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Using Contrasting Cases to Build Metacognitive Knowledge About the Impact of Salient Distracting Features in Physics Problems

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USING CONTRASTING CASES TO BUILD METACOGNITIVE
KNOWLEDGE ABOUT THE IMPACT OF
SALIENT DISTRACTING FEATURES
IN PHYSICS PROBLEMS

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A DISSERTATION
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Student reasoning on physics problems is often context dependent. A possible explanation is that salient distracting features (SDFs) in physics problems may cue students’ “spontaneous” reasoning. This cued reasoning is often accepted without question, even though it may be unproductive and may even preclude the use of relevant knowledge. One possible approach to address such reasoning difficulties is to strengthen students’ metacognitive skills, particularly their metacognitive knowledge. While metacognitive knowledge plays an important role in facilitating effective regulation, little is known about how to build student metacognitive knowledge. This dissertation explores the use of contrasting cases (e.g., a number of cases or instances having the same underlying knowledge across a range of contexts) to build transferable metacognitive knowledge.

The goal of this dissertation is to understand how students can build general metacognitive knowledge (GMK) related to SDFs using contrasting case
instruction. In particular, the GMK targeted in this work involves reflection on how and why SDFs can impact reasoning. Multiple sets of contrasting cases, or contrast pairs, were designed to highlight the GMK via descriptive vignettes of fictional students as they answered physics questions.

In an experiment, in one condition (non-synthesis), college students compared each contrast pair separately. In the other condition (synthesis), students compared the same pairs together. All students received direct instruction on the potential value of reflecting on how SDFs could impact their reasoning, and then took a post-test. Results revealed that synthesis students could generate the GMK while no non-synthesis students did. Analysis also revealed that synthesis students demonstrated greater learning of the GMK on the post-test. Furthermore, no non-synthesis students could apply the GMK, even after being told the knowledge. Together, these findings suggest that synthesis may be an effective approach to building student GMK, while direct instruction on its own is not. More broadly, students may not generate GMK on their own without the appropriate instructional scaffolding.
DEDICATION

To J.J., best of husbands and best of men
ACKNOWLEDGEMENTS

I was lucky during my time at UMaine. My work was supported by the National Science Foundation under grants DUE-1431940, DUE-1245313, and DUE-0962805. I also found an intellectual home with the Physics Education Research Laboratory. Most importantly, I had the two best advisors, Mac Stetzer and Jon Shemwell, who always had my back.
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Chapter 1
INTRODUCTION AND RATIONALE

Over the last thirty years, the Physics Education Research (PER) community has extensively investigated the learning and teaching of physics. In particular, researchers have examined student understanding, reasoning, and problem-solving skills for a variety of physics topics and in different instructional settings. Much of this work has focused on students’ conceptual understanding of introductory topics such as kinematics (Trowbridge & McDermott, 1980, 1981), dynamics (Rosengrant, Van Heuvelen, & Etkina, 2009), work and energy (Lawson & McDermott, 1987) and circuits (McDermott & Shaffer, 1992; Shaffer & McDermott, 1992). These in-depth studies of student conceptual understanding have guided the development and refinement of a variety of effective, research-based instructional materials (McDermott & Shaffer, 2002; Sokoloff & Thornton, 1997). A large body of PER literature indicates that the use of research-based and research-validated instructional materials can lead to improved student understanding (e.g., Thornton & Sokoloff, 1998).

Yet, there is growing evidence that, even after research-based instruction, students often reason inconsistently across multiple problems targeting the same concept. For example, student performance on one question may be quite strong, suggesting that the students possess the requisite knowledge and skills to arrive at a correct answer. However, student performance on a closely related question may be weaker, with many of the same students employing very different lines of reasoning. Perhaps more importantly, students may demonstrate that they possess the relevant conceptual knowledge on one problem but fail to apply that same knowledge on a related problem (Kryjevskaia, Stetzer, & Grosz, 2014). While it is
necessary for students to develop a robust conceptual understanding, these findings suggest that conceptual understanding alone may not be sufficient. Indeed, even after targeted research-based instruction shown to improve student conceptual understanding, student reasoning difficulties have been shown to persist (Kryjevskaia et al., 2014). This suggests that the reasoning process itself is somehow getting in the way of students successfully applying their physics knowledge.

While research has explored several possible mechanisms to account for this phenomenon, most emphasize the context sensitivity of student reasoning (diSessa, 1993; Hammer, Elby, Scherr, & Redish, 2005). Specific information in physics problems may cue different student reasoning approaches, thereby making the reasoning context-sensitive. This information may include surface features such as physics terms, objects, physical configurations, diagrams, graphs, and numerical values, as well as situational and contextual characteristics. While attention to the relevant features and their roles in a given situation can support correct lines of reasoning, certain attractive but irrelevant features frequently cue incorrect lines of reasoning. Researchers argue that these “salient distracting features,” or SDFs (Mamede, Splinter, van Gog, Rikers, & Schmidt, 2012), capture students’ attention and are perceived as relevant (Heckler, 2011; Osman & Stavy, 2006). As a result, SDFs interfere with correct reasoning and may constrain students’ thinking and exploration of more productive reasoning approaches. Thus, if students learned about SDFs, they may be able to avoid the reasoning pitfalls of SDFs and potentially have the opportunity to consider other less salient but relevant features.

To address such reasoning inconsistencies that may be cued by SDFs, current studies in PER, and within science education more broadly, offer little guidance on how to support students as they encounter SDFs. A common approach, which altogether dismisses student thinking about SDFs, is to help
students determine which features are actually relevant (Heller, Keith, & Anderson, 1992) or to redirect students’ attention to the relevant features in the situation (Conati & Vanlehn, 2000; Steif, Lobue, Kara, & Fay, 2010). There are also approaches related to SDF-cued reasoning, such as examining common incorrect lines of reasoning in the context of hypothetical student statements (see, for example, McDermott & Shaffer, 2002). The underlying assumption of these approaches is that incorrect reasoning is due to gaps in student knowledge, rather than a phenomenon in which student reasoning may be cued by an SDF, and the underlying goal is therefore to strengthen student conceptual understanding.

Taken together, these approaches may inhibit the impact of SDFs on student thinking and increase student use of the correct reasoning for particular concepts and problem types. However, because they are context-bound, they neither focus on generalizing the impact of SDFs nor have the affordances to support generalization. Thus, there is more work to be done to explicitly understand how to develop student thinking about the impact of SDFs across multiple contexts.

One potential approach toward a more general disposition in addressing student thinking about the impact of SDFs is to target and enhance student metacognition, or thinking about thinking. Studies have shown that instruction incorporating metacognition improves student conceptual understanding (Grotzer & Mittlefehldt, 2012; Mevarech & Fridkin, 2006; White & Frederiksen, 1998) as well as problem-solving and laboratory skills (Etkina et al., 2010; Schoenfeld, 2009). Metacognition, generally described as having an awareness and understanding of one’s own thinking, is understood to consist of two distinctive components: metacognitive knowledge and regulation (Brown, 1987; Jacobs & Paris, 1987). Metacognitive knowledge refers to one’s knowledge about cognition and cognitive processes, as well as knowledge about general strategies to be used on different cognitive tasks. Metacognitive regulation, or metacognitive skills,
includes processes that monitor, control, and regulate cognition and learning (e.g., planning, monitoring, and evaluation). These two components are closely related, as metacognitive knowledge is believed to facilitate regulation (Kuhn, 2000; Zohar & Barzilai, 2013). In addition, studies show that students who know about different metacognitive strategies are more likely to use them (Pintrich, 2002; Schraw, 1998; White & Frederiksen, 1998; Zepeda, Richey, Ronevich, & Nokes-Malach, 2015).

Research has largely examined metacognition in the context of problem-solving (Schoenfeld, 1987; Taasoobshirazi & Farley, 2013), conceptual understanding (Adler, Zion, & Mevarech, 2015; White & Frederiksen, 1998), and laboratory skills (Etkina et al., 2010; Kung & Linder, 2007) with, proportionally, fewer studies focusing on metacognition within the context of SDFs and their impact on student reasoning. Although researchers suggest the impact of SDFs on reasoning processes can be mitigated by metacognition (Amsel et al., 2008; Thompson, 2009), the role of metacognition in student reasoning in relation to SDFs remains largely unexamined. Additionally, the primary focus of metacognition studies within PER, and more broadly science education, is the development of metacognitive skills, with less attention devoted to developing metacognitive knowledge (Zohar & Barzilai, 2013).

In sum, there is a need for understanding how students can develop general metacognitive knowledge of SDFs and their impact on reasoning processes when approaching a wide range of physics situations. Therefore, in this dissertation, the focus of my investigation is on student general metacognitive knowledge (GMK) of the impact of SDFs, which may be useful for student reasoning in physics. Specifically, the targeted GMK is students’ awareness and understanding of how and why SDFs can impact thinking across multiple contexts. This dissertation contributes to the understanding of how general metacognitive knowledge can be
built. More broadly, it also contributes to the pedagogy surrounding SDFs and SDF-cued reasoning by emphasizing a metacognitive focus.

In the remainder of this chapter, I motivate my investigation from the literature on SDFs and metacognition within physics education. First, I expand on the nature and impact of SDFs, including an illustration of how they may impact student reasoning. Following this, I provide a metacognitive framework and discuss the benefits of a metacognitive focus on student thinking of SDFs. The discussion emphasizes the importance of metacognitive knowledge and reviews approaches to building general metacognitive knowledge. Based on this discussion, I suggest a more robust approach to building generalizable knowledge using instruction involving contrasting cases. Finally, I conclude with a more explicit statement of the purpose of my investigation and provide an overview of the structure of my dissertation.

Illustration of a Salient Distracting Feature in a Physics Question

In a given situation, a certain feature may stand out more than others. This salient feature may or may not be relevant to successfully completing the task. When the salient feature is irrelevant, it can be distracting because learners divert their attention and behavioral resources toward the feature, leaving less opportunity for other relevant features to be considered (Heckler, 2011). In addition, these features may cue incorrect lines of reasoning. Learners often perceive these cued responses as reasonable since the responses are accessible and generated quickly.

As a simple example, consider the physics question in Figure 1.1 (Heckler, 2011). The question presents a position versus time graphs for two cars, and the task is to determine when, if ever, the speeds of the cars are the same. There are
When, if ever, are the speeds of the car the same?

Figure 1.1: Example of an SDF in a physics task. The task is drawn from Heckler (2011). The salient distracting feature is the intersection point which cues the incorrect response that the cars will have the same time at point B.

Several competing features, such as the intersection point, the height of the graph, and the slope. The relevant feature is the slope, which indicates the velocity of the car on a position-time graph. Thus, the correct answer is that the cars have the same speed at time A, when the slopes of two curves are equal. In this task, the salient distracting feature is the intersection point, as has been documented empirically by Heckler (2011). In this task, Heckler found that the intersection point led to the most prevalent incorrect response that the speeds are the same at time B. Although it may be possible that students lack knowledge to arrive at the correct response, Heckler showed that students do have the relevant content knowledge. Students could arrive at the correct response if their responses were delayed (Scaife & Heckler 2010). This indicates that even if students have the relevant content knowledge, the salient distracting feature may interfere with the correct reasoning.
Table 1.1: Definitions of salient distracting features across different domains

<table>
<thead>
<tr>
<th>Study</th>
<th>Term</th>
<th>Definition</th>
<th>salient captures attention</th>
<th>distracting Cues...</th>
<th>perceptual feature</th>
<th>non-perceptual feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>salient distracting features</td>
<td>X</td>
<td>X</td>
<td>incorrect responses</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Heckler (2010)</td>
<td>salient yet scientifically irrelevant features</td>
<td>X</td>
<td>X</td>
<td>incorrect, patterned responses</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Osman and Stavy (2006)</td>
<td>salient irrelevant variable</td>
<td>X</td>
<td>X</td>
<td>intuitive rules</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Elby (2001)</td>
<td>compelling visual attribute</td>
<td>X</td>
<td>X</td>
<td>What-you-see-is-what-you-get</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Mamede et al. (2014)</td>
<td>salient distracting features</td>
<td>X</td>
<td></td>
<td>pattern recognition</td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

Definition of Salient Distracting Features

In the broader literature and a variety of contexts, researchers have used different terminology and definitions to describe salient, irrelevant task features and their impact on decisions and task performance. Table 1.1 summarizes four definitions from different domains, including physics education (Heckler, 2010; Elby, 2001), more broadly in science and mathematics education (Babai, Shalev, & Stavy, 2015; Lubin et al., 2016; Stavy & Tirosh, 2000), and medical education (Mamede, Van Gog, Van Den Berge, Van Saase, & Schmidt, 2014; Mamede et al., 2012). I use the term salient distracting features (SDFs), which is borrowed from Mamede and colleagues (2012), who use the term in the context of clinical reasoning. However, I also draw heavily on Heckler’s work (2010, 2011), as it pertains to physics learning.

I define SDFs to be task-irrelevant features, which are perceived as relevant and cue incorrect lines of reasoning. First, an SDF is salient because it...
immediately stands out and captures attention. Part of its salience includes being
easily and quickly processed to generate a response. Second, distracting relates to
the SDF’s irrelevance to the specified task. Because students perceive SDFs as
relevant, they may not consider actual relevant, but less salient, features. In this
sense, SDFs are distracting. Most importantly, SDFs are also distracting because
they cue incorrect reasoning approaches and therefore interfere with correct
reasoning. Finally, features refer to any information conveyed in a task. They can
be either explicit surface features or implicit contextual and situational features.
Within a physics situation, surface features can include physics terms, variables,
numerical values, or perceptual features in diagrams or graphs.

Within physics education, Heckler argued that many features in physics
questions compete for a student’s attention. He proposed that students’ responses
may be based on features that are processed the fastest and the “most salient and
plausibly relevant features,” which tend to automatically capture attention.
Heckler calls these “salient yet scientifically irrelevant features.” He stresses that
they are not “randomly distracting.” Rather, students perceive them as relevant.
They cue students to use incorrect lines of reason, even though students may have
the knowledge to correctly answer the question. Because attention is directed
toward these salient irrelevant features, Heckler reasons that there is less
opportunity for students to attend to the less salient but relevant features needed
for a correct response.

Along the same lines as Heckler, Elby has made similar arguments for
certain perceptual features in physics questions (Elby, 2000). Elby argues that
“compelling visual attributes” of physics representations (e.g., edges, corners, or
motion) quickly draw attention and tend to cue incorrect reasoning. Specifically,
these attributes cue students to intuitively interpret visual representations in a
simple, literal way, which he refers to as “What-You-See-Is-What-You-Get.” Like
Hecker’s features, Elby’s “compelling visual attributes” function in the same way. They attract students’ attention and cue incorrect responses. However, Elby focuses on specific perceptual features, which cue a certain type of reasoning. Heckler was broader in his inclusion of feature types and the types of reasoning cued. Elby’s “compelling visual features” could be considered to be a subset of Heckler’s “salient yet scientifically irrelevant features.”

Similarly, within mathematics education, Osman and Stavy (2006) also believe that irrelevant task features can interfere with correct reasoning. They use the term “salient irrelevant task features.” They also believe that students can have the knowledge and skills to answer questions correctly, but these features can interfere with correct reasoning. They note that these features capture students’ attention and are quickly processed; moreover, these features can cue intuitive rules, which are implicit and automatic operations that students use to make inferences about a task property. These intuitive rules are essentially heuristics, which allow students to efficiently generate a response based on a task feature, even though it may be irrelevant and lead to incorrect responses.

