Durability of Entrapped Gas in Quasi-saturated Porous Media: Two Geotechnical Perspectives

Babak Mahmoodichanzab
University of Maine, babak.mahmoodi@maine.edu

Follow this and additional works at: https://digitalcommons.library.umaine.edu/etd

Part of the Geotechnical Engineering Commons

Recommended Citation

This Open-Access Thesis is brought to you for free and open access by DigitalCommons@UMaine. It has been accepted for inclusion in Electronic Theses and Dissertations by an authorized administrator of DigitalCommons@UMaine. For more information, please contact um.library.technical.services@maine.edu.
DURABILITY OF ENTRAPPED GAS IN QUASI-SATURATED POROUS MEDIA: TWO GEOTECHNICAL PERSPECTIVES

By

Babak Mahmoodi

M.Sc., University of Tehran, 2016
B.Sc., University of Tehran, 2013

A DISSERTATION

Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy (in Civil and Environmental Engineering)

The Graduate School
The University of Maine
December 2020

Advisory Committee:

Dr. Aaron Gallant, Assistant Professor of Civil and Environmental Engineering, Advisor

Dr. Bill Davids, Professor of Civil and Environmental Engineering

Dr. Shaleen Jain, Professor of Civil and Environmental Engineering

Dr. Ben Mason, Associate Professor of Civil and Construction Engineering

Dr. Carlos Vega-Posada, Professor of Civil and Environmental Engineering
DURABILITY OF ENTRAPPED GAS IN QUASI-SATURATED POROUS MEDIA: TWO GEOTECHNICAL PERSPECTIVES

By Babak Mahmoodi

Dissertation Advisor: Dr. Aaron Gallant


Entrapped gas bubbles in quasi-saturated porous media are of practical importance in the field of science and engineering. Gas bubbles, which may occur naturally or be introduced artificially, significantly influence the mechanical behavior of soil. In this study, the durability of gas is examined from two geotechnical engineering perspectives.

In the field of geotechnics, the artificial introduction of gas is being considered, as it has been widely recognized that entrapped gas, even in nearly-saturated sediments, has an appreciable influence on soil’s mechanical behavior. Entrapped gas bubbles in quasi-saturated sediments significantly increases the pore fluid compressibility and suppresses the generation of positive excess pore water pressure, thus increasing liquefaction resistance. The first topic considers the influence of gas durability as it relates to a novel ground improvement method, induced-partial-saturation (IPS). Recognizing that harnessing the mechanical benefits of gas may offer an economical liquefaction mitigation method with a low carbon footprint—that is relatively non-intrusive and applicable to new and existing civil infrastructure systems—previous studies have successfully demonstrated IPS increases the liquefaction resistance of soil. However, the efficacy of IPS is linked to the long-term persistence of entrapped gas after emplacement. Motivated by the potential benefits of IPS, this research aims to address the salient consideration of gas durability and
its longevity after emplacement—an effort that so far has been secondary to demonstrating
IPS can mitigate liquefaction, and has not yet been meaningfully addressed.

Assessment of gas longevity is challenging, particularly through physical
demonstrations, which cannot be performed practically on time-scales of interest (i.e.
decades). Modeling of the physical and chemical mechanisms that influence the durability
and persistence of entrapped bubbles was undertaken, as it is a practical avenue to
overcome these limitations and provide novel insight. The governing aqueous-phase
advection-diffusion processes and inter-phase gas kinetics associated with bubble
dissolution are simulated in a finite-difference framework. A greater understanding behind
material-dependent tortuosity linked to effective diffusion coefficients used in the modeling
effort are derived from experiments performed at the Advanced Geotechnics Lab in the
civil and environmental engineering department at University of Maine, using equipment
readily available in many geotechnical engineering laboratories. The modeling framework is
also validated with elemental and bench-scale experiments performed by others, and then
extended to address soil resaturation rates under a variety of subsurface conditions.

The numerical modeling framework, which simulated the aqueous-phase gas mobility
contributing to the dissolution of gas, was capable of predicting experimental observations
of gas dissolution from several independent studies—which had varied spatial scales, pore
fluid constituents, gas solubilities, and boundary conditions—under both hydrostatic and
groundwater flow conditions. Under hydrostatic conditions, the thickness of the gassy layer
decays due to a diffusion-induced resaturation front that advances from the boundary of
quasi-saturated sediment. Under seepage conditions, a saturation front progressively
advances downgradient due to imbibing groundwater that acts as a sink. The numerical
study demonstrates that emplaced gas is durable to the extent where diffusion- and
groundwater seepage-induced dissolution should not discourage advancement of IPS, but
will not remain indefinitely. Potential solutions to mitigate the decay of a gassy soil layer
are discussed.
The second topic considers the sediment response to tsunamis loading. Tsunamis are an extreme coastal hazard that cause catastrophic damage and disruption in the nearshore environment. In addition to inundation and flooding, tsunamis are attributed to the formation of deep-seated scour features and erosion in the soil bed, which can be exacerbated by the generation of excess pore water pressures that instigate hydraulic gradients and momentary liquefaction. The pore water pressure response in the soil bed is influenced by: i.) seepage arising from the changing boundary pore water pressure at the soil bed surface and ii.) a partially undrained mechanical response that pressurizes the pore fluid as the soil skeleton deforms under the changing weight of the wave. Shallow nearshore coastal sediments will contain entrapped gas due to tidal fluctuations. Differential pressurization of pore water that instigates groundwater flow is intimately linked to entrapped gas and the associated pore fluid compressibility. Tsunami waves can be in excess of 10 m, generating fluid pressures in the sediment that will compress, and possibly dissolve, air bubbles as the wave height increases. Therefore, from a gas durability perspective, tsunami loading conditions are extreme and relevant to assessment of the sediment under loading imposed by this hazard. Consideration of gas kinetics, and the durability of gas as it relates to the pore fluid compressibility and pore water pressure response has not been addressed.

To address the role of gas durability on the pore water pressure response and liquefaction during tsunami loading, the modeling effort was extended, and gas kinetics were incorporated in a poroelastic seepage-deformation model to examine the durability of gas and pore fluid hardening under shorter-duration, but extreme, loading scenarios where large excess pore water pressures are generated and high hydraulic gradients develop. A tsunami loading event was chosen to demonstrate the influence of gas durability because: a.) duration of the event is on the order of tens of minutes (where earthquakes only last seconds to minutes); b.) tsunamis impose large changes in total stress on the sediment that is linked to the mechanical generation of large excess pore water pressure; which is further
exacerbated by c.) a dynamic boundary pore water pressure imposed at the seabed surface during runup and drawdown of the wave. Results were compared with simpler pore fluid compressibility assumptions (i.e. constant compressibility) to highlight the influence of gas kinetics on the pore water pressure response in the sediment.

Numerical studies indicate that the temporal evolution of excess pore water pressure generated in the sand bed was appreciably influenced by the consideration of gas kinetics and pore fluid hardening. When inter-phase gas exchange is considered to simulate the pore fluid compressibility, stabilizing hydraulic gradients (i.e. infiltration) during runup of a tsunami wave are appreciably less than when a constant pore fluid compressibility (i.e. no compression or dissolution) of the gas is considered. The duration of sustained liquefaction after the wave has receded is significantly less than when a constant pore fluid compressibility was assumed. The maximum depth of liquefaction increases with thickness of a layer where gas is entrapped, but only to an extent. Additionally, the assumed tsunami wave-height time series plays a role in the maximum depth of liquefaction. Notably, when the rate of drawdown is greater, the maximum depth of liquefaction increases.
DEDICATION

This dissertation is dedicated to my parents, who instilled in me the virtue of perseverance and commitment and relentlessly encouraged me to strive for excellence.
I wish to thank my committee members who were more than generous with their expertise and precious time. A special thanks to Dr. Aaron Gallant, my committee chairman for his countless hours of reflecting, reading, encouraging, and most of all patience throughout the entire process. Thank you Dr. Bill Davids, Dr. Shaleen Jain, Dr. Ben Mason, and Dr. Carlos Vega-Posada for agreeing to serve on my committee and for your guidance.

I would like to acknowledge and thank Geosyntec Consultants for their support and encouragement over the final semester of my PhD studies.

Finally I would like to thank all the graduate students, faculty members and staff at the CIE department at the University of Maine who made this journey a unique experience for me, a journey which I will remember and cherish for many years to come.
# TABLE OF CONTENTS

DEDICATION ................................................................................ iii

ACKNOWLEDGEMENTS ............................................................ iv

LIST OF TABLES ........................................................................... ix

LIST OF FIGURES .......................................................................... x

1. INTRODUCTION ....................................................................... 1
   1.1 Induced-Partial-Saturation .................................................... 2
   1.2 Tsunami Loading ............................................................... 4
   1.3 Research Goals and Methodology ........................................... 5
   1.4 Benefits ........................................................................... 7
   1.5 Outline of this dissertation ................................................... 8

2. BACKGROUND ......................................................................... 10
   2.1 Induced-partial-saturation .................................................... 10
      2.1.1 Mechanical Behavior of Gassy Soil ................................. 10
      2.1.2 Previous IPS Studies .................................................. 16
      2.1.3 Gas Longevity ......................................................... 22
   2.2 Tsunami Loading ............................................................... 26
      2.2.1 Pore Water Pressure Response During Tsunami Loading .... 27
      2.2.2 Influence of Entrapped Gas on Pore Water Pressure Response 30
2.3 Theory and the Governing Mechanisms Influencing the Durability of Entrapped Gas

2.3.1 Stability of Entrapped Bubbles

2.3.2 Gas Mobility: Advection and Diffusion

2.3.3 Inter-phase Gas Kinetics

2.3.4 Effective Aqueous-Phase Gas Diffusion Coefficient

3. LABORATORY DETERMINATION OF AQUEOUS-PHASE GAS DIFFUSION COEFFICIENTS AND TORTUOSITY IN NON-PLASTIC SOILS

3.1 Introduction

3.2 Methodology

3.3 Results

3.4 Discussion

4. ASSESSING THE PERSISTENCE OF ENTRAPPED GAS FOR INDUCED-PARTIAL-SATURATION

4.1 Theory

4.1.1 Stability of Entrapped Bubbles

4.1.2 Gas Mobility: Advection and Diffusion

4.1.3 Inter-phase Gas Kinetics

4.2 Model Formulation

4.3 Model Validation

4.3.1 Gas diffusion in sandstone

4.3.2 Gas diffusion in sand
4.3.3 Gas dissolution due to vertical seepage in sand ......................... 76
4.3.4 Horizontal seepage in synthetic sand aquifer .............................. 77
4.4 Hydrostatic Conditions .................................................................. 81
  4.4.1 Effect of Tortuosity Factor and Layering ................................. 86
4.5 Groundwater Seepage Conditions ................................................. 89
4.6 Gas Longevity Coefficient .............................................................. 92
  4.6.1 Gas longevity coefficients for hydrostatic condition ......... 94
  4.6.2 Gas longevity coefficient for groundwater seepage condition .... 96
4.7 Discussion .................................................................................. 97
4.8 Summary and Conclusions ........................................................... 98

5. INFLUENCE OF GAS KINETICS ON LIQUEFACTION TRIGGERING
  DURING TSUNAMI LOADING .......................................................... 102
  5.1 Introduction .............................................................................. 102
  5.2 Background .............................................................................. 104
  5.3 Gas Kinetics............................................................................. 108
  5.4 Model Formulation ................................................................. 112
  5.5 Geometry and Conditions for Numerical Study ...................... 113
  5.6 Numerical Results ................................................................. 115
    5.6.1 Interaction between a quasi-saturated and saturated sand bed .... 116
    5.6.2 Effect of Pore Fluid Compressibility Assumption and Thickness of
          Sediment Containing Entrained Air ........................................... 119
    5.6.3 Influence of Initial Gas Content and Degree of Saturation ........ 123
    5.6.4 Influence of Beach Profile and Tsunami Wave-Height Time Series .... 126
LIST OF TABLES

2.1 Examples of the laboratory elemental tests conducted showing the effectiveness of gas on improving liquefaction resistance of sandy soils. ..... 13

2.2 Examples of tests conducted showing the effectiveness of gas on improving liquefaction resistance of sandy soils. ......................... 21

2.3 A list of experimental studies discussing durability of entrapped gas in porous media. .......................................................... 25

2.4 A literature review on the tortuosity factors measured for aqueous-phase diffusion in saturated porous media (after Shackelford 

3.1 Summary of aqueous-phase gas diffusion experiments, dimensions, and gas properties for experiments conducted with the pressure-decay-method ................................................................. 54

3.2 Summary of the range and average tortuosity factors determined from pressure-decay data for silt, sand, and gravel. ..................... 56

4.1 Summary of parameters used to simulate experiments to validate the numerical model .............................................................. 72

4.2 Longevity coefficient for hydrostatic condition condition. ............... 95

4.3 Longevity coefficient for flow condition........................................ 96

5.1 Assumed modelling parameters .................................................. 116
2.1 Behavior of saturated sands in a drained triaxial compression test.
a.) deviatoric stress vs. axial strain; b.) deviatoric stress vs. void ratio; c.) confining effective stress vs. void ratio (after Kramer et al., 1996). ............................................................... 12

2.2 Behavior of saturated sands in an undrained triaxial compression test. a.) effective confining stress vs. void ratio; b.) stress path for a loose sand c.) axial strain vs. deviatoric stress for a loose sand; d.) axial strain vs. excess pore pressure for a loose sand (after Kramer et al., 1996). ............................................................... 14

2.3 Laboratory test data showing the effect of gas on the liquefaction resistance of: a.) Ottawa sand; b.) Toyoura sand; c.) Tongjiazhi sand; d.) Niigata sand (after Yang et al., 2004). ......................... 15

2.4 Tests used for measuring the effectiveness of induced-partial-saturation on improving liquefaction resistance: a.) triaxial setup used by He & Chu (2014); b.) bench-scale shaking table setup used by Eseller-Bayat et al. (2013a); c.) large-scale shaking table setup used by Kato & Nagao (2020); d.) ground shaker machine used by Flora et al. (2020). ............................................................... 19

2.5 Effect of IPS on the pore water pressure response from different studies: a.) He & Chu (2014); b.) Eseller-Bayat et al. (2013a); c.) Flora et al. (2020). ............................................................... 20
2.6 Nearshore sediments under a tsunami wave: a.) conceptual demonstration of a representative sand column in seabed; b.) behavior of sand column during tsunami runup; c.) behavior of sand column during tsunami drawdown.

2.7 Conceptual illustration of inter-phase gas exchange in porous media.

3.1 Effect of tortuosity on the effective diffusion length \( (L_e > L) \) through porous media.

3.2 Experimental configuration of the pressure decay method.

3.3 The stainless steel high pressure chamber used for diffusion tests.

3.4 A view of the inside of the diffusion chamber containing reconstituted Ottawa sand.

3.5 Boundary conditions during a pressure decay test.

3.6 Dual-channel pneumatic controller (brown box on top shelf).

3.7 Gas bubbles used as a gas source.

3.8 Hydraulic volume/pressure controller.

3.9 Grain size distribution of the soils tested.

3.10 Flasks used for deaeration of soil and sample reconstitution.

3.11 Water deaerator apparatus used for experiments.

3.12 Representative pressure decay observed and simulated from inverse analyses of the effective aqueous-phase diffusion coefficient through: a) water; b) silt; c) sand; d) gravel.
3.13 Aqueous-phase concentration profiles computed from inverse analyses of the effective aqueous-phase gas diffusion coefficient to demonstrate the depth gas penetrated through: a) water only; b) saturated silt; c) saturated sand; and d) saturated gravel........................................... 56

3.14 Images illustrating grain shape: a.) top and side profile view of grave; b.) top and side profile view of sand; c.) top view of silt............. 57

3.15 a) Conceptual illustration of particle shape on tortuosity b) computed local tortuosity factor around an elliptical shape with different dimensions............................................................. 58

4.1 Conceptual IPS scenario and associated groundwater conditions after gas emplacement: a.) liqueifiable layer improved via IPS; b.) aqueous-phase dissolved gas concentrations and water-gas saturation through cross-section A-A soon after gas emplacement. .................. 64

4.2 Conceptual illustration of inter-phase gas transfer and dissolution of an entrapped bubble in soil. ................................................... 68

4.3 Algorithm to couple the advection-diffusion equation and kinetic bubble dissolution for finite difference simulations. ...................... 70

4.4 Experimental configuration of column tests performed to assess gas dissolution and changes in gas content and degree of saturation: a) 1D diffusion tests under hydrostatic conditions by McWhorter et al. (1973) with test configuration based on Adam et al. (1969); b) 1D diffusion test under hydrostatic conditions by Yegian et al. (2007); c.) 1D constant head seepage test by He et al. (2016). ....................... 74
4.5 Comparison between experimental observations and numerical predictions for column tests: (a) temporal changes in normalized gas volume from McWhorter et al. (1973) diffusion experiment; (b) temporal changes in $S_r$ averaged over the height of the column from Yegian et al. (2007) diffusion experiment; (c) temporal changes in $S_r$ averaged over the height of the column from He et al. (2016) seepage experiment.......................... 75

4.6 Experimental configuration, dimensions, boundary conditions, and location of TDR probes in a synthetic aquifer (soil tank) used to simulate groundwater flow and dissolution of entrapped air during the McLeod et al. (2015) study.................................................. 78

4.7 Comparison of numerical results and observations by McLeod et al. (2015) of gas dissolution under flow conditions in a synthetic aquifer: a.) position of the saturation front downgradient at several depths compared to the pore volumes of groundwater introduced; b.) location of the saturation front at the end of the experiments when 19 pore volumes of groundwater were introduced ......................... 80

4.8 a.) Depths and boundary conditions considered to demonstrate gas longevity under hydrostatic conditions for a 1.5 m gassy layer; b.) conceptual changes in thickness and progression of the diffusion-induced saturation front for the gassy layer under hydrostatic conditions with different boundary conditions; c.) depths considered to demonstrate gas longevity and temporal changes in the saturation front under horizontal groundwater seepage conditions........... 82
4.9 Resaturation curves and associated $S_r$ and gassy layer thickness:
a.) influence of depth and gas type on the rate of diffusion-induced resaturation of the gassy layer for boundary condition 1 in Figure 4.8a; b.) influence of the boundary condition on the rate of diffusion-induced resaturation of the gassy layer; c.) influence of bubble radius on the rate of diffusion-induced resaturation of the gassy layer. Note: $S_r$ is computed based on remaining gas content and the initial thickness of gassy layer to provide context for $S_r$ reported during column tests, as it is the thickness of the gassy layer that changes, as indicated by the secondary vertical axis.

4.10 Different soil layering scenarios affecting the gas longevity: (a) homogeneous soil layer; (b) homogeneous soil layer with a top cap; (c) layered soil system.

4.11 Effect of silt lenses, tortuously factor and silt cap on gas longevity under hydrostatic condition.

4.12 Computed pore volumes and temporal evolution of the saturation front based on associated $v_s$ (see Figure 4.8c) with consideration of: a.) the influence of depth and gas type; b.) influence of the initial bubble radius; c.) influence of the initial water-gas saturation of infiltrating groundwater ($\eta_o = 100\%$).

4.13 Conceptual illustration of the advancement of a saturation front for a gassy soil volume: (a) gas dissolution under groundwater seepage conditions; (b) diffusion-induced resaturation under hydrostatic conditions.
5.1 Conceptual illustration of excess pore pressures and groundwater flow in a soil column under tsunami loading at two instances in time: a.) tsunami runup; b.) tsunami drawdown. ................................................. 107

5.2 Changes to the bubble size due to: a.) Changes in pore pressure; b.) gas dissolution and transport. .................................................. 109

5.3 Algorithm incorporating gas kinetics to update pore fluid compressibility when solving the governing poroelasticity equations ............. 113

5.4 Numerical model inputs definitions: a.) soil column geometry; b.) tsunami profiles. ................................................................. 115

5.5 Contours of the response of a soil column with $Z_g = 2m$ and $Z_s = 8m$ with $S_r = 97\%$ under Tsunami 2 : a.) excess pore pressure; b.) excess pore pressure gradient; c.) effective stress ratio. .......................... 118

5.6 Effect of the initial depth of gassy layer on the response of sandy sediments to tsunami 2 loading assuming a constant pore fluid compressibility : a.) depth of full liquefaction; b.) max. depth of full liquefaction. .............................................................. 121

5.7 Effect of the initial depth of gassy layer on the response of sandy sediments to tsunami 2 loading considering compression of gas bubbles: a.) depth of full liquefaction; b.) max. depth of full liquefaction. .............................................................. 121

5.8 Effect of the initial depth of gassy layer on the response of sandy sediments to tsunami 2 loading considering compression, dissolution and transport of gas bubble: a.) depth of full liquefaction; b.) max. depth of full liquefaction. .............................................................. 122
5.9 Contours of the excess pore pressure of a soil column with $Z_g = 2m$
and $Z_s = 8m$ with $S_r = 97\%$ under *Tsunami 2* : a.) Constant
Compressibility; b.) Compression Plus KBD. ......................... 124

5.10 Contours of the hydraulic gradients of a soil column with $Z_g = 2m$
and $Z_s = 8m$ with $S_r = 97\%$ under *Tsunami 2* : a.) Constant
Compressibility; b.) Compression Plus KBD. ......................... 125

5.11 Effect of the initial degree of saturation on maximum depth of
liquefaction during *tsunami 2* loading for different pore fluid
compressibility assumptions: a.) $Z_g = 0\text{ m}$; b.) $Z_g = 2\text{ m}$; c.) $Z_g = 4$
\text{m}; d.) $Z_g = 10\text{ m}$. ......................................................... 129

5.12 Effect of the initial degree of saturation on liquefaction during
*tsunami 2* loading for different pore fluid compressibility assumptions:
a.) $Z_g = 0\text{ m}$; b.) $Z_g = 2\text{ m}$; c.) $Z_g = 4\text{ m}$; d.) $Z_g = 10\text{ m}$. ................. 130

5.13 Effect tsunami wave-height time series on the maximum depth of
momentary liquefaction: a.) $Z_g = 0\text{ m}$; b.) $Z_g = 2\text{ m}$; c.) $Z_g = 4\text{ m}$;
d.) $Z_g = 10\text{ m}$. ................................................................. 131

5.14 Effect gas bubble distribution assumption on the liquefaction depth
during Tsunami 2 loading: a.) $Z_g = 2m$ s; b.) $Z_g = 4m$. ................. 132

A.1 Pressure decay profiles observed and simulated from inverse analyses
of the aqueous-phase diffusion coefficient corresponding to: a) Test
1 (He through water); b) Test 2 (N$_2$ through water; c) Test 3 (N$_2$
through water); d) Test 4 (N$_2$ through Silt). ............................ 153
A.2 Pressure decay profiles observed and simulated from inverse analyses of the aqueous-phase diffusion coefficient corresponding to: a) Test 5 ($N_2$ through Silt); b) Test 6 ($N_2$ through Silt; c) Test 7 ($N_2$ through Sand); d) Test 8 ($N_2$ through Sand).

A.3 Pressure decay profiles observed and simulated from inverse analyses of the aqueous-phase diffusion coefficient corresponding to: a) Test 9 ($N_2$ through Sand); b) Test 10 ($N_2$ through Sand; c) Test 11 ($N_2$ through Sand); d) Test 12 ($N_2$ through Gravel).

A.4 Pressure decay profiles observed and simulated from inverse analyses of the aqueous-phase diffusion coefficient corresponding to: a) Test 9 ($N_2$ through Gravel); b) Test 10 ($N_2$ through Gravel).
CHAPTER 1
INTRODUCTION

Entrapped gas bubbles in quasi-saturated porous media are of practical importance in the fields of science and engineering. Gas bubbles may occur naturally in soil, such as through air trapping during rapid changes in the water table elevation and infiltration (Fayer & Hillel, 1986; Constantz et al., 1988) or generation of biogenic gas (Rad & Lunne, 1994; Rebata-Landa & Santamarina, 2012). Naturally occurring entrapped gas has been studied in the context of: changes in groundwater’s hydraulic behavior (e.g. Christiansen, 1944; Faybishenko, 1995), which impacts drainage as it applies to irrigation (Powers, 1934) and beach morphology (Horn, 2002); “excess air” dissolved in groundwater (Heaton & Vogel, 1981) that influences groundwater dating (Cirpka & Kitanidis, 2001; Holocher et al., 2002; Geistlinger et al., 2005); geomorphological seabed depressions or “pockmarks” on the seabed (e.g. King & MacLEAN, 1970; Kelley et al., 1994; Barnhardt et al., 1997; Rogers et al., 2006; de Vries et al., 2007; Mazzini et al., 2016); instigation of submarine landslides (e.g. Christian et al., 1997; Riboulot et al., 2013), among other topics.

Gas may also be introduced artificially for engineering applications. For example, in the petroleum industry gas is injected into oil reservoirs for storage of gas, maintenance of reservoir pressure, and enhanced recovery of hydrocarbons (Riazi et al., 1994; Riazi, 1996). In environmental engineering air sparging has been adopted to facilitate aerobic in situ bioremediation of soils contaminated with volatile organic compounds (VOCs) and fuels (e.g. Fry et al., 1996, 1997).

In the succeeding sections two topics where the durability of gas is of interest, from a geotechnics perspective, are introduced. The topics deal with a novel method of ground improvement, where the artificial introduction and longevity of gas. The second deals with the tsunami hazard, where naturally occurring entrapped gas can instigate hydraulic gradients and sediment instability.
1.1 Induced-Partial-Saturation

In the field of geotechnics, the artificial introduction of gas is being considered, as it has been widely recognized that entrapped gas, even in nearly-saturated sediments, has an appreciable influence on the mechanical behavior of soil. Entrapped gas bubbles in quasi-saturated sediments significantly increases the pore fluid compressibility (Skempton, 1954; Fredlund, 1976) and suppresses the generation of positive excess pore water pressure in loose granular soils sheared under globally undrained conditions (Sherif et al., 1977; Yoshimi et al., 1989; Grozic et al., 1999, 2000; Okamura et al., 2006). From a geotechnics perspective, the dampening of excess pore water pressure is beneficial, as it increases the liquefaction-resistance of saturated loose granular media. The liquefaction phenomenon, which is the transition of soil from a solid- to fluid-like behavior, results in partial or total loss of soil shear strength, which can have catastrophic consequences.

Yegian et al. (2007) first introduced the expression “induced-partial-saturation” (IPS) to refer to the deliberate introduction of occluded gas bubbles to mitigate liquefaction. In their experiments they generated gas using electrolysis and performed bench-scale shake table tests to demonstrate that IPS increases liquefaction resistance. Several shake table testing experiments followed to demonstrate increased liquefaction resistance with biogenic gas (He et al., 2013), reaction of sodium perborate monohydrate (efferdent) with water to generate oxygen bubbles (Eseller-Bayat et al., 2013a). “Field-scale” shake table tests (3.6 m x 10 m x 4.5 m deep) were also performed by Kato & Nagao (2020), whereby gas bubbles were introduced with a “microbubble generator,” which successfully demonstrated that the excess pore water pressure response is significantly dampened. Lab (Yasuhabara et al., 2008) and field (Okamura et al., 2011) trials have also been performed to demonstrate IPS via air-injection. Some of the most compelling evidence on the efficacy of IPS was revealed in a recent study conducted in Italy (Flora et al., 2020). Directional drilling methods were used to install horizontal well screens and inject air into the ground. A large shaker truck was used to impose vibrations and simulate strong ground motions in the ground. Shaking was
applied to both treated and untreated volumes of soil. In the untreated liquefied soil pore water pressure measurements revealed that the vibrations liquefied the soil. However, in soil containing entrapped gas, excess pore water pressures were reduced by nearly 90%.

Recognizing that harnessing the mechanical benefits of gas may offer an economical liquefaction mitigation method with a low carbon footprint—that is relatively non-intrusive and applicable to new and existing civil infrastructure systems—previous studies successfully demonstrated IPS increases the liquefaction resistance of soil.

Once gas bubbles are entrapped, capillary forces tend to immobilize the bulk flow of gas through pore throats. External forces imposed by the viscous flow of groundwater and buoyancy are relatively small in comparison (Peck, 1969; Fry et al., 1997; Maryshev, 2017). Gas has also has been deomstrated to remain immobile in soil subjected to dynamic excitation (Eseller-Bayat et al., 2013a). However, the efficacy of IPS is linked to the long-term persistence of entrapped gas after emplacement. Infrastructure systems supported on liquefaction-susceptible soils are often designed to operate for decades, even more than a century. Gas is immiscible with pore water as long as the aqueous-phase (i.e. dissolved) concentration is in equilibrium with partial pressures of each gas specie in a bubble (Henry’s law). However, aqueous-phase diffusion can lead to concentrations deficits and dissolution of gas in porous media (e.g. Bloomsburg & Corey, 1964; Adam et al., 1969; McWhorter et al., 1973).

Yegian et al. (2007) performed diffusion tests and monitored changes in a desaturated 1.5 m sand column for 442 days (1.2 years) and observed a limited change in the degree of saturation from 82.1% to 83.9%. Eseller-Bayat et al. (2013a) performed similar experiments on a 1.2 m column of sand and reported changes in the degree of saturation from 82% to 84% after 805 days (2.2. years). These laboratory demonstrations are encouraging, but represent limited subsurface conditions (i.e. shallow depths with low hydrostatic pressures). Some lab studies directed at IPS have demonstrated that gas will remain entrapped under sustained seepage conditions (Eseller-Bayat et al., 2013a), though observations from others
suggest groundwater flow conditions may accelerate the dissolution of gas (e.g. McLeod et al., 2015; He et al., 2016). Field evidence has suggested that entrapped gas may persist for appreciable time-periods. Delayed increases in penetration resistance, on the order of months and years, to verify ground improvement after blast densification have been partially attributed to entrapped gas generated by explosives (Dowding & Hryciw, 1986; Finno et al., 2016; Gallant & Finno, 2017). Okamura et al. (2006) observed that loose sands densified with sand compaction piles (SCPs) contained entrapped gas that was exhausted from a casing pipe (an unintended consequence of the construction method). Frozen sand samples collected months after SCP construction consistently revealed the degree of saturation ranged between 75% and 90% at several sites. Frozen samples were also obtained at three sites where SCPs were installed 4, 8, and 26 years prior, where they measured degrees of saturation between 75-98%, 95-100%, and 92-98% at each site, respectively (though the initial degree of saturation was unknown). However, it may be noted that higher degrees of saturation were observed at sites where the gas would have been entrapped for longer time periods. Based on both laboratory and field observations, gas may be introduced and persist for appreciable periods of time, but studies demonstrating its longevity indicate that it will not last indefinitely.

