the participants in the example above agreed that the hypothetical teacher's decision to ensure logical coherence in the student's proof was mathematically justifiable, the teacher prioritized their duties to the students by inferring that the workload might overwhelm the student and impair their ability to learn. The findings from the justification analysis of the storyboard targeting proof transcription are thus consistent with the findings from the replication study and those reported by Dimmel & Herbst (2020) that secondary mathematics teachers recognize the practice of proof transcription as part of the routine work that students do when they are asked to present a proof to the class. The findings from this analysis also seem to suggest that even when teachers recognize that students are engaging in an unreflective transcription of their proofs, other competing obligations, such as those to particular students or the institution (class time/curriculum coverage), may still have an impact on their instructional choices.

For the storyboard targeting Alternative communication, I analyzed how teachers justify or criticize the hypothetical teachers' actions in the instructional episode depicted in storyboard A. Storyboard A as described in chapter 3 shows an instructional scenario where two students presented proof at the board, in the treatment version, one student writes the proof and the other verbalizes the proof as it is being written, while in the control version, the students took turns transcribing the proof. The findings from the justification coding of this storyboard indicate that participants in the routine version of this storyboard generally referenced individual and interpersonal obligations as reasons for justifying the instructional episodes, with few participants citing institutional or disciplinary obligations. The references were also mostly positive. The finding from this analysis thus supports the claim that secondary mathematics teachers generally expect students to transcribe their proofs when they present at the board. Nevertheless, very few participants provided comments that suggested that they recognized that proof transcription is not disciplinarily accountable. For example, one participant in the routine condition said: "*One student could have put the proof on the board while the other pointed out* *the reasons as progress is made through the proof. Or one student could have written the proof on the board while someone from another pair could have shown their work also*." (Storyboard 27017, Participant 4995). Even in some of these cases, participants still rated the work of the teacher highly. For the breach version of Storyboard A, I found that a large number of participants cited the individual obligation as a reason for justifying the teaching actions depicted in the scenario. However, in comparison to the routine version, more participants explicitly referenced the communication practice depicted in the episodes. For example, one participant in the breach version said: "The *teacher really didn't have any actions other than allow students to present. If this were at the teachers directions then kudos having one student write and the other talk."* (Storyboard 27007, participant 5049).

In general, across both the breach and routine versions of the storyboard targeting proof transcription and the storyboard targeting alternative communication practices, participants' comments largely suggest a stronger sense of obligation towards either the student presenting at the board, or the teacher-student interactions depicted in the episode. While this is not necessarily a bad thing, these findings suggest that conflicting obligations might impact how much opportunities teachers provide for students to develop disciplinary-specific mathematical communication skills. Does this imply that all teachers do not expect students to engage in multimodal presentation of their proofs? Not at all, but these findings do demonstrate the complexity of the reasoning that governs teachers' instructional behaviors. As a result, while one teacher might prioritize the disciplinary obligation, another teacher might prioritize other obligations.

5.2 Implications

The results of this study have implications for both research and education. First, the results of the replication portion of this study support the notion that there is a common foundation of instructional practice upon which progressively unique routines could be constructed. (Dimmel & Herbst, 2020). This notion is further supported by the findings from the second part of the study which investigated teachers' reactions to alternative communication practices that steer students toward a multimodal presentation of their proofs. The findings from this study indicated that although proof transcription is a recognized communication practice during proof presentation there are opportunities for teachers to create value for disciplinary approximate communication practice when students are presenting proofs. The implication for teaching is that rather than concentrating solely on teaching students how to write proofs, teachers could provide opportunities for students to learn multimodal presentation skills by teaching them to generate their proofs using speaking, writing, drawing, and gesturing while providing commentary on their proof strategy as the proof is being generated (Dimmel & Herbst, 2020).