The effects of irrelevant, salient features have been extended beyond science and mathematics education. For instance, in the context of clinical reasoning, Mamede and colleagues (2012, 2014) describe how certain irrelevant features in a patient’s case history are more salient than others because they have strong associations with a particular medical condition. They call these features “salient distracting features,” and such features tend to attract a doctor’s attention and cue pattern recognition, leading to incorrect diagnoses. The authors do not attribute such incorrect diagnoses to a lack of knowledge on the part of the doctor but rather on implicit automatic processes. Similar to both Elby and Stavy et al., Mamede and colleagues describe contextual features that cue a specific type of heuristic.
Theoretical Frameworks for SDFs

Different theoretical frameworks have been used to explain the mechanism behind the impact of SDFs on student reasoning. Researchers have commonly used a class of theories known as dual-process theories of reasoning, which explain people’s decision-making and problem-solving processes in a variety of domains (e.g., Evans, 2003). Dual-process theories can explain the difficulty of overcoming SDFs. There are many dual-process theories put forth by researchers, but to illustrate the impact of SDFs on reasoning, the discussion will be limited to one such theory – the heuristic-analytic (H-A) theory of reasoning and decision-making originally proposed by Evans (2006). According to H-A theory, there are two distinct modes of thinking or processing. These two processes, or systems, are known as the heuristic process and the analytic process; in other dual-process models, they are referred to as system 1 and system 2, respectively (Kahneman, 2011). The heuristic system (system 1) is unconscious, implicit, automatic, quick, and low effort. In contrast, the analytic system (system 2) is conscious, explicit, controlled, slow, and high effort. The heuristic and analytic systems are interdependent and follow a default-intervention structure. For example, when a student is presented with a physics task, the SDF of the situation may cue a heuristic-based response. Generally, a heuristic-based response quickly and easily comes to mind. It can also be characterized as “intuition,” which has been described as pattern recognition (Kahneman, 2011), or it can also be the result of using mental short-cuts. Once the heuristic-based response is formed, the analytic system can intervene. However, the analytic process may not intervene if the student is confident in the initial heuristic-based response. If the analytic process does intervene, it may change or inhibit the default heuristic-based response cued by the SDF. This will depend on the extent to which the response is judged as
satisfactory. However, an analytic intervention does not always result in a rigorous evaluation of a heuristic-based response. The analytic process is also prone to reasoning biases (e.g., confirmation bias), so heuristic-based reasoning cued by the SDF may persist if, for example, the analytic process solely focuses on reasoning that supports the initial heuristic-based response. If a student believes a response is reasonable, the student tends not to consider other less salient but relevant features or to search for alternative explanations.

In addition to dual-process theories, a resource-based framework also can account for the impact of SDFs. A resource-based framework models cognition as consisting of resources, or knowledge elements, which are activated depending on context (diSessa, 1993; Hammer et al., 2005). Furthermore, for a given situation, when a particular resource is activated, other resources may be cued or prevented. Thus, within this framework, the SDFs of a given situation may activate particular cognitive and epistemological resources of a learner, which can lead to incorrect lines of reasoning. Similar to a heuristic-response cued by an SDF in the H-A framework, the activation of resources cued by an SDF can also occur under the surface without a learner’s awareness.

In either framework, SDFs essentially have the same effects on students’ reasoning. Both frameworks also share the view that the impact of SDFs may occur at an implicit/subconscious level.

**Cognitive Approaches Addressing the Impact of SDFs**

Prior studies have addressed the impact of SDFs on student reasoning using a primarily cognitive perspective. There have been two general related approaches. The first approach is strengthening students’ domain knowledge. The second approach is specifically teaching students the relevant variables of a situation,
without the explicit consideration of SDFs. Both approaches stem from a body of studies exploring the processes that influence participants’ visual attention as they complete different tasks. Typically, these studies use eye-tracking methodologies to measure the duration and location of participants’ eye movements. A common finding is that participants’ domain knowledge may guide what feature receives the most attention. For example, Madsen and colleagues (2012) investigated college students’ visual attention during physics problem solving. Using eye tracking, they determined which areas of a diagram in a physics problem received the most attention. Their results show that students who incorrectly answered the problems spent more time looking at irrelevant areas of the diagram in a physics problem containing the SDFs. In contrast, students who answered correctly spent more time looking at the relevant areas of the diagram. The researchers assumed that students who answered correctly had greater domain knowledge compared to students who answered incorrectly. Thus, based on this assumption, they concluded that students’ physics content knowledge played an important role in guiding their attention.

The role of domain knowledge on attentional allocation is also shown in other studies investigating the differences between the eye movements of experts and novices. Research indicates that experts attend to more relevant features and tend to use both conceptual and perceptual strategies based on these relevant features when approaching problems (Feil & Mestre 2010; Jarodzka, Scheiter, Gerjets, & Van Gog 2010). So, accordingly, an approach designed to strengthen students’ domain knowledge could help students focus more on the relevant features and avoid the distraction of the SDFs. This approach has been supported by studies in a variety of contexts that show participants who were initially distracted by irrelevant features of a situation tended to shift attention toward
more relevant features after general instruction on the topic (e.g., Hegarty, Canham, & Fabrikant, 2010).

In addition to strengthening domain knowledge, researchers examining the differences between the eye movements of experts and novices suggest another approach to addressing the impact of SDFs. Because experts spend more time attending to the relevant features of a situation, researchers also argue that cues may help learners recognize and focus on the relevant features (Hegarty, Canham, & Fabrikant, 2010; Jarodzka, Scheiter, Gerjets, & Van Gog, 2010). A study by Madsen and colleagues (2013) offers some evidence of the benefits of providing cues to redirect students’ attention toward relevant features of a situation and away from SDFs. The researchers investigated whether visual cues, in which relevant areas of a diagram were highlighted, improved students’ task performance while solving physics problems on four different concepts. College students, all with the requisite knowledge of the targeted concepts, were split into two conditions. One condition received visual cues, while the other condition did not. Using eye tracking, the results showed that students in the cued condition spent more time more on the relevant areas on a transfer problem for one of the four concepts and provided the correct reasoning. On the other concepts, they also spent significantly less time focusing on irrelevant areas, which contained the SDFs, but they still answered the problems incorrectly. The findings were modest but provide some promise to help students focus on relevant areas of a diagram in a physics problem and ignore the areas containing the SDFs.

Other studies have used a more explicit and direct approach to focus students’ attention to the relevant features of questions containing SDFs. For instance, within a physics context, Heckler and Scaife (2010) conducted a series of experiments to model incorrect answer patterns, cued by irrelevant dimensions of a problem, or SDFs, based on college students’ processing times to complete physics
tasks similar to the task in Figure 1.1. In their experiments, they examined approaches that would mitigate the effects of an SDF on physics graph questions. The researchers argued that students often relied on the feature of the problem that they could process the fastest. In the context of finding an electric field from a voltage versus position graph, they hypothesized that students processed the heights (irrelevant feature) faster than the tangents of the slopes (relevant feature) of the curve. Thus, they reasoned that if students could process slopes as quickly as positions, then students could inhibit their responses based on the height, which cued the incorrect reasoning that electric field is stronger when the height is greater. They found that college students who practiced physics graph questions and received immediate feedback (i.e., the right correct answer) improved their performance on similar problems. They also found that college students given an explicit general rule based on the slope, the relevant feature, on how to find the electric field from a voltage versus position graph also improved their performance on related questions.

These findings suggest that the impact of SDFs on learners’ reasoning approaches may be mitigated by such cognitive approaches as building learners’ content knowledge and helping them to identify the relevant features of situation. Although they may not be specifically targeting SDFs, many classroom practices and pedagogical approaches seem to fall within these approaches and have potential to address SDFs. For example, a general classroom practice is examining common conceptual difficulties, which may be related to SDF-cued reasoning. Many research-based instructional materials and strategies employ this approach, including Tutorials in Introductory Physics (McDermott & Shaffer, n.d.), Peer Instruction and recommended clicker question practices (Mazur, 1997; Beatty, Gerace, Leonard, & Dufresne, 2006), and Interactive Lecture Demonstrations (Sokoloff & Thornton, 1997). In some cases, documented incorrect reasoning
patterns and conceptual difficulties may be cued by an SDF associated with a
given situation. By evaluating the incorrect lines of reasoning, students learn why
the incorrect reasoning cued by the SDF is inappropriate. That is, students learn
the actual relevance of the SDF. For example, in Figure 1.1, the SDF was the
intersection and cued the reasoning that the cars’ speeds were the same at the
intersection. In evaluating this incorrect reasoning, students may learn that the
intersection indicates when the locations of the cars are the same on a
position-time graph. Students strengthen their content knowledge and learn the
meaning of the intersection for position-time graphs. As a result, they learn both
the relevant and irrelevant features of the situation, preventing incorrect lines of
reasoning cued by the SDF in similar situations.

Along with general classroom practices, there are specific pedagogical
strategies to help students distinguish between relevant and irrelevant features of a
situation. Ranking tasks may serve as one such example (O’Kuma, Maloney, &
Hieggelke 2000; Maloney 1987). A ranking task presents a set of variations on a
physical situation. Values for two or more features are given, and students are
asked to rank the variations based on some specified quantity. By ranking,
students need to consider multiple features together and to determine the
relationship among them within the given situation. Through multiple ranking
tasks, students may learn the situations when features are relevant or irrelevant
and the interaction among them.

**Metacognition as a Complement to Cognitive Approaches**

The prevailing approach that is used to address the impact of SDFs on
students’ reasoning is to strengthen students’ content knowledge and
understanding, either broadly or specifically toward SDFs and the associated cued
reasoning. With a stronger and wider knowledge base, the argument is that students will be more likely to resist the allure of SDFs that lead to them down incorrect reasoning paths. However, these approaches are highly context-dependent and may not explicitly support students to generalize the effects of SDFs across multiple contexts within physics, or more generally to outside the domain. Furthermore, current research-validated and research-based instructional materials broadly target students’ conceptual understanding on specific topics without explicit consideration for the SDFs. Although these instructional materials have been shown to lead to improved student understanding (e.g., Thornton & Sokoloff, 1998), which may indirectly also help mitigate the impact of SDFs, an increasing number of studies show that even after research-based instruction, students may still base their reasoning on the SDFs of a situation (Kryjevskaia et al., 2014). Strong content knowledge is necessary and may contribute to overcoming the impact of SDFs, but it may not be enough.

**Metacognitive Regulation on the Impact of SDFs Can Inform Metacognitive Knowledge**

Researchers, using a dual-process theory of reasoning, have argued that metacognitive regulation and reflection could help learners overcome the interference of SDFs on their reasoning and engage in more analytical reasoning (Thompson, 2009; Mamede, Van Gog, Van Den Berge, Van Saase, & Schmidt, 2014). In addition, studies in various contexts show that inhibitory control mechanisms play an important role in overcoming SDF-cued responses (Babai, Shalev, & Stavy, 2015; Lubin et al., 2016). Metacognitive regulation and function supports inhibitory control since metacognition is responsible for control and regulation of one’s thinking. For instance, Heckler and Scaife (2010) found that
withholding students’ responses (3-second delay) reduced the effects of the SDFs in the context of physics questions, and students were more accurate in their responses. The researchers suggested that the imposed delay prompted students to use a different strategy rather than to rely on their initial, implicit response cued by the SDF. Although not argued by the researchers, a potential explanation is that an imposed delay provided an opportunity for metacognitive processes to intervene so that the SDF-cued response could be inhibited. This finding suggests that a “stop and think” strategy could provide an opportunity for metacognitive regulation and reflection to overcome the impact of SDFs.

In another study, Babai, Shalev, and Stavy (2015) provided students with a task-specific warning of the SDF to activate students’ inhibitory control mechanisms to overcome an SDF-cued response. The explicit statement cautioned students about the potential pitfall of a geometry task shown in Figure 1.2. In this task, students are asked to compare the perimeters of two shapes. The correct response is that the shapes have the same perimeter. The SDF is the difference in the areas, which cues the most common incorrect response that shape 1 has greater perimeter because it has greater area. Students were warned of the tendency of using the SDF (area) on task, reminded them to use the relevant feature (perimeter), and encouraged students to overcome this tendency. Sixth grade students who received the warning intervention performed better on the tasks compared to those who did not receive the warning. By providing knowledge about the tendency to use SDFs, students were able to inhibit the SDF and consider the relevant feature of the task to arrive at the correct response by engaging in metacognitive regulation.

These studies demonstrate the benefits of strengthening students’ metacognitive skills to address the impact of SDFs. However, even if students could intentionally control their thinking via metacognitive regulation to inhibit an
Figure 1.2: Example of an SDF in a geometry task. Drawn from Babai et al. (2015), the task is to compare the perimeters of the two shapes. The salient distracting feature is the difference in the areas which cues the most common incorrect response that the shape 1 has a greater perimeter because it has a greater area.

SDF-cued response, students may not have explicit awareness or understanding about how SDFs could influence their thinking (i.e., declarative metacognitive knowledge). Rather, students may be focusing on the content and the correctness of their response. For potentially greater learning beyond the task, along with the development of relevant metacognitive skills, it would be beneficial for students to have general declarative metacognitive knowledge of reflecting on the impact of SDFs. Researchers have argued that metacognitive knowledge can lead to more effective strategy use (Kuhn 2000; Zohar & Barzilai 2013).

Defining Metacognitive Knowledge

Broadly, metacognition is defined as the awareness, understanding, and regulation of one’s own thinking (Brown 1987; Flavell 1979). Metacognition is generally agreed to consist of two distinct components: metacognitive regulation and metacognitive knowledge (Schraw & Moshman 1995). Metacognitive regulation, or metacognitive skills, includes executive processes that monitor, control, and regulate cognition and learning such as planning, monitoring, and evaluation. Metacognitive knowledge refers to the awareness and understanding about cognition and cognitive processes and can also refer to knowledge about strategies for different tasks. These two components are inter-dependent as metacognitive knowledge is believed to facilitate regulation, and metacognitive
regulation can build and strengthen metacognitive knowledge \cite{Brown1987, Kuhn2000, Pintrich2002}.

I follow a common definition shared by many researchers, such as Brown (1987) and Schraw (1998), that divides metacognitive knowledge into three sub-components: declarative, procedural, and conditional. The first sub-component is declarative MK, which is the focus of this study. Declarative MK refers to knowledge about cognition and cognitive processes, which can include knowledge about oneself as a learner. This includes self-knowledge about one’s strengths and weaknesses about how one learns, and what one knows and does not know \cite{Brown1987, Kluwe1987}. In this study, I focus on a specific kind of declarative MK – *knowing that* one can reflect on the nature and characteristics of the factors influencing one’s thinking. Procedural and conditional MK, although not the focus of this study, are the other two sub-components of MK. Generally, procedural MK is *knowing how* to effectively use declarative knowledge, while conditional MK is *knowing why* and *why* to implement declarative and procedural MK \cite{JacobsParis1987}. Specifically, as it applies to this study, the procedural sub-component is *knowing how* to reflect on the factors influencing one’s thinking, while conditional sub-component is *knowing why* and *when* one would reflect. To distinguish MK from metacognitive thinking or regulation, MK does not refer to the actual use of the knowledge.

Another conception of metacognitive knowledge includes knowledge of thinking and learning strategies to complete different tasks \cite{Flavell1979, Flavell1992}. For instance, Flavell and colleagues (1992) separated MK into the three categories: MK of persons, MK of task, and MK of strategy. MK of persons refers to self-knowledge about oneself as a learner and differences with respect to others, which is consistent with the sub-component of declarative MK. MK of task and strategy refer to understanding the nature of task goals and the strategies required
to complete a task, both of which fall across all three sub-components of MK. Other researchers combine the sub-components together. Zohar (2006), in line with Kuhn and colleagues (2005), uses the term meta-strategic knowledge to describe explicit knowledge about the procedures of a thinking strategy used to complete to a task. This encompasses all three sub-components together and includes naming and describing the thinking strategy and explaining when, how, and why the thinking strategy should be used on a task.

The Need for a Study of Metacognitive Knowledge

There have been some studies investigating the role of metacognitive knowledge in overcoming SDFs. They indicate that learners with greater metacognitive knowledge of the nature of SDF-cued responses were more effective in the evaluation and regulation of their reasoning on tasks with SDFs (e.g., Amsel et al., 2008). However, there are few studies that focus on how to build students’ declarative MK. Similarly, in the broader literature in science and mathematics education, many of the studies focus on metacognitive skills, with less emphasis on metacognitive knowledge (Zohar & Barzilai, 2013). Thus, there is an implicit assumption that metacognitive knowledge is easily acquired.