1.2 Tsunami Loading

Tsunamis are an extreme coastal hazard that cause catastrophic damage and disruption in the nearshore environment. In addition to inundation and flooding, tsunamis enhance sediment mobility attributed to the formation of deep-seated scour features and erosion in the soil bed. Takahashi et al. (1995) reported that the 1960 Chilean tsunami resulted in more than 8 m of erosion at the Kesennuma Port. Deep-seated scour and erosion as great as 4 m contributed to the failure of a breakwater during the 1993 Okushiri tsunami in Japan (Kimura et al., 1997; Yeh & Mason, 2014). Erosion, which is attributed to soil bed shear
stresses imposed during inundation and recession of the tsunami wave, can be exacerbated by the development of hydraulic gradients in the sediment at depth (Yeh & Mason, 2014).

The pore water pressure response in the soil bed is influenced by: i.) seepage arising from the changing boundary pore water pressure at the soil bed surface and ii.) a partially undrained mechanical response that pressurizes the pore fluid as the soil skeleton deforms under the changing weight of the wave, which can lead to momentary liquefaction (Young et al., 2009; Abdollahi & Mason, 2019, 2020). When soil bed deformations are considered, differential pressurization of pore water that instigates groundwater flow is intimately linked to entrapped gas and the associated pore fluid compressibility (Mahmoodi et al., 2019; Abdollahi & Mason, 2020). Tsunami waves can be in excess of 10 m (Kundu, 2007; Bryant, 2014), imposing large fluid pressures in the sediment that will compress, and possibly dissolve, air bubbles as the wave height increases. Consideration of gas kinetics, and the durability of gas as it relates to the pore fluid compressibility, during loading conditions imposed by tsunami waves on the sediment has not been addressed.

1.3 Research Goals and Methodology

Motivated in part by the potential benefits of IPS, this research aims to address the salient consideration of gas durability and its longevity after emplacement—an effort that so far has been secondary to demonstrating IPS can mitigate liquefaction. Additionally, this research addresses the influence of gas durability on the seepage-deformation response during loading conditions imposed by the natural tsunami hazard. The research presented herein addresses questions regarding:

- The role of intrinsic soil properties, such as grain size and shape, on aqueous-phase mobility that may lead to diffusion-induced resaturation.

- How the aqueous-phase mobility of gas, under both hydrostatic and groundwater flow conditions, influences the longevity of gas on time-scales of interest for civil infrastructure.
• The durability of gas under mechanical loading and undrained or partially undrained loading scenarios, which is linked to excess pore water pressure generation and development of hydraulic gradients.

Assessment of gas longevity is challenging, particularly through physical demonstrations, which cannot be performed practically on time-scales of interest (i.e. decades). Modeling the physical and chemical mechanisms that influence the durability and persistence of entrapped bubbles is a practical avenue to overcome these limitations and provide novel insight. The governing aqueous-phase advection-diffusion processes and inter-phase gas kinetics associated with bubble dissolution are simulated in a finite-difference framework. A greater understanding behind material-dependent tortuosity linked to effective diffusion coefficients used in the modeling effort are derived from experiments performed at the Advanced Geotechnics Lab in the civil and environmental engineering department at University of Maine, using equipment readily available in many geotechnical engineering laboratories. The modeling framework is also validated with elemental and bench-scale experiments performed by others, and then extended to address soil resaturation rates under a variety of subsurface conditions.

Additionally, the modeling effort is extended and gas kinetics are incorporated in a poroelastic seepage-deformation model to examine the durability of gas and pore fluid hardening under shorter-duration, but extreme, loading scenarios where large excess pore water pressures are generated and high hydraulic gradients develop. A tsunami loading event was chosen to demonstrate the influence of gas durability because: a.) duration of the event is on the order of tens of minutes (where earthquakes only last seconds to minutes); b.) tsunamis impose large changes in total stress on the sediment that is linked to the mechanical generation of large excess pore water pressures; which is further exacerbated by c.) dynamic boundary pore water pressures imposed at the seabed surface during runup and drawdown of the wave. Moreover, large hydraulic gradients and liquefaction can be sustained well after a tsunami wave has receded. Therefore, from a gas
durability perspective, tsunami loading conditions are extreme. Though this topic is not linked to IPS directly, it demonstrates another topic where the durability of gas is relevant in the field of geotechnics and hazard assessment.

1.4 Benefits

The liquefaction phenomenon, which may develop during earthquakes or tsunamis, is common when natural disasters occur. Damaging earthquakes may occur in more than 40 percent of areas included in the continental United States alone (Kavazanjian Jr et al., 1997). Jaiswal et al. (2017) report a steady increase in damage and financial losses due to earthquakes, attributing this trend to (i) population growth in earthquake-susceptible urban areas, (ii) vulnerability of the older building stock, and (iii) increased interdependency of businesses operating throughout the world. The estimated annualized earthquake losses (AEL) in the United States' building stock is $6.1 billion per year, and does not consider disruption of lifeline infrastructure or long-term economic losses affecting businesses. Also, measures like AEL are a long-term average, and the acute financial strain imposed on a region in any given year can be much greater. Much of catastrophic losses associated with extreme events can be attributed to foundation instabilities.

IPS has shown great promise as a method to mitigate liquefaction. Furthermore, it has the potential to improve ground beneath new and aging infrastructure, and over large areas like railroad and highway embankments. In these times of economic austerity, it will be important to accelerate the advancement of ground improvement alternatives like IPS, which have the potential to mitigate seismic hazards at relatively low cost. Previous work has, by in large, focused on scaling early findings that entrapped gas suppresses the generation of excess pore water pressure to demonstrate its applicability in the field. However, adoption of IPS in practice ultimately hinges on reliable methods to evaluate the persistence of gas. This study will provide a novel assessment of gas durability that will meaningfully advance this nascent liquefaction-mitigation method.
Addressing tsunami loading and physics associated with instigation of hydraulic gradients that induce liquefaction can inform future design in areas vulnerable to this rare, but catastrophic natural hazard.

1.5 Outline of this dissertation

This dissertation includes six chapters, including the introduction, background information (Chapter 2), research chapters (3-5), and summary and conclusions. Though background information is provided in chapter 2, each research chapter is written in such a matter that they are self-contained.

Chapter 2 provides relevant background information to the reader that is specific to the focus of this research. The chapter begins with background information on induced partial saturation, including details regarding the mechanical behavior of loose gassy granular sediments. Following, details regarding previous IPS work and evidence regarding gas longevity are discussed and summarized. Background information regarding tsunamis, focusing primarily on the interaction between the soil and this loading condition. A brief discussion regarding the influence of entrapped gas on the pore water pressure response is also provided, but discussed in greater detail in Chapter 5. The chapter closes with pertinent theory regarding the durability of gas, primarily focusing on the physical and chemical mechanisms influencing inter-phase gas exchange and dissolution of entrapped gas. Concepts from other solute transport problems are reviewed and discussed as they relate to the durability of entrapped gas.

Chapter 3 presents the experimental laboratory methodologies and procedures used to investigate the influence of intrinsic grain properties on the effective diffusion coefficient in soil. As there is currently no established method to determine the effective aqueous-phase gas diffusion coefficient and associated tortuosity factor through soil, a pressure-decay-method was adopted to investigate different soil types using equipment available in the Advanced Geotechnics Laboratory at University of Maine. Inverse
numerical analyses were applied to assess the effective tortuosity factor from pressure-decay data, which provided a consistent and efficient assessment of tortuosity associated with aqueous-phase gas diffusion for each soil type. Microscopic imaging of tested material was used to better understand and explain differences between the measured tortuosity factors, which were not always intuitive.

Chapter 4 presents the incorporation of the governing advection-diffusion equation and inter-phase gas kinetics in a finite difference numerical framework. The model is validated with elemental and bench-scale experiments from several independent studies, where some model parameters were informed, in part, by experiments (Ch. 3). The model is then applied to simulate gas longevity under hydrostatic and seepage conditions on time-scales pertinent to civil infrastructure. The influence of depth, gas type, bubble size, and initial aqueous-phase gas concentrations on the evolution and rate of resaturation are investigated. Potential methods to mitigate resaturation of a targeted IPS layer are discussed.

Chapter 5 presents an extension of the modeling effort to incorporate gas kinetics in a poroelastic seepage-deformation model to simulate pore fluid hardening linked to dissolution of gas. Thus, the pore fluid compressibility, which is associated with the mechanical generation of excess pore water pressure, is coupled to the physics governing inter-phase gas exchange. The governing equations are solved using the finite difference method. The response of quasi-saturated sediments and the influence of gas durability under extreme tsunami loading conditions is analyzed. Particular focus is given to the role of bubble kinetics (gas durability) on the pore fluid compressibility, which is linked to differential pressurization of pore water that instigates hydraulic gradients and momentary liquefaction in the sediment.

In Chapter 6 of this dissertation, summary of the research and conclusions are discussed, followed by suggested topics of future research.
CHAPTER 2
BACKGROUND

Inclusion of entrapped gas bubbles in otherwise fully-saturated porous medium alters the mechanical and hydraulic properties of soil. This study focuses on the durability from two different perspectives: i.) the persistence and longevity of gas for a the novel ground improvement method induced-partial-saturation; and ii.) the influence of gas durability on the pore water pressure response and liquefaction triggering under the extreme seepage and mechanical tsunami loading scenario. This chapter provides background information regarding each of the aforementioned considerations, and greater context for studying the influence of gas durability. This chapter then concludes with a summary of the relevant theory for the physical and chemical processes governing the durability of entrapped gas.

2.1 Induced-partial-saturation

Induced-partial-saturation (IPS) is a novel method to improve the liquefaction resistance of loose granular soils by artificially introducing gas. However, temporal changes in “improvement” are linked to maintenance and preservation of entrapped gas in treated soil.

This chapter discusses the mechanical benefits of using gas to improve ground using “critical-state soil mechanics” framework, followed by a review of previous studies of IPS and evidence of gas longevity.

2.1.1 Mechanical Behavior of Gassy Soil

The effect of gas on the mechanical behavior of granular (sandy) soils can be explained using critical-state soil mechanics concepts (Casagrande, 1936; Schofield & Wroth, 1968). Figure 2.1 illustrates the drained volumetric response of loose and dense sand sheared in a triaxial compression test, whereby the effective confining stress remains constant during shear. Figures 2.1a conceptually illustrate the deviatoric stress ($q$) and axial strain ($\varepsilon_a$)
response of loose (contractive) and dense (dilative) sand sheared at the same initial confining stress. Loose sands tend to have a ductile response, where dense sands exhibit “brittle” behavior and post-peak softening upon rupture and formation of a shear band. In both cases the development of shear bands develop locally (Mooney et al., 1998). During shear, loose sands accumulate positive volumetric strains and the void ratio \(e\) decreases, while dense sands exhibit volumetric expansion and the void ratio increases (Casagrande, 1936), as shown in Figure 2.1b. The "critical-state" is the density or void ratio at which soil continues to deform or flow under a constant state of stress, irrespective of initial density. The onset of critical-state may be defined as:

\[
\frac{\partial q}{\partial \varepsilon_a} = \frac{\partial e}{\partial \varepsilon_a} = \frac{\partial p'}{\partial \varepsilon_a} = 0 \tag{2.1}
\]

where \(p' = (\sigma'_1 + \sigma'_2 + \sigma'_3)/3\) is the mean normal effective stress (note that \(\sigma'_2 = \sigma'_3\) in triaxial tests).

The critical state void ratio is dependent on the initial stress-state, where a unique relationship is assumed. For triaxial compression tests, the critical-state void ratio may be defined as a function of the initial confining stress, \(\sigma'_{3c}\), as shown in Figure 2.1c. When the initial state is below this line, soil will exhibit dilative (or dense) behavior; initial states above this line are contractive (loose). Thus, it may be recognized that the same soil sheared at the same initial density may exhibit both “dense” and “loose” behavior, depending on the initial state of stress. Herein, loose and dense sands, and their associated mechanical behavior, are discussed in this context.

The critical-state framework is also useful to discuss the undrained behavior of granular soils. A distinct difference from drained behavior is the volumetric response. Fully-saturated sediments sheared under globally undrained conditions do not accumulate volumetric strain, thus no change in void ratio. Instead, a soil’s tendency to contract or dilate causes excess pore water pressure \((\Delta u)\) to be generated during shear. As shown in
Definitions and Background

Kramer (1996)

Figure 2.1: Behavior of saturated sands in a drained triaxial compression test. a.) deviatoric stress vs. axial strain; b.) deviatoric stress vs. void ratio; c.) confining effective stress vs. void ratio (after Kramer et al., 1996).

Figure 2.2a, positive and negative excess pore water pressure is generated in loose and dense sands, respectively.

In triaxial compression tests, this gives rise to changes in the effective confining stress, which results in higher shear strengths in dense sand relative to the drained conditions; the opposite is true in loose sands. The potential onset of instability (liquefaction) is illustrated in Figure 2.2b, where both the mean normal effective stress and deviatoric stress decrease at a threshold strain level (Figure 2.2c), resulting in a post-peak residual liquefied shear strength (Lade, 1994). The liquefied residual strength coincides with $\Delta u = 0$ (Figure 2.2d). Thus, an alternative definition of critical state for undrained conditions is:
\[
\frac{\partial q}{\partial \varepsilon_a} = \frac{\partial u}{\partial \varepsilon_a} = \frac{\partial \rho'}{\partial \varepsilon_a} = 0
\]  \hspace{1cm} (2.2)

IPS aims to combat the generation of positive excess pore water pressure in liquefaction-susceptible materials by permitting volume change when sheared under globally undrained conditions. In fully-saturated sediments the pore fluid (water) and solid constituents (i.e. grains) are typically assumed incompressible. However, entrapped air or gas in soil below the water table results in a quasi-saturated condition and an associated increase in the pore fluid compressibility (Okamura & Soga, 2006) that permits volume changes (i.e. changes in void ratio) to occur. Therefore, entrapped gas suppresses the generation of positive excess pore water pressure in loose granular soils sheared under globally undrained conditions, and increases the liquefaction resistance (e.g. Sherif et al., 1977; Yoshimi et al., 1989; Xia & Hu, 1991; Grozic et al., 1999, 2000; Ishihara, 2001).

Table 2.1 summarizes studies that have demonstrated the increased shear strength observed from elemental tests performed on nearly-saturated sand specimens.

Table 2.1: Examples of the laboratory elemental tests conducted showing the effectiveness of gas on improving liquefaction resistance of sandy soils.

<table>
<thead>
<tr>
<th>Test type</th>
<th>Material</th>
<th>Initial (S_r)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torsional shear</td>
<td>Ottawa sand</td>
<td>75%-100%</td>
<td>Sherif et al. (1977)</td>
</tr>
<tr>
<td>Torsional shear</td>
<td>Toyoura Sand</td>
<td>71%-100%</td>
<td>Yoshimi et al. (1989)</td>
</tr>
<tr>
<td>Triaxial</td>
<td>Tongjazhi sand</td>
<td>70%-100%</td>
<td>Xia and Hu (1991)</td>
</tr>
<tr>
<td>Triaxial</td>
<td>Ottawa sand</td>
<td>79%-100%</td>
<td>Grozic et al. (2000)</td>
</tr>
<tr>
<td>Triaxial</td>
<td>Niigata sand</td>
<td>73%-100%</td>
<td>Ishihara et al. (2001)</td>
</tr>
<tr>
<td>Triaxial</td>
<td>Toyoura Sand</td>
<td>70%-100%</td>
<td>Okamura and Soga (2006)</td>
</tr>
</tbody>
</table>

Figure 2.3 shows representative results of cyclic triaxial compression tests performed on partially-saturated sands with different initial degrees of saturation. These tests were cyclically loaded under stress-controlled conditions with different cyclic stress ratios (CSR):

\[
CSR = \frac{\Delta \sigma_1}{2\sigma_3} = \frac{\tau_{cyc}}{\sigma_{3c}}
\]  \hspace{1cm} (2.3)
Figure 2.2: Behavior of saturated sands in an undrained triaxial compression test. a.) effective confining stress vs. void ratio; b.) stress path for a loose sand c.) axial strain vs. deviatoric stress for a loose sand; d.) axial strain vs. excess pore pressure for a loose sand (after Kramer et al., 1996).

For Figures 2.3b&d, the liquefaction criteria was defined as $DA = 5\%$, where $DA$ is the double amplitude of strain. For Figures 2.3a&c, liquefaction criteria is defined by the pore water pressure ratio, $r_u=1$, where:

$$r_u = \frac{\Delta u}{\sigma'_{3c}} \quad (2.4)$$

For isotropically consolidated samples $\sigma'_{3c} = \sigma'_{1c} = \sigma'_{vc}$, where $\sigma'_{vc}$ is the consolidated vertical effective stress prior to shear. For each test series the initial B-values prior to shearing are indicated, where the B-value is:
Figure 2.3: Laboratory test data showing the effect of gas on the liquefaction resistance of: a.) Ottawa sand; b.) Toyoura sand; c.) Tongjiazhi sand; d.) Niigata sand (after Yang et al., 2004).
\[ B = \frac{\Delta u}{\sigma_3} = \frac{1}{\beta \frac{\beta_{sk}}{n} + 1} \]  

and \( \beta \) is the compressibility of the pore fluid, \( \beta_{sk} \) is the compressibility of the soil skeleton, and \( n = e/(1 + e) \) is porosity. Lower B-values represent a lower initial degree of saturation (note that B-values between 0.9-1.0 generally indicate full-saturation). Figure 2.3 shows that appreciably larger CSRs are required to initiate liquefaction at the same number of loading cycles for lower B-values (lower degrees of saturation); i.e. liquefaction resistance increases.

2.1.2 Previous IPS Studies

The fundamental mechanical benefits accompanying entrapped gas motivated investigations of potential methods to introduce gas and assess the efficacy of emplacing gas to increase liquefaction resistance. Yegian et al. (2007) first introduced the expression induced-partial-saturation (IPS) to describe the artificial introduction of gas to mitigate liquefaction—terminology adopted in this dissertation.

It has been demonstrated that partial saturation in soils can be induced using different methods, which include:

- Use of physical methods, such as direct injection of gas into soil below water table (e.g. Okamura et al., 2006; He & Chu, 2014; Flora et al., 2020), drainage-recharge methods, whereby gas is entrapped during artificial lowering and raising of groundwater levels (Yegian et al., 2007), or injection of water already containing micron-size bubbles (i.e. microbubbles) (Kato & Nagao, 2020).

- Chemical generation of gas, such as through electrolysis where electrodes send a direct electric current through the groundwater to generate oxygen \((O_2)\) and hydrogen \((H_2)\) gas (Yegian et al., 2007) or introducing efferdent (sodium perborate monohydrate) to react with pore water and generate \(O_2\) gas (Eseller-Bayat et al., 2013a).
• Biological methods, such as microbial reactions to generate $N_2$ gas (Rebata-Landa & Santamarina, 2012; He et al., 2013; DeJong et al., 2014).

Generally, IPS studies have progressively scaled up demonstration of gas’ influence on liquefaction resistance from elemental testing (discussed in the preceding section) to laboratory bench-scale tests and field demonstrations. Figure 2.4 illustrates the scale of different methods used to demonstrate the efficacy of IPS, which range from elemental testing (Figure 2.4a), bench-scale shake table tests (Figure 2.4b), large-scale shake table tests (Figure 2.4c) to field-scale demonstrations where vibrations are imposed by ground shaking machines (Figure 2.4d). Table 2.2 summarizes several laboratory and field studies, their scale, degrees of saturation achieved, and gas specie(s) utilized.

In many cases they report on the improved liquefaction resistance. Generally, implementation of IPS methods have been able to achieve degrees of saturation ($S_r$) greater than 80%. Below the water table this results in a quasi-saturated condition in granular materials, whereby there is a continuous water-phase with occluded gas bubbles. Figure 2.5 shows results of the pore water pressure response from a laboratory elemental tests (Figure 2.5a), laboratory shake table tests (Figure 2.5b), and large-scale field vibration test (Figure 2.5c). As shown, reducing the degree of saturation dampens the generation of excess positive pore pressure. Most notable is the pore water pressure response from field-demonstrations by Flora et al. (2020), where they report injecting air volumes that would result in $S_r$ values greater than 80%. In their study they recorded the pore water pressure response at the same site (i.e. soil conditions). In one trial the soil was untreated (UN), a second trial considered the use of horizontal drains (HDL), and third trial implemented IPS. The comparison in Figure 2.5c illustrates that vibrations from a shaking truck (Figure 2.4d) were able to achieve pore water pressure ratios ($r_u = \Delta u/\sigma'_{vo}$ here) greater than 0.9, a value typically accepted as full or nearly-full liquefaction of the loose material. The horizontal drains decreased the pore water pressure response by a
limited amount, but IPS reduced $r_u$ values to 0.1—indicating it was very effective at suppressing the generation of excess pore water pressure.
Figure 2.4: Tests used for measuring the effectiveness of induced-partial-saturation on improving liquefaction resistance: a.) triaxial setup used by He & Chu (2014); b.) bench-scale shaking table setup used by Eseller-Bayat et al. (2013a); c.) large-scale shaking table setup used by Kato & Nagao (2020); d.) ground shaker machine used by Flora et al. (2020).
Figure 2.5: Effect of IPS on the pore water pressure response from different studies: a.) He & Chu (2014); b.) Eseller-Bayat et al. (2013a); c.) Flora et al. (2020).
Table 2.2: Examples of tests conducted showing the effectiveness of gas on improving liquefaction resistance of sandy soils.

<table>
<thead>
<tr>
<th>Test type</th>
<th>Sample size</th>
<th>Material</th>
<th>Method (Gas type)</th>
<th>Initial Sr</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaking table</td>
<td>21 cm x 33 cm x 42 cm</td>
<td>Ottawa Sand</td>
<td>Drainage-recharge (Air)</td>
<td>86%</td>
<td>Yegian et al. (2007)</td>
</tr>
<tr>
<td>Shaking table</td>
<td>22 cm x 33 cm x 42 cm</td>
<td>Ottawa Sand</td>
<td>Electrolysis ($O_2$ and $H_2$)</td>
<td>96%</td>
<td>Yegian et al. (2007)</td>
</tr>
<tr>
<td>Shaking table</td>
<td>19 cm x 30 cm x 49 cm</td>
<td>Ottawa Sand</td>
<td>Efferdent ($O_2$)</td>
<td>40%-90%</td>
<td>Esseler-Bayat et al. (2013)</td>
</tr>
<tr>
<td>Shaking table</td>
<td>30 cm x 30 cm x 45 cm</td>
<td>Ottawa Sand</td>
<td>Biogas ($N_2$)</td>
<td>80%-100%</td>
<td>He et al. (2013)</td>
</tr>
<tr>
<td>Triaxial</td>
<td>7 cm x 10 cm</td>
<td>Ottawa Sand</td>
<td>Gas injection (CO$_2$)</td>
<td>87%-100%</td>
<td>He and Chu (2014)</td>
</tr>
<tr>
<td>Shaking table</td>
<td>360 cm x 480 cm x 1000 cm</td>
<td>Toyoura sand</td>
<td>Mircobubbled water (Air)</td>
<td>90%</td>
<td>Kato and Nagao (2020)</td>
</tr>
<tr>
<td>Triaxial</td>
<td>5 cm x 10 cm</td>
<td>Toyoura sand</td>
<td>Mircobubbled water (Air)</td>
<td>90%-100%</td>
<td>Kato and Nagao (2020)</td>
</tr>
<tr>
<td>Field shaking plates</td>
<td>Field scale</td>
<td>Local sand</td>
<td>Gas injection (Air)</td>
<td>80%</td>
<td>Flora et al. (2020)</td>
</tr>
</tbody>
</table>
2.1.3 Gas Longevity

Once gas bubbles are entrapped, capillary forces tend to immobilize the bulk flow of gas through pore throats. External forces imposed by the viscous flow of groundwater and buoyancy are relatively small in comparison (Peck, 1969; Fry et al., 1997; Maryshev, 2017). It has also been shown that entrapped gas can remain immobile under strong ground shaking and seepage conditions Eseller-Bayat et al. (2013a). However, the longevity of entrapped gas must be demonstrated for adoption of IPS in practice, but the time-scales of interest (decades or longer) are challenging to address.

Though the longevity of entrapped gas in porous media has not been demonstrated on these time-scales, several experimental studies—often performed outside the context of IPS—provide insight behind the persistence of entrapped gas. With this in mind, a review of previous studies are summarized in Table 2.3. They have been categorized based on the “flow condition," where no flow indicates the longevity of gas was demonstrated under hydrostatic conditions and flow indicates seepage was imposed on the porous media.

McWhorter et al. (1973) studied the longevity of entrapped air from small-scale sandstone cores and observed resaturation of a core with an initial \( S_r = 81\% \) after 30 days, attributing it to diffusion. Yegian et al. (2007) performed diffusion tests by measuring changes in \( S_r \) in a 1.5 m column of sand for 442 days (1.2 years) and observed a limited change in \( S_r \) (82.1% to 83.9%). Eseller-Bayat et al. (2013a) performed an identical experiment on a 1.2 m column of sand and reported a change in \( S_r \) from 82% to 84% after 805 days (2.2 years). Though encouraging, the subsurface conditions simulated in these tests (i.e. shallow depths with low hydrostatic pressures) are limited. He et al. (2016) performed 1D diffusion tests on desaturated sand (with biogenic nitrogen gas) for 10 days and saw no change from the initial \( S_r = 88\% \). Okamura et al. (2006) offered some of the most compelling field evidence for the persistence of entrapped gas. Their study revealed that soil densified with sand compaction piles (SCPs) contained entrapped gas that was exhausted from a casing pipe (an unintended consequence of the construction method).
Frozen sand samples collected months after SCP construction consistently revealed $S_r$ ranged between 75% and 90% at several sites. Frozen samples were also obtained at three sites where SCPs were installed 4, 8, and 26 years prior, where they observed higher values of $S_r$ between 75-98%, 95-100%, and 92-98% at each site, respectively. However, the actual conditions (i.e. no flow vs. flow) at this site are unknown.

Under flow conditions Eseller-Bayat et al. (2013a), performed 1D vertical seepage tests through desaturated sand columns (1.2 m long) with an initial $S_r \approx 85\%$. Seepage was imposed under hydraulic gradients ranging from 0.05-0.5 for 30 hour intervals, and they effectively saw no change in gas content for all conditions tested. Additionally, they performed separate experiments where the column was exposed to dynamic base excitation as great as 0.99g (g is the gravitation acceleration coefficient). No change in $S_r$ was observed after more than 10,000 cycles of excitation at 0.99g was applied. In contrast, He et al. (2016) performed 1D vertical seepage tests and observed that a 1 m column of sand with an initial $S_r = 89\%$ became saturated after approximately 4 days when flow was maintained with a hydraulic gradient of 0.1. McLeod et al. (2015) studied entrapped air dissolution and the resaturation of sediments in a large (1.8 m x 2.4 m x 6 m) sand tank (synthetic aquifer) where horizontal seepage was imposed on quasi-saturated sediments (initial $S_r$ ranged from 81-86%). They monitored the evolution of gas dissolution for 344 days and observed a distinct wedge-shaped resaturation front, where resaturation of the sand extended further downgradient at greater depths; indicating entrapped gas at depth is more susceptible to resaturation under horizontal flow conditions.

These observations suggest that (a) gas may be introduced and persist for appreciable periods of time and (b) that dissolution of entrapped gas and resaturation of the soil may be anticipated in the field. Though the dissolution of entrapped gas is slow, infrastructure systems operate for decades, and often more than a century. Demonstration of gas longevity on these time-scales has been a practical limitation of physical experiments. Modeling the mechanisms that influence the persistence of entrapped bubbles is a practical
avenue to address concerns regarding gas longevity. In the context of IPS, the objective of this study is to provide a novel assessment of gas persistence by explicitly considering the governing physical and chemical processes associated with aqueous-phase gas mobility (advection-diffusion) and dissolution (inter-phase gas kinetics). The aforementioned governing mechanisms are discussed in greater detail at the end of this chapter.
Table 2.3: A list of experimental studies discussing durability of entrapped gas in porous media.

<table>
<thead>
<tr>
<th>Duration</th>
<th>Flow condition</th>
<th>Sample size</th>
<th>Material</th>
<th>Method (Gas type)</th>
<th>Initial Sr</th>
<th>Final Sr</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 days</td>
<td>No Flow</td>
<td>4.52 cm core</td>
<td>Sand stone</td>
<td>Drainage-recharge (Air)</td>
<td>81%</td>
<td>100%</td>
<td>McWhorther et al. (1973)</td>
</tr>
<tr>
<td>26 years</td>
<td>Unknown</td>
<td>Field scale</td>
<td>Local sand</td>
<td>Gas injection (Combustion gas)</td>
<td>75%-90%</td>
<td>&lt;98%</td>
<td>Okamura et al. (2006)</td>
</tr>
<tr>
<td>442 days</td>
<td>No Flow</td>
<td>1.5 m sand column</td>
<td>Ottawa Sand</td>
<td>Drainage-recharge (Air)</td>
<td>82%</td>
<td>84%</td>
<td>Yegian et al. (2007)</td>
</tr>
<tr>
<td>805 days</td>
<td>No Flow</td>
<td>1.2 m sand column</td>
<td>Ottawa Sand</td>
<td>Efferdent (O₂)</td>
<td>82%</td>
<td>84%</td>
<td>Esseler-Batat et al. (2013)</td>
</tr>
<tr>
<td>9 days</td>
<td>Flow</td>
<td>1.2 m sand column</td>
<td>Ottawa Sand</td>
<td>Efferdent (O₂)</td>
<td>85%</td>
<td>86%</td>
<td>Esseler-Batat et al. (2013)</td>
</tr>
<tr>
<td>344 days</td>
<td>Flow</td>
<td>1 m x 1.8 m x 5.25 m</td>
<td>Ottawa Sand</td>
<td>Drainage-recharge (Air)</td>
<td>81%-86%</td>
<td>81%-100%</td>
<td>McLeod et al. (2015)</td>
</tr>
<tr>
<td>10 days</td>
<td>No Flow</td>
<td>1 m sand column</td>
<td>Ottawa Sand</td>
<td>Biogas (N₂)</td>
<td>88%</td>
<td>88%</td>
<td>He et al. (2016)</td>
</tr>
<tr>
<td>5 days</td>
<td>Flow</td>
<td>1 m sand column</td>
<td>Ottawa Sand</td>
<td>Biogas (N₂)</td>
<td>88%</td>
<td>100%</td>
<td>He et al. (2016)</td>
</tr>
</tbody>
</table>
2.2 Tsunami Loading

While entrapped gas bubbles and their persistence are crucial to IPS as a liquefaction mitigation method, there are situations where naturally occurring entrapped gas can be problematic. A second perspective in this study on the durability of gas is directed at quasi-saturated nearshore sediments during tsunami loading. Context for studying the durability of gas for this problem is provided herein.