To create such opportunities for students to develop multimodal presentation skills, teachers might need to move away from the formalizations of the two-column proof to designing proof activities for students that inevitably result in multimodal exchanges. This is significant because previous studies on proof presentations have shown evidence that students have difficulties with the formalization of proofs but provide sound arguments when they are required to orally present their proofs (Stylianides, 2019; Soto-Johnson, 2012; Campbell et al, 2020). Other studies e.g., Kokushkin, 2022 have shown that using hand gestures when constructing, reading, or presenting proofs, supports students' cognitive offloading. Hence, creating opportunities for students to use multimodal presentations in presenting their mathematical arguments might help students with difficulties associated with proving activities.

As an example, Dimmel (2014) suggests a non-written mode of communication whereby a teacher assigns a proof task that requires students to present proofs orally in such a way that the classroom community would understand without writing the claims and justifications on the board. They argued that this might prompt the student to think about how to talk clearly while gesturing at a diagram when presenting their proofs. Such work would highlight and enable students to practice oral and gestural techniques of mathematical communication. Another proof task that could help students develop a multimodal mode of communicating proofs could be a task that requires students to present proof to the class using unlabeled diagrams. The goal of the activity would be for students to be able to formulate a mathematical argument while coordinating conceptual language, pointing, and possibly other movements.

Because the concept of proofs is difficult to teach and learn (Stylianides, 2019), a large number of mathematics education research studies on proof have tended to focus on identifying and correcting students' or teachers' conceptual or psychological deficiencies about proof (Dimmel, 2014). While it's crucial to comprehend how students view proofs, other disciplinary elements of argumentation practices, like the way arguments are presented, are just as crucial. Thus, the current study opens the way for future studies on how students communicate in mathematics classrooms. An important area for future research would be to investigate whether the instructional norm of transcription is also recognized in other domains of mathematics instruction such as algebra, precalculus, or calculus. Another research implication that would be worth investigating relates to investigating students' mathematical communication practices at the elementary or middle school level. For example, what sort of expectations do middle school mathematics teachers have for how their students communicate their mathematical ideas? Since mathematical communication is emphasized as one of the curriculum standards for students in grades k–12 by NCTM, Common Core, and other education standards. Research on existing classroom communication practices with the goal of helping students develop disciplinary-approximate standard of communication is important and should be encouraged.

In this research, I explored how participants' reference their professional obligations as a way of justifying or criticizing the hypothetical communication practices depicted in the instructional scenarios. It may be useful to understand the rationale for teachers' tacit acceptance of proof transcription by investigating how institutional factors such as teachers' years of geometry teaching experience might influence their instructional choices.

5.3 Conclusions

I return to the overarching question of this thesis: How do students learn discipline-specific mathematical communication practices? It appears that students' presentation of proofs is not within the proximity of the standards of the discipline of mathematics and US secondary mathematics teachers do not have such expectations of student's written work. Yet, evidence from this study suggests that there are opportunities for teachers to create value for disciplinary approximate communication practices when students present their proofs. This is significant because by investigating normative patterns of interactions in the classroom, this study sheds light on the way mathematics is taught, and by extending the research to investigating teachers' reactions to alternative communication practices, this study provides an insight into whether secondary mathematics teachers find value for instructions that are designed to help students develop effective mathematical communication skills and how teachers' four professional obligations can impact their willingness to deviate from the norm of proof transcription towards a more disciplinary approximate style of communication. Therefore, this study contributes to works of literature that can help inform mathematics education reform initiatives.

The experimental approach of testing teachers' recognition of hypothesized instructional norms or practices allows for a way to examine shared classroom instructional practices and lends evidence to the notion that teaching is a socially embedded activity. By investigating specific teaching practices, this study provides an opportunity to meet teachers where they are thereby providing insights into their routine instructional practices and the complexities of enacting instructional change. This is significant because it provides an understanding of the rationality that produces, regulates, and sustains mathematics instruction which in turn, can help inform professional development programs aimed at reforming instructional practices, and help shape the development of mathematics instruction and teachers' education programs.

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