For instance, specifically within physics education, the work is largely focused on supporting metacognitive regulation and reflection in order to develop student conceptual understanding (e.g., White & Frederiksen, 1998), as well as problem-solving and laboratory skills (Etkina et al., 2010; Kung & Linder, 2007; Singh, 2004; Singh & Haileselassie, 2010). For example, in studies by Singh and Haileselassie (2010) and Singh (2004), web-based tutorials were created for introductory physics courses. These tutorials were designed to scaffold student problem solving. The metacognitive aspect of the tutorials involved regulation,
specifically monitoring and evaluation supported by reflection. Students were
given reflective questions after completing physics problems. The reflective
questions were designed to help students “think about what they learned by
solving the problem and how it helps them restructure, extend and organize their
knowledge” (Singh & Haileselassie, 2010, p.1). However, the effectiveness of the
tutorials was not measured in terms of the quality of their reflections or the
metacognitive knowledge students may have acquired, but rather it was measured
solely on students’ content understanding and problem solving strategies.

In another study by Etkina and colleagues, the Investigative Science
Learning Environment (ISLE) materials incorporated design labs and reflection
with an explicit focus on developing scientific abilities in introductory physics
(Etkina et al., 2010). The design aspect of the labs engaged and scaffolded
students in metacognitive regulation via planning, monitoring, and evaluating. In
addition, the ISLE materials included rubrics for students, which provided
knowledge about evaluation strategies for problem solving. For example, unit
analysis (i.e., unit analysis determining whether a physics equation is physically
possible by checking if each term has the same units) was one such evaluation
strategy. It was hoped that students would utilize these strategies to monitor their
thinking. Furthermore, students could use the rubrics to facilitate metacognitive
reflection and self-assessment, although this was not required. The intervention led
to greater learning and scientific abilities, but because it was not the focus of the
investigation, students’ metacognitive skills and declarative MK were not
measured.

Some of these studies do address building metacognitive knowledge but
without much sophistication, as knowledge is told to students to be applied and
practiced later (Adler, Zion, & Mevarech, 2015; Etkina et al., 2010; Heller et al.,
1992; Mevarech, 1999; Zepeda, Richey, Ronevich, & Nokes-Malach, 2015). For
instance, Zepeda and colleagues (2015) developed 6th graders’ declarative and procedural metacognitive knowledge on the regulation strategies of planning, monitoring, and evaluating using direct instruction. Over the course of four weeks, in the experimental condition, students were given packets on each strategy and on the integration of all three, both in domain-general (puzzle problems) and domain-specific (physics problems) contexts. Each packet was self-guided and provided an explanation, worked examples, and practice with the targeted metacognitive skill. In the control condition, students completed similar packets that omitted the explanations and worked examples of the metacognitive skills. The researchers found the students in the experimental condition had a modest gain in their declarative metacognitive knowledge of metacognitive strategies and improved their strategy use.

In addition to direct instruction, other pedagogical approaches for teaching MK include reflection, discussion, and presentation of metacognitive knowledge (Roll, Aleven, McLaren, & Koedinger 2007, 2011; Zohar & Barzilai, 2013). For example, Roll and colleagues (2007, 2011) used a combination of pedagogical approaches (e.g., feedback, self-assessment, instructional video, and discussion) to support middle school students’ metacognition (with a particular focus on help-seeking strategies) in a computer-based learning environment while learning geometry. Help-seeking is metacognitive because it involves knowing what one understands or does not understand (i.e., metacognitive knowledge) so that one can improve learning (i.e., metacognitive regulation). Their approach was rooted in a learning-by-doing approach, in which students practiced help-seeking strategies and received feedback to build metacognition. Immediate individualized feedback was explicit declarative MK given to students based on an analysis of patterns in their log files, which recorded the hints students requested or the glossary searches students conducted. For example, a student who abused hints
could receive the following message: “By clicking through hints you may solve the problem faster, but you will not learn, and you may have similar problems the next time you encounter a similar problem.” Self-assessment questions asked students to reflect on their learning and understanding when solving a problem (e.g., Do you think that you can solve a similar problem without hints?) Lastly, students also built aspects of metacognitive MK through a video-based class discussion about effective and poor help-seeking behavior and the value of help-seeking.

Taken together, as useful as these approaches may help support the building of MK, these studies do not use learning strategies that might increase the transferability of the generated metacognitive knowledge. The paucity of studies that engage deeply with metacognitive knowledge seems to highlight a missed opportunity for informing pedagogical practices to increase learning of the targeted metacognitive knowledge, especially as it pertains to SDFs.

Building General Metacognitive Knowledge Using Contrasting Cases

An alternative approach to constructing general and transferable metacognitive knowledge is through the use of contrast. In a variety of domains, research has shown that the use of contrasts can help learners discern the relevant features that make a concept distinctive (Schwartz & Bransford, 1998). Within instruction, a number of cases or instances, called contrasting cases, can be used to indicate a principle in the form of a commonality that is present across a range of contexts. Contrasting case instruction has been effective in increasing the learning and transfer of diverse physics concepts such as rates (Schwartz, Chase, Oppezzo, & Chin, 2011), Faraday’s law (Kuo & Wieman, 2016), Shemwell, Chase, & Schwartz, 2015), and projectile motion (Chin, Chi, & Schwartz, 2016) when compared to other instructional approaches.
Purpose of the Dissertation

The literature offers many effective approaches to help students inhibit an SDF-cued response when completing different tasks. The prevalent approaches take a cognitive perspective, which includes strengthening students’ content and helping students learn the relevant and irrelevant features of the tasks. These approaches are helpful and can mitigate the impact of SDFs on student reasoning, but they are generally targeted to a specific task or topic. Therefore, they do not serve to generalize the impact of SDFs across multiple contexts. In addition, SDFs can still impact students’ reasoning even though they may have the requisite knowledge and skills to successfully complete the task.

Broadly, studies on metacognition focus on developing metacognitive skills through direct instruction paired with practice, or strategy training. Students practice the MK in many contexts over a period of time, which could contribute to developing more general metacognitive knowledge. Research shows that a “tell-and-practice” approach is of limited value for the development of conceptual knowledge (Schwartz et al., 2011). This suggests that a similar approach may also be of limited value for the development of metacognitive knowledge. Although there are studies that focus on building metacognitive knowledge, they do not leverage the current research base to develop general metacognitive knowledge.

These concerns underscore the need to understand how to develop generalized metacognitive knowledge. Thus, the goal of this study is to understand how students can build general metacognitive knowledge about the impact of SDFs on reasoning so that it is applicable both within and outside the physics domain. Specifically, my focus is declarative general metacognitive knowledge as defined as students’ understanding of reflecting on why and how reasoning can be influenced by SDFs when answering physics problems. To build this targeted metacognitive
knowledge, I used contrasting case instruction, which has been shown to support the development of robust knowledge in diverse domains. In particular, I drew upon “inventing with contrasting cases” (Schwartz et al., 2011) and derivative inductive activities (Shemwell et al., 2015) to create a task called synthesis, in which students looked across sets of contrasting cases simultaneously to synthesize and construct a general explanation that fits all sets. In addition, to build generalized knowledge, I used multiple contexts for the cases, including contexts across different physics topics and an everyday situation to illustrate the applicability of the targeted general metacognitive knowledge. Thus, the central research question guiding this dissertation is: How can students build general metacognitive knowledge (GMK) about SDFs via synthesis?

In the next chapter, I discuss the methods employed in a study that investigated whether synthesis can help students build general metacognitive knowledge. Broadly, students in the experiment either engaged in synthesis or not. In Chapter 3, I present the quantitative results of the experiment, and in Chapter 4, I discuss student learning of the GMK and provide insights on how students can build metacognitive knowledge. In Chapter 5, I present a qualitative analysis of students’ processes within synthesis and describe key productive processes that may support student building of the GMK. In Chapter 6, I discuss efforts in designing and implementing an online synthesis activity to build student GMK for classroom instruction. Finally, in Chapter 7, I summarize the findings and provide suggestions for future research, along with the implications for instruction.
Chapter 2
METHODS

The primary goal of the study was to understand how students may be supported in building general metacognitive knowledge about their reasoning processes when solving physics problems. To this end, the study was designed with two main purposes. The first purpose was to examine how synthesis, a form of contrasting case instruction, could be an effective approach for learning the targeted GMK. Broadly, synthesis presents students with multiple cases that share the same underlying idea and are in different contexts. It also provides a structure for students to search for commonalities and use contrast to abstract and generalize the underlying idea across the cases. The second purpose was to highlight the productive processes students used during the synthesis that supported the building of the targeted GMK. This is addressed in Chapter 5. The design also allowed for an examination of whether including a context outside of the physics domain within synthesis would impact students’ generalizations and abstraction of the GMK.

This chapter is organized into five sections. This first section describes the instructional approach to build student GMK called synthesis. The next section provides an overview of the design of experiment, which broadly investigates the learning of the GMK via synthesis, as well as the specific research questions. The third section describes the participants and context in which the study took place. The fourth section discusses the procedure, including the instructional materials. The final section describes the measures, as well as the associated coding and procedures for analysis.
Learning the General Metacognitive Knowledge via Synthesis

Informed by contrasting case instruction (Schwartz et al., 2011; Shemwell et al., 2015), a task called synthesis was designed to help students learn the GMK. Broadly, in synthesis, students were provided with cases in multiple contexts that shared the same underlying idea. These cases were also juxtaposed side-by-side with cases in same contexts characterized by the absence of the underlying idea. Students could learn the idea through searching for the commonality across the cases with the presence of the idea and through contrasting them to the cases with the absence of it.

Synthesis involved six cases placed in a $3 \times 2$ matrix (Figure 2.1). Each case was a vignette describing the approach of a hypothetical student to a situation containing an SDF. Each row represented a contrast pair, which consisted of two vignettes of hypothetical students considering the same situation, with one student using the targeted GMK and the other not using the GMK. Taking the rows together, there was contrast across the columns while each column was characterized by the same underlying idea, or invariant. The task directive was to compare the approaches of the hypothetical students in one column to those of students in the other and to state the defining characteristic of the approach in the metacognitive column.

To illustrate how learning can occur in synthesis, consider Figure 2.1. Students saw the $3 \times 2$ matrix in the dashed box. In this matrix, within each column, the cases shared the underlying idea, or invariant. In column B, the hypothetical students demonstrated the targeted GMK, while those in column A did not. However, column A had less structure, as it is defined in relation to column B. Students may learn the GMK by searching for the invariant across the three cases in different contexts within column B. The relevant portion of the
vignette is underlined in the figure. Across these cases, students could induce and abstract the GMK. Andy, Dana, and Fred all thought about why the SDF in each situation impacted thinking. In contrast to column B, the hypothetical students in column A did not think about why and how the SDF can impact thinking. There was an absence of the targeted GMK in the approaches of the hypothetical students in column A, which were in the same varied contexts as in column B.

The structure of the matrix provided students with two affordances to build understanding of the GMK: invariance and contrast. There was invariance within each column. In addition, there was also an invariant contrast, both held locally within each row and more globally across the columns. The combination of both contrast and invariance could help students differentiate the approaches and understand the defining characteristic of the metacognitive approach in column B.

**Design and Instructional Overview**

Broadly, the study was an experiment comparing the learning of the targeted GMK from synthesizing multiple contrast pairs (*synthesis* condition) to sequentially processing the same contrast pairs (*non-synthesis* condition). Students participated in one-on-one think-aloud interviews with no interruptions. Figure 2.2 provides an overview of the design, which consisted of four phases: (1) processing

---

**Figure 2.1: Structure of the matrix used for synthesis**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does not think about why and how SDFs impact thinking</td>
<td>Thinks about why and how SDFs impact thinking</td>
</tr>
<tr>
<td>Blair ignores the coefficients of static friction, and uses Newton’s 2nd law. She thinks Andy made a mistake using the coefficients.</td>
<td>Andy uses the coefficients of static friction and realizes he should have used Newton’s 2nd law. He thinks about why he used the coefficients.</td>
</tr>
<tr>
<td>Corey uses steeper slope to determine speed. He realizes he is wrong and decides to always use energy conservation.</td>
<td>Dana uses energy conservation to determine the speed. She wonders why steeper slope was tempting for others to use.</td>
</tr>
<tr>
<td>Ellen is tempted by the intersection point but recalls that the slope indicates the speed.</td>
<td>Fred is drawn to the intersection point but recalls that the slope indicates the speed. He thinks about why he focused on the intersection.</td>
</tr>
</tbody>
</table>

---

Figure 2.1: Structure of the matrix used for synthesis
paired contrasts sequentially, (2) synthesizing the same paired contrasts, (3) listening to a scripted lecture about the GMK, and (4) completing post-assessment items.

In phase 1, the purpose was to determine if students could spontaneously abstract and generalize the GMK without looking across multiple contrast pairs and without an explicit prompt to generalize. All students sequentially processed three contrast pairs. These were the same three that later appeared in synthesis. The task directive for each contrast pair had identical structure to the task directive for synthesis. Students were prompted to state how the approach of the hypothetical student on the right (metacognitive) differed from the one on the left (not metacognitive). Student responses from each individual contrast pair served as data sources \( a-c \) in Figure 2.2. For each contrast pair, students may generate GMK for the particular situation, but the knowledge is likely to be context-bound. However, through the comparisons, students may be able to generalize and abstract the GMK. Thus, as a screening question, students were asked to identify any commonalities of approaches across the three contrast pairs immediately after they examined and discussed the third contrast pair. The question was, “Are there any similarities across the approaches of any of the students?” Student responses from the pre-screening question served as data source \( d \).

Only students in the synthesis group completed phase 2. The purpose of this phase was to provide students with the opportunity to look across all contrast pairs simultaneously, with an explicit prompt to generalize across all pairs. The task directive was to explain how the approaches in column B were different than the approaches in column A. The prompt was the following: “In general, how are students in column B approaching the situation differently from the way that students in column A are approaching it?” As described earlier, students may learn the GMK by using a combination of commonalities and contrasts between the
Figure 2.2: Overview of study design

Phase 1
Processing contrast pairs sequentially

Phase 2
Synthesizing contrast

Phase 3
Scripted lecture about the GMK

Phase 4
Assessment

Synthesis
$n = 37$

No synthesis
$n = 19$

Sources of data

Assisted synthesis

Pre-screening question:
Finding commonalities across the all cases

Defining SDF and its impact on thinking using the paired contrasts as examples

Four assessment items: two recognition and two application items
cases to describe how column B (metacognitive) differed from the other column (not metacognitive). Student responses were data source \( e \).

After students completed the phase 2 synthesis task independently and arrived at a final response, the interviewer decided whether to provide students with assistance in the synthesis depending on how far students progressed in their understanding of the GMK. Broadly, the interviewer helped students by using three different types of prompting. The first type of prompt directed students to explain or elaborate their idea of the GMK based upon the cases (e.g., “How does your generalization for column B apply to the cases?”). The second type of prompt directed students to find the commonalities among two or more cases within a column (e.g., “How is Andy similar to Dana?”). The last approach prompted students to contrast two or more cases across the columns (e.g., “How is Andy different from Blair?”). Students generated additional ideas about the GMK with assistance. These articulated ideas served as data source \( f \).

Synthesis is a form of contrasting case instruction that can prepare students for future learning, such as that from direct instruction (Schwartz & Martin, 2004). Ideally, in the first two phases, students would have built early knowledge of the GMK, which may not have been well structured. Thus, following synthesis, in phase 3, the interviewer provided students with an overview of the GMK. To give students the same information, the interviewer used a script to explain SDFs and to define metacognitive thinking about SDFs and their impact on one’s thinking, using the contrast pairs as examples. The purpose of this phase was twofold. First, the lecture could provide students with a way to organize their understanding of the GMK and enhance the learning from synthesis, revealing any learning in the synthesis conditions. Second, this phase could show that the learning of the GMK was not trivial. If students could learn the GMK from the
scripted lecture without engaging in synthesis, then the synthesis and non-synthesis conditions would perform similarly on the post-test items.