Tsunamis are an extreme coastal hazard that cause catastrophic damage and disruption in the nearshore environment. In addition to inundation and flooding, tsunamis enhance sediment mobility attributed to the formation of deep-seated scour features and erosion in the soil bed. Takahashi et al. (1995) reported that the 1960 Chilean tsunami resulted in more than 8 m of erosion at the Kesennuma Port. Deep-seated scour and erosion as great as 4 m contributed to the failure of a breakwater during the 1993 Okushiri tsunami in Japan (Kimura et al., 1997; Yeh & Mason, 2014). Erosion, which is attributed to soil bed shear stresses imposed during inundation and recession of the tsunami wave, can be exacerbated by the development of hydraulic gradients in the sediment at depth (Yeh & Mason, 2014). Recognition of the importance of groundwater flow and prediction of the pore water pressure response instigated by tsunami loading has lead to more robust models that account for both i.) seepage arising from the changing boundary pore water pressure at the soil bed surface and ii.) pore fluid pressurization linked to deformation of the soil skeleton under the changing weight of the wave (Young et al., 2009; Abdollahi & Mason, 2019, 2020).

When soil bed deformations are considered, differential pressurization of pore water that instigates groundwater flow, liquefaction triggering and enhanced scour during tsunami loading is intimately linked to the pore fluid compressibility (Mahmoodi et al., 2019; Abdollahi & Mason, 2020). Air entrainment near the phreatic surface (Heaton & Vogel, 1980; Faybishenko, 1995; Holocher et al., 2002), which fluctuates continuously in nearshore sandy sediments due to the tides, increases the bulk compressibility of pore fluid.
relative to fully-saturated sediments (Skempton, 1954). Previous numerical studies considering tsunami loadings have accounted for gas entrapment by selecting a pore fluid compressibility greater than that of water (Young et al., 2009; Abdollahi & Mason, 2019), but implicitly assumed uniform entrainment of air irrespective of depth. This has also been assumed in other studies examining the pore water pressure response for non-solitary waves (e.g. Yamamoto et al., 1978; Okusa, 1985; Tsai, 1995). However, gas entrapment is likely isolated to nearshore sediments in the swash zone (Turner, 1993; Baldock et al., 2001; Horn, 2002; Steenhauer et al., 2011) where oscillating tide levels desaturate and resaturate sediments, a requisite condition for air entrapment. Thus, a multi-layered system likely exists, whereby a quasi-saturated soil bed overlies fully-saturated sediments below the low-tide elevation. Aside from the assumed distribution of gas, previous studies also implicitly assume gas content is constant throughout tsunami loading. Tsunami waves can be in excess of 10 m (Kundu, 2007; Bryant, 2014), imposing large fluid pressures in the sediment that will contract, and possibly dissolve, air bubbles as the wave height increases. Thus, pore fluid compressibility changes throughout this dynamic loading process.

2.2.1 Pore Water Pressure Response During Tsunami Loading

Tsunamis are long period waves (hundreds of meters) that impose relatively uniform, though dynamic, bed loads and total stress to the sediment. A unique feature of tsunami loading is that fluid pressure in the sediment is governed by both i.) seepage and ii.) the soil skeleton’s tendency to contract (or expand) under the changing weight of the wave, which is referred to herein as mechanical generation of excess pore water pressure. With respect to the second consideration, the duration of tsunami loading is on the order of minutes and likely invokes a partially undrained response in sand beds (Abdollahi & Mason, 2019); i.e. mechanical generation of pore pressure as the porous soil skeleton deforms under the weight of the wave. Therefore, both considerations are necessary to
adequately describe changes in pore water pressures associated with destabilizing seepage mechanisms.

Mass conservation of pore fluid in a deformable porous material may be described using Biot (1941) formulation for a 1D poroelastic material (Verruijt, 1969):

\[
\frac{\alpha}{\partial t} + S \frac{\partial P_e}{\partial t} = \frac{k_h}{\gamma_w} \frac{\partial^2 P_e}{\partial z^2}
\]  

(2.6)

where \( \varepsilon \) is volumetric strain, \( \alpha \) is Biot’s coefficient, \( S \) is storativity, \( P_e \) is excess pore water pressure, \( k_h \) is hydraulic conductivity, \( \gamma_w \) is the unit weight of water, \( z \) is depth, and \( t \) is time. Biot’s coefficient is defined as:

\[
\alpha = 1 - \frac{\beta_s}{\beta_m}
\]  

(2.7)

where \( \beta_s \) and \( \beta_m \) are the compressibility of the soil particles and porous medium (soil skeleton) under changes in effective stress, respectively; soil particles are assumed incompressible and \( \alpha = 1 \) under practical stress levels, which is applicable to tsunami loading. Storativity is defined as:

\[
S = n\beta + (\alpha - n)\beta_s
\]  

(2.8)

where \( n \) is porosity and \( \beta \) is the compressibility of the pore fluid, which is influenced by the degree of saturation and gas durability (focus of this study). From Terzaghi’s one-dimensional consolidation theory, volumetric strain of the soil skeleton is:

\[
\frac{\partial \varepsilon}{\partial t} = -m_v \frac{\partial \sigma_z}{\partial t} = -m_v(\frac{\partial \sigma_z}{\partial t} - \alpha \frac{\partial P_e}{\partial t})
\]  

(2.9)

where \( m_v \) is the compression modulus of the soil skeleton, and \( \sigma_z \) and \( \sigma_z' \) are the total and effective vertical stress, respectively. Confined compressibility of the soil skeleton is computed as:

\[
m_v = \frac{1 - 2\nu}{2(1 - \nu)G}
\]  

(2.10)
where $\nu$ is Poisson’s ratio and $G$ is the shear modulus. Substituting equation 2.9 into equation 2.6 and rearranging the terms yields:

$$\frac{\partial P_e}{\partial t} = \frac{\alpha m_v}{S + \alpha^2 m_v} \frac{\partial \sigma_z}{\partial t} + \frac{k_h}{\gamma_w(S + \alpha^2 m_v)} \frac{\partial^2 P_e}{\partial z^2}$$

(2.11)

where the first term on the right hand side describes the mechanical generation of excess pore water pressure and $\partial \sigma_z / \partial t$ is the changing total stress imposed by the weight of a tsunami wave. The second term in equation 2.11 accounts for temporal changes in excess pore water pressure due to seepage.

The effective stress in the sediment can be computed using two different approaches, but with the same outcome. In the first method the generated excess pore pressure gradients are used to computed the effective stress at each depth $z$ below seabed as shown below:

$$\sigma'_z = \int_{-z}^{0} (\gamma' - i \gamma_w) dz$$

(2.12)

where $\gamma'$ is the effective unit weight of soil, $\gamma_w$ is the unit weight of water, and $i$ is the generated excess pressure gradient (variable with depth and positive for upward flow).

The second approach is using Terzaghi’s effective stress definitions where the changes in effective stress is simply defined as the difference between the applied total stress and the generated excess pore pressure. According to this method, at depth $z$ below seabed the effective stress can be computed based on the weight of a tsunami wave and the generated excess pore pressure as shown below:

$$\sigma'_z = \sigma'_{zo} + d\sigma_z - P_e = \gamma' z + \gamma_w h - P_e$$

(2.13)

where $\sigma'_{zo}$ is the initial effective stress, and $h$ is the height of the wave. It can be mathematically shown that using either of equations 2.12 or 2.13 results in the same effective stress values during a tsunami loading. Based on equation 2.12, if there is a sustained positive excess pore pressure gradients below seabed, sediments are prone to full or partial liquefaction. Similarly, according to equation 2.13 if the magnitude of the excess pore pressure
pore pressure at any point in time during tsunami loading is larger than total stress imposed by the weight of the wave, sediments may experience full or partial liquefaction.

### 2.2.2 Influence of Entrapped Gas on Pore Water Pressure Response

In fully-saturated sediments, the pore fluid (water) is nearly incompressible ($\beta \approx 0$) such that $S \approx 0$ and $(\alpha m_v)/(S + \alpha^2 m_v)$ in the first term on the right hand side of equation 2.11 is 1. Thus, it may be recognized that any change in excess pore water pressure corresponds to changes in total stress imposed by the weight of the wave ($\partial P_e/\partial t = \partial \sigma_z/\partial t$); the final term in equation 2.11 is zero to satisfy mass conservation of the pore fluid (i.e. no seepage). By inspection, this also implies no change in effective stress (equations 2.12 and 2.13) or deformation of the soil skeleton (equation 2.9).

However, when entrapped air (even small amounts) constitute a portion of the pore fluid, there is a significant departure from the preceding assumption that the pore fluid is incompressible (Fredlund, 1976), where $\beta \neq 0$ and $S \neq 0$. Thus, in quasi-saturated sediments with entrained air $\partial P_e/\partial t \neq \partial \sigma_z/\partial t$, following well-established observations that entrapped gas suppresses the mechanical generation of excess pore water pressure in globally undrained soils with a tendency to contract (e.g. Skempton, 1954; Sherif et al., 1977; Yoshimi et al., 1989; Grozic et al., 1999, 2000; Tsukamoto et al., 2002; Okamura & Soga, 2006; Yegian et al., 2007; Fredlund et al., 2012; He et al., 2013; Kato & Nagao, 2020, among others). When mechanical generation of excess pore pressure in the quasi-saturated layer does not correspond to the changes in total stress, a pressure head differential arises at the surface, initiating seepage.

The preceding discussion emphasizes the necessity of air entrainment to motivate groundwater flow and changes in effective stress, though the depth where destabilizing gradients develop cannot be fully appreciated without explicitly considering where air is initially entrapped (i.e. in the intertidal zone). Differential pressurization of pore fluid near
the interface of saturated and quasi-saturated sediments will also arise from differences in pore fluid compressibility, also instigating seepage.

Figure 2.6 conceptually illustrates the pore water pressure response and groundwater seepage anticipated in a multi-layered system (quasi-saturated layer overlying fully-saturated sediment) at two different stages of tsunami loading—runup and drawdown. During tsunami runup (Figure 2.6b), the differential pressure head at the surface, linked to the dampened mechanical generation of excess pore pressure in the quasi-saturated layer and increasing height of the wave, causes infiltration. However, mechanically generated excess pore water pressures in the underlying fully-saturated layer (due to the weight of the wave) enforces upward seepage near the interface of the two-layered system. As the wave height increases during runup, pressurization of the quasi-saturated bed arises, to a large extent, from groundwater seepage.

Similarly, mechanical dissipation of excess pore pressure in the quasi-saturated layer does not correspond to the diminishing weight of the wave during drawdown (assuming entrained air still exists). Dissipation is governed again, in part, by seepage-induced diffusion of excess pore water pressure. The pressure head at the ground surface, and changes in mechanically-induced excess pore water pressure in the saturated layer corresponding to changes in total stress, diminish more rapidly as the wave height decreases (than depressurization of the quasi-saturated layer). Thus, seepage is reversed (Figure 2.6c) during drawdown. Depending on the direction of flow, seepage forces may have a stabilizing (during runup) or destabilizing (during drawdown) influence on the sediment (i.e. increase or decrease effective stress) throughout different stages of tsunami loading. During drawdown, when upward seepage is anticipated, body forces applied to the soil skeleton reduce vertical effective stress, which can enhance scour (Tonkin et al., 2003) or cause momentary liquefaction (Abdollahi & Mason, 2019; Mahmoodi et al., 2019; Abdollahi & Mason, 2020).
Figure 2.6: Nearshore sediments under a tsunami wave: a.) conceptual demonstration of a representative sand column in seabed; b.) behavior of sand column during tsunami run-up; c.) behavior of sand column during tsunami drawdown.

Motivated by uncertainties associated with the aforementioned assumptions previously discussed regarding the distribution of gas and the pore fluid compressibility, the objective of this study is to examine a.) the influence of gas distribution and b.) the kinetics of entrapped bubbles on the pore water pressure response. More specifically, the kinetics of entrapped bubbles in a quasi-saturated soil layer is incorporated in a coupled seepage-deformation finite difference framework to more faithfully address the role of pore fluid hardening (i.e. changing pore fluid compressibility) arising from compression and dissolution of gas.
2.3 Theory and the Governing Mechanisms Influencing the Durability of Entrapped Gas

In this section the theory governing the durability of entrapped gas bubbles in porous media is discussed. Following are the mechanisms considered for the assessment of gas durability.

2.3.1 Stability of Entrapped Bubbles

According to Henry’s law, gas is immiscible at the water-bubble interface when the aqueous-phase (i.e. dissolved) concentration of a gas specie, \( i \), is:

\[
C_{aq,i,eq} = \frac{P_i}{H_i}
\]

(2.14)

where \( P_i \) is the partial pressure of a gas specie inside the bubble and \( H_i \) is Henry’s solubility coefficient, which is a function of pressure and temperature. Therefore, the equilibrium, or maximum total dissolved gas concentration is:

\[
C_{eq}^{aq} = \sum C_{i,eq}^{aq}
\]

(2.15)

According to Dalton’s Law the partial gas-phase pressure is:

\[
P_i^g = \frac{C_i^g}{C^g} P^g
\]

(2.16)

where \( C_i^g \) and \( C^g \) are the individual and total gas-phase concentrations and \( P^g \) is the total gas pressure in a bubble, evaluated as:

\[
P^g = P_{atm} + P_w + P_c
\]

(2.17)

where \( P_{atm} \) is atmospheric pressure, \( P_w \) is the pore water pressure, and \( P_c \) is capillary pressure arising from surface tension of the pore fluid and curvature of the bubble.

Assuming a spherical shape for an entrapped bubble:
\[ P_\text{c} = \frac{2\sigma_w^w}{r_b} \]

where \( r_b \) is the bubble radius and \( \sigma_w^w \) is the unit interface surface tension of the fluid; in coarse-grained soils \( P_c \) is small relative to \( P_{\text{atm}} \) and \( P_w \) at depth. Rad & Lunne (1994) introduced the expression “water-gas saturation” (\( \eta \)) to describe the degree to which water’s molecular pore structure is saturated with dissolved gas:

\[ \eta = \frac{\sum C_{aq}^i H_i}{P_{\text{atm}} + P_w} = \frac{TDGP}{P_{\text{atm}} + P_w} \]

where TDGP is the total dissolved gas pressure (note \( C_{aq}^i = \sum C_{aq}^i \) is the total dissolved gas concentration). Groundwater is “supersaturated” (\( \eta > 100\% \)) in the presence of gas due to capillary pressure associated with the bubbles.

### 2.3.2 Gas Mobility: Advection and Diffusion

Under hydrostatic conditions diffusion governs the aqueous-phase mobility of entrapped gas. Aqueous-phase concentration gradients exist within and outside the quasi-saturated IPS zone after gas is introduced. Aqueous-phase concentrations will diffuse until they are in equilibrium with partial atmospheric gas pressures at the ground surface (Christiansen, 1944; Bloomsburg & Corey, 1964; Adam et al., 1969; McWhorter et al., 1973; Faybishenko, 1995; Fry et al., 1995). The steady-state molecular flux in the water column is proportional to the aqueous-phase concentration gradient:

\[ J_{d,i} = -D_i^* \theta_w \nabla C_{aq}^i \]

where \( \theta_w = n S_r \) is the volumetric water content, \( n \) is soil porosity, and \( D_i^* \) is the effective aqueous-phase gas diffusion coefficient.

For non-hydrostatic conditions, \( C_{aq}^i \) is influenced by the bulk flow of groundwater that transports dissolved gas. The advective flux may be expressed as:
\( J_{a,i} = \theta_w v_s C_i^{aq} \) \hspace{1cm} (2.21)

where \( v_s \) is the seepage velocity associated with hydrogeologic conditions and naturally occurring hydraulic gradients. For larger average seepage velocities, grain-scale effects may cause non-uniform interstitial seepage rates and mixing locally, also known as mechanical (or hydrodynamic) dispersion. This is typically accounted for in \( J_d \) as:

\( J_{d,i} = -D_h \theta_w \nabla C_i^{aq} \) \hspace{1cm} (2.22)

where \( D_h = D^* + D_m \) and \( D_m \) is the semi-empirical mechanical dispersion coefficient. The Peclet number, \( P_e \), is defined as:

\[ P_e = \frac{u_s d_p}{D_i} \] \hspace{1cm} (2.23)

where \( d_p \) is the average particle diameter and \( u_s \) is the average interstitial velocity, which may be approximated as \( v_s \). For laminar flow and where \( P_e \) is less than 0.5 to 1, then \( D_h \) may be assumed to equal \( D^*_i \) (Perkins et al., 1963). This is applicable for most naturally occurring groundwater conditions in granular soils (i.e. hydraulic gradients between 0.01 and 0.2).

Mass conservation of a non-reactive gas solute is assessed with the readily applied advection-diffusion equation:

\[ \frac{\partial C_i^{aq}}{\partial t} \theta_w = -\nabla (J_{a,i} + J_{d,i}) = \theta_w [ -\nabla (v_s C_i^{aq}) + \nabla (D_i \nabla C_i^{aq}) ] \] \hspace{1cm} (2.24)

Equation 2.24 is valid away from the quasi-saturated layer. However, where entrapped gas exists, a source term must be considered.

### 2.3.3 Inter-phase Gas Kinetics

The kinetic bubble dissolution (KBD) model introduced by Holocher et al. (2003) was developed to understand the formation of “excess air” and its influence on aqueous-phase
transport dynamics. Understanding gas exchange was motivated by environmental applications, such as the use of tracers for groundwater dating and contaminant transport (e.g. Cirpka & Kitanidis, 2001) or the effectiveness of bioremediation through oxygen injection (e.g. Fry et al., 1995). The KBD model is adopted for the source term because it offers a good description of inter-phase gas exchange between the bubble and groundwater, as well as the flexibility to account for individual gasses (e.g. air is approximately 80% nitrogen and 20% oxygen with other trace gasses).

Figure 2.7 conceptually illustrates the inter-phase gas transfer associated with bubble dissolution. The rate of bubble dissolution depends on the aqueous-phase concentration deficit and mass transfer coefficient, \( k_{g,i} \). The molecular flux at the bubble-water interface is:

\[
J_{g,i} = k_{g,i}(C_{aq}^i - C_{aq,eq}^i)
\]  

(2.25)

Film theory, which assumes gas diffuses over a stagnant film surrounding the surface of the bubble (Cussler, 2009), is used to define \( k_{g,i} \):

\[
k_{g,i} = \frac{D_i}{\delta_{eff}} = D_i \left( \frac{1}{r_b} + \frac{1}{\delta} \right)
\]  

(2.26)

where \( r_b \) is the bubble radius and \( \delta_{eff} \) and \( \delta \) are the effective diffusion distance and static film thickness, respectively. In coarse grained soils \( r_b \) is sufficiently large such that \( r_b >> \delta \) and \( \delta_{eff} \approx r_b \).

For seepage conditions, groundwater infiltrating a quasi-saturated soil presents a potential sink for the gas, assuming there is a concentration deficit at the bubble-water interface. This is illustrated in Figure 2.7. Epstein & Plesset (1950) derived an expression for \( k_{g,i} \) as a function of the surface contact time, \( t_c \), groundwater has with the bubble:

\[
k_{g,i} = D_i \left( \frac{1}{r_b} + \frac{1}{\sqrt{\pi D_i t_c}} \right) = D_i \left( \frac{1}{r_b} + \sqrt{\frac{v_s}{\pi D_i t_c}} \right)
\]  

(2.27)
Note that Equation 2.27 reduces to equation 2.26 when $v_s = 0$. The expression in equation 2.25 becomes:

$$J_{g,i} = D_i \left( \frac{1}{r_b} + \sqrt{\frac{v_s}{\pi D_i 2r_b}} \right) \left( C_{aq,i}^{eq} - C_{aq,i}^{eq} \right)$$ (2.28)

The molar rate of inter-phase gas exchange ($dm_i/dt$) for a bubble is dependent on the surface area of the bubble, $A_{sb}$. For an assumed spherical bubble:

$$\frac{dm_i}{dt} = -A_{sb} J_{g,i} = -4\pi r_b^2 k_{g,i} (C_{aq,i}^{eq} - C_{aq,i}^{eq})$$

$$= -4\pi r_b^2 D_i \left( \frac{1}{r_b} + \sqrt{\frac{v_s}{\pi D_i 2r_b}} \right) \left( C_{aq,i}^{eq} - \frac{P_g}{H_i} \right)$$ (2.29)

The change in the aqueous-phase concentration in a volume of water, $V_w$, contained in an elemental soil volume, $V$, depends on the number of bubbles, $n_b$, and volume of gas,
\[ V_g = nV(1 - S_r) \]. Accordingly, the source term due to dissolution of homogeneous spherical bubbles is:

\[
\frac{dC_{aq}^i}{dt} = \frac{n_b}{V_w} \frac{dm_i}{dt} = \frac{n_b A_{sb}}{nV(S_r)} J_{g,i} = \frac{3(1 - S_r)}{r_b S_r} J_{g,i} \tag{2.30}
\]

Adding the source term due to the presence of entrapped gas bubbles in equation 2.24 provides a complete description of gas mass conservation:

\[
\frac{\partial C_{aq}^i}{\partial t} \theta_w = -\nabla (J_a + J_d) + \frac{n_b A_{sb}}{nV(S_r)} J_{g,i} \tag{2.31}
\]

For soils with uniform spherical bubbles, equation 2.31 may also be expressed as:

\[
\frac{\partial C_{aq}^i}{\partial t} = -\nabla (v_s C_{aq}^i) + \nabla (D_h \nabla C_{aq}^i) + \frac{3(1 - S_r)}{r_b S_r} J_{g,i} \tag{2.32}
\]

For IPS, it is relevant to consider both the advective and diffusive flux of gas within a system, which will be influenced by the natural hydrogeologic conditions (soil type and natural gradients) that exist. For the case of the response of gassy sediments under tsunami loading, generation of excess pore water pressure in the system instigates seepage. Therefore, it is appropriate to account for changes in dissolved groundwater gas concentrations that may influence the aqueous-phase concentration deficit influencing the dissolution (or exsolution) of gas.

### 2.3.4 Effective Aqueous-Phase Gas Diffusion Coefficient

A critical parameter in addressing the influence of aqueous-phase diffusion on the durability of gas is the effective diffusion coefficient. Aqueous-phase gas diffusion is driven by dissolved gas concentration gradients. Fick’s first law for the steady-state 1D molecular flux, \( J_i \), may be expressed as:

\[
J_i = -D_{aq}^i r \theta_w \frac{\partial c_{aq}^i}{\partial z} \tag{2.33}
\]
where $D_i^{aq}$ is the bulk diffusion coefficient of a gas solute, $\partial c_i^{aq}/\partial z$ is the aqueous-phase concentration gradient of gas, $i$. The volumetric water content, $\theta_w = nS_r$ where $n$ and $S_r$ are porosity and degree of saturation, relates the volume of water (solvent) available to convey gas through an element of soil. The tortuosity factor, $\tau < 1$, is conceptually associated with the sinuous path and effective length ($L_e$) a molecule of gas travels in the direction of diffusion (where $L$ is the apparent length of the diffusion path). Thus the tortuosity is $L_e/L > 1$. It is influenced by geometry of the soil fabric, including particle shape, grain size distribution, packing, channel constrictions, etc. (Figure 2.7). The effective aqueous-phase diffusion coefficient through porous media is expressed as:

$$D_i^* = \tau D_i^{aq} \quad (2.34)$$

The tortuosity factor may be derived for 1D molecular flow by considering the residence time, $t$, through an element of soil of length $L$ as:

$$t = \frac{L(\theta_w)}{J_i(\Delta c_i^{aq})} \quad (2.35)$$

Adopting a capillary model, equation 2.35 is expressed as:

$$t = \frac{L_e}{J_{c,i}(\Delta c_i^{aq})} \quad (2.36)$$

where $J_{c,i} = (J_i/\theta_w)/(L_e/L)$ is molecular flux for the capillary model. Thus equation 2.33 may be expressed as:

$$J_i = -D_i^{aq}\theta_w \left( \frac{\Delta c_i^{aq}}{L} \right) \left( \frac{L}{L_e} \right)^2 \quad (2.37)$$

where $\tau = (L/L_e)^2$ is the tortuosity factor (Epstein, 1989). By performing experiments on spherical glass beads, Carman (1937) observed that the effective flow path had an average angle of 45° around the glass beads in the direction of flow (i.e. $L_e/L = \sqrt{2}$), yielding a tortuosity factor of $\tau = (L/L_e)^2 = 0.5$. In their study of gas (non-aqueous) diffusion through soil, Penman (1940) suggested a tortuosity factor of $\tau = 0.66$. Marshall (1959) suggested that pore (i.e. grain) size may be ignored for diffusion, such that the tortuosity
factor is a function of grain packing (porosity), and \( \tau = \sqrt{n} \). Weissberg (1963) adapted Maxwell’s formula for electrical conductivity through a porous medium to the diffusion problem, which also yields a porosity-dependent tortuosity factor of \( \tau = [1 + 0.5(1 - n)]^{-1} \).

The tortuosity factor may also inherently account for the influence of “dead end” pores (i.e. effective porosity) and other sources that impede aqueous-phase gas mobility. Thus, \( \tau \) is effectively a material impedance parameter (Moldrup et al., 2001); for practical considerations, \( \tau \) may account for all elements contributing to diffusion-driven mobility.

Aqueous-phase diffusion of chemical solutes has been widely studied in the field of environmental geotechnics, largely in the context of contaminant mobility through low-permeability barriers, which has been summarized well by Shackelford (2014). (Shackelford & Daniel, 1991) summarized the tortuosity factors for molecular diffusion though soil (mostly fine-grained soils) and reported tortuosity factors ranging between 0.01 and 0.84, thus highlighting the importance of material-specific measurements for diffusion parameters of different solutes (see Table 2.4).
Table 2.4: A literature review on the tortuosity factors measured for aqueous-phase diffusion in saturated porous media (after Shackelford & Daniel, 1991).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Soil Type</th>
<th>Solute</th>
<th>(D^*(10^{-10} m^2/s))</th>
<th>(D_0(10^{-10} m^2/s))</th>
<th>(\tau_{min})</th>
<th>(\tau_{max})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clarke and Graham (1968)</td>
<td>sand</td>
<td>36Cl(^-)</td>
<td>5.6 5.6</td>
<td>20.3</td>
<td>0.28 0.28</td>
<td></td>
</tr>
<tr>
<td></td>
<td>loam</td>
<td>7.1</td>
<td>7.1</td>
<td>20.3</td>
<td>0.35 0.35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>clay</td>
<td>6.1</td>
<td>6.1</td>
<td>20.3</td>
<td>0.30 0.30</td>
<td></td>
</tr>
<tr>
<td>Gillham et al. (1984)</td>
<td>sand-bentonite</td>
<td>7</td>
<td>7</td>
<td>20.3</td>
<td>0.34 0.49</td>
<td></td>
</tr>
<tr>
<td>P.B. Barraclough and Tinker (1981)</td>
<td>silty clay loam; sandy loam</td>
<td>1.6</td>
<td>4.4</td>
<td>20.3</td>
<td>0.08 0.22</td>
<td></td>
</tr>
<tr>
<td>Crooks and Quingley (1984)</td>
<td>silty clay</td>
<td>6</td>
<td>10</td>
<td>20.3</td>
<td>0.30 0.49</td>
<td></td>
</tr>
<tr>
<td>Rowe et al. (1988)</td>
<td>clay till</td>
<td>4.4</td>
<td>10</td>
<td>20.3</td>
<td>0.22 0.49</td>
<td></td>
</tr>
<tr>
<td>Shakelford (1988a)</td>
<td>clay</td>
<td>1.5</td>
<td>4.7</td>
<td>20.3</td>
<td>0.07 0.23</td>
<td></td>
</tr>
<tr>
<td>P.B. Barraclough and Tinker (1981)</td>
<td>silty clay loam; sandy loam</td>
<td>3.7</td>
<td>6.4</td>
<td>20.1</td>
<td>0.18 0.32</td>
<td></td>
</tr>
<tr>
<td>P.B. Barraclough and Tinker (1982)</td>
<td>soil cores (field)</td>
<td>Br(^-)</td>
<td>4.8</td>
<td>9.9</td>
<td>20.1</td>
<td>0.24 0.49</td>
</tr>
<tr>
<td>Shakelford (1988a)</td>
<td>kaolinite</td>
<td>1.5</td>
<td>87</td>
<td>39.1</td>
<td>0.29 0.49</td>
<td></td>
</tr>
<tr>
<td>Gillham et al. (1984)</td>
<td>sand-bentonite</td>
<td>8</td>
<td>17</td>
<td>39.1</td>
<td>0.09 0.18</td>
<td></td>
</tr>
<tr>
<td>Phillips and Brown (1968)</td>
<td>kaolinite</td>
<td>3H(^+)</td>
<td>5.3</td>
<td>10.9</td>
<td>93.1</td>
<td>0.06 0.12</td>
</tr>
<tr>
<td>Rowe et al. (1988)</td>
<td>clay till</td>
<td>K(^+)</td>
<td>6.3</td>
<td>7</td>
<td>19.6</td>
<td>0.32 0.36</td>
</tr>
<tr>
<td>Shakelford (1988a)</td>
<td>kaolinite</td>
<td>Na(^+)</td>
<td>2.5</td>
<td>3.5</td>
<td>13.3</td>
<td>0.19 0.26</td>
</tr>
<tr>
<td>Crooks and Quingley (1984)</td>
<td>clay till</td>
<td>Ca(^{2+})</td>
<td>4.8</td>
<td>5.7</td>
<td>13.3</td>
<td>0.36 0.43</td>
</tr>
<tr>
<td>Rowe et al. (1988)</td>
<td>clay</td>
<td>Ca(^{2+})</td>
<td>3.8</td>
<td>3.8</td>
<td>7.93</td>
<td>0.48 0.48</td>
</tr>
<tr>
<td>Shakelford (1988a)</td>
<td>kaolinite</td>
<td>Cd(^{2+})</td>
<td>3.2</td>
<td>7.6</td>
<td>7.17</td>
<td>0.45 1.06</td>
</tr>
<tr>
<td>Ellis et al. (1970a)</td>
<td>clay</td>
<td>Cu(^{2+})</td>
<td>4.3</td>
<td>4.2</td>
<td>7.33</td>
<td>0.57 0.57</td>
</tr>
<tr>
<td>Ellis et al. (1970b)</td>
<td>kaolinite</td>
<td>Fe(^{2+})</td>
<td>1</td>
<td>1</td>
<td>7.19</td>
<td>0.14 0.14</td>
</tr>
<tr>
<td>Ellis et al. (1970a)</td>
<td>kaolinite</td>
<td>Fe(^{3+})</td>
<td>0.16</td>
<td>0.44</td>
<td>6.04</td>
<td>0.03 0.07</td>
</tr>
<tr>
<td>Gillham et al. (1984)</td>
<td>kaolinite</td>
<td>Mn(^{2+})</td>
<td>4.5</td>
<td>4.5</td>
<td>6.88</td>
<td>0.65 0.65</td>
</tr>
<tr>
<td>Mott and Nye (1968)</td>
<td>clay</td>
<td>Sr(^{2+})</td>
<td>5</td>
<td>20</td>
<td>7.94</td>
<td>0.63 2.52</td>
</tr>
<tr>
<td>Ellis et al. (1970a)</td>
<td>kaolinite</td>
<td>Zn(^{2+})</td>
<td>5.1</td>
<td>5.1</td>
<td>7.15</td>
<td>0.71 0.71</td>
</tr>
<tr>
<td>Shakelford (1988a)</td>
<td>clay</td>
<td>Zn(^{2+})</td>
<td>3.5</td>
<td>10</td>
<td>7.15</td>
<td>0.49 1.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.5</td>
<td>25</td>
<td>7.15</td>
<td>0.21 3.50</td>
</tr>
</tbody>
</table>
Aqueous-phase gas diffusion through porous media has received less attention relative to other solute or gaseous-phase diffusion problems (e.g. Penman, 1940; Van Bavel, 1952; Moldrup et al., 2000; Neale et al., 2000; Moldrup et al., 2001; Werner et al., 2004), and there is no established method to assess effective diffusion coefficients in porous media for dissolved gas. Challenges include: i.) the fluid conveying dissolved gas must remain stagnant, as small perturbations may appreciably influence results; ii.) aqueous-phase diffusion is slow and previous experiments examining diffusion-induced dissolution of entrapped gas from porous media (e.g. Bloomsburg & Corey, 1964; Adam et al., 1969; McWhorter et al., 1973; Yegian et al., 2007; Eseller-Bayat et al., 2013a) lasted months or years and did not yield material-specific tortuosity factors; and iii.) dissolved gas concentrations are sensitive to changes in pressure and temperature. In the absence of aqueous-phase gas experiments, tortuosity factors derived from other diffusion problems would need to be adopted. In the published literature, the reported tortuosity factor are mainly focused around fine-grained soils with ions used a a tracer, which can vary significantly (see Table 2.4).

Though measurement of diffusion coefficients are not always routine and often difficult to obtain, they should be determined for diffusion problems of interest (Cussler, 2009). If gas is entrapped, molecular transport will be the main process controlling the durability of gas bubbles in porous media. However despite this, in the context of gas durability, molecular gas transport has not received noticeable attention as discussed in the following. Particularly with respect to diffusion processes that may influence the longevity of gas for IPS, it will be important to have a reliable and efficient methods to determine the effective aqueous-phase gas diffusion coefficient and tortuosity in different materials.
CHAPTER 3
LABORATORY DETERMINATION OF AQUEOUS-PHASE GAS DIFFUSION COEFFICIENTS AND TORTUOSITY IN NON-PLASTIC SOILS

3.1 Introduction

Diffusion of dissolved (aqueous-phase) gas in porous media is of practical importance in science and engineering. In soil, aqueous-phase diffusion is attributed to oxygen deficiency (e.g. Cook & Knight, 2003; Neira et al., 2015), changes in hydraulic behaviour (e.g. Faybishenko, 1995), “excess air” (Heaton & Vogel, 1981) that influences groundwater dating (e.g. Holocher et al., 2003), air sparging and bioremediation (Fry et al., 1996), and oil recovery for petroleum applications (e.g. Zhang et al., 2000; Sheikha et al., 2005; Unatrakarn et al., 2011). The topic is increasingly relevant to existing and emerging geotechnical engineering applications. Diffusion of gas can affect temporal changes in penetration resistance and verification of ground improvement after blast densification (e.g. Dowding & Hryciw, 1986; Finno et al., 2016; Gallant & Finno, 2017). The rate of soil carbonation, which cements and improves the mechanical performance of weak materials (Yi et al., 2013b,a; Cai et al., 2015; Fasihnikoutalab et al., 2017), depends on diffusion of carbon dioxide through soil. Induced-partial-saturation (IPS), or soil desaturation, is an emerging method to increase the liquefaction resistance of loose granular soil (Yegian et al., 2007; Yasuhara et al., 2008; Okamura et al., 2011; He et al., 2013; Eseller-Bayat et al., 2013a; Kato & Nagao, 2020), but ultimately hinges on the persistence and associated diffusion of entrapped gas.

Effective diffusion coefficients and soil-dependent tortuosity factors are needed to address aqueous-phase gas mobility. The tortuosity is associated with the sinuous path and effective length, $L_e$, a molecule of gas diffuses over length, $L$, as shown in Figure 3.1. Tortuosity, $L_e/L > 1$, is influenced by intrinsic properties such as particle shape, grain size
The steady-state 1D molecular flux, $J_i$, of gas $i$ for aqueous-phase gas diffusion in soil is:

$$J_i = -D_i^{aq} \tau \theta_w \frac{\partial c_i^{aq}}{\partial z} = -D_i^{aq} \theta_w \left( \frac{\Delta c_i^{aq}}{L} \right) \left( \frac{L}{L_e} \right)^2 \quad (3.1)$$

where $D_i^{aq}$ is the aqueous-phase diffusion coefficient of gas in a pore fluid (e.g., water), $c_i^{aq}$ is the dissolved concentration of gas $i$ in groundwater, $\theta_w = nS_r$ is the volumetric water content (solvent), where $n$ and $S_r$ are porosity and degree of saturation, respectively. The tortuosity factor is the intrinsic material impedance parameter of the soil (Moldrup et al., 2001), where $\tau = (L/L_e)^2$ for a capillary model (Epstein, 1989). Thus, the effective aqueous-phase diffusion coefficient is:

$$D_i^* = \tau D_i^{aq} \quad (3.2)$$

Aqueous-phase gas diffusion through porous media has received less attention relative to other solute or gaseous-phase diffusion problems, and there is no established method to assess effective diffusion coefficients in porous media. Challenges include: i.) the fluid conveying dissolved gas must remain stagnant, as small perturbations may appreciably influence results; ii.) aqueous-phase diffusion is slow and previous experiments examining diffusion-induced dissolution of entrapped gas from porous media (e.g., Bloomsburg & Corey, 1964; Adam et al., 1969; McWhorter et al., 1973; Yegian et al., 2007; Eseller-Bayat
et al., 2013a) lasted months or years and did not yield material-specific tortuosity factors; and iii.) dissolved gas concentrations are sensitive to changes in pressure and temperature. In the absence of aqueous-phase gas experiments, tortuosity factors derived from other diffusion problems would need to be adopted (e.g. Carman, 1937; Penman, 1940; Weissberg, 1963; Marshall, 1959; Shackelford & Daniel, 1991; Olesen et al., 1999; Moldrup et al., 2000, 2001; Olesen et al., 2001). Measurement of diffusion coefficients are not always routine and often difficult to obtain, but should be determined for diffusion problems of interest (Cussler, 2009).

This study examines diffusion coefficients and tortuosity factors derived explicitly from aqueous-phase gas diffusion experiments in soil using a measurement method that relies only on common geotechnical laboratory apparatus.

3.2 Methodology

Riazi (1996) introduced the pressure-decay-method (PDM) to assess diffusion of methane through oil for petroleum applications, which has since been applied by others in the oil and gas industry (e.g. Zhang et al., 2000; Sheikha et al., 2005; Unatrakarn et al., 2011). The method eliminates cumbersome steps, such as extraction of pore fluid and gas chromatography testing that introduce potential errors. The PDM was adopted because it could be modified for saturated porous media with experimental equipment readily available in many geotechnical laboratories.

Figure 3.2 shows the experimental configuration. All equipment was manufactured by Global Digital Systems (GDS) Instruments. A stainless steel pressurised diffusion chamber (Figure 3.3) (inside diameter 150 mm) contained reconstituted saturated soil (as shown in Figure 3.4) underlying a film of water and a pressurised inert gas cavity, with thicknesses $H_s$, $H_w$, and $H_a$, respectively. As gas dissolved and diffused through water (i.e. decreasing the moles of gas in $H_a$), pressure in the system decayed according to the ideal gas law.
Temporal changes in system pressure due to gas dissolution was governed by Fick’s second law:
\[
\frac{\partial c_{aq}}{\partial t} = D^* \frac{\partial^2 c_{aq}}{\partial z^2} = \frac{J}{\theta_w} \frac{\partial c_{aq}}{\partial z}
\]  (3.3)

The effective diffusion coefficient was determined by performing 1D numerical inverse analyses from pressure decay data in the diffusion chamber system. The dissolved gas concentration at the gas-water interface (boundary condition) was calculated as:
\[
c_{aq}^{eq} = \frac{P_g}{H}
\]  (3.4)

where \(P_g\) is the changing partial gas pressure above the water film and \(H\) is Henry’s solubility coefficient. The schematics of the model geometry and the associated boundary conditions (BCs) is shown in Figure 3.5.

The governing partial differential equation (i.e. equation 3.3) was solved in MATLAB R2017a using a second-order central finite difference method, applying the Crank-Nicholson
scheme for unconditional stability (Crank & Nicolson, 1947; LeVeque, 2007). The FD discretization is shown below:

$$\frac{c_{aq}^{i,n+1} - c_{aq}^{i,n}}{\Delta t} = D^* \frac{c_{aq}^{i,n+1} - 2c_{aq}^{i,n} + c_{aq}^{i,n+1} - 2c_{aq}^{i,n+1} + c_{aq}^{i,n+1}}{2\Delta z^2}$$

(3.5)

Where $c_{aq}^{i,n}$ represents the aqueous-phase dissolved gas concentration at point $i$ within the soil column at time $n$.

A grid-size of 0.5 mm and a time-step of 10 s were used. The volumetric water content of each soil was measured, thus inverse analyses were performed to identify the tortuosity factor in the soil column that results in agreement with the measured decay of pressure in the system. A single-variable iterative optimisation scheme employing Newton’s method was adopted to fit the experimental results and to assess the effective aqueous-phase gas diffusion coefficient, and thus the tortuosity factor (i.e. diffusion coefficient of gas through water only was assumed to be known).
Figure 3.4: A view of the inside of the diffusion chamber containing reconstituted Ottawa sand.

Figure 3.5: Boundary conditions during a pressure decay test.
At the gas-liquid interface, using mass transfer balance equation, we can link the gas pressure change to the diffusive flux at the surface using Fick’s 1st law and Boyle’s ideal gas law as follows:

\[
\frac{V}{Z_g RT} \frac{dP_g}{dt} = -D^* n A \left( \frac{dc_{aq}}{dz} \right)_{z=0}
\]

Where, \( V \) is the volume of gas in the pressure chamber, \( Z_g \) is the gas compressibility factor, \( R \) is the universal gas constant, \( T \) is the absolute temperature and \( A \) is the cross-sectional area of the pressure cell.

After soil reconstitution, but prior to pressurisation of the chamber, the gas of interest was circulated through the top of the cell under low pressure for approximately 20 seconds to evacuate air and then pressurised to around 800 kPa within 60 seconds using a dual channel pneumatic pressure controller (Figure 3.6) (2 MPa max pressure) fed by a gas bottle (Figure 3.7).

![Figure 3.6: Dual-channel pneumatic controller (brown box on top shelf).](image)

The chamber was then hermetically sealed and isolated from the pneumatic controller. A new style (V2) standard hydraulic pressure/volume controller (Figure 3.8) (max pressure 3 MPa) was hydraulically connected with water at the bottom of the chamber to measure pressure decay arising from diffusion-induced gas dissolution, but locked to prevent volume change. Data was logged with a GDS dynamic control system.
Figure 3.7: Gas bubbles used as a gas source.

Figure 3.9 shows the grain size distributions for three soils (silt, sand and gravel) considered. The sand and silt were Ottawa sand and Sil-Co-Sil 40 provided by the US Silica Company and gravel was acquired locally. To isolate volumetric water content from tortuosity, soils were reconstituted with an attempt to achieve the same porosity. Sand specimens first were poured into a flask full of water and any entrapped air bubbles were extracted using a combination of heat and vacuum (Figure 3.10). Reconstitution of Ottawa sand specimens were conducted using wet-pluviation method. In this method the flask containing fully saturated granular soil is inverted into a container full of water (in this case the diffusion chamber). To avoid particle segregation, the top of flask was placed near the top of the soil column as it was slowly raised out of the diffusion chamber. For all of the diffusion experiments performed, water initially filling the diffusion chamber was deaerated using a deaerator device (Figure 3.11) located in the laboratory.
Silt specimens were reconstituted using wet-pluviation in 10 mm layers until suspended silt was no longer visible; a vacuum was applied at the top of the chamber between deposition of each layer to minimise dissolution of any gas in the deaired water.

Gravel specimens were dry pluviated and then saturated by introducing deaired water from the bottom of the chamber while applying a vacuum from the top. A thin film of water (5-6 mm thick) was left overlying the soil in all cases to ensure full submergence. Table 4.2 summarises dimensions and properties for each experiment, where a minimum of three tests were performed on each soil type. Three tests were also performed with water only, using both helium and nitrogen gas to compare diffusion coefficients interpreted from the PDM (without soil) with published values (Ferrell & Himmelblau, 1967a,b). Nitrogen was used for all tests with soil.
3.3 Results

The adequacy of the PDM to assess aqueous-phase gas diffusion coefficients was verified by demonstrating that the diffusion coefficient of a particular gas could be predicted without soil for two gasses. The interpreted diffusion coefficients for nitrogen and helium with water only were $1.96 \times 10^{-9}$ $m^2/s$ and $6.24 \times 10^{-9}$ $m^2/s$, which are in excellent agreement with published values (see Table 4.2). Figure 3.12a shows representative pressure-decay data for an experiment with water only. Thus, the time required for gas to
diffuse through the thin water film (with a known thickness) and begin penetrating the soil could reliably be predicted using the diffusion coefficient of nitrogen. Once dissolved gas reaches the soil, the rate of diffusion and pressure-decay in the system was governed, in part, by the lower volumetric water content and tortuous path that the dissolved gas takes as it penetrates the soil. Figures 3.12b-d show representative pressure-decay data for each soil type considered. The computed start of diffusion through soil is indicated, which coincides with a notable decrease in the rate that pressure decays. However, it is also important to note that, aside from differences associated with diffusion through soil, the
Table 3.1: Summary of aqueous-phase gas diffusion experiments, dimensions, and gas properties for experiments conducted with the pressure-decay-method

<table>
<thead>
<tr>
<th>Test #</th>
<th>Gas</th>
<th>$H$ (atm × m$^3$/mol)</th>
<th>$D_{aq}^m$ (m$^2$/s)</th>
<th>Soil Type</th>
<th>Porosity, n</th>
<th>$H_a/H_w/H_s$ (mm)</th>
<th>Duration (hrs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>He</td>
<td>2.5</td>
<td>$6.3 \times 10^{-9}$</td>
<td>N/A</td>
<td>N/A</td>
<td>6/1/10/0</td>
<td>119/0</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>Silt</td>
<td>0.40</td>
<td>3/6/116</td>
<td>74</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9.5/5/110.5</td>
<td>42</td>
</tr>
<tr>
<td>7</td>
<td>N$_2$</td>
<td>1.5</td>
<td>$2 \times 10^{-9}$</td>
<td>Silt</td>
<td>0.40</td>
<td>8/4/113</td>
<td>46</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td>Sand</td>
<td>0.39</td>
<td>5/4/116</td>
<td>47</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.5/7/113.5</td>
<td>94</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td>Gravel</td>
<td>0.41</td>
<td>2/8/115</td>
<td>24</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Tests performed on water column only (i.e. no soil).

rate and magnitude of pressure decay is also governed by the initial pressure in the system and the thickness of the gas cavity ($H_a$).

To illustrate the interpreted depth penetrated by the gas, Figure 3.13 shows the computed dissolved concentrations at depth from inverse analyses at different points in time. The gas penetration depth relative to $D_{50}$ from the grain-size distribution curves at the end of the test are also indicated to demonstrate that the gas penetrated several grain thicknesses over the testing period. Table 3.2 summarises the range and average material-dependent tortuosity factors interpreted for each soil type, yielding consistent results for each soil type with the PDM. The interpreted range for tortuosity factors were within 10-30% of the average depending on soil type. The largest range was observed for gravel, which was also the soil type where diffusion-induced dissolution penetrated the fewest grains.

The average tortuosities for silt, sand, and gravel were 0.09, 0.39, and 0.15, respectively. The lower tortuosity factor interpreted for gravel (relative to sand) is not intuitive. Images of each soil were taken to examine the role of particle shape on the interpreted tortuosity factors. Figure 3.14 provides top and side profile views of the gravel and sand. The same
camera lens could not be used for silt, thus only a top view of silt with microscopic images were obtained. The images illustrate that the subangular gravel particles (Figure 3.14a) are more elliptical than the substantially more spherical subrounded Ottawa sand grains (Figure 3.14b). Many of the gravel grains are also oriented (packed) such that the thickness (smallest dimension of the 3D particle) is transverse to the bulk direction of 1D vertical diffusion (side view Figure 3.14a), which would contribute to a greater effective length and lower tortuosity factor. Though side profile views of silt could not be obtained, the particles appear more platy than the sand and gravel (Figure 3.14b).

Simplified porosity (held constant in this study) relationships for tortuosity (e.g. Marshall, 1959; Weissberg, 1963) neglect the influence of intrinsic soil properties such as
Figure 3.13: Aqueous-phase concentration profiles computed from inverse analyses of the effective aqueous-phase gas diffusion coefficient to demonstrate the depth gas penetrated through: a) water only; b) saturated silt; c) saturated sand; and d) saturated gravel.

Table 3.2: Summary of the range and average tortuosity factors determined from pressure-decay data for silt, sand, and gravel.

<table>
<thead>
<tr>
<th>Soil Tested</th>
<th>No. Tests</th>
<th>Tortuosity Factor, $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silt</td>
<td>3</td>
<td>0.08-0.10 (avg. = 0.09)</td>
</tr>
<tr>
<td>Sand</td>
<td>5</td>
<td>0.35-0.40 (avg. = 0.39)</td>
</tr>
<tr>
<td>Gravel</td>
<td>3</td>
<td>0.10-0.20 (avg. = 0.15)</td>
</tr>
</tbody>
</table>

particle shape. Figure 3.15 illustrates the influence of grain shape on tortuosity (Figure 3.15a) and the local tortuosity factor around the perimeter of an elliptical grain with an aspect ratio of 3 and 1 (Figure 3.15b). With an aspect ratio of 3, the tortuosity factor is approximately 0.1 (similar to gravel), where an aspect ratio of 1 would have a local tortuosity factor of 0.4 (similar to sand). These shapes are not dissimilar from the gravel and sand grains shown in Figure 3.14a and Figure 3.14b. This idealised description of tortuosity in Figure 3.15 does not address the full complexity of grain shape and
out-of-plane flow-paths associated with grain packing and particle contacts, but does lend
credence to the lower tortuosity factor determined for gravel relative to sand. Though these
tortuosity factors are not dissimilar from those reported for other diffusion problems, the
results emphasise that factors other than porosity influence the rate of aqueous-phase gas
diffusion and efficacy of the PDM to consistently and efficiently appraise the soil fabric’s
influence on tortuosity for aqueous-phase gas diffusion.

Figure 3.14: Images illustrating grain shape: a.) top and side profile view of grave; b.) top
and side profile view of sand; c.) top view of silt.

3.4 Discussion

Aqueous-phase gas diffusion is slow, and thus diffusion-dependent processes on spatial-
and time-scale relevant to existing and emerging geotechnical applications (e.g. prediction
of the persistence and diffusion of gas after induced-partial-saturation) are challenging and
often impractical to assess through physical demonstrations. Numerical analyses, which
will require reliable methods to identify effective diffusion coefficients, are likely a more
practical approach in many instances. There is currently a scarcity of data and
methodologies available to demonstrate the tortuosity factor associated with aqueous-phase
gas diffusion through soil. This study examined material-dependency of tortuosity factors
for aqueous-phase gas diffusion in three soils with the pressure-decay-method, using
Figure 3.15: a) Conceptual illustration of particle shape on tortuosity b) computed local tortuosity factor around an elliptical shape with different dimensions.

equipment readily available in many geotechnical engineering laboratories. The PDM and inverse numerical analyses provided an efficient assessment of tortuosity factors, which were determined over the course of days with the PDM by imposing relatively high dissolved gas concentration gradients in a pressurised diffusion chamber—a limitation of experiments performed under low ambient pressures.

The PDM was able to capture the influence of grain shape and packing, demonstrated by consistent measurements for each soil type. This was elaborated on further with images of the soil grains. However, it should be recognised that deviation from the spherical particle assumption likely introduces anisotropy associated with aqueous-phase gas diffusion; thus larger effective diffusion coefficients should be expected in the direction orthogonal to the dominant orientation of grain thickness. Though the experimental configuration prohibited testing of diffusion in the orthogonal direction, higher tortuosity factors are anticipated. This follows well-established observations, where hydraulic conductivity in soil is often greater in the direction parallel to the deposition or bedding plane.
CHAPTER 4
ASSESSING THE PERSISTENCE OF ENTRAPPED GAS FOR INDUCED-PARTIAL-SATURATION

In this section the persistence of entrapped gas in porous media is formulated under both hydrostatic and groundwater flow conditions. This formulation includes the physical and chemical processes governing the transport of aqueous-phase dissolved gas considering the inter-phase gas exchange between the pore fluid and entrapped gas phase.

Longevity of entrapped gas can be simulated by solving the above-mentioned governing equations given the appropriate boundary and initial conditions.

Entrapped air or gas in soil below the water table results in a quasi-saturated condition and an associated increase in the pore fluid compressibility (Okamura & Soga, 2006). The gas suppresses the generation of positive excess pore water pressure in loose granular soils sheared under globally undrained conditions, and thus increases the liquefaction resistance (e.g. Sherif et al., 1977; Yoshimi et al., 1989; Grozic et al., 1999, 2000). Tsukamoto et al. (2002) appreciated the need to identify parameters that may be used to indicate degrees of saturation \( S_r \) below unity to evaluate the liquefaction susceptibility of partially or nearly saturated soils. Pietruszczak et al. (2003) later demonstrated through numerical simulations that lowering \( S_r \) may be an alternative method to mitigate liquefaction. It was recognized that proving this postulation valid might offer an economical liquefaction mitigation method with a low carbon footprint that is non-intrusive and applicable to new and existing civil infrastructure systems. The mechanical benefits and accompanying increase in liquefaction resistance associated with gas emplacement motivated several studies that investigated potential methods to introduce gas. Yegian et al. (2007) introduced the expression “induced-partial-saturation” (IPS) to describe the deliberate introduction of gas to mitigate liquefaction—terminology adopted in this article. They
generated gas via electrolysis and performed bench-scale shake table tests to demonstrate that IPS increases liquefaction resistance.

Bench-scale shake table testing was also performed by others to demonstrate increases in liquefaction resistance with biogenic gas (He et al., 2013) or via reaction of sodium perborate monohydrate (efferdent) with water to generate oxygen bubbles (Eseller-Bayat et al., 2013a). Kato & Nagao (2020) introduced gas with a microbubble generator and demonstrated increases in liquefaction resistance with large-scale (3.6 m x 10 m x 4.5 m deep) shake table tests. Yasuhara et al. (2008) and Okamura et al. (2011) demonstrated IPS via air-injection in lab- and field-scale trials, respectively. Practical efforts, including development of models to predict the excess pore water pressure response of quasi-saturated sands (e.g. Eseller-Bayat et al., 2013b), have contributed to the advancement of IPS. However, less attention has been given to the salient consideration of gas durability and its persistence after emplacement.

Once gas bubbles are entrapped, capillary forces tend to immobilize the bulk flow of gas through pore throats. External forces imposed by the viscous flow of groundwater and buoyancy are relatively small in comparison (Peck, 1969; Fry et al., 1997; Maryshev, 2017). Gas also remains immobile under strong ground shaking Eseller-Bayat et al. (2013a). However, Yegian et al. (2007) recognized that the long-term persistence of entrapped gas must be demonstrated for adoption of IPS in practice. They performed diffusion tests by measuring changes in $S_r$ in a 1.5 m column of sand for 442 days (1.2 years) and observed a limited change in $S_r$ (82.1% to 83.9%). Eseller-Bayat et al. (2013a) performed an identical experiment on a 1.2 m column of sand and reported a change in $S_r$ from 82% to 84% after 805 days (2.2 years). Though encouraging, the subsurface conditions simulated in these tests (i.e. shallow depths with low hydrostatic pressures) are limited.

Delayed increases in penetration resistance, on the order of months and years, to verify ground improvement after blast densification have been partially attributed to entrapped
gas generated by explosives based on lab (Dowding & Hryciw, 1986) and field (Finno et al., 2016; Gallant & Finno, 2016, 2017) experiments.

However, Okamura et al. (2006) offered some of the most compelling field evidence for the persistence of entrapped gas. Their study revealed that soil densified with sand compaction piles (SCPs) contained entrapped gas that was exhausted from a casing pipe (an unintended consequence of the construction method). Frozen sand samples collected months after SCP construction consistently revealed $S_r$ ranged between 75% and 90% at several sites. Frozen samples were also obtained at three sites where SCPs were installed 4, 8, and 26 years prior, where they observed higher values of $S_r$ between 75-98%, 95-100%, and 92-98% at each site, respectively.

These observations suggest that (a) gas may be introduced and persist for appreciable periods of time and (b) that dissolution of entrapped gas and resaturation of the soil may be anticipated in the field; but have provided limited context regarding the circumstances influencing gas persistence. Though the dissolution of entrapped gas is slow, infrastructure systems operate for decades, and often more than a century. Demonstration of gas longevity on these time-scales has been a practical limitation of physical experiments. Modeling the mechanisms that influence the persistence of entrapped bubbles is a practical avenue to address concerns regarding gas longevity. In the context of IPS, the objective of this study is to provide a novel assessment of gas persistence by explicitly considering the governing physical and chemical processes associated with aqueous-phase gas transport (advection-diffusion) and dissolution (inter-phase gas kinetics). These mechanisms are incorporated in a finite difference framework to simulate soil resaturation under hydrostatic and groundwater seepage conditions. The model framework is validated with elemental column and bench-scale tests and then extended to address the rate of soil resaturation under different subsurface conditions and for time-scales relevant to civil infrastructure.

From an IPS application perspective, the durability of entrapped gas bubbles are important in long-term. But there are other situations that the changes in the degree of
saturation in a relatively short period of time can be very important. One example of this situation is a case of gassy coastal sediments undergoing a tsunami wave as discussed in the following. In this section the persistence of entrapped gas in porous media is formulated under both hydrostatic and groundwater flow conditions. This formulation includes the physical and chemical processes governing the transport of aqueous-phase dissolved gas considering the inter-phase gas exchange between the pore fluid and entrapped gas phase.

4.1 Theory

4.1.1 Stability of Entrapped Bubbles

According to Henry’s law, gas is immiscible at the water-bubble interface when the aqueous-phase (i.e. dissolved) concentration of a gas specie, $i$, is:

$$C_{i,eq}^{aq} = \frac{P_i^g}{H_i}$$

(4.1)

where $P_i^g$ is the partial pressure of a gas specie inside the bubble and $H_i$ is Henry’s solubility coefficient, which is a function of pressure and temperature. Therefore, the equilibrium, or maximum total dissolved gas concentration is:

$$C_{eq}^{aq} = \sum C_{i,eq}^{aq}$$

(4.2)

According to Dalton’s Law the partial gas-phase pressure is:

$$P_i^g = \frac{C_i^g}{C^g} P^g$$

(4.3)

where $C_i^g$ and $C^g$ are the individual and total gas-phase concentrations and $P^g$ is the total gas pressure in a bubble, evaluated as:

$$P^g = P_{atm} + P_w + P_c$$

(4.4)
where $P_{atm}$ is atmospheric pressure, $P_w$ is the pore water pressure, and $P_c$ is capillary pressure arising from surface tension of the pore fluid and curvature of the bubble.

Assuming a spherical shape for an entrapped bubble:

$$P_c = \frac{2\sigma_i^w}{r_b} \tag{4.5}$$

where $r_b$ is the bubble radius and $\sigma_i^w$ is the unit interface surface tension of the fluid; in coarse-grained soils $P_c$ is small relative to $P_{atm}$ and $P_w$ at depth. Rad & Lunne (1994) introduced the expression “water-gas saturation” ($\eta$) to describe the degree to which water’s molecular pore structure is saturated with dissolved gas:

$$\eta = \sum C_{aq}^i H_i = \frac{TDGP}{P_{atm} + P_w} \tag{4.6}$$

where TDGP is the total dissolved gas pressure (note $C_{aq}^i = \sum C_{aq}^i$ is the total dissolved gas concentration). Groundwater is “supersaturated” ($\eta > 100\%$) in the presence of gas due to capillary pressure associated with the bubbles.