In the final phase, students completed four assessment items designed to probe their learning of the GMK. The items presented students with situations containing an SDF, and all items were in different contexts. There were two types of items: recognition and application. In a recognition item, students were asked to identify the GMK among different options of hypothetical student thinking about a situation. Broadly, recognition items presented four different ways of thinking about a situation, with only one of them illustrating the targeted GMK. Students were required to identify which hypothetical students demonstrated the GMK and explain how they could tell. Students indicated their understanding of the GMK by explaining and differentiating the metacognitive thinking from other ways of thinking. Recognition items were included because they were typically less difficult than application items, such that they were likely to be the most sensitive in detecting any learning of the GMK.

There were two application items, where students were required to explain how the GMK was applicable to a situation that contained an SDF. In the situation, a hypothetical person’s thinking was impacted by the SDF. The application item had two parts. The first part was “unprompted application.” Students were given a general prompt, which did not refer to the GMK (e.g., “What should a student do to make sure he or she fully understands the reasoning involved and learns as much as possible from this question?”), to see if they could spontaneously apply the GMK. Spontaneous application of the GMK to the situation would suggest strong learning of the GMK. The second part was “prompted application,” and the prompt directed students to apply the GMK to the situation (e.g., “How can reflecting on the role of alluring features in this particular case assist your friend?”).
The assessment items included situations both within and outside of the physics domain. Indeed, each type of item (recognition and application) was posed both in a physics context and in an everyday context. In the physics context, the GMK is within the problem-solving process, in which there is an accepted correct response. In the everyday context, the GMK is within a decision-making process, in which there is an appropriate response based on an articulated preference or goal. Taken together, these items across different contexts measured how well students were able to abstract and generalize the GMK. Everyday contexts may indicate greater abstraction and generalization of the GMK. This is because if the learning of the GMK was primarily within the physics context, students may have abstracted the GMK only within physics domain and not beyond.

The synthesis group was further split into two groups, which were either given a third contrast pair in an everyday context or given a third contrast pair in another physics context (i.e., no everyday context). For the synthesis without the everyday context group, the paired contrasts were all within the physics domain but were presented in different physical contexts, as previously discussed and shown in Figure 2.2. The no synthesis group used these same contrasts pairs. In the synthesis with an everyday context group, the last row of the matrix was replaced with a contrast pair featuring an everyday situation involving grocery shopping. This everyday situation was parallel to the physics situation in that it contained an SDF and described metacognitive thinking about the impact of the SDF. However, the everyday situation extended the GMK in context and process. As previously mentioned, within the physics situations, the GMK was specific to the problem-solving process. Within the everyday situation, the GMK was within a decision-making process. It was hypothesized that the inclusion of a contrast pair within an everyday contrast may lead to greater abstraction and generalization of the GMK beyond the physics context.
The primary goal was to compare the learning of the GMK between the synthesis and no synthesis groups. However, if the synthesis groups were to outperform the no synthesis group on the post-test items, there may be two alternative explanations stemming from the study design. The first is that the synthesis group saw the same contrast pairs twice (phases 1 and 2), whereas the no synthesis condition only saw them once (phase 1). With a second exposure to the same contrast pairs, synthesis students had greater time on task to process the contrast pairs, which could have contributed to greater learning. If so, then synthesis may not have been the primary reason for the stronger performance. The time-on-task factor may have been eliminated altogether if the synthesis conditions did not initially process the contrast pairs separately (phase 1) and synthesized the pairs only (phase 2). However, this may have introduced the alternative explanation that the non-synthesis and synthesis conditions differed in cognitive load. Processing the three contrast pairs and then considering all three simultaneously would be more difficult and impose a greater cognitive load compared to considering each pair one-by-one. This would have been a weaker design, as the unequal cognitive load across conditions cannot be easily ruled out.

With the present study design, two additional measures were included to rule out time on task as a factor. The first measures are student responses from the individual contrast pairs (data sources $a-c$). The second measure is the pre-screening question, which occurred immediately after phase 1 (data source $d$). These measured how well students were able to generalize and abstract the GMK when processing each pair individually when not prompted to generalize and after processing all the contrast pairs when prompted to generalize. If these measures show that students do not generalize the GMK, then it may be argued that students would likely not generalize with a second exposure to the cases.
The second possible alternative explanation is that synthesis students received assistance immediately after completing synthesis on their own. The assistance may have helped students learn the GMK as well. To check against this possibility, data source was included, which consisted of students’ responses during synthesis. If students learned from the assistance, as measured by their responses from synthesis, then their responses would be correlated to their post-test performance. Thus, if students’ responses with assistance and post-test performance were not related, then the learning from assistance would be contributing a negligible amount to students’ learning of the GMK.

Finally, there were two mediating variables that could help indicate whether or not the learning of the GMK occurred within synthesis. The first was students’ responses to synthesis without assistance. The second variable was the quality of students’ progress during synthesis from start to end, which measured a relevant change in students’ ideas towards the GMK. Both of these represented the quality of student learning within synthesis. Thus, if synthesis helped students learn the GMK, or mediated the learning of the GMK, then there should be a relationship between students’ learning within synthesis and students’ post-test performance.

**Research Questions**

The study was designed to answer the central research question: How can students build general metacognitive knowledge (GMK)? In particular, I sought to determine how synthesis could support student learning of the GMK. Specifically, the targeted GMK was an awareness of reflecting on how and why salient distracting features can impact thinking when solving physics problems. Therefore, I asked the following research questions:

1. To what extent does synthesis help students learn the GMK?
2. To what extent does an inclusion of an everyday context within synthesis help students to abstract and to generalize the GMK?

3. To what extent is the learning of the GMK located within synthesis?

4. How can students productively use the affordances of synthesis (invariance and contrast) to build the GMK?

Participants and Context

Context

The study was conducted at the University of Maine in an introductory calculus-based electromagnetism course, which was the second semester of a two-course introductory physics sequence, in Spring 2015. A total of 184 students were enrolled in the course. Generally, students were first-year engineering majors. The class was comprised of approximately of 80% males and 20% females. Data were collected in the last four weeks of the semester.

Participants

Fifty-six students were recruited to participate in the study. Approximately 73% of participants were males and 27% were female. The average final exam scores of participants were slightly higher than the course average, 64% vs. 60%, respectively. As an incentive to participate, students were given a small amount of extra credit as well as 10 USD compensation.

Participant Recruitment

To recruit participants, I made an announcement at the beginning of lecture. The announcement provided a general and vague overview of the study’s topic as student reasoning approaches in physics. I described that students would
participate individually, and they would review and compare sample student approaches to qualitative, conceptual physics questions, involving topics from their first-semester mechanics course. To encourage participation, I emphasized to students that the answers would be provided for them as I was mostly interested in their thinking about different reasoning approaches. I directed students to the posted announcement on the course webpage. The announcement contained a link to an online scheduler, through which students could view open one-hour time slots and schedule an interview up to 3 weeks in the future.

Assignment to Conditions

Initially, before recruiting participants, a list was prepared for a total of sixty participants. The participants were numbered from 1 to 60 and randomly assigned one of the three conditions, with equal numbers across the conditions. In the order of the time of the interview, students were assigned the next participant number and associated condition on the list.

Procedure

Students individually participated in audio-recorded, one-on-one think-aloud interviews. Although there were officially no time limits and students were encouraged to take as much as they needed, the sessions were each kept under one hour. On average, students completed the study in 40 minutes. Synthesis students, on average, took 10 minutes longer than non-synthesis students. I was the sole interviewer, and I followed a scripted protocol (included in Appendix A) and took notes on a laptop computer during each student’s think-aloud interview. The following sections provide an overview and description of instructional tasks and the protocol procedure.
Overview of Instructional Tasks

There were two instructional tasks for the learning of the targeted GMK. The first task was the individual contrast pairs. All students considered the three contrast pairs one at a time. The second task was the synthesis matrix. Only students in the synthesis condition completed the synthesis matrix.

**Contrast pairs.** Students processed three contrast pairs sequentially in phase 1. These same pairs were used later in the synthesis matrix. Two cases were presented side-by-side in a contrast pair. Each case presented the problem context, the hypothetical student's written response to the problem, and the vignette of the hypothetical student's approach, which was in a framed textbox. Each case was printed on a single 8.5” × 11” sheet of paper and then attached side-by-side to form the contrast pair. The task directive was provided on a separate piece of paper, and each task directive had identical structure: “How is [student B] approaching the situation differently from the way that [student A] is approaching it?”

**Synthesis.** Only students in the synthesis group received the 3 × 2 matrix containing the same vignettes of the three paired contrasts presented in phase 1. The matrix spanned the space of a single sheet of legal size paper in a landscaped orientation, with the task directive placed directly below the matrix. The row number is parallel to the order that students processed the contrast pairs in the prior phase. The first three rows of Table 2.1 comprised the full contents of the 3 × 2 matrix for the synthesis without an everyday context subgroup (problem contexts shown Fig. 2.2). For the other synthesis sub-group, synthesis with an everyday context, the third row was swapped out for the everyday context. The task directive had identical structure as the individual contrast pairs. The prompt was the following: “How are the students in column B approaching the situation differently from the way that the students in column A are approaching it?”
Table 2.1: Hypothetical students’ vignettes for the contrast pairs and matrix

<table>
<thead>
<tr>
<th>Order</th>
<th>Subject matter</th>
<th>Absence of GMK</th>
<th>Presence of GMK</th>
<th>GMK</th>
<th>Irrelevant contrasts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Dynamics</td>
<td>• Blair ignores the values for $\mu$.</td>
<td>• Andy uses $f = \mu N$ to calculate static friction.</td>
<td></td>
<td>Presence/absence of correctness</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• She correctly applies Newton’s 2nd law.</td>
<td>• He checks his answer with Blair and realizes he should have used Newton’s 2nd Law instead.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• She checks her answer with Andy who got a different answer.</td>
<td>• He wonders why he used $\mu$.</td>
<td></td>
<td>Absence/presence of equations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• She thinks that Andy made a mistake by using $\mu$.</td>
<td>• He decides that having a number for $\mu$ made him think he needed to use it.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Energy conservation</td>
<td>• Corey thought that steeper implies faster, which was wrong.</td>
<td>• Dana uses energy conservation and gets the right answer.</td>
<td></td>
<td>Dana thinks about why others were drawn to use the slope.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• He realizes energy conservation gives the right answer.</td>
<td>• A lot of students answered incorrectly because they thought that steeper implies faster.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• He decides to always use energy conservation.</td>
<td>• She thinks about why the steeper slope is tempting.</td>
<td></td>
<td>Absence/presence of correctness</td>
</tr>
<tr>
<td>3P</td>
<td>Kinematics</td>
<td>• Ellen notices the intersection point.</td>
<td>• Fred’s eyes are drawn to the intersection point.</td>
<td></td>
<td>Fred thinks about why he was drawn to the intersection point.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• She thinks the intersection point could be the answer.</td>
<td>• He then looks at the slope and realizes the velocity is the same when the slope is the same.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• She recalls that the slope corresponds to the velocity and gets the right answer.</td>
<td>• He decides that he wasn’t clear about what needed to be the same and focused on the first thing that he saw – the intersection.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3E</td>
<td>Grocery shopping</td>
<td>• Ellen is in the cereal aisle and searches for Honey Nut O’s.</td>
<td>• Fred is grocery shopping and looks for Honey Nut O’s.</td>
<td></td>
<td>Fred thinks about why he was drawn to the Fruity O’s.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• She picks up a box of Naturally Flavored Fruity O’s, but puts it back down.</td>
<td>• A box of Naturally Flavored Fruity O’s jumps out at him, and he is tempted to choose it over Honey Nut O’s.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• She puts a box of Honey Nut O’s in her cart.</td>
<td>• He thinks maybe it’s because adjectives like “naturally flavored” and “fruity” make the cereal seem healthy.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Synthesis task directive: How are the students in column B approaching the situation differently from the way that the students in column A are approaching it?
Contrast Pairs Used. An example of a contrast pair is shown in Figure 2.3. This was the first contrast pair students considered, which was in the physics context of dynamics. The problem context presented two identical blocks, on different surfaces. The prompt stated that the same horizontal force is applied to both boxes, and both boxes remain at rest. A correct response requires students to recognize that the acceleration is zero in both cases and therefore the static friction forces on both boxes must equal the identical applied forces via Newton’s second law. Thus, the friction force on each box is the same, and therefore equal to one another. In this problem, the SDF is the difference in coefficients of static friction, which can lead to the most prevalent incorrect response that the friction force on box A is less than the friction on box B. The SDF can cue two possible incorrect reasoning approaches to arrive at this response. One is by comparing the coefficients of static friction, and the other approach is by using those coefficients to compare the (maximum) values of the force of static friction via $f = \mu N$. In this contrast pair, the targeted GMK is demonstrated through Andy’s approach in case b. He focuses on the coefficient of static friction, the SDF, which cues him to use $f = \mu N$. When he realizes he is wrong, he thinks about his reasoning and what cued his approach. In contrast, in case a, Blair ignores the values for $\mu$. When she checks her answer with Andy, she does not think about why the SDF is attractive and how it may have impacted Andy’s reasoning. The task directive was to identify how Andy and Blair approach the same situation differently. The relevant difference is the absence/presence of the targeted GMK. However, there are two other irrelevant differences, which are the absence/presence of equations and the presence/absence of correctness. Andy’s reasoning is equation-based and incorrect, but highlights the targeted GMK. Blair’s reasoning is correct, but she does not demonstrate the targeted GMK.
Suppose the coefficient of static friction between box A and the floor is 0.4, as shown at right. The coefficient of static friction between box B and a different floor is 0.6. \( m_A = m_B = 10 \text{ kg} \)

A horizontal 30 N force is applied to each box, and both boxes remain at rest. Is the magnitude of the friction force exerted on box A greater than, less than, or equal to that exerted on box B? Explain.

(a) Blair

**Written Response:** Equal. The boxes have the same mass and same applied force. Boxes remain at rest and since they aren’t any other horizontal forces acting the applied forces, both must remain the same.

**Vignette:**
- Blair ignores the values for \( \mu \).
- She correctly applies Newton’s 2nd law.
- She checks her answer with Andy who got obtained a different answer.
- She thinks that Andy made a mistake by using \( \mu \).

(b) Andy

**Written Response:**

\[
\begin{align*}
    f_A &= (0.4)(100) = 40 \text{ N} \\
    f_B &= (0.6)(100) = 60 \text{ N}
\end{align*}
\]

Box B has greater frictional force because it has a greater coefficient of friction.

**Vignette:**
- Andy uses \( f = \mu N \) to calculate static friction.
- He checks his answer with Blair and realizes he should have used Newton’s 2nd Law.
- He wonders why he used \( f = \mu N \).
- He decides that having a number for \( \mu \) made him think he needed to use it.

**Figure 2.3:** Sample contrast pair

The details for the other contrast pairs are summarized in Tables 2.2 and 2.1. Broadly, Table 2.2 describes the situation contexts presented in each contrast pair, while Table 2.1 provides the complete vignettes for each pair. More specifically, Table 2.2 describes the aspects of the situation contexts that made the GMK applicable. It summarizes the situation contexts with the corresponding appropriate and potentially SDF-cued responses to the situations. The first column of the table indicates the order in which the pairs were presented to students. The second column describes the situation context considered by the hypothetical students. The situation contexts were drawn from different topics within the physics domain, such as force, energy conservation, and kinematics. The third column provides a possible correct response to the situation (i.e., physics question or everyday decision) from a hypothetical student. Within the physics contexts, the correct responses were based on the relevant features of the
situation. The last column provides a typical incorrect SDF-cued response from a hypothetical student who was, in fact, impacted by the SDF.