Figure 4.1 shows the hypothetical pore fluid conditions in a liquefiable layer before and shortly after gas is introduced via IPS. Prior to gas introduction, naturally occurring total dissolved gas concentrations are generally in equilibrium with the partial pressure of atmospheric gasses ($C_{aq} \approx C_{aq}^{atm}$); though small variations may arise due to water table fluctuations and entrapment of air pockets in the vandose zone Heaton & Vogel (1981); Yager & Fountain (2001), microbial processes like denitrification Weymann et al. (2008); Rebata-Landa & Santamarina (2012), and oxygen-reduction Champ et al. (1979); Heaton & Vogel (1980). Regardless, $C_{aq}$ is approximately constant and $\eta$ decreases with depth as pore water pressures increase. Once gas is introduced, it dissolves until $C_{aq}$ increases from $\approx C_{aq}^{atm}$ to $C_{eq}^{aq}$, and emplaced gas is quasi-stable until aqueous-phase mobility causes $C_{aq}$ to decrease below $C_{eq}^{aq}$ and the gas begins to dissolve.
Figure 4.1: Conceptual IPS scenario and associated groundwater conditions after gas emplacement: a.) liquefiable layer improved via IPS; b.) aqueous-phase dissolved gas concentrations and water-gas saturation through cross-section A-A soon after gas emplacement.

4.1.2 Gas Mobility: Advection and Diffusion

Under hydrostatic conditions diffusion governs the aqueous-phase mobility of entrapped gas. By inspection of Figure 4.1b, aqueous-phase concentration gradients exist within and outside the quasi-saturated IPS zone after gas is introduced. Aqueous-phase concentrations will diffuse until they are in equilibrium with partial atmospheric gas pressures at the ground surface Christiansen (1944); Bloomsburg & Corey (1964); Adam et al. (1969);
McWhorter et al. (1973); Faybishenko (1995); Fry et al. (1995). The steady-state molecular flux in the water column is proportional to the aqueous-phase concentration gradient:

$$J_{d,i} = -D_i^* \theta_w \nabla C_{aq}^i$$

(4.7)

where $\theta_w = \eta S_r$ is the volumetric water content, $\eta$ is soil porosity, and $D_i^*$ is the effective aqueous-phase gas diffusion coefficient:

$$D_i^* = \tau_e D_i$$

(4.8)

where $D_i$ is the diffusion coefficient of a gas specie in the pore fluid and $\tau_e$ is the effective tortuosity factor (less than 1). Tortuosity conceptually accounts for the actual travel distance through a porous medium and is influenced by the soil’s fabric, particle-shape, and grain size distribution (i.e. geometry). A commonly adopted value of $\tau_e = 0.5$ was originally proposed by Carman (1937) for granular porous media, and based on geometric considerations of spherical glass beads. Effective tortuosity may also account for the influence of “dead end” pores, anisotropy associated with the soil fabric, and other sources that may impede (or enable) aqueous-phase gas mobility. For practical considerations, $\tau_e$ may account for all elements contributing to mobility Shackelford & Daniel (1991), and in this way is ultimately a “conveyence” factor.

For non-hydrostatic conditions, $C_{aq}^i$ is influenced by the bulk flow of groundwater that transports dissolved gas. The advective flux may be expressed as:

$$J_{a,i} = \theta_w v_s C_{aq}^i$$

(4.9)

where $v_s$ is the seepage velocity associated with hydrogeologic conditions and naturally occurring hydraulic gradients. For larger average seepage velocities, grain-scale effects may cause non-uniform interstitial seepage rates and mixing locally, also known as mechanical (or hydrodynamic) dispersion. This is typically accounted for in $J_d$ as:
where \( D_h = D^* + D_m \) and \( D_m \) is the semi-empirical mechanical dispersion coefficient. The Peclet number, \( P_e \), is defined as:

\[
P_e = \frac{u_s d_p}{D_i}
\]  

(4.11)

where \( d_p \) is the average particle diameter and \( u_s \) is the average interstitial velocity, which may be approximated as \( v_s \). For laminar flow and where \( P_e \) is less than 0.5 to 1, then \( D_h \) may be assumed to equal \( D_i^* \). Perkins et al. (1963). This is applicable for most naturally occurring groundwater conditions in granular soils (i.e. hydraulic gradients between 0.01 and 0.2).

Mass conservation of a non-reactive gas solute is assessed with the readily applied advection-diffusion equation:

\[
\frac{\partial C_{aq,i}}{\partial t} = -\nabla (J_{a,i} + J_{d,i}) = \theta_w [-\nabla (v_s C_{aq,i}) + \nabla (D_i^* \nabla C_{aq,i})]
\]  

(4.12)

Equation 4.12 is valid away from the quasi-saturated layer. However, where entrapped gas exists, a source term must be considered.

### 4.1.3 Inter-phase Gas Kinetics

The kinetic bubble dissolution (KBD) model introduced by Holocher et al. (2003) was developed to understand the formation of “excess air” and its influence on aqueous-phase transport dynamics. Understanding gas exchange was motivated by environmental applications, such as the use of tracers for groundwater dating and contaminant transport (e.g. Cirpka & Kitanidis, 2001) or the effectiveness of bioremediation through oxygen injection (e.g. Fry et al., 1995). The KBD model is adopted for the source term because it offers a good description of inter-phase gas exchange between the bubble and groundwater,
as well as the flexibility to account for individual gasses (e.g. air is approximately 80% nitrogen and 20% oxygen with other trace gasses).

Figure 4.2 conceptually illustrates the inter-phase gas transfer associated with bubble dissolution. The rate of bubble dissolution depends on the aqueous-phase concentration deficit and mass transfer coefficient, $k_{g,i}$. The molecular flux at the bubble-water interface is:

$$J_{g,i} = k_{g,i}(C_{aq}^i - C_{i,eq}^{aq}) \quad (4.13)$$

Film theory, which assumes gas diffuses over a stagnant film surrounding the surface of the bubble Cussler (2009), is used to define $k_{g,i}$:

$$k_{g,i} = \frac{D_i}{\delta_{eff}} = D_i \left(\frac{1}{r_b} + \frac{1}{\delta}\right) \quad (4.14)$$

where $r_b$ is the bubble radius and $\delta_{eff}$ and $\delta$ are the effective diffusion distance and static film thickness, respectively (Figure 4.2). In coarse grained soils $r_b$ is sufficiently large such that $r_b >> \delta$ and $\delta_{eff} \approx r_b$.

For seepage conditions, groundwater infiltrating a quasi-saturated soil presents a potential sink for the gas, assuming there is a concentration deficit at the bubble-water interface. Epstein & Plesset (1950) derived an expression for $k_{g,i}$ as a function of the surface contact time, $t_c$, groundwater has with the bubble:

$$k_{g,i} = D_i \left(\frac{1}{r_b} + \frac{1}{\sqrt{\pi D_i t_c}}\right) = D_i \left(\frac{1}{r_b} + \sqrt{\frac{v_s}{\pi D_i 2r_b}}\right) \quad (4.15)$$

Note that Equation 4.15 reduces to equation 4.14 when $v_s = 0$. The expression in equation 4.13 becomes:

$$J_{g,i} = D_i \left(\frac{1}{r_b} + \sqrt{\frac{v_s}{\pi D_i 2r_b}}\right) (C_{aq}^i - C_{i,eq}^{aq}) \quad (4.16)$$

The molar rate of inter-phase gas exchange ($dm_i/dt$) for a bubble is dependent on the surface area of the bubble, $A_{sb}$. For an assumed spherical bubble:
The change in the aqueous-phase concentration in a volume of water, $V_w$, contained in an elemental soil volume, $V$, depends on the number of bubbles, $n_b$, and volume of gas, $V_g = nV(1 - S_r)$. Accordingly, the source term due to dissolution of homogeneous spherical bubbles is:

$$
\frac{dC_{aq}}{dt} = \frac{n_b}{V_w} \frac{dm_i}{dt} = \frac{n_b A_{sb}}{nV(S_r)} J_{g,i} = \frac{3(1 - S_r)}{r_b S_r} J_{g,i}
$$

(4.18)

Adding the source term due to the presence of entrapped gas bubbles in equation 4.12 provides a complete description of gas mass conservation:

$$
\frac{\partial C_{aq}}{\partial t} \theta_w = -\nabla (J_a + J_d) + \frac{n_b A_{sb}}{nV(S_r)} J_{g,i}
$$

(4.19)

For soils with uniform spherical bubbles, equation 4.19 may also be expressed as:
\[
\frac{\partial C_{aq}^i}{\partial t} = -\nabla (v_s C_{aq}^i) + \nabla (D_h \nabla C_{aq}^i) + \frac{3(1 - S_r)}{r_b S_r} J_{g,i} \tag{4.20}
\]

4.2 Model Formulation

In this study, the governing partial differential equation (i.e. equation 4.20) was solved in MATLAB R2017a using the second order central finite difference method and the Crank-Nicholson scheme for unconditional stability Crank & Nicolson (1947); LeVeque (2007). The FD discretization of equation 4.20 is shown below:

\[
\frac{C_{aq}^{i,n+1} - C_{aq}^{i,n}}{\Delta t} = -\frac{v}{4\Delta Z} (C_{aq}^{i,n+1} - C_{aq}^{i,n+1} + C_{aq}^{i,n} - C_{aq}^{i-1,n}) \\
+ \frac{D}{2\Delta Z^2} (C_{aq}^{i,n+1} - 2C_{aq}^{i,n+1} + C_{aq}^{i,n+1} + C_{aq}^{i-1,n} - 2C_{aq}^{i,n} + C_{aq}^{i+1,n}) \\
- \frac{3(1 - S_r)D_0}{r_b S_r} \left( \frac{1}{r_b} + \sqrt{\frac{v}{2\pi r_b D_0}} \right) \frac{c_{aq}^{i,n+1} + c_{aq}^{i,n}}{2} \\
+ \frac{3(1 - S_r)D_0}{r_b S_r} \left( \frac{1}{r_b} + \sqrt{\frac{v}{2\pi r_b D_0}} \right) \frac{P_g^i}{H_i} \tag{4.21}
\]

The finite difference solution solves for one independent variable, \( C_{aq}^{i,n} \), at each time step, \( n \), using linear matrix calculations and updates dependent variables (\( r_b \), \( S_r \) and \( P_g \)) accordingly. \( j \) denotes the node number and \( i \) represents the gas type. Figure 4.3 illustrates the algorithm that couples the kinetic dissolution of gas bubbles to solve initial boundary value problems.

Solving the coupled behavior between aqueous-phase gas transport and KBD is achieved through an iterative scheme, as the inter-phase flux between the entrapped gas and groundwater depends not only on \( C_{aq}^{i,n} \), but also on \( P_g^i \) and \( r_b \). The change in \( r_b \) is related to the moles of gas, \( m \), that dissolve into solution:

\[
dr_b = \frac{3RT}{4\pi r_b} \left( \frac{1}{4\sigma_w + 3r_b(P_{atm} + P_w)} \right) dm \tag{4.22}
\]

The new bubble radius for each succeeding time step (\( n + 1 \)) is \( r_b^{n+1} = r_b^n + dr_b \). To avoid accumulating error, and possible overestimation of \( dr_b \) and associated inter-phase gas exchange, a sub-stepping routine was adopted to estimate the average bubble radius, \( r_b^* \), for
Figure 4.3: Algorithm to couple the advection-diffusion equation and kinetic bubble dissolution for finite difference simulations.

Each time step (see Figure 4.3). An initial guess of $r^* = r_b^n$ for each time step is applied to solve the governing equation for $C_{aq,i}^{n+1}$ and $r_b^{n+1}$. The sub-stepping routine then initiates by updating $J_{g,i}$ with $r^* = 0.5(r_b^n + r_b^{n+1})$ until a tolerable error of $\left| (r_b^{new} - r_b^{prev}) / r_b^{new} \right| < 0.01$ is achieved.

The degree of gas saturation for an assumed spherical bubble is:

$$S_g = \frac{n_b 4 \pi r_b^3}{\eta V} = 1 - S_r$$

(4.23)

where $n_b$ is the number of gas bubbles in an elemental soil volume, and dependent on the initial degree of gas saturation and bubble radius at the start of any simulation. As the computed radius of entrapped bubbles diminish due to inter-phase gas exchange, $r_b$ becomes very small and $P_c$ becomes very large. Thus, all bubbles in a finite difference cell were assumed to collapse once $S_r = 99\%$. All simulations in this study are 1D with a
grid-size of 0.01 m. A time-step of 5 s and 10 s was applied for seepage and hydrostatic conditions, respectively.

4.3 Model Validation

To validate the numerical platform’s adequacy to assess gas longevity and temporal changes in $S_r$ under hydrostatic and groundwater flow conditions, experimental results from 1D elemental column tests performed by McWhorter et al. (1973), Yegian et al. (2007), and He et al. (2016) on quasi-saturated porous media (sandstone and Ottawa sand) are compared with numerical predictions. Model predictions are also compared with experimental results from a bench-scale aquifer designed to observe patterns of entrapped air dissolution (McLeod et al., 2015), which provided spatial measurements of gas content and greater insight into the influence of depth under seepage conditions.

These experiments were chosen because the size of specimens range considerably, both hydrostatic and seepage conditions are considered, their boundary conditions are well defined, and their measurements can be readily compared with model predictions. Table 4.1 summarizes the model parameters, including specimen dimensions, pore fluid properties, initial $S_r$, and porosity for each porous medium. Details regarding each experiment and comparison with computed results are presented herein.
Table 4.1: Summary of parameters used to simulate experiments to validate the numerical model

<table>
<thead>
<tr>
<th>Reference</th>
<th>Test</th>
<th>Porous Medium</th>
<th>Fluid</th>
<th>Gas</th>
<th>Length (m)</th>
<th>Initial $S_r$</th>
<th>Porosity (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>McWhorter et al. (1973)</td>
<td>Diffusion</td>
<td>Sandstone Core #2</td>
<td>Soltrol Oil$^2$</td>
<td>Air</td>
<td>$45.2 \times 10^{-2}$</td>
<td>0.81</td>
<td>0.18</td>
</tr>
<tr>
<td>Yegian et al. (2007)</td>
<td>Diffusion</td>
<td>Sandstone Core #5</td>
<td>Soltrol Oil$^2$</td>
<td>Air</td>
<td>$45.6 \times 10^{-2}$</td>
<td>0.73</td>
<td>0.26</td>
</tr>
<tr>
<td>He et al. (2016)</td>
<td>Vertical Seepage</td>
<td>Ottawa Sand</td>
<td>Water$^3$</td>
<td>$N_2$</td>
<td>1.00</td>
<td>0.88</td>
<td>0.40</td>
</tr>
<tr>
<td>McLeod et al. (2015)</td>
<td>Horizontal Seepage</td>
<td>Ottawa Sand</td>
<td>Water$^3$</td>
<td>Air</td>
<td>5.25</td>
<td>0.81-0.86</td>
<td>0.37</td>
</tr>
</tbody>
</table>

1. Donaldson et al. (1976) interpreted $\tau_e$ from molecular diffusion of gas and electrical conductivity for several sandstone specimens with similar porosities in an independent study, and reported $\tau_e$ ranged between 0.07 and 0.3. $\tau_e = 0.1$ was adopted for numerical simulations of diffusion in sandstone cores. Due to the substantially spherical shape of Ottawa sand grains (Zheng & Hryciw, 2016), $\tau_e = 0$ was adopted for all Ottawa sand simulations.

2. In Soltrol oil the surface tension is $\sigma_w = 23 \times 10^{-6}$ kN/m. The bulk solubility and diffusion of air is $H_{air} = 120$ atm $\times$ m$^3$/mol and $D = 15 \times 10^{-9}$ m$^2$/s, respectively. These parameters were also adopted by McWhorter et al. (1973).

3. In water the surface tension is $\sigma_w = 72 \times 10^{-6}$ kN/m. For all cases where entrapped air was the gas type, the bubble was assumed to consist of 80% $N_2$ and 20% $O_2$. The solubility of $N_2$ is $H_{N_2} = 1508$ atm $\times$ m$^3$/mol and $O_2$ is $H_{O_2} = 769$ atm $\times$ m$^3$/mol. The diffusion coefficient of $N_2$ and $O_2$ are $D_{N_2} = 2 \times 10^{-9}$ m$^2$/s and $D_{O_2} = 2.9 \times 10^{-9}$ m$^2$/s.

4. For sandstone an initial $r_b = 0.03$ mm was estimated from the bubbling pressures reported by McWhorter et al. (1973) using equation 4.5. For Ottawa sand an initial $r_b = 0.03$ mm was also adopted. Kato & Nagao (2020) reported that the radii of occluded bubbles entrapped in poorly graded silica sand ranged between 0.005 and 0.05 mm. Mahabadi et al. (2018) also adopted bubble sizes ranging between 0.005 to 0.05 mm in a pore network study on poorly graded Mallik Sand, which had a similar grain size distribution as Ottawa sand. The initial number of bubbles, $n_b$, were computed for each finite difference cell by rearranging equation 4.23.
4.3.1 Gas diffusion in sandstone

Figure 4.4a shows the experimental configuration for the column diffusion tests on sandstone. McWhorter et al. (1973) reported changes in gas volume due to diffusion-induced resaturation of two quasi-saturated sandstone cores with a diameter of 25 mm and lengths of 45.2 (core #2) and 45.6 mm (core #5). The cores were initially oven dried and weighed to evaluate porosity, and then wrapped in aluminium sheeting (fixed in place with an epoxy resin) to enforce 1D upward vertical diffusion during the experiment. The top of the core was not sealed (boundary condition atmospheric pressure of air). The bottom of the core was sealed, except for a small opening to allow infiltration of pore fluid (soltrol oil) into the core. The core was first placed in a plastic box, causing infiltration of the fluid through the small opening at the bottom, which partially saturated the core. Once soltrol oil infiltrated to the top of the core, it was reweighed to determine the initial amount of entrapped gas. The core was then returned to the box to begin the diffusion experiment and removed periodically to measure changes in weight due to diffusion of entrapped gas and accompanying infiltration of soltrol oil through the bottom.

Figure 4.5a compares computed and observed changes in the volume, $\Delta V$, of soltrol normalized by the initial gas volume, $V_o$, due to aqueous-phase diffusion of entrapped gas. For the simulation an initial $r_b = 0.03$ mm was estimated from the bubbling pressures reported by McWhorter et al. (1973) using equation 4.5. They also adopted $\tau_e=0.5$ based on an assumed spherical shape of sandstone particles (Carman, 1937), which provided strong agreement with experimental results in the absence of KBD (not shown). However, when KBD was considered, the rate of diffusion and associated changes in $\Delta V/V_o$ are overestimated (Figure 4.5a).

In an independent study, Donaldson et al. (1976) interpreted $\tau_e$ from molecular diffusion of gas and electrical conductivity for several sandstone specimens with similar porosities, and reported $\tau_e$ ranged between 0.07 and 0.3. This is likely attributed to cementitious bonding between particles and a substantial deviation from the simplified
spherical particle assumption. When $\tau_e=0.1$ was adopted—a more faithful representation of tortuosity in sandstone—computed results show excellent agreement with experimental results using the same $r_b$. The model also captured the influence of initial $S_r$, simulating longer resaturation times for core #5, which had a greater initial volume of gas.

It is worth noting that “homogenous” dissolution of bubbles in the core was not predicted by the model (nor implied by the experimental results). Rather, a diffusion-induced saturation (or dissolution) front that propagates from the top to the bottom of the specimen is predicted. However, spatial measurements of gas dissolution throughout the core were not available (true for all other column tests).

**4.3.2 Gas diffusion in sand**

For all remaining numerical demonstrations (column tests and the bench-scale aquifer), the porous medium was Ottawa sand. Due to the substantially spherical shape of Ottawa
Figure 4.5: Comparison between experimental observations and numerical predictions for column tests: (a) temporal changes in normalized gas volume from McWhorter et al. (1973) diffusion experiment; (b) temporal changes in $S_r$ averaged over the height of the column from Yegian et al. (2007) diffusion experiment; (c) temporal changes in $S_r$ averaged over the height of the column from He et al. (2016) seepage experiment.
sand grains (Zheng & Hryciw, 2016), $\tau_e = 0.4$ was adopted. Kato & Nagao (2020) reported that the radii of occluded bubbles entrapped in poorly graded silica sand ranged between 0.005 and 0.05 mm. Mahabadi et al. (2018) also adopted bubble sizes ranging between 0.005 to 0.05 mm in a pore network study on poorly graded Mallik Sand, which had a similar grain size distribution as Ottawa sand. Thus, a uniform initial $r_b = 0.03$ mm was adopted. The influence of $r_b$ is investigated later in a numerical study.

Yegian et al. (2007) investigated long-term diffusion of entrapped air in Ottawa sand on a significantly larger specimen (Figure 4.4b) than McWhorter et al. (1973). A plexiglass cylinder with diameter and length of 100 mm and 1,510 mm, respectively, was prepared by wet-pluviating sand to initially saturate the soil column. A “drainage-recharge” method, whereby water was drained from the column and subsequently refilled through the bottom, created a quasi-saturated condition. They noted that entrapped air bubbles ($N_2$ and $O_2$ gas) were uniformly distributed. The column was weighed to determine the amount of entrapped gas and the initial $S_r$. Temporal changes in $S_r$ were monitored by the level of standing water at the top of the column, which was sealed to prevent evaporation.

Figure 4.5b, shows negligible changes in $S_r$ (averaged over the height of the specimen) were observed and predicted over 450 days. Though strong agreement arose from this comparison, and illustrates that diffusion is slow, this time-scale does little to “excite” the model. However, it is worth noting that in an identical experiment, Eseller-Bayat et al. (2013a) observed gas bubbles were lost at the top of the specimen, which was predicted by the model.

### 4.3.3 Gas dissolution due to vertical seepage in sand

He et al. (2016) investigated the influence of groundwater flow on the persistence of gas using an experimental configuration similar to Yegian et al. (2007), as shown in Figure 4.4c. However, they generated gas bubbles ($N_2$ gas) using a microbial reaction in a plexiglass cylinder with a diameter and length of 70.4 mm and 1,000 mm, respectively. The
initial $S_r$ was determined by measuring changes in the height of the water column as gas was generated. After gas was introduced, a constant head of 100 mm was applied to the specimen, inducing groundwater flow. A seepage velocity of 0.001 m/s was assumed based on a reported hydraulic gradient of 0.1, average hydraulic conductivity of 0.004 m/s and $n = 0.4$. Changes in gas content, thus the average $S_r$ averaged over the height of the specimen, were determined by weighing the specimen periodically. Water entering the column was assumed to have dissolved gas concentrations in equilibrium with the atmosphere; n.b. they were not recirculating water saturated with dissolved gas, which would reduce the rate of dissolution.

As shown in Figure 4.5c, computed results show excellent agreement with observations, thus indicating that the numerical method was capable of capturing the rate of gas bubble dissolution and temporal changes in $S_r$ averaged over the height of the column. Notably, the persistence of gas is significantly influenced by vertical groundwater flow—where the rate of dissolution is largely dependent on the seepage velocity and associated hydraulic gradient.

### 4.3.4 Horizontal seepage in synthetic sand aquifer

Measurements from the 1D column tests considered changes in gas content averaged over the length of a specimen, but provided no measurements regarding temporal changes spatially throughout the specimen. McLeod et al. (2015) performed relatively large bench-scale experiments, simulating horizontal groundwater seepage through quasi-saturated Ottawa sand in a synthetic aquifer (soil tank). Figure 4.6 shows the experimental configuration, dimensions, and locations of time domain reflectometry (TDR) probes used to measure temporal changes in entrapped air content throughout the soil tank. The soil tank was prepared by initially dry-pluviating sand. Water was subsequently percolated from the surface, entrapping air until the soil tank was filled. The initial $S_r$ increased slightly from 0.81 to 0.86 ($S_g = 0.19$ to 0.14) at the top and bottom, respectively,
due to differences in hydrostatic pressure. Air was uniformly distributed at each depth based on visual observation and initial TDR probes.

Figure 4.6: Experimental configuration, dimensions, boundary conditions, and location of TDR probes in a synthetic aquifer (soil tank) used to simulate groundwater flow and dissolution of entrapped air during the McLeod et al. (2015) study.

Horizontal groundwater flow was imposed by applying a constant head along the influent boundary for 344 days until 19 pore volumes of water were introduced. Though the system is 3D, given the uniform boundary conditions it may be expected to behave like a stack of horizontal 1D columns (Klump et al., 2008). Therefore, the numerical analysis was performed assuming groundwater flow through an assembly of 1D horizontal columns at depths where TDR probes were performed, thus vertical diffusion was neglected. This assumption was assumed reasonable based on the duration of the experiment and the 1D diffusion tests performed by Yegian et al. (2007) and simulated in this study. McLeod et al. (2015) reported changes in $S_g$ based on the number of pore volumes of groundwater introduced and discharged from the system at each depth. Therefore, an average linear seepage velocity resulting in 19 pore volumes introduced over 344 days was assumed ($v_s = 3 \times 10^{-6} \text{m/s}$). Groundwater was not recirculated and *infiltrating* groundwater at the influent boundary was assumed to have initial dissolved gas concentrations in equilibrium with atmospheric conditions (i.e. decreasing initial $\eta$ with depth).
Figure 4.7a compares the computed and observed position of the saturation front downgradient at five depths as a function of the pore volumes introduced. There is strong agreement in all cases, though a slight underestimation of the saturation front position at the greatest depth (1.65 m) after the saturation front progressed 2 m downgradient. The simulation and observations illustrate that the rate of gas dissolution increased with depth. Even though the initial $S_g$ at depth is lower (accounted for in the simulation), this is mainly attributed to the influence of pore water pressure and the associated higher solubility and capacity for pore water to serve as a gas sink as “fresh” groundwater with lower water-gas saturation infiltrates the system.

Figure 4.7b shows the observed 2D pattern of $S_g$ and the distinct wedge-shaped saturation front observed and predicted at the end of the experiment. While the numerical simulation accurately predicts the saturation front, there were changes in gas content downstream not predicted by the model. This may be due to the 3D nature of the soil tank and heterogeneous dissolution of gas, which could promote fingering (i.e. development of localized preferential flow paths) near the saturation front. Though not shown, McLeod et al. (2015) reported $S_g$ approximately half way through the experiment when the saturation front was further upstream, and gas contents downstream of the saturation front did not deviate significantly from their initial values. Therefore, fingering in relatively homogeneous coarse grained soils is likely localized near the saturation front when gas is initially distributed uniformly.

Result above illustrated the numerical framework’s potential to capture the: i.) rate of gas dissolution and resaturation under hydrostatic and seepage conditions for different experimental scales; ii.) evolution of the saturation front under horizontal seepage conditions; and iii.) influence of different porous media and solubility associated with different fluids (soltrol oil vs. water). This provides the foundation to explore a broader range of conditions that may be applicable to IPS. In a numerical study, the relative influence of depth, bubble size, gas type, boundary conditions, and the initial
Figure 4.7: Comparison of numerical results and observations by McLeod et al. (2015) of gas dissolution under flow conditions in a synthetic aquifer: a.) position of the saturation front downgradient at several depths compared to the pore volumes of groundwater introduced; b.) location of the saturation front at the end of the experiments when 19 pore volumes of groundwater were introduced
aqueous-phase gas concentration of groundwater outside of a quasi-saturated layer are assessed. The objective of this numerical study is not to provide a comprehensive assessment of all possible conditions that may exist, but to highlight some of the conditions and assumptions that influence the predicted longevity of gas.

Unless stated otherwise, all simulations assume the entrapped gas is air (80% $N_2$ and 20% $O_2$), parameters adopted for the Ottawa sand demonstrations earlier, $n = 0.44$ and an initial $S_r = 82\%$. Within the gassy layer, initial aqueous-phase concentrations were in equilibrium (i.e. $C_{aq}^{eq}$) with gas bubbles. Initial dissolved gas concentrations outside the gassy layer were in equilibrium with partial gas pressures from the atmosphere.

4.4 Hydrostatic Conditions

Yegian et al. (2007) is one of the most influential studies regarding IPS, and their experiment for long-term diffusion (1.5 m specimen) has been one of the leading indicators that the presence of gas can be relied on after emplacement. To provide more context, an extensively wide gassy soil layer 1.5 m thick was considered to numerically demonstrate the longevity of gas under hydrostatic conditions (i.e. 1D diffusion-induced resaturation). This is not meant to imply IPS might only “treat” a liquefiable layer 1.5 m thick, but to demonstrate the rate that a gassy layer decays under varying assumptions. Figure 4.8a shows the depths and boundary conditions considered; i.e. one-way diffusion arising from an impermeable bottom boundary and two-way diffusion arising from an infinitely thick soil layer through which gas can diffuse. The top of the water table is subjected to atmospheric conditions. As mentioned previously, diffusion in soil with a homogeneous distribution of bubbles does not result in homogeneous dissolution of gas throughout a layer, but the progressive advancement of a saturation front and changes in thickness to the gassy soil layer. Figure 4.8b illustrates the predicted temporal evolution of the gassy layer thickness subjected to different boundary conditions.
Figure 4.8: a.) Depths and boundary conditions considered to demonstrate gas longevity under hydrostatic conditions for a 1.5 m gassy layer; b.) conceptual changes in thickness and progression of the diffusion-induced saturation front for the gassy layer under hydrostatic conditions with different boundary conditions; c.) depths considered to demonstrate gas longevity and temporal changes in the saturation front under horizontal groundwater seepage conditions.
Figure 4.9a shows resaturation curves and compares the temporal changes in $S_r$ and the thickness of the gassy layer at depths of $Z = 0$, 5, and 10 m (top of the soil column is at 0, 5 and 10 m below water table) for boundary condition 1 (BC1 in Figure 4.8a, b). The $S_r$ is computed based on the initial thickness of the layer, and shown to provide context for $S_r$ reported for column tests on diffusion. For $Z = 0$ m (akin to Yegian et al. (2007) experiment) a change in layer thickness of approximately 1 m was predicted over 100 years (full saturation after 160 years). At a depth of 5 m the gassy layer decreased in thickness by 1.3 m in 100 years and was fully saturated after 125 years, while for $Z = 10$ m, full saturation was predicted after 98 years. This highlights the durability of entrapped air bubbles, even at depth, and lends merit to the conclusion by Yegian et al. (2007) that air bubbles can persist over appreciable time-scales.

Figure 4.9a also shows the effect of gas type, assuming all bubbles were initially $O_2$ gas. For instance, Yegian et al. (2007) and Eseller-Bayat et al. (2013a) used electrolysis and efferdent to generate $O_2$ gas and increases liquefaction resistance in their IPS experiments. However, the higher solubility of $O_2$ ($\approx 2$ times greater than $N_2$) resulted in full saturation of the 1.5 m layer of gassy soil in less than 34 years for all depths. Though it has been recognized that $N_2$, the primary constituent of air, is the most promising gas specie for IPS because of its low solubility (e.g. Yasuhara et al., 2008; Okamura et al., 2011; He et al., 2013; Kato & Nagao, 2020), the computed prediction clearly demonstrates its impact on the rate of diffusion-induced resaturation.

The computed predictions also indicate that depth was less influential for $O_2$ gas bubbles. This is linked to the low partial pressure of $O_2$ (relative to $N_2$) in the atmosphere—i.e. boundary condition at the water table elevation. Early on in a simulation, larger aqueous-phase gas concentration gradients exist in the water column (for $Z > 0$), which accelerates the rate of diffusion. However, once dissolved gas concentrations buildup and a pseudo-steady-state condition is established (i.e. approximately constant $\partial C_{eq}^{aq}/\partial Z$ in Figure 4.1b), the rate of diffusion is largely controlled by the flux and partial
pressure of gasses at the water table elevation (i.e. atmospheric conditions). Note that the slope of the resaturation curves for air in Figure 4.9a are nearly parallel after some time, and their separation is largely controlled by the larger concentration gradients that exist in the water column during early stages of resaturation.

Figure 4.9b compares the predicted evolution of resaturation arising from one- vs. two-way diffusion for entrapped air bubbles. As illustrated in Figure 4.1, the aqueous-phase concentration of gas outside the gassy layer is initially in equilibrium with atmospheric conditions, thus concentration deficits can instigate downward diffusion and an upward propagating saturation front at the lower boundary of the gassy layer (BC2 in Figure 4.8b). Full saturation of the depth interval beginning at $Z = 0$ m was predicted to occur in 78 years (vs. 160 years for BC1) with entrapped air bubbles. BC2 is a likely condition to exist in many, if not most, circumstances. Assuming BC2, a resaturation time of 160 years is more representative of a thicker gassy layer. Based on symmetry and the difference in resaturation times computed for BC1 and BC2 with the 1.5 m assumption, a gassy layer $\approx 3$ m thick would be required to achieve full saturation in 160 years (assuming BC2). While the boundary condition assumption is less influential than gas type, it has an appreciable influence on the persistence of gas.

Figure 4.9c illustrates the relative influence of bubble size. The homogeneous $r_b$ assumed in previous parametric studies was 0.03 mm, and $r_b = 0.015$ and 0.06 mm were considered for comparison (boundary condition 1 and $Z = 0$ m are assumed). As shown in Figure 4.9c, the influence of $r_b$ is significant. Full saturation for the smallest bubble radius was achieved after 91 years, though the largest $r_b$ did not result in full saturation until 450 years. Additionally, it is interesting to note the undulations in the resaturation curves (more pronounced with smaller bubble radii), which are predicted due to the progressive collapse and increased capillary pressure due to bubble dissolution near the saturation front.
Figure 4.9: Resaturation curves and associated $S_r$ and gassy layer thickness: a.) influence of depth and gas type on the rate of diffusion-induced resaturation of the gassy layer for boundary condition 1 in Figure 4.8a; b.) influence of the boundary condition on the rate of diffusion-induced resaturation of the gassy layer; c.) influence of bubble radius on the rate of diffusion-induced resaturation of the gassy layer. Note: $S_r$ is computed based on remaining gas content and the initial thickness of gassy layer to provide context for $S_r$ reported during column tests, as it is the thickness of the gassy layer that changes, as indicated by the secondary vertical axis.
The initial distribution and size of \( r_b \) (assumed uniform in this study) are challenging to assess, or even control. The initial size of the entrapped gas depends on many factors, including but not limited to, the method and rate of gas injection, grain size distribution, and depth below water table. Thermodynamic gas systems will attempt to minimize the gas-phase boundary area around a bubble to achieve a quasi-stable condition within the gassy layer. Therefore, internal diffusion after emplacement, particularly near the dissolution front, may cause bubbles to redistribute and re-nucleate within larger pores (Geistlinger et al., 2005), decreasing the capillary pressure associated with faster dissolution rates. These grain-scale effects could be beneficial from the perspective of gas persistence for IPS, but not accounted for with the KBD assumption (i.e. constant number of bubbles) applied here.

It’s important to recognize that IPS does not need to be limited to a liquefiable layer. From a design perspective, it would be advantageous to extend the gassy zone above and below the liquefiable layer, creating a “sacrificial” thickness that increases the longevity of gas in the targeted soil. As demonstrated by the diffusion simulations, extension of the gassy zone by only one or two meters may be sufficient. In that same vein, the presence of an soil layer with a low effective diffusion coefficient as discussed below can increase the longevity of gas.

4.4.1 Effect of Tortuosity Factor and Layering

Figure 4.10 illustrates different scenarios concerning an Induced-Partial-Saturation zone in a 1D situation. Figure 4.10a shows a case of homogeneous soil layer with an impermeable bottom boundary. The durability of entrapped gas in this case depends on the magnitude of the upward diffusive flux. In this case one of the main parameters governing the longevity of entrapped gas is the effective diffusion coefficient or more specifically, the tortuously factor of soil. As a part of this study, a parametric analysis is conducted on the effect of the tortuously factor on the durability of entrapped gas bubbles.
under 1D hydrostatic conditions. For this scenario, the column geometry used corresponds to \( Z=0 \) m and BC1 (see Figure 4.8).

Figure 4.10b illustrates a similar case with addition of a silt cap at the top. This is a scenario where a liquefiable sandy layer is capped with a fine grained (normally silty) material. Due to the differences between the tortuosity factor of sandy and silty material, presence of the top cap can influence the longevity of entrapped gas bubbles underneath. For this scenario a 0.5 m thick silt cap is added to the column geometry used in the previous scenario.

Figure 4.10c shows a case of layered soil system within which different layers have different tortuosity factors. This scenario can occur in alluvial sediments. For example, presence of silt lenses in between sandy material can alter the diffusion flux and ultimately the durability of entrapped gas bubbles. For this scenario, 1.5 cm thick silt lenses are placed in between 4.5 cm thick sand layers. The column length, depth and boundary conditions is the same as the first scenario.

The results of the numerical study on the effect of tortuosity factor and soil heterogeneity is shown in Figure 4.11. The tortuosity factor directly affects the molecular flux within the porous media, hence an important parameter in the context of gas durability under hydrostatic condition. Going from the tortuosity factor of 0.5 to 0.1
improves the durability by more than 100%. Low tortuosity factors can also play a significant role in layered material. Figure 4.11 also shows the relative effect of having a 0.5 m thick silt cap ($\tau = 0.1$) on top of a sandy 1D soil ($\tau = 0.4$) column. We can see that this silt cap slows down the diffusion process from the beginning (more linear increase in the degree of saturation) and after 100 years, the behavior of the capped sandy soil is almost equivalent to a homogeneous soil (no cap) with $\tau = 0.2$.

Another important observation was the relative effect of a thin 1.5 cm silt lenses ($\tau = 0.1$) at 4.5 cm vertical intervals within a sandy deposit ($\tau = 0.4$). Although the weighted average of the tortuosity factor in this case was 0.325, but due to the nonlinear nature of the diffusion problem, the influence of the silt lenses on the durability of entrapped gas in a sandy deposit is equivalent to having a homogeneous soil layer with $\tau = 0.2$. Basically, the presence of silt lenses can improve the gas longevity by 30% (for this specific configuration).

Figure 4.11: Effect of silt lenses, tortuously factor and silt cap on gas longevity under hydrostatic condition.
4.5 Groundwater Seepage Conditions

For comparison, the time required to saturate 1.5 m of sand under horizontal groundwater seepage conditions was considered (Figure 4.8c). Similarly, this is not meant to imply a soil layer treated via IPS would only extend 1.5 m laterally. Similar to diffusion, a practical approach to combat seepage-induced resaturation could be to incorporate a “sacraficial” thickness along the perimeter of a large gassy soil volume. It’s important to note that vertical diffusion is neglected for the horizontal seepage analyses, implying that either: a.) a cap or impermeable boundary has prevented gas escape; or b.) a vertical saturation front attributed to diffusion has not advanced to the depth under consideration for seepage analyses.

Figure 4.12 shows the evolution of the saturation front based on the pore volumes of groundwater passed through the 1.5 m horizontal sand column (Figure 4.8c) for a variety of subsurface conditions. Natural hydraulic gradients typically range between 0.001 and 0.2 (Cedergren et al., 1967; Sudicky, 1986; LeBlanc et al., 1991; Devlin & McElwee, 2007); for coarse-grained soils, \( v_s \) may be expected to range between 0.001 and 0.5 m/day. During this parametric study, \( v_s \) was varied between 0.01 and 0.3 m/day to test the relative influence of seepage velocity on inter-phase gas exchange (equation 4.16), which revealed that seepage velocity had a negligible influence on the computed pore volumes of fluid required to achieve the same position of the saturation front. Therefore, Figure 4.12 also indicates the temporal evolution of the saturation front based on two seepage velocities \( (v_s = 0.01 \text{ and } 0.3 \text{ m/day}). \)

Figure 4.12a illustrates the influence of depth and dissolution of gas as groundwater infiltrates the gassy layer. The computed results reemphasize the influence of depth (i.e. greater concentration deficits) on the persistence of gas under groundwater flow conditions, as the number of pore volumes needed to move the saturation front 1.5 m were 115, 22, and 15 pore volumes at \( Z = 0, 5, \text{ and } 10 \text{ m}, \) respectively. For \( v_s = 0.01 \text{ m/day} \) it would take 50, 9 and 5.5 years for the same corresponding depths and less than 2 years for all
depths when $v_s = 0.3$ m/day. This highlights the importance of understanding the hydrogeologic conditions associated with ground improved by IPS. For comparison, emplacing $O_2$ bubbles is also shown, which required 64 pore volumes to advance the saturation front 1.5 m at $Z = 0$ m, or 26 years and 1 year at $v_s = 0.01$ and 0.3 m/day.

Similar to hydrostatic conditions, the initial $r_b$ has a significant influence on the number of pore volumes required to advance the saturation front (Figure 4.12b). Though the initial $S_r$ is the same, the initial capillary pressure computed with smaller bubbles increases the equilibrium aqueous-phase concentration (capacity of groundwater to behave as a sink). The initial equilibrium-state bubble size(s) after emplacement ultimately depends on the soil’s grain-size distribution. However, with a reasonable selection of a representative initial $r_b$ for the poorly graded Ottawa sand, the saturation front was accurately predicted for the synthetic aquifer experiments performed by (McLeod et al., 2015). In more complex systems, particularly where soil’s exhibit a wider distribution of particle sizes, it may be necessary to simulate a distribution of bubble sizes, which was not considered in this study.

The examination of groundwater flow highlights its influence relative to hydrostatic conditions regarding the persistence of gas. However, it should not be concluded that hydrogeologic conditions imposing groundwater flow might necessarily preclude IPS from consideration as a ground improvement alternative. For instance the sacrificial thickness of a gassy layer could be increased, or cutoff walls could be employed around the perimeter of foundation soils to prevent groundwater outside the IPS zone from infiltrating—but may be less economical. Alternatively, a maintenance system, whereby gas is periodically introduced near the perimeter of the targeted foundation soil, could be instituted to increase the dissolved gas concentration of imbibing water, which would decrease the concentration deficit and amount of inter-phase gas transfer (equation 4.13 and Figure 4.2).

To further illustrate this concept, Figure 4.12c shows the evolution of the saturation front assuming a hypothetical maintenance system is capable of “saturating” groundwater with dissolved gas (i.e. $\eta_o = 100\%$ in equation 4.6). Therefore, it is only the relatively
Figure 4.12: Computed pore volumes and temporal evolution of the saturation front based on associated $v_s$ (see Figure 4.8c) with consideration of: a.) the influence of depth and gas type; b.) influence of the initial bubble radius; c.) influence of the initial water-gas saturation of infiltrating groundwater ($\eta_o = 100\%$).
small capillary pressure from the bubbles that drive inter-phase gas exchange and accompanying dissolution of the bubble as groundwater infiltrates the system. This condition increases the pore volumes of groundwater and the time required to advance the saturation front relative to the assumed condition where $C_{aq}$ is initially $C_{atm}^{aq}$ (i.e. comparison to Figure 4.12a). For depths of 0, 5, and 10 m, the predicted number of pore volumes required to saturate a length of 1.5 m increases 0%, 750%, and 1300% (50, 74, and 88 years for $v_s = 0.01$ m/day), respectively. Interestingly, the longevity of gas at depth is greater, which is not immediately intuitive. Although the same initial $S_r$ was assumed, the moles of gas required to achieve the same initial $S_r$ is greater at depth, thus dissolution driven solely by capillary pressures from the same initial bubble size takes longer. This numerical demonstration highlights the practical benefits of developing such a maintenance system, though higher groundwater seepage velocities may call for other methods to defend against dissolution (e.g. cutoff wall) or preclude IPS from hydrogeologic conditions where naturally high hydraulic gradients exist.

4.6 Gas Longevity Coefficient

The numerical study of gas longevity demonstrated that: i.) gas, particularly air, has the potential to persist for appreciable periods of time, even at depth; ii.) naturally occurring hydraulic gradients driving groundwater flow introduce a sink with the potential to drive a saturation (i.e. gas dissolution) front at the perimeter of an IPS zone more rapidly than hydrostatic conditions; and iii.) selection of any IPS method to generate or introduce gas (e.g. air vs. $O_2$) are important considerations. Figure 4.13 illustrates the conceptual progression of gas dissolution, where imbibing groundwater under horizontal seepage conditions would create a predominantly wedge-shaped saturation front (Figure 4.13a); a more homogeneous advancement (Figure 4.13b) due to diffusion-induced resaturation is expected under hydrostatic conditions.
Assessment of the durability of entrapped gas bubbles for an IPS application is a major design consideration and providing simple and practical assessment tools can help to promote the IPS method for liquefaction mitigation. In this context, the "Longevity Coefficient", a parameter for gas durability estimation will be developed based on the data provided by numerical parametric studies. Basically, the longevity of entrapped gas bubbles can be correlated to the rate of movement of the saturation front.

Under hydrostatic condition, the Longevity Coefficient will have units of meter/year while under a horizontal seepage condition, it has units of velocity/velocity. For the first case, the longevity coefficient will indicate the rate of movement of the saturation front. However, for the second case, the longevity coefficient conveys the ratio of the movement rate of saturation front to the seepage velocity.

Tables 4.2 and 4.3 provide a summary of the calculated longevity coefficients under hydrostatic and horizontal seepage conditions, respectively. As previously mentioned, the definition of the longevity coefficient is slightly different for hydrostatic vs. flow condition.
For hydrostatic condition, the gas longevity depends on gas-water properties (e.g. gas type and bubble size) and porous media properties (e.g. depth below water table, diffusion boundary conditions and tortuosity factor). The gas type encompasses the effects of gas solubility and its bulk diffusion in pore fluid. For the case of horizontal seepage (flow) condition, the longevity of gas depends mostly on gas-water properties (e.g. gas type and bubble size) and porous media properties (e.g. depth below water table and the initial concentration of the effluent pore fluid). Another important factor (which is not a basic characteristics of either gas or the porous media ) is the seepage velocity within the porous media.

Since under hydrostatic condition, the effective parameters are within a certain range and to some degree controllable, the longevity coefficient is calculated and reported in units of m/year. However, under flow situation in order to take the flow velocity out of the equation, the longevity coefficient is calculated unit-less.

4.6.1 Gas longevity coefficients for hydrostatic condition

The effect of different parameters on the gas longevity has been discussed in the previous section. Here, we'll discuss the ranges of the longevity coefficient and how it can be used for design and durability assessments. For all the different combinations of the factors considered including the depth below water table (Z), initial bubble radius ($r_b$), the tortuosity factor ($\tau$) and the boundary condition (BC), we see that the longevity coefficient for Air changes between 0.005 and 0.05 (m/year). Among the different factors and parameters we can see that the changes in the boundary condition and gas type has the largest impact on the longevity coefficient. Fortunately, these are the parameters that can be identified and/or controlled for an IPS application. The tortuously factor can also be measured or estimated to some degree. Now the only remaining parameters are the initial bubble radius and the depth below water table. For a gassy zone, the depth of gas entrapment and the initial bubble radius can vary from one point to another. Therefore a
Table 4.2: Longevity coefficient for hydrostatic condition condition.

<table>
<thead>
<tr>
<th>Z (m)</th>
<th>r_b(µm)</th>
<th>BC</th>
<th>τ</th>
<th>Longevity Coefficient (m/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Air</td>
</tr>
<tr>
<td>Effect of Z</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>1</td>
<td>0.4</td>
<td>0.011</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>1</td>
<td>0.4</td>
<td>0.013</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>1</td>
<td>0.4</td>
<td>0.015</td>
</tr>
<tr>
<td>Effect of BC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>2</td>
<td>0.4</td>
<td>0.019</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>2</td>
<td>0.4</td>
<td>0.037</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>2</td>
<td>0.4</td>
<td>0.050</td>
</tr>
<tr>
<td>Effect of r_b</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>15</td>
<td>1</td>
<td>0.4</td>
<td>0.017</td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>1</td>
<td>0.4</td>
<td>0.011</td>
</tr>
<tr>
<td>0</td>
<td>60</td>
<td>1</td>
<td>0.4</td>
<td>0.005</td>
</tr>
<tr>
<td>Effect of τ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>1</td>
<td>0.1</td>
<td>0.005</td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>1</td>
<td>0.2</td>
<td>0.007</td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>1</td>
<td>0.3</td>
<td>0.009</td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>1</td>
<td>0.4</td>
<td>0.010</td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>1</td>
<td>0.5</td>
<td>0.012</td>
</tr>
<tr>
<td>Effect of Silt Cap</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>1</td>
<td>0.4</td>
<td>0.007</td>
</tr>
<tr>
<td>Effect of Silt Lenses</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>1</td>
<td>0.1-0.4</td>
<td>0.007</td>
</tr>
</tbody>
</table>

A convenient approach could be using an average bubble radius with an average entrapment depth. Now we can simplify the longevity coefficient as follows:

- For cases where we have air as the entrapped gas in a sandy material (i.e. \( \tau = 0.4 \)) in which diffusion only takes place at the top boundary (i.e. BC1), the longevity coefficient of 0.13 (m/year) can be used.

- If the entrapped gas is Oxygen, the longevity coefficient is multiplied by 3.

- For cases where the diffusion can take place in two directions (i.e. BC2), the longevity coefficient should be multiplied by 2.5.

- If the soil deposit containing entrapped gas bubbles has silt lenses or a silt cap, the longevity coefficient should be multiplied by 0.5.
Table 4.3: Longevity coefficient for flow condition.

<table>
<thead>
<tr>
<th>$Z$ (m)</th>
<th>$r_b$ ($\mu$m)</th>
<th>$\eta$ (%)</th>
<th>Longevity Coefficient (-)</th>
<th>Air</th>
<th>Oxygen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect of $Z$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>100</td>
<td>0.008</td>
<td>0.016</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>68</td>
<td>0.045</td>
<td>0.077</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>50</td>
<td>0.067</td>
<td>0.125</td>
<td></td>
</tr>
<tr>
<td>Effect of $\eta = 100%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>100</td>
<td>0.008</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>100</td>
<td>0.006</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>100</td>
<td>0.005</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Effect of $r_b$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>15</td>
<td>100</td>
<td>0.056</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>100</td>
<td>0.008</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>60</td>
<td>100</td>
<td>0.004</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

4.6.2 Gas longevity coefficient for groundwater seepage condition

Similar to the case of hydrostatic condition, we can first look at the ranges of the longevity coefficients under flow condition. For different combinations of bubble radius ($r_b$), initial dissolved gas concentration in the effluent pore fluid (i.e. $\eta$) and depth below water table ($Z$), the longevity coefficient for air ranges between 0.004 and 0.067. We notice that compared to the hydrostatic condition, the relative effect of the depth on the longevity coefficient under flow condition is significant. This effect is responsible for a wedge-shaped advancement of the saturation front. We can simplify the longevity coefficient under flow condition as below:

- For cases where we have air as the entrapped gas in porous media, where the concentration of the effluent flow is in equilibrium with the atmosphere, the longevity of gas can be estimated using the longevity coefficient of 0.045.

- If the entrapped gas is Oxygen, the longevity coefficient is multiplied by 2.

- In cases where the effluent flow has $\eta = 100\%$, the longevity coefficient should be divided by 7.
4.7 Discussion

As discussed, adopting approaches to mitigate decay of a gassy layer—such as spatial extension of the gassy layer and incorporation of a sacrificial thickness, continued maintenance of water-gas saturation near the perimeter, use of cutoff walls, or combination thereof—are potential avenues to alleviate these concerns.

This study considered time-scales of interest for civil infrastructure, and the methodologies applied in this study may be used to improve long-term predictions of gas persistence. However, the numerical predictions considered idealized subsurface conditions and the modeling approach was validated against controlled, relatively small-scale, experiments. Hydrogeologic conditions in the field are more complex, and temporal fluctuations of the water table, changes in groundwater temperature and associated solubility, soil layering, homogeneity and initial distribution of bubbles, among other things, will be important to consider in future work. Also, it may be necessary to incorporate the buoyant mobility and redistribution of bubbles as they shrink (e.g. near the saturation front) using pore network models or similar (e.g. Zhao & Ioannidis, 2011; Mahabadi et al., 2018). Field-scale long-term monitoring programs with accompanying methods to track the saturation front are needed to demonstrate that reliable model predictions of gas longevity can be achieved—which is principal to the advancement of IPS and should be incorporated in future studies. However, this numerical study has provided greater context for the evolution of saturation fronts that may be anticipated and the methodologies applied can better inform future monitoring programs to assess gas longevity.

The longevity assessment approach presented in this study considers a 1D situation where the diffusion only takes place in a vertical direction (for hydrostatic condition) and the advection-diffusion only takes places in a horizontal direction (for flow condition). Obviously, more complex scenarios require more complex analysis.

The diffusion part of the advective-diffusional flux taking place during a flow condition is negligible, especially for the flow rates considered, hence the effect of tortuosity factor and
soil layering was not considered in the longevity coefficient under flow condition. For gas durability estimation of a gassy zone with the known initial thickness (e.g. L), depending on the gas and soil properties and the boundary conditions, the thickness of the gassy zone after T years will be $L^* = L - T \times LC$, where $L^*$ is the reduced gassy layer thickness and $LC$ is the longevity coefficient.

For a similar case but under a seepage condition with flow velocity $V$, after T years, the thickness of the gassy layer will be reduced to: $L^* = L - T \times V \times LC$.

Based on the numerical results shown above, while under flow condition the saturation front moves is a relatively constant rate, under hydrostatic condition the saturation front moves faster than average early on but once there is enough accumulation of dissolved gas in pore fluid surrounding the gassy zone, the diffusion approaches a steady state condition. Therefore, in short term, the proposed longevity coefficients might overestimate the gas durability.

4.8 Summary and Conclusions

Induced-partial-saturation is a novel method to suppress the generation of excess pore water pressure and increase the liquefaction resistance of loose granular materials. Methodologies to assess and demonstrate the persistence of entrapped gas, which is linked to the pore fluid compressibility and mechanical benefits associated with IPS, was the focus of this study. Temporal changes and progression of a saturation front were simulated with the governing advection-diffusion equation and consideration of inter-phase gas kinetics to simulate temporal changes in bubble radius and associated dissolved gas concentrations surrounding the bubble. The governing equation and accompanying finite difference scheme were validated with experimental results and then extended to larger time-scales relevant for civil infrastructure systems in a numerical parametric study.

Based on the results of this study:
• The model was capable of predicting independent experimental observations—which had varied spatial scales, pore fluid constituents, gas solubilities, and boundary conditions—under both hydrostatic and groundwater flow conditions.

• Under 1D hydrostatic conditions (i.e. infinitely wide gassy soil volume), the thickness of the gassy layer can decay due to a diffusion-induced resaturation front that can advance at both the top and bottom boundary. The methodologies presented in this study may be applied for a variety of gassy soil thicknesses, initial degrees of saturation, gas type, and bubble sizes. When a 1.5 m thick layer desaturated with air (initial \( r_b = 0.03 \text{ mm} \)) with an initial \( S_r = 82\% \) extended from the ground surface \((Z = 0 \text{ m})\) was considered, full saturation was predicted after 160 and 78 years for one- vs. two-way diffusion, respectively. When the top of the layer extended from \( Z = 5 \) and 10 m, resaturation times were 125 and 99 years for one-way diffusion, and decreased to 42 and 30 years for two-way diffusion. Thus, both the boundary condition and depth significantly influence gas longevity.

• Under horizontal flow conditions, a saturation front progressively advances downgradient due to imbibing groundwater that acts as a sink, resulting in dissolution of gas bubbles. The rate that the saturation front progresses depends on the pore volumes of groundwater introduced and the associated seepage velocity, as well as depth. At \( Z = 0, 5, \) and 10 m, the predicted saturation front progressed 1.5 m through a quasi-saturated layer (initial \( S_r = 82\% \) and \( r_b = 0.03 \text{ mm} \)) after 115, 22, and 15 pore volumes were introduced; this corresponded to 50, 9, and 5.5 years when \( v_s = 0.01 \text{ m/day} \). Thus, groundwater flow conditions have the potential to attack the perimeter of a quasi-saturated soil volume more rapidly, that rate of which is highly dependent on the local hydrogeologic conditions and naturally occurring hydraulic gradient that exists.
• As the initial bubble radius decreases, saturation rates increase under both hydrostatic and groundwater flow conditions. Therefore, characterization of a representative initial bubble size of entrapped gas is an important consideration.

• Similarly, when entrapped gas has a higher solubility ($O_2$ examined in this study), the rate of resaturation increase significantly for both hydrostatic and groundwater flow conditions. Therefore, IPS methods that generate or introduce low solubility gasses should be preferred; this has been recognized and reemphasized in this study.

• A relatively uniform advancement of the saturation front may be expected under hydrostatic conditions while a predominantly wedge-shaped saturation front may be anticipated where a natural hydraulic gradient driving groundwater flow exists.

Incorporating a sacrificial thickness to a targeted volume of loose soil is a potential avenue to increase the durability and extend the life of an IPS system. For groundwater flow conditions, it was demonstrated that a potential maintenance scheme, whereby gas is periodically introduced near the perimeter to saturate flowing groundwater with dissolved gas, significantly decreases the rate that the saturation front advances at the perimeter of a gassy soil volume. For a typical $v_s = 0.01 \text{ m/day}$, it was found that the saturation front would advance 1.5 m in 52, 74, and 88 years at depths $z = 0, 5,$ and $10 \text{ m}$, respectively. This may be a practical solution to increase the durability of an IPS system.

• Offering a simple and practical tool for gas durability assessment can be useful and help promote the IPS method. For this purpose, the Gas Longevity Coefficient was introduced based on the results of the numerical studies conducted. Under hydrostatic condition, the Longevity Coefficient will have units of meter/year while under a horizontal seepage condition, it has units of velocity/velocity.

• This study offers a range for the Gas Longevity Coefficient under both Hydrostatic and Flow conditions. Having the major parameters known, such as gas type, average
bubble size and the average depth of entrapment, the gas longevity coefficient can be estimated and used for a simple and practical assessment of the gas durability under the given conditions.
CHAPTER 5
INFLUENCE OF GAS KINETICS ON LIQUEFACTION TRIGGERING DURING TSUNAMI LOADING

5.1 Introduction

Tsunamis are an extreme coastal hazard that cause catastrophic damage and disruption in the nearshore environment. In addition to inundation and flooding, tsunamis enhance sediment mobility attributed to the formation of deep-seated scour features and erosion in the soil bed. Takahashi et al. (1995) reported that the 1960 Chilean tsunami resulted in more than 8 m of erosion at the Kesennuma Port. Deep-seated scour and erosion as great as 4 m contributed to the failure of a breakwater during the 1993 Okushiri tsunami in Japan (Kimura et al., 1997; Yeh & Mason, 2014). Erosion, which is attributed to soil bed shear stresses imposed during inundation and recession of the tsunami wave, can be exacerbated by the development of hydraulic gradients in the sediment at depth (Yeh & Mason, 2014). Recognition of the importance of groundwater flow and prediction of the pore water pressure response instigated by tsunami loading has lead to more robust models that account for both i.) seepage arising from the changing boundary pore water pressure at the soil bed surface and ii.) pore fluid pressurization linked to deformation of the soil skeleton under the changing weight of the wave (Young et al., 2009; Abdollahi & Mason, 2019, 2020).

When soil bed deformations are considered, differential pressurization of pore water that instigates groundwater flow, liquefaction triggering and enhanced scour during tsunami loading is intimately linked to the pore fluid compressibility (Mahmoodi et al., 2019; Abdollahi & Mason, 2020). Air entrainment near the phreatic surface (Heaton & Vogel, 1980; Faybishenko, 1995; Holocher et al., 2002), which fluctuates continuously in nearshore sandy sediments due to the tides, increases the bulk compressibility of pore fluid relative to fully saturated sediments (Skempton, 1954). Previous numerical studies
considering tsunami loadings have accounted for gas entrapment by selecting a pore fluid compressibility greater than that of water (Young et al., 2009; Abdollahi & Mason, 2019), but implicitly assumed uniform entrainment of air irrespective of depth. This has also been assumed in other studies examining the pore water pressure response for non-solitary waves (e.g. Yamamoto et al., 1978; Okusa, 1985; Tsai, 1995). However, gas entrapment is likely isolated to nearshore sediments in the swash zone (Turner, 1993; Baldock et al., 2001; Horn, 2002; Steenhauer et al., 2011) where oscillating tide levels desaturate and resaturate sediments, a requisite condition for air entrainment. Thus, a multi-layered system likely exists, whereby a quasi-saturated soil bed overlies fully-saturated sediments below the low-tide elevation. Aside from the assumed distribution of gas, previous studies also implicitly assume gas content is constant throughout tsunami loading. Tsunami waves can be in excess of 10 m (Kundu, 2007; Bryant, 2014), imposing large fluid pressures in the sediment that will contract, and possibly dissolve, air bubbles as the wave height increases. Thus, pore fluid compressibility changes throughout this dynamic loading process.