Table 2.1 includes explanations of how the GMK was applied within each problem context and displays the full vignettes of the hypothetical students. The first column matches the same problem contexts as in Table 2.2. In the next two columns, the vignettes are shown. As indicated in the column headings, the same contrast of the absence/presence of the GMK held across every pair. There was an absence of GMK in the cases on the left while there was a presence of GMK in the cases on the right. How the hypothetical student in the right cases within each contrast pair used the GMK is explained to the right of the vignettes. All of the hypothetical students in the right cases shared the same underlying approach: they thought about why they were drawn to the SDF. Finally, in the last column, other irrelevant contrasts across the cases are summarized, such as the absence/presence of equations or correctness. The contrast pairs were designed to vary in the number of irrelevant contrasts, which may impact the difficulty students encounter when attempting to articulate the GMK. The contrasts pairs decreased in the number of irrelevant contrasts as students received them.

For one of the synthesis sub-conditions (*synthesis with an everyday context*), the last paired contrast (3P) was replaced with one in the context of grocery shopping (3E). These paired contrasts were not isomorphic in context or structure. For example, the physics problems contexts had well-defined correct responses, while the everyday problem context did not. More broadly, however, the everyday problem context had appropriate responses based on one’s goals and preferences. In addition, within the vignettes, the actions taken by the hypothetical students across the contexts were not identical. Nonetheless, the everyday context shared the same invariant contrast (GMK absence/presence) with the replaced physics context, and neither contained irrelevant contrasts.
Table 2.2: Situation contexts for the contrast pairs

<table>
<thead>
<tr>
<th>Order</th>
<th>Subject matter</th>
<th>Problem context</th>
<th>Appropriate response (Relevant feature in bold)</th>
<th>SDF-cued response (SDF in bold)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Dynamics</td>
<td>Suppose the coefficient of static friction between box A and the floor is 0.4, as shown at right. The coefficient of static friction between box B and a different floor is 0.6, as shown below right. $m_A = m_B = 10$ kg. A horizontal 30 N force is applied to each box, and both boxes remain at rest. Is the magnitude of the friction force exerted on box A greater than, less than, or equal to that exerted on box B? Explain.</td>
<td>Because each box remains at rest, the acceleration is zero and thus the net force on the box must be zero. The static friction force and the horizontal force must be equal in magnitude in order to have no net force. Since the same 30 N horizontal force is applied to each box, the magnitudes of the friction forces on both boxes must also be equal.</td>
<td>The friction force is determined by $f = \mu N$. They boxes have the same normal force because they have the same mass. Then the friction force exerted on box A is less than that on B because coefficient of static friction on box A and the floor is less than that for box B.</td>
</tr>
<tr>
<td>2</td>
<td>Energy</td>
<td>A young girl wants to select one of the (frictionless) playground slides illustrated below to give her the greatest possible speed when she reaches the bottom of the slide. Which should she choose?</td>
<td>Since the girl starts from rest and has the same change in height on all three slides, conservation of total mechanical energy says that the girl will have the same kinetic energy, and therefore the same speed, when she reaches the bottom of all three slides.</td>
<td>Slide 1 will have the fastest speed at the bottom of the slide because it is steepest.</td>
</tr>
<tr>
<td>3P</td>
<td>Kinematics</td>
<td>The motions of two cars are described by the position vs. time graphs shown at right. When, if ever, are the speeds of the cars the same?</td>
<td>On a position versus time graph, the slope at any point indicates the speed. Thus, the cars have the same speeds when they have the same slopes at point A.</td>
<td>The intersection point is where the cars will have the same speeds.</td>
</tr>
<tr>
<td>3E</td>
<td>Grocery shopping</td>
<td>Ellen and Fred are grocery shopping for Honey Nut O’s, but they are tempted to buy the Naturally Flavored Fruity O’s.</td>
<td>The goal was to buy Honey Nut O’s, not Fruity O’s.</td>
<td>Fruity O’s is tempting to buy because it is naturally flavored and fruity, which makes it seem like it is healthier.</td>
</tr>
</tbody>
</table>
Instructional Procedure

The start of the interview involved typical activities, such as putting students at ease, informing students about the study, and requesting research permission. Because students were likely not familiar with a think-aloud method, students completed a short training session. The interviewer told students that in a think-aloud, they should say whatever they are thinking when they first see the task and until they reach an answer. In addition, the interview instructed students not to plan what they say or try to explain what they are saying, thinking, or doing. Students completed two simple think-aloud tasks as practice.

Once familiar with the think-aloud protocol, students processed the three contrast pairs independently in phase 1. These contrast pairs contained the same vignettes that the synthesis group saw in the next phase. Students were given both the contrast pair and task directive at the same time. On average, students processed all three contrast pairs in 6 minutes. Once students indicated they were done and had reached an answer for a given pair, the interviewer took both sheets from the students and gave the students the next contrast pair and task directive. Table 2.1 shows the order of the contrast pairs that students processed. Students in the non-synthesis condition and synthesis sub-condition, synthesis without an everyday context, processed the same three contrast pairs. For students in the other synthesis sub-group, synthesis with an everyday context, the last pair was replaced with an everyday context in a grocery shopping scenario.

After students completed all three contrast pairs, the interviewer gave students another sheet containing the pre-screening question: “Are there any similarities in the approaches used between any of the students?” Students did not have access to the contrast pairs when they were answering this question.
Only students in the synthesis conditions completed phase 2, which involved synthesis. Students in the non-synthesis condition went straight to phase 3. On average, students completed the synthesis phase in about 5 minutes, not including the practice exercise. At the start of phase 2, the interviewer told students that they would be looking across all the hypothetical students’ vignettes together. To familiarize students with the synthesis process, students were asked to complete a practice $3 \times 2$ synthesis matrix, which was in an everyday context (Appendix ??). Part of this practice also instructed students to verbalize their general procedure for arriving at their responses. When students finished, the interviewer provided a brief explanation of the practice synthesis task and a sample procedure.

After the practice, the interviewer gave students the $3 \times 2$ synthesis matrix, which contained the same vignettes that students saw in the prior phase. She took structured notes on students’ ideas of the GMK as they synthesized the cases. Based on how far students progressed in building their ideas about the targeted GMK, the interviewer decided in the moment on whether or not to assist students and also selected the level of assistance provided, as needed. (In total, seven students were not assisted because the interviewer judged, at the time, that their progress and final definition of the GMK were adequate.) While assisting the students, the interviewer provided no explicit verbal feedback about the correctness of their responses and restricted herself to prompts for students to find commonalities between cases, to contrast cases, and to elaborate their ideas.

In the next phase, the interviewer used a scripted lecture to provide students with an overview of the GMK. The entire phase lasted approximately 5 minutes. The overview consisted of three parts: defining SDFs, explaining the impact of SDFs, and describing the benefits of reflecting on the impact of SDFs. The scripted response for each part is provided in Table 2.3. During this overview, SDFs were referred to as “alluring features.”
First, the interviewer called the targeted GMK “thinking about the role of alluring features,” and connected it to a similar idea that each student mentioned in the prior phases. In the scripted response, an alluring feature was defined as something in a problem that automatically captures attention. The interviewer then prompted students to identify the alluring feature within each contrast pair. For the non-synthesis group, she used the individual contrasts pairs, while using the matrix for the synthesis group. In the second part, she described the impact of SDFs on one’s own thinking. She then prompted students to describe how the SDF impacted Andy’s thinking before providing a scripted explanation (see Fig. 2.3). In the final part, the interviewer said that it was a good idea to think about the role of alluring features and asked students to provide a reason why. She then provided her own scripted explanation that highlighted two key points. First, alluring features can occur without the students’ awareness. Second, although an alluring feature in a situation may not impact the students’ thinking in that moment, it may under different circumstances. Thus, students should be aware of alluring features and think about how they may impact their thinking as a means to overcome the associated distractions. At the end, the interviewer asked students to share any thoughts about the GMK.

In the final phase, students completed four post-test items that assessed their learning of the GMK. Each item was presented on a separate sheet of paper, and the interviewer gave the student each item one by one, taking back the prior item before giving next one. Starting with the two items within the physics context, students completed the application item (unprompted), then the recognition item, and returned to the application item (prompted application). Then students completed the items in the everyday context in the same sequence.

Because the scheduled interviews spanned across 3-week period, it was important to limit the potential communication between students over the course
Table 2.3: Scripted lecture overviewing the general metacognitive knowledge

<table>
<thead>
<tr>
<th>Description</th>
<th>Script</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name of the targeted GMK</td>
<td>Thinking about the role of alluring features</td>
</tr>
<tr>
<td>Definition of an alluring feature</td>
<td>An alluring feature is something in the problem that automatically captures our attention.</td>
</tr>
<tr>
<td>Role of an alluring feature</td>
<td>Alluring features cue us on how we reason or think about the question in a certain way and may lead us down an incorrect reasoning path.</td>
</tr>
<tr>
<td>Explanation of how an alluring feature impacted Andy</td>
<td>The coefficients cued Andy to use the equation and even though he knew Newton’s 2nd law, he wasn’t able to think about it because he was focused on the coefficients.</td>
</tr>
<tr>
<td>Explanation on why to think about the role of alluring features</td>
<td>Because alluring features captures all our attention and gives us tunnel vision, we can’t see anything else, and we’re might not even be aware that it’s happening. Even if it’s not an alluring feature for us in the moment it might be later on and impact how we respond to a different question. Being on the lookout for alluring features and taking some perspective on why someone is attracted by the alluring feature and what type of tempting reasoning it cues, can help us avoid and get out of these unintentional thinking traps.</td>
</tr>
</tbody>
</table>

of the study, which might prevent the inclusion of a student’s participation. Thus, at the start of the interview, the interviewer asked students to share what they have heard about the study, and at the end, the interviewer asked students not to discuss the details of the study, such as the procedures and tasks, with other students in the class.

**Instrumentation and Data Analysis**

This section broadly describes the measures with their coding scheme and analysis procedures. First, I discuss the overarching analysis framework that was used to code all measures. Then, I summarize the sources of data and measures. Next, I discuss the measures, starting with the learning process measures followed by outcome measures. Finally, the section ends with the procedures for data analysis.
Analysis Framework

I analyzed student responses to all measures using a three-level from 0 to 2 (Table 2.4). Broadly, the levels describe the reflective thinking involving an SDF-cued action, either the approach to solving a physics problem (physics context) or to deciding on a purchase (everyday context). The scale levels (0, 1 and 2) measured the extent to which a student articulated the elements of the GMK, with level 2 representing the targeted GMK. If students articulated the targeted GMK (level 2), their responses described thinking or reflection that focused on the thinking process that led to an SDF-cued action. For example, student responses that described reflecting about why a distraction occurred would fall within this level. In level 1 responses, students focused on the action itself. These responses typically described understanding why an SDF-cued approach was incorrect. Finally, in some responses, no elements of the thinking connected to the targeted GMK were articulated, and these responses were categorized as level 0, which indicated the absence of any elements of the GMK. A typical level 0 response focused on understanding the correct answer, and dismissed the SDF-cued approach altogether.

In summary, responses with levels greater than zero contained elements of the GMK. Both levels 1 and 2 focused on thinking that is related to an action. An action refers broadly to an SDF-cued approach to solving a physics problem or deciding on a purchase. In level 1 responses, the focus or object of reflection is the action itself, whereas in level 2 responses, the focus or object of reflection is the thinking process behind the action. Both are valuable, but only level 2 responses fully capture the targeted GMK.

The levels corresponds to a three-point scale (0, 1, and 2) measuring the extent of metacognitive reflection. The levels were ordinal and not necessarily
Table 2.4: Three-level metacognitive analysis framework

<table>
<thead>
<tr>
<th>Level</th>
<th>Label</th>
<th>Object of thinking</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No elements of the GMK</td>
<td>---</td>
<td>Focus of thinking is on the correct approaches to the situation (e.g., <em>What is the correct approach to this situation?</em>)</td>
</tr>
<tr>
<td>1</td>
<td>Some elements of the GMK</td>
<td>an action to self-regulate to guide or improve one’s action</td>
<td>Focus of thinking is on the appropriateness of an SDF-cued response (e.g., <em>Why is the SDF incorrect to use for this situation?</em>)</td>
</tr>
<tr>
<td>2</td>
<td>Full GMK</td>
<td>the thinking that led to an action to be aware of one’s cognitive processes</td>
<td>Focus of thinking is on how the SDF influenced a choice (e.g., <em>Why was I attracted to the SDF in this situation?</em>)</td>
</tr>
</tbody>
</table>

interval; that is, the difference between levels 0 and 1 is not the same as that between levels 1 and 2. Where exactly the intermediate level 1 lies between levels 0 and 2 is not known. However, what is known is that level 1 shares elements in common with level 2, the focus of the targeted GMK.

**Connections to Metacognition.** The three-level scale maps directly onto the definitions of metacognitive reflection established in the literature. The highest level (level 2) is consistent with the most common and accepted definition of metacognition, in which the objects of reflection are thinking processes (Flavell [1979]). More specifically, the object of reflection of the targeted GMK is the thinking process cued by the SDF. The purpose of this reflection on the thinking process is to bring the automatic cognitive processing that led to an action to the surface, thereby allowing a student to be aware of how he or she thinks. Awareness and understanding of one’s thinking are considered key components of metacognition (Brown [1987], Kuhn [2000]).

The intermediate level (level 1) describes reflecting on an action, which lies on the “fuzzy border” between metacognition and cognition. Reflecting on an action takes on many forms such as evaluation, which can refer to many different types of activities. For instance, self-explanation, a strategy students explain the
content material or their own problem solving steps to themselves, is a form of evaluation and widely accepted as a metacognitive activity (Aleven & Koedinger, 2002). Zepeda, Richey, Ronevich, and Nokes-Malach (2015) defined evaluation as assessing a solution to a problem, determining if the solution meets a goal, and evaluating the best strategies used to arrive at a solution. For Grotzer and Mittlefehldt (2012, p. 82), metacognitive thinking consisted of three dimensions: intelligibility (e.g., “Does this explanation make sense to me?”), plausibility (e.g., “Do I think that the explanation is a possible explanation?”), and wide applicability (e.g., “Can I apply the explanation beyond the contexts in which I have learned it?”). Many researchers consider evaluation as a marker of metacognitive thinking (e.g., White, 1992; Veenman, 2012). However, not all agree. For example, Barzilai and Zohar (2014) argued that evaluation is cognitive when the focus is on correctness of one’s understanding, thoughts, or ideas.

Flavell (1979) suggested that the intent behind reflection can help one ascertain whether that reflection is metacognitive or cognitive. Reflection is metacognitive when the intent is to monitor cognitive progress, whereas reflection is cognitive when the intent is to make cognitive progress. From level 1 responses, the intent accompanying the reflection about an action is difficult to infer. Moreover, because this level only captures some elements of the thinking related to the targeted GMK, it is not clear whether these responses are metacognitive or cognitive. However, this distinction was not critical, as the intervention was designed to target the knowledge of the metacognitive thinking represented by level 2.

**Sources of Data**

The primary sources of data were audio recordings of students’ think-aloud responses throughout the different phases. Figure 2.2 indicates when data were
collected. The four post-test items in phase 4 were outcome measures and assessed how well students learned the GMK. Data collected from phases 1 and 2 were used as process measures, which provided information of the learning processes that occurred during each phase. These included students’ responses to the three contrast pairs, pre-screening question, synthesis, and assisted synthesis, and all measured the extent of students’ abstraction and generalization of the GMK.

Generally, all of the measures were analyzed how well students articulated the targeted GMK. However, students’ entire synthesis responses were further coded on whether students’ ideas of the GMK changed and moved closer to the targeted GMK, an indication of building understanding of the GMK. All measures, as well as their coding, are discussed in the following sections and are summarized in Table 2.5.

Instrumentation

This section describes the measures and coding in greater detail, as well as provides examples of student responses. The learning process data are discussed first, and then the outcome measures follow.

Process measures. In total, there were seven process measures. Four were from phase 1, which included student responses to each of the contrast pairs and the pre-screening question. The last three measures were from phase 2, and included students’ responses in synthesis and assisted synthesis. Descriptions of the contrast pairs and synthesis were discussed in prior section on instructional materials. The discussion here is limited to the coding of students’ responses. Refer to Tables 2.1 for the contrast pairs and matrix used for synthesis.