Motivated by uncertainties associated with the aforementioned assumptions regarding the distribution of gas and the dynamic pore fluid compressibility, the objective of this study is to examine a.) the influence of gas distribution and b.) the kinetics of entrapped bubbles on the pore water pressure response. More specifically, the kinetics of entrapped bubbles in a quasi-saturated soil layer is incorporated in a coupled seepage-deformation finite difference framework to more faithfully address the role of pore fluid hardening arising from compression and dissolution of gas. Both the initial thickness of a quasi-saturated layer and the initial gas content are considered to understand the susceptibility of nearshore sandy sediments to momentary liquefaction and seepage gradients linked to enhanced scour.
5.2 Background

Tsunamis are long period waves (hundreds of meters) that impose relatively uniform, though dynamic, bed loads and total stress to the sediment. A unique feature of tsunami loading is that fluid pressure in the sediment is governed by both i.) seepage and ii.) the soil skeleton’s tendency to contract (or expand) under the changing weight of the wave, which is referred to herein as mechanical generation of excess pore water pressure. With respect to the second consideration, the duration of tsunami loading is on the order of minutes and likely invokes a partially undrained response in sand beds (Abdollahi & Mason, 2019); i.e. mechanical generation of pore pressure as the porous soil skeleton deforms under the weight of the wave. Therefore, both considerations are necessary to adequately describe changes in pore water pressures associated with destabilizing seepage mechanisms.

Mass conservation of pore fluid in a deformable porous material may be described using Biot (1941) formulation for a 1D poroelastic material (Verruijt, 1969):

\[ \alpha \frac{\partial \varepsilon}{\partial t} + S \frac{\partial P_e}{\partial t} = k_h \frac{\partial^2 P_e}{\partial z^2} \]  

(5.1)

where \( \varepsilon \) is volumetric strain, \( \alpha \) is Biot’s coefficient, \( S \) is storativity, \( P_e \) is excess pore water pressure, \( k_h \) is hydraulic conductivity, \( \gamma_w \) is the unit weight of water, \( z \) is depth, and \( t \) is time. Biot’s coefficient is defined as:

\[ \alpha = 1 - \frac{\beta_s}{\beta_m} \]  

(5.2)

where \( \beta_s \) and \( \beta_m \) are the compressibility of the soil particles and porous medium (soil skeleton) under changes in effective stress, respectively; soil particles are assumed incompressible and \( \alpha = 1 \) under practical stress levels, which is applicable to tsunami loading. Storativity is defined as:

\[ S = n\beta + (\alpha - n)\beta_s \]  

(5.3)
where \( n \) is porosity and \( \beta \) is the compressibility of the pore fluid (focus of this study). From Terzaghi’s one-dimensional consolidation theory, volumetric strain of the soil skeleton is:

\[
\frac{\partial \varepsilon}{\partial t} = -m_v \frac{\partial \sigma'_z}{\partial t} = -m_v \left( \frac{\partial \sigma_z}{\partial t} - \alpha \frac{\partial P_e}{\partial t} \right) \tag{5.4}
\]

where \( m_v \) is compressibility of the soil skeleton, and \( \sigma_z \) and \( \sigma'_z \) are the total and effective vertical stress, respectively. Confined compressibility of the soil skeleton is computed as:

\[
m_v = \frac{1 - 2\nu}{2(1 - \nu)G} \tag{5.5}
\]

where \( \nu \) is Poisson’s ratio and \( G \) is the shear modulus. Substituting equation 5.4 into equation 5.1 and rearranging the terms yields:

\[
\frac{\partial P_e}{\partial t} = \frac{\alpha m_v}{S + \alpha^2 m_v} \frac{\partial \sigma_z}{\partial t} + \frac{k_h}{\gamma'_w(S + \alpha^2 m_v)} \frac{\partial^2 P_e}{\partial z^2} \tag{5.6}
\]

where the first term on the right hand side describes the mechanical generation of excess pore water pressure and \( \partial \sigma_z / \partial t \) is the changing total stress imposed by the weight of a tsunami wave. The second term in equation 5.6 accounts for temporal changes in excess pore water pressure due to seepage. The effective stress in the sediment can be computed using two different approach but with the same outcome. In the first method the generated excess pore pressure gradients are used to computed the effective stress at each depth \( z \) below seabed as shown below:

\[
\sigma'_z = \int_{-z}^{0} (\gamma' - i\gamma_w) \, dz \tag{5.7}
\]

where \( \gamma' \) is the effective unit weight of soil, \( \gamma_w \) is the unit weight of water, and \( i \) is the generated excess pressure gradient (variable with depth and positive for upward flow).

The second approach is Using Terzaghi’s effective stress definitions where the changes in effective stress is simply defined at the difference between the change in total applied stress and the generated excess pore pressure. According to this method at depth \( z \) below seabed the effective stress can be computed based on the weight of a tsunami wave and the generated excess pore pressure as shown below:
\[ \sigma'_z = \sigma'_{zo} + d\sigma_z - P_e = \gamma'z + \gamma_w h - P_e \]  

(5.8)

where \( \sigma'_{zo} \) is the initial effective stress, and \( h \) is the height of the wave. It can mathematically shown that using either of equations 5.7 or 5.8 results in the same effective stress values during a tsunami loading. Based on equation 5.7, if there is a sustained large excess pore pressure gradients below seabed, sediments are prone to full or partial liquefaction. Similarly, according to equation 5.8 if the magnitude of the excess pore pressure at any point in time during tsunami loading is larger than total stress imposed by the weight of the wave, sediments may experience full or partial liquefaction.

In fully-saturated sediments, the pore fluid (water) is nearly incompressible \((\beta \approx 0)\) such that \( S \approx 0 \) and \((\alpha m_v)/(S + \alpha^2 m_v)\) in the first term on the right hand side of equation 5.6 is 1. Thus, it may be recognized that any change in excess pore water pressure corresponds to changes in total stress imposed by the weight of the wave \((\partial P_e/\partial t = \partial \sigma_z/\partial t); \) the final term in equation 5.6 is zero to satisfy mass conservation of the pore fluid (i.e. no seepage). By inspection, this also implies no change in effective stress (equations 5.7 and 5.8) or deformation of the soil skeleton (equation 5.4).

However, when entrained air (even small amounts) constitute a portion of the pore fluid, there is a significant departure from the preceding assumption that the pore fluid is incompressible (Fredlund, 1976), where \( \beta \neq 0 \) and \( S \neq 0 \). Thus, in quasi-saturated sediments with entrained air \( \partial P_e/\partial t \neq \partial \sigma_z/\partial t \), following well-established observations that entrapped gas suppresses the mechanical generation of excess pore water pressure in globally undrained soils with a tendency to contract (e.g. Skempton, 1954; Sherif et al., 1977; Yoshimi et al., 1989; Grozic et al., 1999, 2000; Tsukamoto et al., 2002; Okamura & Soga, 2006; Yegian et al., 2007; Fredlund et al., 2012; He et al., 2013; Kato & Nagao, 2020, among others). When mechanical generation of excess pore pressure in the quasi-saturated layer does not correspond to the changes in total stress, a pressure head differential arises at the surface, initiating seepage.
The preceding discussion emphasizes the necessity of air entrainment to motivate groundwater flow, though the depth where destabilizing gradients develop cannot be fully appreciated without explicitly considering where air is initially entrapped (i.e. in the intertidal zone). Differential pressurization of pore fluid near the interface of saturated and quasi-saturated sediments will also arise from differences in pore fluid compressibility, also instigating seepage.

Figure 5.1 conceptually illustrates the pore water pressure response and groundwater seepage anticipated in a multi-layered system (quasi-saturated layer overlying fully-saturated sediment) at two different stages of tsunami loading—runup and drawdown. During tsunami runup (Figure 5.1a), the differential pressure head at the surface, linked to the dampened mechanical generation of excess pore pressure in the quasi-saturated layer and increasing height of the wave, causes infiltration. However, mechanically generated excess pore water pressures in the underlying fully-saturated layer (due to the weight of the wave) enforces upward seepage near the interface of the two-layered system. As the wave height increases during runup, pressurization of the quasi-saturated bed arises, to a large extent, from groundwater seepage.

Figure 5.1: Conceptual illustration of excess pore pressures and groundwater flow in a soil column under tsunami loading at two instances in time: a.) tsunami runup; b.) tsunami drawdown.
Similarly, mechanical dissipation of excess pore pressure in the quasi-saturated layer does not correspond to the diminishing weight of the wave during drawdown (assuming entrained air still exists). Dissipation is governed again, in part, by seepage-induced diffusion of excess pore water pressure. The pressure head at the ground surface, and changes in mechanically-induced excess pore water pressure in the saturated layer corresponding to changes in total stress, diminish more rapidly as the wave height decreases (than depressurization of the quasi-saturated layer). Thus, seepage is reversed (Figure 5.1b) during drawdown. Depending on the direction of flow, seepage forces may have a stabilizing (during runup) or destabilizing (during drawdown) influence on the sediment (i.e. increase or decrease effective stress) throughout different stages of tsunami loading. During drawdown, when upward seepage is anticipated, body forces applied to the soil skeleton reduce vertical effective stress, which can enhance scour (Tonkin et al., 2003) or cause momentary liquefaction (Abdollahi & Mason, 2019; Mahmoodi et al., 2019; Abdollahi & Mason, 2020).

5.3 Gas Kinetics

Biot (1941) defined the pore fluid compressibility as:

$$\beta = S_r \beta_w + (1 - S_r) \beta_a$$  \hspace{1cm} (5.9)

where $\beta_w$ is the bulk compressibility of water and $\beta_a$ is the compressibility of entrapped air. In quasi-saturated granular sediments with relatively high degrees of saturation ($S_r > 80 - 85\%$), gas exists as occluded bubbles (Fredlund et al., 2012). The gas compressibility is defined as:

$$\beta_a = \frac{1}{P^g}$$  \hspace{1cm} (5.10)

where $P^g$ is the absolute gas pressure:

$$P^g = P_{atm} + P_w + P_c$$  \hspace{1cm} (5.11)
where $P_{atm}$ is atmospheric pressure, $P_w$ is the total pore water pressure, and $P_c = (2\sigma_t^w/r_b)$ is capillary bubble pressure ($\sigma_t^w$ is the unit interface surface tension of water).

Figure 5.2 illustrates the evolution of an entrapped bubble volume due to changes (in this case increases) in excess pore water pressure. As $P_e$ increases, gas bubbles are compressed and the gas volume diminishes according to Boyle’s law (Figure 5.2a). Additionally, when gas is not assumed to be immiscible with the pore water, the volume of entrapped gas further decreases due to inter-phase gas exchange and dissolution in accordance with Henry’s law (Figure 5.2b). Thus, it may be recognized that the pore fluid compressibility is dynamic and likely hardens throughout tsunami loading. Previous studies applying a constant pore fluid compressibility implicitly assume the degree of saturation remains constant.

![Figure 5.2: Changes to the bubble size due to: a.) Changes in pore pressure; b.) gas dissolution and transport.](image)

The initial volume of gas, $V_{go}$, for an elemental soil volume, $V$, containing spherical bubbles is:

$$V_{go} = \frac{4}{3} \pi \sum_{n=1}^{n_b} r_b^3$$

(5.12)

where $n_b$ and $r_b$ are the number and radius of entrained bubbles. According to Boyle’s law, the change in gas volume corresponding to changes in gas pressure, $\Delta V_g^I$, is:

$$\Delta V_g^I = \Delta P_g^a \beta_a V_{go} = \frac{\Delta P_g}{P_g} V_{go}$$

(5.13)
where $V_{g0}$ is the initial volume of gas and $\Delta P^g = P_e + dP_c$. Withstanding the assumption that gas is immiscible with pore water, a kinetic bubble dissolution (KBD) model, which has been considered for environmental applications (Holocher et al., 2002) and to address the persistence of entrapped gas for induced-partial-saturation and liquefaction mitigation (Mahmoodi & Gallant, 2020a), was adopted to simulate pore fluid hardening arising from gas dissolution. The diffusive flux and rate of inter-phase gas exchange at the bubble interface for a gas, $i$, is governed in part by a “concentration deficit” of dissolved gas in the pore water:

$$J_{g,i} = D_i \left( \frac{1}{r_b} + \sqrt{\frac{\nu_s}{\pi D_i 2r_b}} \right) (C_{aq,i} - C_{aq,i,eq})$$  \hspace{1cm} (5.14)

where $D_i$ is the aqueous-phase (dissolved) diffusion coefficient of a gas, $\nu_s$ is the interstitial seepage velocity, $r_b$ is the bubble radius for an assumed spherical bubble, $C_{aq,i}$ is the dissolved concentration of gas, and $C_{aq,i,eq}$ is the equilibrium dissolved concentration according to Henry’s law:

$$C_{i,eq} = \frac{P^g_i}{H_i}$$  \hspace{1cm} (5.15)

where $H_i$ is the solubility coefficient and $P^g_i$ is the partial gas pressure in a bubble:

$$P^g_i = \frac{m^g_i}{m^g} P^g$$  \hspace{1cm} (5.16)

where $m^g_i/m^g$ is the mole fraction of gas $i$ and $m^g$ is the total moles of gas in an entrapped bubble. Following, the inter-phase exchange of gas is:

$$dm_i = -(A_{sb} J_{g,i}) dt = -4\pi r_b^2 D_i \left( \frac{1}{r_b} + \sqrt{\frac{\nu_s}{\pi D_i 2r_b}} \right) (C_{aq,i} - \frac{P^g_i}{H_i}) dt$$  \hspace{1cm} (5.17)

where $A_{sb}$ is the surface area of an assumed spherical bubble. The change in bubble radius linked to inter-phase gas exchange is:

$$dr_b = \frac{3RT}{4\pi r_b} \left( \frac{1}{4\sigma_i^w + 3r_b(P_{atm} + P_w)} \right) dm$$  \hspace{1cm} (5.18)
where \( dm \) is the total change in moles due to inter-phase exchange of all gas species (\( \Sigma dm_i \)) and the new bubble radius, \( r_b^* \), is:

\[
r_b^* = r_b + dr_b
\] (5.19)

The corresponding change in gas volume due to inter-phase gas exchange, \( \Delta V_{g}^{II} \) is:

\[
\Delta V_{g}^{II} = V_{go} \left[ \left( \frac{r_b^*}{r_b} \right)^3 - 1 \right]
\] (5.20)

The total change in gas volume arising from changes in pore fluid pressure:

\[
\Delta V_g = \Delta V_g^I + \Delta V_g^{II}
\] (5.21)

The new degree of saturation, \( S_r^* \) may be computed as:

\[
S_r^* = 1 - \frac{V_{go} + \Delta V_g}{nV}
\] (5.22)

Generation of excess pore water pressure in the system instigates seepage. Therefore, it is appropriate to account for changes in dissolved groundwater gas concentrations that may influence the aqueous-phase concentration deficit influencing the dissolution (or exsolution) of gas. For an assumed homogeneous bubble size in an element of soil, this may be accomplished by considering the mass conservation of gas using the readily applied advection-diffusion equation:

\[
\frac{\partial C_{aq}^i}{\partial t} = -v_s \frac{\partial C_{aq}^i}{\partial z} + D_{i} \tau \frac{\partial^2 C_{aq}^i}{\partial z^2} + \frac{3(1 - S_r)}{r_b S_r} J_{g,i}
\] (5.23)

where \( \tau \) is the tortuosity factor and the seepage velocity is:

\[
v_s = \frac{k_h i_h}{n S_r} = \frac{k_h}{n S_r \gamma_w} \frac{1}{\partial z} \frac{\partial P_e}{\partial z}
\] (5.24)

The first term in equation 5.23 accounts for the aqueous-phase transport of dissolved gas due to seepage and the third term (source term) considers the inter-phase gas exchange of entrained bubbles. The second term considers the diffusion of gas arising from dissolved
gas concentration gradients, which will have a negligible influence on the dissolution of gas on time-scales relevant to tsunami loading (e.g. Mahmoodi & Gallant, 2020a,b), but included here for completeness.

5.4 Model Formulation

The partial differential equation governing generation of excess pore water pressure and deformation in the soil skeleton (equation 5.6) was solved in MATLAB R2017a using the second-order central finite difference method and the Crank-Nicholson scheme for unconditional stability (Crank & Nicolson, 1947; LeVeque, 2007). The discretized form of equation 5.6 used in the finite difference formulation is shown below:

\[
\frac{k_h}{\alpha \gamma_w} \left( \frac{P_e}{\alpha + \alpha m_v} \right) \frac{(P_e)_{i+1}^{n+1} - (P_e)_i^n}{\Delta t} = \\
\frac{(P_e)_{i+1}^{n+1} - 2(P_e)_i^{n+1} + (P_e)_{i-1}^{n+1}}{2\Delta z^2} + m_v \frac{\partial \sigma_z}{\partial t} \tag{5.25}
\]

For all cases in this study, \( \Delta t = 0.1 \text{ s} \) and \( \Delta z = 0.02 \text{ m} \) is considered.

Figure 5.3 illustrates the algorithm that incorporates gas kinetics to update pore fluid compressibility during a simulation. After each time step, \( n \), the bubble radius is used to compute the initial gas volume, degree of saturation, and corresponding pore fluid compressibility (equation 5.9). The governing poroelasticity equation is first solved to compute changes in excess pore water pressure and change in gas volume corresponding to Boyle’s law (i.e. \( \Delta V_g^I \)); a new bubble radius, \( r_b^I \), is also computed. Following, gas mass conservation (equation 5.23) is considered to compute inter-phase gas transfer (and \( \Delta V_g^{II} \)) using an “average” bubble size over the time-step (see Figure 5.3). The final gas volume and corresponding bubble size is then computed (i.e. equation 5.21). A sub-stepping routine is then initiated and the governing poroelasticity equation is solved using a pore fluid compressibility computed with an \( S_r \) corresponding to \( r_b = 0.5(r_b^n + r_b^{n+1}) \) until the computed excess pore water pressure converges and a tolerable error is achieved.
Figure 5.3: Algorithm incorporating gas kinetics to update pore fluid compressibility when solving the governing poroelasticity equations

### 5.5 Geometry and Conditions for Numerical Study

The pore water pressure response and liquefaction triggering were studied numerically under different assumed initial conditions to assess the influence of the thickness of a layer where entrained gas exists, different initial degree of saturation, and assumption regarding gas kinetics. Figure 5.4a illustrates the assumed soil conditions, which includes a 10 m thick column of sand underlain by an impermeable layer (e.g. rock or clay). Within the sand layer different assumed thicknesses where gas is entrained are considered ($Z_g = 1$ m, 2 m, 4 m, 10 m). To elucidate the role of gas kinetics on the pore water pressure response and liquefaction triggering, three pore fluid compressibility assumptions are compared:
• The first assumption is referred to herein as "Constant Compressibility," where the pore fluid compressibility remains constant, an assumption considered in previous studies assessing response of the seabed under tsunami loading (e.g. Young et al., 2009; Abdollahi & Mason, 2019, 2020).

• The second assumption is referred to as "Compression Only," where the pore fluid compressibility is influenced by compression/expansion of the gas only (i.e. Boyle’s Law and $\Delta V_g = \Delta V_g^I$).

• The third assumption is referred to as "Compression Plus KBD" where both compression/expansion and inter-phase gas transfer are considered (i.e. $\Delta V_g = \Delta V_g^I + \Delta V_g^{II}$).

Six initial degree of saturation in the quasi-saturated gassy layer investigated (85%, 90%, 95%, 97%, 98%, and 99%).

Three different idealized tsunami profiles are also considered (Figure 5.4b). The tsunami profiles are derived from the shallow-water-wave equations proposed by Carrier et al. (2003) for a plane beach geometry. Using the non-dimensional parameters, Carrier et al. (2003) derived nonlinear shallow water wave equations as:

$$\frac{\partial \hat{\eta}}{\partial \hat{t}} + \frac{\partial}{\partial \hat{x}} [\hat{\nu} (\hat{x} + \hat{\eta})] = 0$$  \hspace{1cm} (5.26)

$$\frac{\partial \hat{\nu}}{\partial \hat{t}} + \hat{\nu} \frac{\partial \hat{\nu}}{\partial \hat{x}} + \hat{\eta} \frac{\partial \hat{\eta}}{\partial \hat{x}} = 0$$  \hspace{1cm} (5.27)

where, $\hat{\nu} = \frac{\nu}{\sqrt{g \psi L}}$, $\hat{\eta} = \frac{\eta}{\psi L}$, $\hat{x} = \frac{x}{L}$, and $\hat{t} = t \sqrt{\frac{g \psi}{L}}$ are the non-dimensional parameters. $\nu$ is the depth averaged horizontal velocity, $\eta$ is the water surface departure from its quiescent position, $\psi$ is the plane beach slope, and $L$ is the length scale which can be assumed as the distance from shoreline to the middle of the initial wave condition (Abdollahi, 2017).

The tsunami profiles assume slopes ranging from 1/100 to 3/100 for length scale of $L=40$ km.
Loss of effective stress and liquefaction can affect the post liquefaction generation of excess pore pressures as the liquefied soil is highly deformable and can undergo large volumetric strains. This condition is reflected in the numerical simulations via assignment of residual shear modulus (e.g. 50 kPa) to liquefied layers. The shear modulus recovers once the excess positive pore pressures dissipate and effective stress levels increase.

The model assumptions regarding sediments saturated and quasi-saturated hydraulic conductivity, nonlinear shear modulus and entrapped gas bubble properties are summarized in Table 5.1.

![Figure 5.4: Numerical model inputs definitions: a.) soil column geometry; b.) tsunami profiles.](image)

### 5.6 Numerical Results

In this section the results of numerical studies on the response of sand beds to tsunami loading are presented. The numerical experiments are designed to investigate and
Table 5.1: Assumed modelling parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative permeability</td>
<td>$K_r$</td>
<td>$[\frac{\theta_s - \theta_r}{\theta_s - \theta_r}]^n$</td>
<td>Faybishenko (1995)</td>
</tr>
<tr>
<td>Shear modulus</td>
<td>$G$</td>
<td>$A'(\sigma_r')^{n'}$</td>
<td>Lu &amp; Kaya (2014)</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>$\nu$</td>
<td>0.25</td>
<td>-</td>
</tr>
<tr>
<td>Soil initial porosity</td>
<td>$\eta$</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>Reference stress</td>
<td>$\sigma_r$</td>
<td>1 atm</td>
<td>-</td>
</tr>
<tr>
<td>Fitting parameter</td>
<td>$n$</td>
<td>1</td>
<td>McLeod et al. (2015)</td>
</tr>
<tr>
<td>Fitting parameter</td>
<td>$n'$</td>
<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td>Fitting parameter</td>
<td>$A'$</td>
<td>500</td>
<td>-</td>
</tr>
<tr>
<td>Gravitational acceleration</td>
<td>$g$</td>
<td>9.806 m/s$^2$</td>
<td>-</td>
</tr>
<tr>
<td>Saturated hydraulic conductivity</td>
<td>$K_s$</td>
<td>1e-5 m/s</td>
<td>-</td>
</tr>
<tr>
<td>Saturated water content</td>
<td>$\theta_s$</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>Residual water content</td>
<td>$\theta_r$</td>
<td>0.2</td>
<td>McLeod et al. (2015)</td>
</tr>
<tr>
<td>Initial bubble radius</td>
<td>$r_{b0}$</td>
<td>30 µm</td>
<td>Mahmoodi &amp; Gallant (2020a)</td>
</tr>
<tr>
<td>Tortuosity factor for sandy material</td>
<td>$\tau$</td>
<td>0.4</td>
<td>Mahmoodi &amp; Gallant (2020b)</td>
</tr>
</tbody>
</table>

1. In water the surface tension is $\sigma^{w}_t = 72 \times 10^{-6}$ kN/m. The entrapped gas assumed to be air, consisting of 80% $N_2$ and 20% $O_2$. The solubility of $N_2$ is $H_{N_2} = 1508$ atm $\times m^3/mol$ and $O_2$ is $H_{O_2} = 769$ atm $\times m^3/mol$. The diffusion coefficient of $N_2$ and $O_2$ are $D_{N_2} = 2 \times 10^{-9}$ m$^2$/s and $D_{O_2} = 2.9 \times 10^{-9}$ m$^2$/s.

demonstrate: i.) the interaction between a quasi-saturated and saturated sand bed under tsunami loading; ii.) the influence of the pore fluid compressibility assumption on sustained momentary liquefaction; and iii.) the influence of the assumed initial gas content (degree of saturation) and tsunami wave-height time series.

5.6.1 Interaction between a quasi-saturated and saturated sand bed

The stability of sediments under tsunami loading requires a complete understanding of the temporal and spatial generation of excess pore water pressure in the seabed. Differential generation of excess pore pressure within the soil column results in hydraulic gradients linked to groundwater seepage. The accompanying increase or decrease of effective stress in the sediment depends on the seepage direction. To first demonstrate the physics associated with the interaction between a quasi-saturated and saturated sand bed, a soil column with $Z_g = 2$ m for the tsunami 2 wave-height time series is first considered.
A value of $S_r = 97\%$ and constant pore fluid compressibility assumption were adopted—which were considered in previous numerical studies investigating the response of the seabed to tsunami loading, but air was assumed to be entrained throughout the entire soil column (Young et al., 2009; Abdollahi & Mason, 2019, 2020).

Sediment instability and enhanced scour are exacerbated when positive hydraulic gradients generate upward flow and decrease effective stresses in the sediment. Figure 5.5 shows the computed excess pore water pressure (Figure 5.5a), excess pore water pressure gradient (Figure 5.5b), and normalized effective stress, $\sigma'_v/\sigma'_vo$ (Figure 5.5c). Temporal changes at depth within the gassy layer are attributed to mechanically- and seepage-induced generation of excess pore water pressure under the changing total stress and boundary pore water pressure at the seabed surface. The excess pore water pressure at the top of the soil column corresponds to height of the wave (i.e. $P_e = \gamma_wh$). During runup of the tsunami wave ($t < 17.5$ minutes), the excess pore water pressure generally decreases with depth within the surfacial layer containing entrained air ($z < 2$ m) due to the dampened mechanical generation of excess pore water pressure associated with a higher pore fluid compressibility.

However, excess pore water pressure begins to increase near the interface of the quasi-saturated and fully-saturated layer at $z = 2$ m. This response is associated with mechanically generated excess pore water pressure in the fully-saturated layer, which is not suppressed due to the presence of entrained gas, but upward vertical diffusion of excess pore pore water pressure—which, in addition to seepage from the seabed surface, contributes to pressurization of the quasi-saturated sand layer. As illustrated in Figure 5.5b, this results in a positive excess pore water pressure gradient, indicating upward seepage, near the interface of the fully- and quasi-saturated layers at $z = 2$ m. A negative hydraulic gradient is observed at the seabed surface during runup, indicating downward vertical seepage. Thus, groundwater infiltrates from both the top and bottom of the quasi-saturated layer during runup. However, the weight of the wave has a stabilizing
effect and effective stress increases throughout the entire soil column during runup, irrespective of the seepage direction (see Figure 5.5c).

![Figure 5.5: Contours of the response of a soil column with $Z_g = 2m$ and $Z_s = 8m$ with $S_r = 97\%$ under Tsunami 2: a.) excess pore pressure; b.) excess pore pressure gradient; c.) effective stress ratio.](image)

During drawdown ($17.5 < t < 22$ minutes) total stresses and boundary pore water pressures imposed at the surface begin to decrease. As the wave height recedes, excess pore water pressure does not dissipate as quickly as the wave height diminishes, and seepage within the gassy layer is reversed after the wave has receded ($t > 22$ minutes). The
development of destabilizing hydraulic gradients is a function of how quickly the wave height recedes, thickness of the quasi-saturated layer, and hydraulic conductivity in the sand bed. Destabilizing positive excess pore pressure gradients (Figure 5.5b) decreases effective stress near the surficial layer (Figure 5.5c) can result in enhanced scour and erosion (e.g. Tonkin et al., 2003; Yeh & Mason, 2014) and trigger momentary liquefaction (i.e. development of a critical gradient and loss of effective stress).

5.6.2 Effect of Pore Fluid Compressibility Assumption and Thickness of Sediment Containing Entrained Air

For the same tsunami wave-height time series (tsunami 2) and initial degree of saturation ($S_r = 97\%$), the different pore fluid compressibility assumptions are tested to evaluate the influence of gas durability on liquefaction triggering. The thickness of sediment where entrained air is assumed to exist is also tested to evaluate its influence on the maximum depth of momentary liquefaction.

Figures 5.6-5.8 illustrate the evolution of the depth where momentary liquefaction triggering occurs (primarily towards the end and after tsunami drawdown). Figure 5.6 shows how the maximum depth of instability changes for different values of $Z_g$ when the pore fluid compressibility remains constant throughout the tsunami loading. Recall that this assumption, along with the chosen initial degree of saturation, is the same as previous studies (e.g. Young et al., 2009; Abdollahi & Mason, 2019). For reference, it is also noted that $Z_g = 10$ m is equivalent to the assumption by Abdollahi & Mason (2019). As shown in Figure 5.6a, the maximum depth of full liquefaction approaches the depth where entrained air exists for $Z_g = 1$ and 2 m, and continues to expand up until $Z_g = 4$ m. Thus, increasing the thickness of the gassy layer pushes the flow reversal point (see Figure 5.1) deeper. However, for $Z_g = 10$ m, the maximum depth of liquefaction does not continue to increase, even decreasing marginally, but is similar to $Z_g = 4$ m.
The maximum depth of liquefaction for each case are illustrated in Figure 5.6b. The limits of momentary liquefaction expand for a short duration after the wave has receded (≈1-2 minutes). Thereafter, diffusion of excess pore water pressures are linked to temporal changes (decreases) in the thickness of liquefied sediment, not mechanical dissipation due to unloading of the wave. For the $Z_g = 10$ m scenario all sediment is pressurized to a large extent by infiltration during runup, as excess pore water pressures are dampened by the gas. Stated another way, shallower quasi-saturated sediments are not pressurized by seepage and infiltration from an underlying fully-saturated layer where mechanically induced excess pore water pressures are greater—which during drawdown leads to greater expansion of the depth where momentary liquefaction is triggered. This further highlights the interaction of the quasi-saturated and fully-saturated sand bed.

In Figures 5.7 and 5.8 temporal changes and the maximum depth of momentary liquefaction are illustrated for the “compression only” and “compression plus KBD” assumptions, respectively. In both cases similar trends are observed for each assumption. Though the maximum depth of momentary liquefaction is similar, the time that liquefaction is sustained at the ground surface is significantly reduced in both instances relative to the constant compressibility assumption. This difference in the computed response arises from the influence of gas kinetics and pore fluid hardening throughout the tsunami loading process.

To facilitate a better understanding of gas kinetics on the pore water pressure response, Figure 5.9 compares the computed excess pore water pressures and hydraulic gradients for the constant compressibility and compression plus KBD pore fluid compressibility assumptions. During runup of the tsunami wave excess pore water pressure in the quasi-saturated layer ($Z_g < 2$ m) continues to increase in both instances. However, when compression plus KBD is considered to simulate pore fluid hardening, there is an exceptional difference in the hydraulic gradients instigated during this stage of the loading process. At $t = 800$ seconds, groundwater seepage and infiltration from the seabed surface
Figure 5.6: Effect of the initial depth of gassy layer on the response of sandy sediments to tsunami loading assuming a constant pore fluid compressibility: a.) depth of full liquefaction; b.) max. depth of full liquefaction.

Figure 5.7: Effect of the initial depth of gassy layer on the response of sandy sediments to tsunami loading considering compression of gas bubbles: a.) depth of full liquefaction; b.) max. depth of full liquefaction.

abruptly stops (Figure 5.10). It is at this stage of tsunami loading that pressurization of the pore fluid has caused all gas to dissolve. Therefore, a constant compressibility assumption does not adequately account for the durability (or lack thereof) of the gas and its contribution to pore fluid compressibility. Thus, for the compression plus KBD assumption, moving forward in time the excess pore water pressure response corresponds to changes in total stress imposed by the increasing weight of the wave (i.e. the sediment is fully saturated and \( P_e \) is mechanically generated). This is the period of time where runup velocity of the wave is greatest. It may be recognized from a sediment transport and scour
Figure 5.8: Effect of the initial depth of gassy layer on the response of sandy sediments to tsunami loading considering compression, dissolution and transport of gas bubble: a.) depth of full liquefaction; b.) max. depth of full liquefaction.

perspective, the stabilizing influence of groundwater infiltration is lost for a period of time, which is not the case for the constant compressibility assumption.

During the initial period when drawdown begins (after $t = 1080$ seconds or 18 minutes) the excess pore water pressures are still elevated such that gas remains dissolved and excess pore water pressures dissipates, primarily, due to the diminishing total stress and mechanical relief of excess pore water pressure. However, towards the end of drawdown ($t = 1250$ seconds or 20.8 minutes) excess pore water pressures reduce enough such that inter-phase gas exchange begins to occur (i.e. dissolved gas exsolves from solution) and the pore fluid compressibility begins increasing. Therefore, changes in excess pore water pressure are no longer linked solely to the diminishing weight of the wave. Following, the rate of excess pore water pressure dissipation in the quasi-saturated layer decreases appreciably slower than that boundary pore water pressure at the seabed surface. From $t = 1250-1320$ seconds (20.8-22 minutes), which is the end of the drawdown phase, positive hydraulic gradients quickly develop, inducing momentary liquefaction (see Figure 5.10).

Similar to the constant compressibility assumption, the thickness of liquefied sediment decreases due to dissipation of excess pore water pressure after drawdown. The time that liquefaction was sustained is significantly less for the compression plus KBD pore fluid
compressibility assumption because of the difference in residual excess pore water pressure not just in the quasi-saturated layer, but also the fully saturated layer, after the tsunami had ended \((t \geq 22\) minutes). When compression plus KBD was considered, which were both fully-saturated for a significant portion of the drawdown stage, excess pore pressure dissipated more uniformly, as it was governed in large part by the mechanical relief associated with the diminishing weight of the wave (see differences in Figure 5.9). For the constant compressibility assumption the residual excess pore water pressure in the fully saturated layer was greater after drawdown, and facilitated a sustained upward hydraulic gradient in the seabed sediment for an appreciably longer time period.

### 5.6.3 Influence of Initial Gas Content and Degree of Saturation

Previous examination of the pore water pressure response considered only degree of saturation \((S_r = 97\%\)). In this section the influence of initial gas content is elaborated on by examining a wide range of \(S_r\) values. The initial degree of saturation chosen in this (and previous) studies stems from experiments performed at the Tsunami Wave Basin at the O.H. Hinsdale Wave Research Laboratory (Young et al., 2009). However, other experiments unrelated to tsunami loading have demonstrated that drainage and/or subsequent recharge of sand beds can result in residual degrees of saturation between 80-85\% (e.g. Yegian et al., 2007; Eseller-Bayat et al., 2013a; McLeod et al., 2015). Thus, similar entrapment of gas may be anticipated in the nearshore sediments in the intertidal zone due to tidal fluctuations (Turner, 1993; Baldock et al., 2001; Horn, 2002; Steenhauer et al., 2011). Here the influence of \(S_r\) ranging from 85-99\% is tested to examine the influence of the pore fluid compressibility assumption on the maximum depth of liquefaction triggering and sustained liquefaction.

Figure 5.11 illustrates the maximum depth of liquefaction triggering for each pore fluid compressibility assumption and gassy layer thicknesses considered \((Z_g = 1, 2, 4, \text{ and } 10\) m). For both the constant compressibility and compression only assumptions, there is a limited
difference in the computed maximum depth of liquefaction. However, when compression plus KBD is considered in the evaluation of pore fluid compressibility, the maximum depth of liquefaction is significantly reduced for higher degrees of saturation ($S_r > 97\%$). This follows preceding observations and contrasts previously discussed (for constant compressibility vs. compression plus KBD assumption) regarding pore fluid hardening and changes in excess pore water pressure. As the degree of saturation increases (volume of entrained air decreases), hardening of the pore fluid due to gas dissolution causes the previously saturated bed to become fully-saturated earlier on during the runup stage, and
Figure 5.10: Contours of the hydraulic gradients of a soil column with $Z_g = 2m$ and $Z_s = 8m$ with $S_r = 97\%$ under Tsunami 2: a.) Constant Compressibility; b.) Compression Plus KBD.

gas is not reintroduced until later on during the drawdown phase—alleviating the differential pressurization of the pore fluid throughout tsunami loading and decreased maximum depths of momentary liquefaction. This highlights how subtle difference in the initial $S_r$ can have a significant impact on the maximum depth of liquefaction triggering in nearly saturated sediments. However, for the conditions examined in this study, full consideration of gas kinetics has a limited influence on the predicted depth of momentary liquefaction for $S_r \leq 97\%$. 

125
However, it may be observed that the time liquefaction is sustained at the ground surface is still influenced by the pore fluid compressibility assumption for values of $S_r \leq 97\%$ (Figure 5.12).

5.6.4 Influence of Beach Profile and Tsunami Wave-Height Time Series

The Carrier et al. (2003) solutions for the wave-height time series is dependent on a number of assumed conditions, including slope of the beach. With regards to momentary liquefaction and the tsunami profile used during an analysis, there are two important characteristics worth noting: i.) the maximum height of the wave, which dictates the maximum total stress and boundary seabed pressure during the event; and ii.) the tsunami duration and rate of drawdown. Importance of the first consideration is more readily apparent, but the latter is less straightforward. A more abrupt temporal change in the tsunami wave-height creates a greater pressure differential between the boundary pore water pressure and excess pore water pressure generated in the sediment during drawdown. Therefore, a faster drawdown rate has the potential to instigate larger upward hydraulic gradients and create a more severe condition. As shown in Figure 5.4b, faster drawdown rates (i.e. $\Delta h/\Delta t$) are also associated with greater wave heights. Tsunami 1, which has the lowest wave-height, also has the lowest drawdown rate, while tsunamis 2 and 3 have greater heights a faster drawdown rates, but a shorter duration.

Figure 5.13 shows the computed results for all three tsunamis for four values of $Z_g$. Pore fluid compressibility is only computed with compression plus KBD. For depths where entrained air exists at shallow depths ($Z_g =1$ and 2 m) there is a limited difference in the computed max depth where momentary liquefaction occurs for all values of $S_r$. Similar to previous demonstrations, as $S_r$ increases above 97%, the maximum depth of liquefaction is significantly reduced (also true for $Z_g =4$ and 10 m, but greater) due to the durability of the gas. For lower degrees of saturation where $Z_g =4$ and 10 m, the maximum depth of liquefaction is notably smaller for tsunami 1 (lower wave height drawdown rate) than
tsunamis 2 and 3, which do not exhibit as great a difference. Though the max wave height of tsunami 3 is 6 m greater, the difference in the computed depth of liquefaction does not increase significantly. Thus, it may be concluded that the drawdown rate plays a greater role in the expansion of the depth where momentary liquefaction occurs.

5.6.5 Influence of varying $S_r$ with depth

All previous demonstrations assumed that the initial degree of saturation (i.e. volume of entrapped air) was constant with depth. However, if the same volume of air is entrained in nearshore sediments as the tide height increases, hydrostatic pore water pressures will reduce the initial volume of gas (i.e. $S_r$ varies with depth). This is investigated here, where the initial bubble size in the quasi-saturated layer is computed as follows (which is derived based on Boyle’s gas law):

$$r^3(z_w) = \frac{3n_{tot}RT}{4\pi P_g}$$

(5.28)

where $z_w$ is depth below water table and $n_{tot}$ is the total number of moles assumed to be initially entrained in each bubble as the tide level rises. Note that the depth where entrained air actually exists is assumed to be a function of tidal fluctuations.

Figure 5.14 shows the computed depth of maximum liquefaction for $Z_g = 2$ and 4 m and the compression plus KBD pore fluid compressibility assumption (note that $S_r$ indicates the value of $S_r$ at the top of the quasi-saturated layer. As shown, the assumed distribution of gas does not have a significant influence on the computed response, as the gas does not compress significantly under these hydrostatic pressures.

5.7 Discussion

Response of coastal sediments to tsunami loading is affected by the assumed geometry and initial conditions. A 2-layer system is not only more faithful to the existing condition of the coastal sediments, it also can potentially reveal a more critical situation not shown by a 1-layer uniform geometry. As shown in this study, increasing the thickness of the
gassy layer does not always result in a deeper sediment instability. In other words, there is a critical thickness of gassy layer where the instabilities are the deepest and also the time-window where sediments are susceptible to erosion is the largest.

The initial degree of saturation in the gassy zone is another important factor that can influence the depth of projected instabilities. However, a more important factor is keeping track of the changes in the degree of saturation during tsunami loading and updating the pore fluid compressibility accordingly, especially for a high degree of saturation. For \( S_r > 97\% \) gas bubbles will most likely undergo compression and dissolution to a point where the gassy zone becomes fully saturated even for a short period of time. This is a more realistic approach for conducting a seepage-deformation analysis, and will probably result in a more accurate instability patterns compared to a more conventional assumption of assigning a constant pore fluid compressibility.

The volume of entrapped gas bubbles in a swash zone can be affected by the hydrostatic pore pressure. Basically, before a tsunami event the initial degree of saturation of a gassy soil layer is expected to slightly increase with depth as the deeper gas bubbles experience a small compression due to a higher hydrostatic pore pressures. It means that compared to a uniform assumption for \( S_r \), for a similar initial degree of saturation at the seabed level, in reality the average degree of saturation can be slightly smaller. At illustrated in the previous section, for thin gassy layer (e.g. \( Z_g = 2 \) m), smaller average degree of saturation as a result of natural compression results in a slight increase in the computed maximum depth of instability at high degrees of saturation. However, for the case of thicker gassy layer (e.g. \( Z_g = 4 \) m), at low degree of saturation assuming a constant \( S_r \) overestimates the depth of instability but at higher \( S_r \) assuming a variable initial degree of saturation yields a deeper instability.
Figure 5.11: Effect of the initial degree of saturation on maximum depth of liquefaction during tsunami loading for different pore fluid compressibility assumptions: a.) $Z_g = 0$ m; b.) $Z_g = 2$ m; c.) $Z_g = 4$ m; d.) $Z_g = 10$ m.
Figure 5.12: Effect of the initial degree of saturation on liquefaction during tsunami 2 loading for different pore fluid compressibility assumptions: a.) $Z_g = 0$ m; b.) $Z_g = 2$ m; c.) $Z_g = 4$ m; d.) $Z_g = 10$ m.
Figure 5.13: Effect tsunami wave-height time series on the maximum depth of momentary liquefaction: a.) $Z_g = 0$ m; b.) $Z_g = 2$ m; c.) $Z_g = 4$ m; d.) $Z_g = 10$ m.
Figure 5.14: Effect gas bubble distribution assumption on the liquefaction depth during Tsunami 2 loading: a.) $Z_g = 2 \text{ m}$; b.) $Z_g = 4 \text{ m}$. 
CHAPTER 6
SUMMARY AND CONCLUSIONS

Entrapped gas bubbles in quasi-saturated porous media are of practical interest in many fields of science and engineering. A summary of many fields dealing with this topic was provided in the introduction of this dissertation. This study focused on the practical importance of gas durability in geotechnics, motivated by the recognition that occluded gas bubbles suppress the generation of excess pore water pressure and increase the liquefaction resistance of loose saturated granular soils. Induced-partial-saturation is a nascent approach to mitigate the liquefaction phenomenon by artificially introducing gas into the ground. Previous efforts to advance IPS have focused, by in large, on developing methods to introduce gas, which has been scaled from elemental- and bench-scales to field application. However, the salient consideration of gas durability, and its persistence under time-scales of interest for civil infrastructure systems (decades), had not been meaningfully addressed. An obstacle in dealing with this issue is that gas cannot be monitored for decades (at this time) prior to widespread adoption in practice—which is unlikely to occur without a reliable understanding of gas longevity. To overcome this obstacle, a chemical- and physics- based numerical modeling framework was developed to assess, for the first time, the durability and longevity of entrapped gas as it applies to IPS. This effort provided novel insight and is a substantial advancement in the understanding of gas durability and persistence for geotechnical applications.

One of the challenges with the numerical modeling of gas durability in porous media is determination of the input values for the relevant soil and gas properties. Among all the input parameters considered, the effective diffusion coefficient was found to be less known, especially for non-plastic soils. Therefore, as a part of this study the effective diffusion coefficient and tortuosity factor for different soil types were experimentally measured using the apparatus commonly found in most geotechnical laboratories. The modeling effort was
also extended to incorporate the physics governing gas kinetics in a poroelastic seepage-deformation model to simulate pore fluid hardening that is associated with the dissolution of gas. This effort was undertaken in part to demonstrate the durability of gas under extreme loading conditions, and to demonstrate the influence of inter-phase gas exchange and more faithfully account for the dynamic pore fluid compressibility under extreme loading conditions. The response of quasi-saturated sediments, and the influence of gas durability, under extreme tsunami loading conditions was analyzed to elucidate the role of bubble kinetics on differential pressurization of pore water at depth in the sediment that instigates hydraulic gradients and momentary liquefaction in the sediment. Results were compared with simpler pore fluid compressibility assumptions (i.e. constant compressibility and compression only) to highlight its influence. This topic demonstrates another area of research in the field of geotechnics where the durability of gas is relevant.

6.1 Major findings from Aqueous-Phase Gas Diffusion Experiments

Motivated by uncertainties regarding the effective diffusion coefficient in soil for aqueous-phase gas, and the applicability of tortuosity factors for transport of other solutes, a pressure-decay-method was adopted to determine the effective diffusion coefficient through three different soils (gravel, sand, and silt). The following observations were made:

- The pressure-decay-method, in combination with inverse numerical analyses, was capable of yielding consistent tortuosity factors for all three soil types tested. Additionally, the experimental procedures yielded efficient determination of the tortuosity factor for each soil over the course of hours or days.

- Average tortuosity factors of 0.15, 0.39, and 0.08 were determined from the experiments performed on gravel, sand, and silt, respectively. The lower tortuosity factor for gravel (relative to sand) was not intuitive. Imaging of each soil type revealed significant differences in grain-shape. Notably, gravel grains were more
elliptical than sand grains, which would contribute to a greater effective diffusion length (i.e. tortuosity). The silt grains were more similar, even more platy, than the gravel grains, which lends merit to the conclusion that grain-shape significantly influences the effective tortuosity factor.

- Measurement of diffusion coefficients are not always routine and often difficult to obtain, but should be determined for diffusion problems of interest. The pressure-decay-method, which was performed using equipment readily available in many geotechnical engineering laboratories, may be adopted to efficiently determine the effective tortuosity and aqueous-phase gas diffusion coefficient for site-specific soils in the future—parameter needed to simulate aqueous-phase gas mobility and gas longevity.

6.2 Major Findings Concerning the Durability of Gas for IPS

- The numerical modeling framework, which simulated the aqueous-phase gas mobility contributing to the dissolution of gas, was capable of predicting experimental observations of gas dissolution from several independent studies—which had varied spatial scales, pore fluid constituents, gas solubilities, and boundary conditions—under both hydrostatic and groundwater flow conditions.

- Under 1D hydrostatic conditions (i.e. infinitely wide gassy soil volume), the thickness of the gassy layer decays due to a diffusion-induced resaturation front that advances from the boundary of quasi-saturated sediment. The methodologies presented in this study may be applied for a variety of gassy soil thicknesses, initial degrees of saturation, gas type, and bubble sizes. When a 1.5 m thick layer desaturated with air (initial $r_b = 0.03$ mm) with an initial $S_r = 82\%$ extended from the ground surface ($Z = 0$ m) was considered, full saturation was predicted after 160 and 78 years for one- vs. two-way diffusion, respectively. When the top of the layer extended from
Z = 5 and 10 m, resaturation times were 125 and 99 years for one-way diffusion, and decreased to 42 and 30 years for two-way diffusion. Thus, both the boundary condition and depth influence gas longevity.

- Under horizontal flow conditions, a saturation front progressively advances downgradient due to imbibing groundwater that acts as a sink, resulting in dissolution of gas bubbles. The rate that the saturation front progresses depends on the pore volumes of groundwater introduced and the associated seepage velocity, as well as depth. At Z = 0, 5, and 10 m, the predicted saturation front progressed 1.5 m through a quasi-saturated layer (initial $S_r = 82\%$ and $r_b = 0.03\ mm$) after 115, 22, and 15 pore volumes were introduced; this corresponded to 50, 9, and 5.5 years when $v_s = 0.01\ m/day$. Thus, groundwater flow conditions have the potential to attack the perimeter of a quasi-saturated soil volume more rapidly, that rate of which is highly dependent on the local hydrogeologic conditions and naturally occurring hydraulic gradient that exists. Therefore, seepage into a gassy soil volume and/or recharge of the gassy soil volume needs to be mitigated and/or maintenance systems that recharge the layer with gas are warranted.

- As the initial bubble radius decreases, saturation rates increase under both hydrostatic and groundwater flow conditions. Therefore, characterization of a representative initial bubble size of entrapped gas, which is dependent in part on the pore size distribution, is an important consideration.

- Similarly, when entrapped gas has a higher solubility (O$_2$ examined in this study), the rate of resaturation increase significantly for both hydrostatic and groundwater flow conditions. Therefore, IPS methods that generate or introduce low solubility gasses should be preferred; this has been recognized and reemphasized in this study.

- A relatively uniform advancement of the saturation front may be expected under hydrostatic conditions while a predominantly wedge-shaped saturation front may be
anticipated where a natural hydraulic gradient driving groundwater flow exists. Incorporating a sacrificial thickness to a targeted volume of loose soil is a potential avenue to increase the durability and extend the life of an IPS system. For groundwater flow conditions, it was demonstrated that a potential maintenance scheme, whereby gas is periodically introduced near the perimeter to saturate flowing groundwater with dissolved gas, significantly decreases the rate that the saturation front advances at the perimeter of a gassy soil volume. For a typical $v_s = 0.01 \text{ m/day}$, it was found that the saturation front would advance 1.5 m in 52, 74, and 88 years at depths $Z = 0$, 5, and 10 m, respectively. This may be a practical solution to increase the durability of an IPS system.

- Offering a simple and practical tool for gas durability assessment can be useful and help promote the IPS method. For this purpose, the Gas Longevity Coefficient was introduced based on the results of the numerical studies conducted. Under hydrostatic condition, the Longevity Coefficient will have units of meter/year while under a horizontal seepage condition, it has units of velocity/velocity.

- This study offers a range for the Gas Longevity Coefficient under both hydrostatic and flow conditions. Having the major parameters known, such as gas type, average bubble size and the average depth of entrapment, the gas longevity coefficient can be estimated and used for a simple and practical assessment of the gas durability under the given conditions.

### 6.3 Major Findings From Tsunami Simulations

Motivated by uncertainties regarding the durability of gas, which is linked to the pore fluid compressibility and instigation of hydraulic gradients and momentary liquefaction in sand beds during tsunami loading, the following major findings were:
• The entrapment of gas and increased pore fluid compressibility in surficial sand beds instigates stabilizing and destabilizing hydraulic gradients during runup and drawdown of the tsunami wave, respectively. During and after drawdown, this can lead to momentary liquefaction that is sustained after the wave has receded. For granular sand beds underlain by an “impermeable” layer at 10 m, the maximum depth of liquefaction generally increased with thickness of the gassy layer to 4 m. When the thickness of soil containing entrapped gas extended to depths greater than 4 m, the maximum depth of liquefaction decreased slightly. Thus, the assumed thickness of soil containing entrapped is an important consideration when understanding the interaction between quasi-saturated and fully-saturated sand beds and the depth of momentary liquefaction triggered by tsunami loading.

• The pore fluid compressibility assumption (constant vs. dynamic) influences the excess pore water pressure generated during tsunami loading. When the pore fluid compressibility was simulated with consideration of inter-phase gas exchange, pore fluid hardening significantly influence the maximum depth of liquefaction for low initial gas contents and high initial degrees of saturation ($S_r > 97\%$). At lower degrees of saturation the pore fluid compressibility assumption did not have a large impact on the maximum depth of liquefaction.

• The temporal evolution of excess pore water pressure generated in the sand bed was appreciably influenced by the pore fluid compressibility assumption. When gas kinetics were considered, increased pore water pressure can lead to full dissolution of the gas during tsunami runup. Thus, there is a period of time during runup and drawdown where the entire sand column is fully-saturated and generation of excess pore water pressure corresponds to the changing total stress imposed by the weight of the wave. This leads to a difference in the pore water pressure response to that observed for the constant pore fluid compressibility assumption during this same
stage of tsunami loading. As the tsunami wave recedes during drawdown, gas
exsolves from solution and contributes to greater differential pressurization of the
pore water (opposed to when gas is dissolved) at depth during the end of the
tsunami, which can still trigger momentary liquefaction.

- When inter-phase gas exchange is considered to simulate the pore fluid
  compressibility, stabilizing hydraulic gradients (i.e. infiltration) during runup of a
tsunami wave are appreciably less than when a constant pore fluid compressibility
  (i.e. no compression or dissolution) of the gas is considered.

- When inter-phase gas exchange is considered to simulate the pore fluid
  compressibility, the duration of sustained liquefaction after the wave has receded is
  significantly less, irrespective of initial degrees of saturation considered in this study
  ($S_r = 85-99\%$).

- The assumed tsunami wave-height time series plays a role in the maximum depth of
  liquefaction. Notably, when the rate of drawdown is greater, the maximum depth of
  liquefaction is greater. For the same initial wave length, the rate of drawdown (and
  wave height), is greater for beach profiles with a greater slope.

6.4 Research Limitations and Recommendations for Future Work

This study has provided an assessment of gas durability using a numerical framework
that explicitly considers aqueous-phase gas mobility and inter-phase gas exchange, under
both in situ conditions and mechanical loading scenarios. However, there are limitations
with the current framework that could be expanded on.

- In this study, the experiments conducted for the measurement of the tortuosity factor
  of non-plastic soils only considered diffusion in the direction normal to the soil
deposition plane. For understanding the effect of soil anisotropy on the durability of
entrapped gas, future work could repeat these tests for cases where diffusion is predominantly taking place parallel to the soil deposition plane.

- The entrapped gas bubbles were assumed to be homogeneous (initially) and uniformly distributed within the porous media. The attempt was made to use a bubble size (volume and surface area) that would be representative of all bubbles. In reality, the bubble size is unlikely to be uniform, particularly within well-graded sediments (i.e. with a wide grain-size distribution). Future work should attempt to address the adequacy of this assumption on different soils.

- The buoyant mobility of gas was not considered in this study (i.e. bubbles remained entrapped). As bubbles begin to dissolve, it is likely that they will begin to migrate upward through pore throats until they again expand under decreasing hydrostatic pressure and are again entrapped. It is unclear what influence this may have on the persistence of entrapped gas. It may be appropriate to expand this modeling framework using pore network models or similar. This is relevant to addressing both the long-term durability of gas for IPS and temporal changes in the pore fluid compressibility throughout tsunami loading.

- Another important consideration for numerical assessment of the longevity of entrapped gas is the method used for gas generation. Active gas generating agents such as microbial activity may continue to generate gas versus other methods of IPS implementation (e.g. forced air injection). This could be readily expanded on in the existing numerical framework presented here by adding an additional source term in the advection-diffusion equation.

- To date there is a lack of long-term field-scale monitoring of gas content and changes in the degree of saturation. Field-scale long-term monitoring programs with accompanying methods to track the saturation front are needed to demonstrate that reliable model predictions of gas longevity can be achieved—which is principal to the
advancement of IPS and should be incorporated in future studies. This study has provided greater context for the evolution of saturation fronts that may be anticipated. Thus, the methodologies applied to predict the evolution of a saturation front in this study can be used to better inform future monitoring programs by targeting measurements at the perimeter of a gassy soil volume where resaturation is first anticipated. With this understanding, it may be possible to gather meaningful field data under practical time-constraints in a future study.

- In that same vain, it is not yet well-understood how the uniformity of gas after emplacement will affect the liquefaction resistance of targeted soil. It’s likely that isolated “pockets” of soil with entrapped gas may initially exist after emplacement, and then redistribute with time. Future field-scale studies should address the necessity of initial gas uniformity, and subsequent redistribution of gas after emplacement.

- Similar challenges exist with respect to field demonstrations of the pore water pressure response in sand beds during tsunami loading. The exact timing and location where a tsunami will occur is virtually impossible to predict. However, it is conceivable that large laboratory experiments could be arranged to simulate the total stresses and boundary pore water pressures imposed on sand beds during tsunami loading. Current wave tanks used to simulate solitary waves cannot correctly scale the aforementioned loading conditions. Developing experiments that can, would significantly advance the understanding of the pore water pressure response, and influence of pore fluid compressibility associated with the durability of gas, for this natural hazard.

- Additionally, the tsunami loading model adopted in this study did not address the potential influence of erosion and changes in the weight of the overlying sediment on the excess pore water pressure response. Future experiments, where scaling of the
total stress and boundary pore water pressures are scaled appropriately, could also incorporate the flow velocity of the wave to observe its effect on the excess pore water pressure response and erosion of sediment.
REFERENCES


Figure A.1: Pressure decay profiles observed and simulated from inverse analyses of the aqueous-phase diffusion coefficient corresponding to: a) Test 1 ($He$ through water); b) Test 2 ($N_2$ through water); c) Test 3 ($N_2$ through water); d) Test 4 ($N_2$ through Silt).
Figure A.2: Pressure decay profiles observed and simulated from inverse analyses of the aqueous-phase diffusion coefficient corresponding to: a) Test 5 ($N_2$ through Silt); b) Test 6 ($N_2$ through Silt); c) Test 7 ($N_2$ through Sand); d) Test 8 ($N_2$ through Sand).
Figure A.3: Pressure decay profiles observed and simulated from inverse analyses of the aqueous-phase diffusion coefficient corresponding to: a) Test 9 ($N_2$ through Sand); b) Test 10 ($N_2$ through Sand; c) Test 11 ($N_2$ through Sand); d) Test 12 ($N_2$ through Gravel).
Figure A.4: Pressure decay profiles observed and simulated from inverse analyses of the aqueous-phase diffusion coefficient corresponding to: a) Test 9 (N₂ through Gravel); b) Test 10 (N₂ through Gravel).
BIOGRAPHY OF THE AUTHOR

Babak Mahmoodi Chanzab was born 1990 in Ardabil, Iran. At the age of 12, he was selected by the National Organization for Development of Exceptional Talents to receive special education and after 7 years he graduated from NODET with a high school diploma in Math & Physics. Babak later pursued his passion for mathematics and physics by attending the college of engineering at the University of Tehran and receiving his B.S. in Civil Engineering (2013) and M.S. in Geotechnical Engineering (2016). While at graduate school, Babak started working as a geotechnical engineer for a geotechnical specialty contractor and was able to gain valuable experience on the design and construction of large-scale civil engineering projects including ground improvement methods and soil retaining structures.

For his graduate work at the University of Tehran, he studied the seismic behavior of concrete face rockfill dams. The result of his work was presented at multiple conferences and was later employed as the state of practice for the design of CFR dams.

On August, 2017 Babak joined the University of Maine to pursue his PhD degree in Geotechnical engineering. His PhD work revolved around the interaction between an entrapped gas phase and fluid saturated porous media. Using state of art laboratory apparatus and development of computer codes, he was able to address some of the current important issues with regard to the short-term and long-term behavior of gassy granular sediments.

Babak likes attempting new challenges and would like to expand his knowledge on the application of AI in Geotechnical Engineering. On his free time he likes to go on road trips and hike the great outdoors. Babak is a candidate for the Doctor of Philosophy degree in Civil and Environmental Engineering from the University of Maine in December 2020.