Contrast pairs. All students considered three contrast pairs one at a time during phase 1. Within each contrast pair, the GMK was contextualized to the particular situation. For each contrast pair, students were required to explain how
Table 2.5: Summary of measures

<table>
<thead>
<tr>
<th>Source (Figure 2.2)</th>
<th>Measure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Process measures (7 total items)</td>
<td>Measures are focused on students abstraction and generalization of the GMK</td>
</tr>
<tr>
<td>a-c</td>
<td>Three contrast pairs</td>
<td>Measures the degree that students abstracted and generalized the GMK from each separate pair</td>
</tr>
<tr>
<td>d</td>
<td>Pre-screening question</td>
<td>Measures the degree that student can abstract and generalize the GMK when prompted</td>
</tr>
<tr>
<td>e</td>
<td>Synthesis (final response)</td>
<td>Measures how well students abstracted and generalized the GMK when looking across all pairs together</td>
</tr>
<tr>
<td>e</td>
<td>Synthesis (entire response)*</td>
<td>Measures whether students’ ideas of the GMK moved towards the GMK during the synthesis process</td>
</tr>
<tr>
<td>f</td>
<td>Assisted synthesis (final response)</td>
<td>Measures how well students abstracted and generalized the GMK with assistance</td>
</tr>
<tr>
<td></td>
<td>Outcome measures (4 post-test items)</td>
<td>Measures are focused on student instantiation of the GMK to the given situation</td>
</tr>
<tr>
<td>g-h</td>
<td>Two recognition</td>
<td>Measures how well students can differentiate GMK from other ways of thinking in a situation containing an SDF in a physics or everyday context</td>
</tr>
<tr>
<td>i-j</td>
<td>Two application</td>
<td>Measures how well students can apply the GMK spontaneously in a situation with an SDF in a physics or everyday context</td>
</tr>
<tr>
<td></td>
<td>Unprompted application</td>
<td>Measures how well students can apply the GMK when directly prompted in a physics or everyday context</td>
</tr>
</tbody>
</table>

student B (metacognitive) approached the situation differently from student A (non-metacognitive). Although not explicitly prompted to abstract and generalize the GMK, the pairs provided students for spontaneous generalization. Thus, contrast pairs measured the extent to which students spontaneously abstracted and generalized the GMK from its context. To be coded as a level 1 or 2, student responses were required to remove the particulars of the contextualized GMK in
each case. This meant that student responses were coded as a zero even if they articulated the contextualized GMK for that particular situation. For example, for the third contrast pair, a student saying that Fred was thinking about why he was drawn to the intersection or to Fruity O’s would be coded as 0. Students were required to describe the approach in more general terms, or to extend, connect, or relate the contextualized GMK to other situations. That is, a student would need to go beyond Fred’s distraction of intersection or Fruity O’s to a more general case, namely articulation of the GMK.

**Pre-screening question.** Immediately after students finished processing the three contrast pairs individually, students were asked, “Are there any similarities between the approaches used by any of the students?” Their responses measured how well they could abstract and generalize the GMK without explicit prompting or synthesis. Table 2.6 displays sample student responses for each coded level. At level 2, student responses articulated the thinking about why a certain feature or reasoning was attractive. At level 1, student responses articulated thinking about why an answer was incorrect. At level 0, students did not describe a reflective approach.

**Synthesis.** Student responses during synthesis and assisted synthesis were coded using the three-level analysis framework. The last row of Table 2.6 shows sample student responses for the three levels. At level 2, a student described how the hypothetical students in column B were thinking about how an answer occurred. At a level 1, a student described column B as thinking about why the tempting choice was wrong. At a level 0, a student did not describe any reflective thinking about the choice.

The purpose was to determine how each student’s ideas of the GMK progressed from beginning to end. More specifically, the focus was on whether a student’s ideas moved closer to the targeted GMK. The responses were coded as 0
Table 2.6: Coding and sample students responses for pre-screening and synthesis measures

<table>
<thead>
<tr>
<th>Level Label</th>
<th>0 No elements</th>
<th>1 Some elements</th>
<th>2 Full GMK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-screening question</td>
<td>A lot of the students used a more a qualitative approach at first to the problem</td>
<td>Some of them thought back afterwards on why they got the answer wrong.</td>
<td>Half the time maybe the students didn't go back and critically think about why they were attracted to the wrong answer.</td>
</tr>
<tr>
<td>Synthesis</td>
<td>Students in column A are kinda just looking at more of the big picture and getting too specific, whereas all the people in column B are using really specific things.</td>
<td>They look back on wrong answers that were tempting to them and try to decide why and figure out why they were wrong.</td>
<td>There's a greater self-analysis, perhaps. B, there's a bit more working backwards on how an answer occurred to you.</td>
</tr>
</tbody>
</table>

Outcome measures. Four items assessed students’ learning of the GMK. Two were recognition items, and two were application items. For each type, there was one in a physics context and one in an everyday context. Student responses were coded using the three-level framework outlined in Table 2.4 and coded at the highest level that was articulated. Provided below is further description and details for each item type.

Recognition items. The two recognition items required students to differentiate metacognitive thinking from the other ways of thinking for given a situation containing an SDF. There was one recognition item in a physics context and another recognition item in an everyday context, and they had the same general format. Table 2.7 summarizes the problem contexts. Both presented a situation containing a SDF and four different students’ approaches to the situation. The task directive was to identify the student(s) using the GMK and to provide an explanation.
For example, the physics recognition item presented a problem, drawn from Frank (2009), in which two identical balls were thrown straight up at different speeds and asked students to compare the times each ball would take to reach its maximum height. The correct response, which was provided to students, is that the ball thrown with greater speed will take longer to reach its maximum height. Because both balls have the same acceleration due to gravity and are at rest at their respective maximum heights, the ball with greater speed will have a greater change in velocity and thus require a greater time interval to reach its maximum height. However, the difference in speeds serves as the SDF in this problem, and can cue the incorrect reasoning that faster implies quicker, so that the ball thrown at greater speed will reach its maximum height in a shorter amount of time.

Of the four approaches demonstrated by the four hypothetical students provided, one was impacted by the SDF and was metacognitive about its impact (Bobbi). One was tempted by the SDF but then thought about another feature (David). Two were not affected by the SDF and did not reflect (Amanda and Chris). Students were prompted to identify the student reflecting on the impact of the SDFs and to explain how they could tell. Similarly, the everyday recognition item was in the context of purchasing new headphones, and also followed the same format. Of the four hypothetical students deciding between the headphones, one was impacted by the SDF (headphone’s popularity) and was metacognitive about its impact (Cathy). Another was tempted by an SDF (prior experience) but then thought about another feature (Ally). Two were not reflective in their decision-making (Brian and Danny).

Coding of recognition items. Sample student responses are shown in Table 2.8. At the highest level, students were required to select the correct hypothetical student (but could include others) and provide the correct description. At the highest level, student responses included describing Bobbi as thinking about what
Table 2.7: Post-test recognition items

<table>
<thead>
<tr>
<th>Scenario context</th>
<th>Physics</th>
<th>Everyday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four students consider a physics question. They all eventually reach the correct response. Their initial approaches are provided for the following question:</td>
<td></td>
<td>Four people are buying new wireless headphones. They are deciding between two headphones that have roughly the same price. The approaches the individuals use to select their headphones are provided.</td>
</tr>
<tr>
<td>A student throws a ball straight up in the air and times how long it takes the ball to reach its maximum height. Afterwards, the student throws the same ball with a greater initial speed.</td>
<td></td>
<td>Audiophile On-Ear Bluetooth</td>
</tr>
<tr>
<td>Compared to the first throw, will the amount of time taken for the ball to reach its maximum height be greater, less, or the same?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Choices**

A. Amanda thinks the second ball has greater change in speed so it will take more time to reach its maximum height.
B. Bobbi focuses on the initial speed and thinks that a greater speed means less time. She wonders why she focused on the speed.
C. Chris thinks the second ball travels higher because it was thrown with a greater initial speed and will take longer.
D. David is tempted to say that the ball with greater speed would take less time but remembers that acceleration is constant.

A. Ally previously owned the Audiophile headphones and is tempted to purchase them again. However, she really likes the powerful bass from the Urban Bass.
B. Brian finds the Audiophiles more comfortable and lighter than the Urban Bass.
C. Cathy is inclined to buy the Urban Bass but prefers the Audiophile's sound quality. She thinks that she wanted Urban Bass because they are popular with her friends.
D. Danny likes the sleek design of the Urban Bass. He also thought they felt sturdier than the Audiophile headphones.

**Task Directive**

Describe which student(s) are reflecting on role of alluring features.
Table 2.8: Coding and sample students responses for all post-test items

<table>
<thead>
<tr>
<th>0</th>
<th>No understanding</th>
<th>1</th>
<th>Partial understanding</th>
<th>2</th>
<th>Full understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Recognition</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Physics</td>
<td>David is one of the students who was drawn in by the alluring features because he's thinking about its maximum height and he's not necessarily thinking about the time.</td>
<td>Bobbi and David realize that it's tempting to use the initial speed and the fact that it's greater, but they think about why it's an incorrect way to think about it.</td>
<td>Bobbi is the only one that thinks about why she was led to use the equation ( v = \frac{d}{t} ) over ( t ), or considered the relationship.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Everyday</td>
<td>Cathy's really the only one who's falling, or who's almost falling, for the alluring feature of the Urban Bass.</td>
<td>Cathy and Ally both go back. They think about why am I deciding to get these as oppose to the other ones. Brian and Danny really don't think about that. They just say I'm getting these. That's it. I'm not going to think about it anymore.</td>
<td>Brian and Danny don't describe why, but they don’t reflect on why they’re allured to that whereas Ally and Cathy do describe that and reason their way through buying which headphones they prefer.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Application</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Physics</td>
<td>If they can immediately point out the alluring feature and realized it is the problem in this situation and that it's going to persuade them in the wrong way, then can immediately look the other way, and go to the right application.</td>
<td>By reflecting on the alluring features, you kinda understand what you did wrong and what you shouldn’t do for next time.</td>
<td>You can have a much deeper understanding of it if you're open minded and you understand why you are you are wrong and what lead you to that answer, rather than just being, like, I got it wrong.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Everyday</td>
<td>I would tell her that she was allured from what she originally planned and wanted to do over winter break, which was swimming on the sandy beach, and she was allured probably by the new modern hotel.</td>
<td>I think that if your friend reflects on these alluring features and looks at them more and how they don’t apply to what they really want, they would see that option A is best for them.</td>
<td>So by reflecting on, on the alluring features and going back and thinking, OK, the reason why I want Option B is because of these basically buzz words, but realizing you want Option A, is Option A the better option?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

led her to use an equation and Cathy as thinking about why she was tempted to purchase the headphones. At level 1, students’ responses focused on thinking about why the choice was incorrect for the situation, such as the evaluating why a higher speed did not imply a shorter time or considering why one set of headphones was chosen over another. At the lowest level, students often identified and explained which hypothetical students(s) were affected by the SDF.
Application items. The two application items used scenarios in which the GMK was applicable. Students first had the opportunity to spontaneously apply the GMK, but were then given another opportunity to apply the GMK when explicitly prompted. Table 2.9 summarizes the application items. The application item in the physics domain is in second column, while the one in the everyday domain is in the third column. The first row displays the scenario context. The next rows show the prompts, and the last row provides the correct response using the targeted GMK.

As an example of an application item, students were presented a scenario involving a work-energy task drawn from the literature (Hestenes & Wells, 1992; Lawson & McDermott, 1987). The work-energy task requires a comparison of the final kinetic energies of two pucks; one puck is four times more massive than the other, and they are pushed from rest with equal forces across a frictionless table to the finish line. The kinetic energies are the same because of the work-energy theorem (or energy law). Since both pucks have the same force applied over the same distance, the same amount of work, and thus the same kinetic energies. The SDF is the difference in masses, which cues a prevalent incorrect answer that the lighter mass will have greater speed and therefore higher kinetic energy. In the presented scenario, a clicker question about this task was posed in a physics lecture. Participants were told that a correct response is that both pucks arrive at the finish line with the same amount of kinetic energy because of the work-energy theorem. They were also given a histogram showing a hypothetical distribution of clicker responses and told that many students incorrectly concluded that lighter puck would have more kinetic energy. In the interview, students first received a general prompt (unprompted application). The general prompt asked students to explain how to help a student learn as much as possible from the situation. Later, students were prompted to apply the GMK (prompted application). They were
Table 2.9: Post-test application items

<table>
<thead>
<tr>
<th>Subject domain</th>
<th>Physics</th>
<th>Everyday</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scenario context</strong></td>
<td>Many students incorrectly answered the following question: The diagram depicts two pucks on a frictionless table. Puck II is four times as massive as puck I. Starting from rest, the pucks are pushed across the table by two equal forces. Which puck will reach the finish line with greater kinetic energy? A. Puck I B. Puck II C. Both will have the same. D. There is not enough information to decide.</td>
<td>Your friend is planning a week’s vacation for winter break. Your friend plans to spend most of the time outdoors swimming and enjoying the sandy seashore. Your friend has currently narrowed the options down to two places that are reasonably priced and asks for your advice. The websites give only a limited amount of information about the two options.</td>
</tr>
<tr>
<td></td>
<td>The correct response is provided to students.</td>
<td></td>
</tr>
<tr>
<td><strong>Unprompted application</strong></td>
<td>What should a student do to make sure he or she fully understands the reasoning involved and learns as much as possible from this question?</td>
<td>What would you tell your friend to help your friend make the best decision between these two vacation options?</td>
</tr>
<tr>
<td><strong>Prompted application</strong></td>
<td>Describe how reflecting on the role of alluring features can enhance student understanding of the reasoning involved.</td>
<td>How can reflecting on the role of alluring features in this particular case assist your friend?</td>
</tr>
<tr>
<td><strong>Correct response</strong></td>
<td>The student can reflect on why they were drawn to the difference in mass and how that impacted their thinking.</td>
<td>The friend can think about why they were drawn to option B and how that impacted their decision, even though the friend preferred option A.</td>
</tr>
</tbody>
</table>

![Diagram](attachment://Diagram.png)
prompted to explain how “reflecting on the role of alluring features” could help the hypothetical student with reasoning for the problem. A response demonstrating the targeted GMK would describe thinking about why the difference in masses was attractive and how it might impact student thinking.

The everyday application item was administered in a similar format to the physics application item. The presented scenario involved a friend making a decision between two vacation spots. The friend’s vacation goal was to visit a sandy beach and to swim. However, although only one option meets this goal (Option A), the friend leans toward the other option (Option B). The SDF is the framing of the two vacation options stemming from the adjectives, which may have influenced the friend to think that option has better amenities. In the interview, students first received a general prompt and then a targeted prompt, analogous to those in the physics application item. A response demonstrating the GMK would advise the friend to think about why the friend was attracted to Option B. For instance, a student said that the friend could think: “Why I want Option B is . . . because of these, basically, buzz words.”

Coding of application items. The coding framework applied to the recognition items was also applied to the application items. Both unprompted and prompted responses were coded. Sample student responses to the targeted prompt are shown in Table 10. At the highest level, students described reflection on why the difference in masses or option was tempting. At the intermediate level, students describing understanding why using the difference in masses in this fashion was incorrect or whether option met the friend’s goals. At the lowest level, students typically described how the hypothetical student was affected by the SDF or strategies to avoid the SDF.
Procedures for Data Analysis

To start, I assigned each student an ID. I transcribed complete think-aloud interviews and then put the verbal outputs for each measure into a spreadsheet by student ID and condition. Before any coding, student IDs were randomly sorted, and the IDs and experimental conditions were removed.

Inter-rater reliability and internal consistency of post-test items.

To establish inter-rater reliability, for all items, I and another graduate student independently coded a 20% random sample of student responses for each item. Inter-rater agreement was greater than 80% for all measures. After inter-rater agreement was established, I coded the remaining data alone.

Together, the four post-test items (using prompted application scores) were internally consistent. The alpha value (Cronbach) is 0.71, which is an acceptable value to indicate that all four items measured the same construct (DeVellis 2016; Bland & Altman 1997). This suggests that together, on the same scale, the items were reliably measuring student understanding of the GMK.

Process and outcome measures. The general coding procedure for both process and outcome measures was similar. For process measures, the first step was to identify all statements that generalized the approach of the particular situation. Statements were defined as consisting of phrases that reflected a single idea (Ericsson & Simon 1993). Generalized statements related the approach to previously encountered situations, extended the approach to a larger range of situations, or removed the contextual details of the approach to describe a more global one. These statements were then coded using the three-level framework. The highest score for any statement within a student’s response was used.

Similarly, for the outcome measures, the first step was to identify all statements that generalized or contextualized the GMK for the particular situation. These
Table 2.10: Coding scheme and sample students responses for relevant change in ideas during synthesis

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
<th>Sample student ideas for column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No change or no relevant change</td>
<td>“data-oriented” → “more quantitative”</td>
</tr>
<tr>
<td>1</td>
<td>Relevant change</td>
<td>“more analytical” → “reconsiders wrong answer”</td>
</tr>
</tbody>
</table>

statements were also coded using the three-level framework, and the highest score for any statement was used.

**Change in relevant ideas in synthesis.** Students’ entire responses during synthesis were analyzed to determine whether there was a relevant change in their ideas. For each student’s response, the first step was to identify when the student was making a generalized statement, which was defined as phrases that described what an entire column represented and reflected a single idea (Ericsson & Simon, 1993). If there was only one generalized statement for each column, I interpreted this as no change in definition (score 0). If there were multiple generalization statements and each statement gained greater definition the previous, I interpreted this as a relevant change (score 1). There were two common relevant changes. One was when subsequent generalized statements moved toward describing reflecting thinking. The other was when each subsequent generalized statement became less vague.

**Comparison between groups.** To compare learning across conditions, I performed a One-Way Analysis of Variance (ANOVA) with the statistical significance level at 0.05. As the dependent variable, I summed the four post-test items to one numerical score as the measure of student learning of the GMK. Only scores from when students were prompted to use the GMK were used. (Unprompted application scores are addressed in the next chapter.) I assumed
that the total post-test item scores were interval, although each post-test item was scored on an ordinal scale, which is not necessarily interval. However, with statistical testing, any small differences can be enhanced by stretching the scale to equal intervals when it is not. Therefore, using two different two-level models, I performed additional statistical tests to ensure that any statistical significance resulted from actual differences and not an artifact of an interval scale assumption.

Figure 2.4 displays the two-level models used for additional analyses. Because the exact placement of the intermediate level (some elements of the GMK) may lie anywhere on the scale between 0 and 2, the models either placed the intermediate level at its lower (near 0) or upper limit (near 2) on the scale from 0 to 2. One model, called the GMK model, represented the lower bound placement of the intermediate level. This model was conservative and collapsed the intermediate level (some elements of the GMK) with the lowest level (no elements of the GMK). On the post-test items, students either articulated full understanding of the GMK (score 1) or did not (score 0). The other two-level model, called the reflective thinking model, represented the upper bound placement of the intermediate level and collapsed the intermediate level with the highest level (full GMK). Students either articulated reflective thinking (score 1) or did not (score 0) on the post-test items.

The total post-test items score (maximum 4) from these models were used in additional statistical tests to compare between conditions. If the same results held across all three models (i.e., statistically significant higher means in the synthesis conditions), this would suggest that learning of the GMK was robust, and the exact placement of the intermediate level anywhere along the a 0-2 scale does not affect the results. Thus, it could be concluded that the three-level model, although ordinal, essentially acted as an interval scale and was thus valid to use for statistical testing to compare the learning of the GMK between conditions.
Figure 2.4: Collapsed three-level model for analysis into two-level models

**Three-level model**

<table>
<thead>
<tr>
<th>Level</th>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No elements of the GMK</td>
<td>No reflecting thinking</td>
</tr>
<tr>
<td>1</td>
<td>Some elements of the GMK</td>
<td>Reflecting about an action</td>
</tr>
<tr>
<td>2</td>
<td>Full GMK</td>
<td>Reflecting about the thinking that led to an action</td>
</tr>
</tbody>
</table>

**Two-level GMK Model**

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Other</td>
</tr>
<tr>
<td>1</td>
<td>Reflecting about the thinking that led to an action</td>
</tr>
</tbody>
</table>

**Two-level Reflective Thinking Model**

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No reflecting thinking</td>
</tr>
<tr>
<td>1</td>
<td>Reflecting about an action</td>
</tr>
</tbody>
</table>
Chapter 3
RESULTS

The central research question that guided the work in this dissertation is the following: How can synthesis help students learn GMK? To this end, I investigated student learning of the targeted GMK with and without synthesis. As a preview of my findings, I show that students who synthesized multiple contrast pairs were better able to recognize and apply the GMK than students who processed the same pairs without synthesis. Thus, synthesis helped students better abstract and generalize the GMK. As an auxiliary question, within synthesis, I also examined whether including an everyday context would improve student learning of the GMK compared to contexts exclusively in the physics domain. The results show that there was no difference between the synthesis conditions.

In this chapter, I begin with a descriptive overview of student responses on the post-test items and report on basic analysis between conditions for each item. Next, I give the results using finer analysis of the data and check for the statistical significance of the differences between conditions. Following this, I provide evidence to rule out two potential alternative explanations for the results that stem from the study design. I also provide additional evidence locating the learning within synthesis. Finally, I offer additional analyses that address the validity of using the total score as interval data.

Descriptive Overview of Student Performance on Post-test Items

This section provides an overview of student understanding of the GMK across all items and then summarizes the results for each item type: recognition
Table 3.1: Highest level attained by students across the four post-test items

<table>
<thead>
<tr>
<th></th>
<th>0 No elements</th>
<th>1 Some elements</th>
<th>2 Full GMK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synthesis</td>
<td>5% (2)</td>
<td>27% (10)</td>
<td>68% (25)</td>
</tr>
<tr>
<td>No synthesis</td>
<td>26% (5)</td>
<td>47% (9)</td>
<td>26% (5)</td>
</tr>
<tr>
<td>Total</td>
<td>12% (7)</td>
<td>34% (19)</td>
<td>54% (30)</td>
</tr>
</tbody>
</table>

Note: Frequencies are given in parentheses.

and application. All post-test items were coded using the three-level framework described earlier. For simplicity, synthesis conditions are combined.

Highest Level Reached by Students

Table 3.1 displays the highest level of GMK understanding that students demonstrated across any of the four post-test items. As a whole, nearly 90% of students articulated some understanding of the GMK at least once, with approximately half demonstrating full understanding and one third demonstrating partial understanding. The table also shows that the distributions were different between conditions. By condition, almost all synthesis students provided evidence of either full or partial understanding of the GMK compared to three-fourths of the non-synthesis students. More synthesis students demonstrated full understanding, while the non-synthesis students typically exhibited partial understanding.

Frequencies of Students’ Articulation of the Targeted GMK on Items

Table 3.2 summarizes how often students articulated a full understanding of the GMK across the four post-test items. Overall, synthesis students articulated the GMK on two or more items three times more often than non-synthesis students, approximately 30% versus 10%, respectively. One-tenth of the synthesis
Table 3.2: Frequency of articulating the full GMK across the four post-test items by condition

<table>
<thead>
<tr>
<th></th>
<th>No items</th>
<th>1 item</th>
<th>2 items</th>
<th>3 items</th>
<th>4 items</th>
</tr>
</thead>
<tbody>
<tr>
<td>No synthesis</td>
<td>74% (14)</td>
<td>16% (3)</td>
<td>10% (2)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Synthesis</td>
<td>33% (12)</td>
<td>38% (14)</td>
<td>19% (6)</td>
<td>5% (2)</td>
<td>5% (2)</td>
</tr>
<tr>
<td>Total</td>
<td>46% (26)</td>
<td>31% (17)</td>
<td>14% (8)</td>
<td>5% (3)</td>
<td>4% (2)</td>
</tr>
</tbody>
</table>

Note: Frequencies are given in parentheses.

students were able to demonstrate the GMK on three or more items while none of the non-synthesis students did.

Results by Item Type

**Recognition.** Figure 3.1 shows the distribution of scores on the two recognition items for each condition. One recognition item was in a physics context, whereas the second item was in an everyday context. Overall, across both contexts, more synthesis students were able to recognize and explain the GMK in situations containing an SDF than the non-synthesis students. The item in an everyday context appeared to be easier for students than the item in a physics context, with more students demonstrating full understanding.

**Application.** For each application item, students were initially not prompted to apply the GMK (*i.e.*, unprompted application), and later, they were prompted to apply the GMK (*i.e.*, prompted application). The unprompted application measured student spontaneous application of the GMK, while the prompted application measured student application of the GMK when directed. Across both items, there was a floor effect for the unprompted application. No students spontaneously expressed full understanding of the GMK on either item. Within the everyday context, no students articulated partial understanding either.
Within the physics context, a quarter of the students articulated partial understanding, although there was no variation between the conditions.

Figure 3.2 shows the distribution of scores across conditions by application item for the prompted application. The application items were generally more difficult items than the recognition items, with fewer students articulating full understanding. As with the recognition items, synthesis students demonstrated greater understanding of the GMK compared to the non-synthesis students. However, unlike the recognition items, none of the non-synthesis students were able to articulate full understanding on either application item. Here, the everyday context appeared to be the more difficult item.

**Comparison of Learning Between All Three Conditions**

To compare between conditions, students’ total scores on the four post-test items were used. However, the total included only students’ prompted scores from the application items since there was a floor effect with the unprompted responses. The following analyses assumed the total item scores as interval data, although
In Figure 3.3, the mean total scores of the post-test items are shown by condition. Both synthesis conditions had similar means, which were greater than that for the non-synthesis condition. In a one-way analysis of variance (ANOVA), differences between the means were statistically significant, $F(2, 53) = 4.755, p = 0.013$. Results also held when the unprompted application items were used instead. Post hoc comparisons using the Tukey HSD test indicated that both synthesis conditions had significantly higher means than the no synthesis condition. However, there was no statistical difference between the two synthesis conditions. The difference between synthesis and non-synthesis groups corresponds to an effect size of 0.92 standard deviations.

Within the synthesis conditions, students performed similarly on the post-test items. In addition within the synthesis task, there was no difference in the distribution the responses for GMK between conditions (Table 3.3) or the
Figure 3.3: Mean total scores of the four post-test items by condition

Table 3.3: Distribution of scores for student response to the $3 \times 2$ matrix by synthesis condition

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No elements</td>
<td>Some elements</td>
<td>Full GMK</td>
</tr>
<tr>
<td>Without everyday context</td>
<td>50% (9)</td>
<td>44% (8)</td>
<td>6% (1)</td>
</tr>
<tr>
<td>With everyday context</td>
<td>58% (11)</td>
<td>42% (8)</td>
<td>0 (0)</td>
</tr>
<tr>
<td>Total</td>
<td>54% (20)</td>
<td>43% (16)</td>
<td>3% (1)</td>
</tr>
</tbody>
</table>

Note: Frequencies are given in parentheses.

number of students who had a change in relevant ideas towards the GMK (37% without everyday context versus 21% with everyday context).

To What Extent is the Learning of the GMK Located Within Synthesis?

The results support that the learning of the GMK is located within synthesis. However, stemming from the study design, the results could potentially be attributed to two other factors. The first is assistance on synthesis. Student learning of the targeted GMK could have occurred with the interviewer, not
necessarily from the engagement with synthesis. The second is time-on-task, as the synthesis conditions had additional time to process the same contrast pairs than the non-synthesis condition. The synthesis conditions had two exposures to the same contrast pairs, while the no-synthesis condition had one. By design, measures obtained from student responses from the three individual contrast pairs, pre-screening question, and synthesis checked for these effects.

**Assisted synthesis.** Table 3.4 shows the distributions of student responses in synthesis with and without assistance. If synthesis with or without assistance supported the learning of the GMK, then there would be a positive relationship between student responses with or without assistance and total item scores. Using a Pearson’s correlation, the correlation between students’ responses with assistance and total item scores was small and not statistically significant, explaining 5% of the variance, \( r^2 = 0.053, p = 0.594 \). However, students’ responses without assistance and total item scores explained 14% of the variance, \( r^2 = 0.142, p = 0.021 \). Although small, there was a statistically significant positive relationship between students’ responses without assistance and total item scores. Thus, independent engagement in synthesis supported students’ learning, with negligible effects from the assistance of the experimenter.

### Table 3.4: Distribution of scores from synthesis with and without assistance

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No assistance</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((n = 37))</td>
<td>54% (20)</td>
<td>43% (16)</td>
<td>3% (1)</td>
</tr>
<tr>
<td><strong>With assistance</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((n = 30))</td>
<td>7% (2)</td>
<td>80% (24)</td>
<td>13% (4)</td>
</tr>
</tbody>
</table>
Table 3.5: Distribution of scores for pre-screening and synthesis measures

<table>
<thead>
<tr>
<th></th>
<th>0 No elements</th>
<th>1 Some elements</th>
<th>2 Full GMK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contrast pairs</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Prescreening question</td>
<td>93% (52)</td>
<td>5% (3)</td>
<td>2% (1)</td>
</tr>
<tr>
<td>Synthesis</td>
<td>54% (20)</td>
<td>43% (16)</td>
<td>3% (0)</td>
</tr>
</tbody>
</table>

**Time on task.** There are two key pieces of evidence that suggest another exposure to the same contrast pairs would unlikely have made a significant difference for no-synthesis students in learning the GMK. First, no students abstracted and generalized the GMK when processing the three contrast pairs sequentially. Second, immediately after considering the contrast pairs, in the pre-screening question in which students were prompted to describe any commonalities between the cases, only 7% of students articulated some (3 students) or full elements of the GMK (1 student). If students did not abstract and generalize the first time with the contrast pairs, it is unlikely that they would have done so during a second time. Thus, non-synthesis students would likely not have benefited from another exposure with the same contrast pairs. Table 3.5 displays the distribution of student scores for the individual contrast pairs, pre-screening question, and synthesis.

**Mediating variables.** Two mediating variables located the learning within synthesis. The first variable is whether students’ responses during synthesis related to the GMK (without assistance). As reported earlier, students’ responses in synthesis explained 14% of the variance on the total item scores, \( r^2 = 0.142, p = 0.02 \). The second factor is whether there was a change in students’ relevant ideas toward the GMK. Approximately 30% of synthesis students had a relevant change
in ideas that moved closer to the targeted GMK. Having a relative change in ideas explained 22% of the variance on total post-test scores, $r^2 = 0.215$, $p = 0.002$.

Taken together, all of these factors suggest that it was the synthesis process, not the assistance from the experimenter or additional time on task, that helped students build an understanding of the GMK.

**Using the Ordinal Data as Interval**

The statistical analyses assumed that the total post items scores were on an interval scale, although each post-test item was scored on a three-level ordinal scale. This violates an equal interval assumption of the statistical tests performed. To address this concern, I conducted additional analyses using two-level models (see Fig. 2.4), which did not violate the equal interval assumption, and show that the results are robust.

Table 3.6 shows the total score means and standard deviations for the different two-level scales by condition. As seen in the table, for both models, the means for the synthesis conditions were descriptively higher than non-synthesis condition. Additionally, a one-way ANOVA showed that the effect of condition was significant at the 5% level for the GMK model, $F(2, 53) = 3.818$, $p = 0.028$, as well as for the reflective thinking model, $F(2, 53) = 3.950$, $p = 0.025$. Post hoc pairwise comparisons revealed that there was no difference between the synthesis conditions. For the reflective thinking model, both synthesis conditions were significant compared to the non-synthesis condition. However, for the GMK model, one synthesis condition was not statistically significant compared to the non-synthesis condition ($p = 0.102$); the effect size was 0.86. Taken together, these analyses are consistent with those reported using the three-level model. In the three-level model, the intermediate level (some elements of the GMK) was placed
Table 3.6: (Standard deviations are presented in parentheses)

<table>
<thead>
<tr>
<th>Synthesis</th>
<th>GMK model (max 4)</th>
<th>Reflective thinking model (max 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without everyday context</td>
<td>1.00 (1.03)</td>
<td>2.56 (1.10)</td>
</tr>
<tr>
<td>With everyday context</td>
<td>1.16 (1.26)</td>
<td>2.53 (1.50)</td>
</tr>
<tr>
<td>No synthesis</td>
<td>0.32 (0.58)</td>
<td>1.47 (1.39)</td>
</tr>
</tbody>
</table>

halfway on the scale from 0 (no elements of the GMK) and 2 (full GMK).

However, the precise placement of the intermediate level is unknown. Placing the intermediate level at 0 in the reflective thinking model or at 2 for the GMK model provided insights about the extent to which its placement (and the assumption of the three-level model as interval) may impact the results. These additional analyses with the two-level models indicate that placing the intermediate level equidistant from the lower and upper levels is not unreasonable, as the results were robust when the intermediate level was placed at either of the two limits. Thus, assuming the three-level model as interval data may be appropriate and valid for the purposes of these analyses.
Chapter 4
DISCUSSION

The goal of this study was to understand how students could develop general metacognitive knowledge (GMK) for physics learning, in particular GMK about thinking about how and why salient distracting features (SDFs) of a situation can impact reasoning. In this study contrasting case instruction was used to build student GMK. It utilized three sets of contrasting cases, or contrast pairs, which were presented side-by-side in order to illustrate the absence and presence of the targeted GMK using vignettes that summarized the approaches of hypothetical students on different qualitative physics situations. Broadly, the investigation examined whether structuring the contrast pairs sequentially or together could support the learning of the SDF-related GMK. In one condition (non-synthesis), students compared the three contrast pairs separately. In another condition (synthesis), students also compared the same contrast pairs together and synthesized the pairs to describe the defining characteristic of the metacognitive approach. Afterwards, both conditions received a scripted lecture about the GMK, and then completed four post-test items. In addition, the synthesis conditions were further split up, with half of the students considering pairs exclusively within the physics context and the other half considering two pairs in the context of physics and one pair in an everyday context.

Overall, the results show that synthesis students outperformed the non-synthesis students on the post-test items. The results suggest that the learning of the GMK was located within synthesis, although by study design, there were two potential explanations that may account for student learning. The first alternative explanation is that synthesis students had more time to process the
contrast pairs, having two exposures to the same cases while the non-synthesis
only had one. However, the results show that none of the students generalized the
GMK within each given pair or across the pairs when they processed the pairs
separately. Additionally, even when prompted to search for a commonality across
the cases after all three pairs had been examined individually, only 7% of students
could generate elements of the GMK. However, it is possible that students could
not see the connection between the pairs due to the order in which they received
them. Each contrast pair differed in the number irrelevant differences (e.g.,
absence/presence of correctness and absence/presence of equations). Students
processed the pairs in the order of decreasing number of irrelevant features. Thus,
the last pair had no irrelevant differences, such that students could discriminate
the GMK more easily compared to the other pairs. If, instead, students had
received the pairs in the reverse order, could this have helped the students perceive
the GMK in subsequent pairs and connect them? Other studies suggest that this
would be unlikely. They have shown that students tend to consider cases
independently and do not spontaneously generalize across related cases
(Loewenstein, Thompson, & Gentner, 2003; Rittle-Johnson & Star, 2007; Shemwell
et al., 2015). Thus, another exposure to the same contrast pairs would not have
likely helped non-synthesis students better learn the GMK.

The second alternative explanation for the synthesis students’ enhanced
learning involves the assistance provided at the end of the synthesis. Immediately
after students completed synthesis on their own, I assisted them with synthesis. As
expected, with my assistance, more students could arrive at general reflective
thinking. However, the results show that there was a weak relationship between
the students’ responses with assistance and post-test performance, and thus,
assistance with synthesis had a negligible effect on students’ learning of the GMK.
Nevertheless, it is surprising that assistance did not help students learn the GMK,
as prompting to compare and contrast cases has been shown to be productive for learning within contrasting case instruction (Alfieri, Nokes-Malach, & Schunn, 2013; Roll et al., 2011). While there may be many possible explanations, one possible explanation is that assistance hindered students’ productive struggle to build the GMK. Chapter 5 sheds some light on this explanation.

There were also two mediating variables that attributed the learning of the GMK to synthesis: students responses from synthesis (without assistance) and a change in students’ relevant ideas during synthesis. Both of these had statistically significant relationships to students’ post-test performances, which suggests that the learning during synthesis improved student learning of the GMK. Taken together, engagement in synthesis was the primary explanation for student learning.

What Did Synthesis Students Learn?

Compared to the non-synthesis students, synthesis students had greater learning of the GMK, and more generally, they developed a greater general awareness of reflecting about SDF-cued thinking, which includes evaluating its appropriateness. Specifically, synthesis students could differentiate and explain how the GMK was distinct from the other thinking approaches. In addition, when prompted, they could also apply the GMK to new situations. By contrast, the non-synthesis students, who processed the same contrast pairs separately, tended to focus on the particular contextual details within each pair, and thus, they did not generalize the approach beyond a given pair or across the pairs. Even after direct instruction about the GMK, non-synthesis students did not to recognize or apply the GMK in new situations as well as synthesis students.
Students, however, could not apply the GMK unprompted. They did not see how the GMK would apply to a novel situation that contained an SDF. In retrospect, it was not surprising that students did not see the GMK as relevant to the given situation. One possible reason may be that the prompts for the application items did not have a strong enough cue for students to apply the GMK. This may have been contributed to the framing of prompts. Within the physics context, the prompt was framed around “understanding the reasoning involved” and “learning,” while the everyday context was framed around “making the best decision.” These are distinct thinking processes. Perhaps if the prompt were framed around “thinking” in general, this would better cue students to see that the GMK as applicable. Nevertheless, when prompted to apply the GMK, synthesis students were more readily able than the non-synthesis students.

Within synthesis, including an everyday context did not help students further abstract or generalize the GMK compared to contexts exclusively within the physics domain. It also did not improve student learning of the GMK. Perhaps the varied physics contexts spanning across multiple topics were sufficient for students to build general knowledge about reflecting on the impact of SDFs. In addition, the post-test measures may not have been sensitive enough to measure the difference in the learning between the conditions. The easiest and hardest items appeared to be the recognition and application items, respectively, in the everyday context. Thus, these items did not capture the variance between the conditions, which made any differences difficult to measure.

Despite the synthesis students’ stronger post-test performance compared to the non-synthesis students, one concern may be their low performance overall. Approximately 70% of synthesis students articulated the targeted GMK but only 30% did so more than once across the four post-test items. One possibility for the low frequency rate could be that the post-test items were difficult. Another
possibility is students’ conflation of the targeted GMK with a closely related reflective approach. Along with the GMK, students also described reflection that focused on the content, or specifically why the SDF-cued thinking was inappropriate for the situation.

Three reasons may have contributed to students’ conflation of the two closely related reflective approaches. The first concerns the cases within the synthesis process, where the learning occurred — specifically the type of contrast used between the cases. During synthesis, only one student discriminated the focus of the reflection, while less than half differentiated between the absence and presence of reflective thinking, in particular reflection that focused on content, or specifically why the SDF-cued thinking was inappropriate for the situation. This may be due to the absence/presence contrast highlighting whether the fictional students did or did not demonstrate the targeted GMK. This contrast did not adequately highlight the nuanced differences between the reflective thinking related to the SDF-cued action, as suggested by students’ responses during synthesis. Rather, the absence/presence contrast could also be more generally described as an absence/presence of reflective thinking. Consequently, on the post-test items, many students conflated the targeted GMK with the closely related reflective approach that focused on content. For novices, the nuanced differences between the reflective thinking approaches can become apparent through contrast (Gibson & Gibson, 1955). Thus, one way to facilitate greater differentiation during synthesis, and thus improve student overall performance on the post-test items, could be to contrast these two reflective approaches in order to highlight their nuances. For example, juxtaposing cases that highlight reflection about the content (i.e., why the SDF is incorrect) with those highlighting reflecting about thinking (i.e., why the SDF is tempting) may help students
differentiate the focus of metacognitive reflection. This approach was explored in a subsequent study described in Chapter 6.

Second, outside of synthesis, another explanation could be that the direct instruction about the GMK did not adequately help students distinguish between different reflective approaches. In the scripted lecture, I explained SDFs and their impact on thinking, then described reflection on how and why SDFs impact thinking as a beneficial approach to address SDFs. Given the nature of SDF-cued reasoning is often inappropriate to a given situation, it was reasonable that students may have interpreted the focus of the reflection to be SDF-cued thinking itself, rather than reflecting on its cause, after direct instruction. Perhaps the particular focus of the reflection may be too subtle to address via direct instruction, particularly if students did not progress far in synthesis. One way to explore this possibility further would be to measure students’ ideas of the GMK before and after direct instruction.

Finally, reflecting on the content may be natural for students. Students may have interpreted reflection in the way that they believed was most valuable for them. Most likely, introductory physics college students’ primary goal may be content understanding, as the learning within a physics course depends on conceptual understanding and successful problem solving. Thus, reflecting on the content and one’s content understanding has immediate value. By contrast, metacognitive reflection on the impact of SDFs on thinking processes may not seem to have immediate benefits for students. Given that the intervention was a one-shot activity with a short timeline, follow-up instruction that emphasizes the value of the GMK and the further learning of the GMK may be needed. One option would be to integrate questions about GMK within physics problem solving to help increase its relevance as well as to provide practice in applying the GMK.
How Did Synthesis Students Learn the GMK?

Synthesis students learned the GMK by abstracting and generalizing the GMK across multiple contrast pairs in varied contexts. By contrast, the non-synthesis students, who processed the same contrast pairs separately, could not see the connection across the related pairs even when prompted to look for commonalities after processing all pairs. These results are consistent with prior studies in contrasting case instruction. For instance, Shemwell and colleagues (2014) showed that college students who were prompted to find an explanation that fit across multiple cases with the same shared underlying physics principle better learned the principle compared to students who considered the cases individually. How students can abstract and generalize the GMK during synthesis is described in Chapter 5.

Direct instruction, or explicitly telling students the GMK, also played an important role in synthesis students’ learning. Before direct instruction, only one student articulated the GMK after engaging in synthesis. After direct instruction, nearly 70% of synthesis students articulated the GMK compared to 25% of non-synthesis students. On its own, direct instruction was not enough to help students build the GMK, since it did not help non-synthesis students as well as synthesis students. This effect was as intended by design. The study was designed with a “preparation for future learning” (PFL) approach (Schwartz & Bransford, 1998), which uses direct instruction to enhance students’ learning after they have engaged in an inductive activity, such as synthesis. The study, however, was not designed to measure a PFL effect. Rather, the PFL approach was used to enhance the effect of synthesis and reveal the learning of the synthesis students. Thus, exactly what the synthesis students learned from direct instruction more than non-synthesis students is unclear, but the results do suggest that engaging in
synthesis prepared students to learn more from direct instruction. Chapter 5, which analyzes students’ processes during synthesis, reveals more on the PFL effect. Analyses show that even though synthesis students did not fully articulate the GMK, students knew more than what they outwardly expressed. Most likely, direct instruction provided language that students could use to make sense of their learning during synthesis. In particular, it provided language about reflecting thinking, as well as SDFs and their potential impact on thinking.

Another possible explanation for student learning may be due to their change of relevant ideas of the GMK during synthesis. Students who had a change in relevant ideas were more able to recognize and apply the GMK. Building GMK is non-trivial and may take effort and agency for students to improve their idea, which may lead to greater learning. Chapter 5 also offers insight into this explanation.

**Building Metacognitive Knowledge via Synthesis**

The primary results show that synthesis, paired with direct instruction, can be an effective approach to build students understanding of the GMK. More importantly, it shows that direct instruction on its own may not be an effective method. This was supported by the fact that non-synthesis students could not recognize and apply the GMK as successfully as the synthesis students. This outcome was not surprising, as a “tell and practice” approach has been shown as ineffective in building student conceptual knowledge. However, within this study, “practice” was not integrated into the study design. A future study could determine whether adding a “practice” phase could enhance learning of the GMK, and compare the learning between “tell and practice” and “synthesis, tell, and practice.”
Compared to other effective pedagogical ways to build GMK (e.g., discussion, presentation, and training) synthesis is a time-efficient, which is an especially important consideration for college physics instructors. Thus, synthesis may be a viable approach to building student GMK within an authentic college physics course, along with follow-up instruction, such as practice problems, to strengthen learning. Supporting synthesis within a college classroom is explored in Chapter 6.

**Explicit Instruction Needed to Build Metacognitive Knowledge**

To some extent, considering contrast pairs sequentially exemplifies implicit instruction of GMK, which may represent many typical classroom activities in which learning GMK is not the primary goal. Although the contrast pairs were designed with the explicit goal of building student GMK, students were more likely to miss the GMK when processing the contrast pairs separately compared to processing the contrast pairs together. Even when prompted to generalize across the contrast pairs immediately after processing the pairs sequentially, few students could generalize. These findings suggest that without explicit prompting to generalize and appropriate structure of the contrast pairs, students are unlikely to develop GMK on their own. Other studies also stress the importance of explicit instruction GMK during instruction. For instance, White and Fredriksen (1998) and Zohar and David (2008) demonstrated that addressing GMK explicitly during instruction led to greater strategic use, especially for low-academic achieving students.
Metacognitive Knowledge Can Facilitate Regulation

An underlying assumption of this study was that metacognitive knowledge informs metacognitive regulation and, thus, targeting students’ metacognitive knowledge may lead to greater strategic application of that knowledge. Yet, many studies focus on developing student metacognitive regulation, or strategic use, and thus overlook the importance of developing metacognitive knowledge, implicitly assuming that metacognitive knowledge is easily acquired. This was not the case in this study. Rather, in this study, students who were aware that they could reflect about the impact of SDFs on their thinking were believed to be more likely to be able to apply the GMK, and ultimately, to use it to regulate their thinking processes when encountering situations with SDFs. The findings support this assumption to some extent and highlights the importance of building metacognitive knowledge. Synthesis students who understood the GMK applied it when prompted, while non-synthesis students who did not understand the GMK did not. Whether synthesis students could use the knowledge to regulate their thinking more effectively than non-synthesis students was not the focus of this study. However, the findings may be mapped onto strategy training, in which students are provided with a description of the strategy and examples and then given practice to apply the strategy. With strategy training, students used the strategy when prompted (e.g., Bielaczyc, Pirolli, & Brown, 1995; Aleven & Koedinger, 2002). Similarly, when prompted to apply the GMK to regulate their thinking, synthesis students may be more likely to use it more effectively than the non-synthesis students. One would speculate after multiple exposures and practice applying the GMK, students may be able to use it to regulate thinking when prompted, or even spontaneously use it, and thus improve their reasoning in
situations with an SDF. Thus, metacognitive knowledge may be a promising approach to facilitate more effective metacognitive regulation.
Chapter 5
HOW STUDENTS CAN BUILD KNOWLEDGE VIA SYNTHESIS

In Chapter 4, synthesis was shown to be an effective method to building student general metacognitive knowledge (GMK) about reflecting on the impact of salient distracting features (SDFs) in physics questions. In particular, results revealed that students' who had a change in relevant ideas of the GMK during synthesis also had greater learning of the GMK, as measured by post-test performance. In this chapter, I examine student processes during synthesis to shed further understanding on how students can build GMK, in particular, in how students’ ideas changed. This chapter supplements the findings in Chapter 4 by providing insight on how students could learn the GMK through abstraction and generalization. In addition, this chapter also contributes to the contrasting case instruction literature. Thus, I shift the focus from GMK to contrasting case instruction. In particular, my focus is to understand how students can productively use both affordances of synthesis, contrast and invariance, to build their knowledge during synthesis. Within the contrasting case literature, studies that focus on students’ processes while engaging in the inductive activity are limited. Understanding how students’ build knowledge with contrasting cases is valuable because it may provide insights on the design and pedagogy of contrasting case instruction.

To start, I provide a brief overview of the contrasting case instruction literature, which is primarily comprised of experimental studies. Next, I provide an overview of synthesis, a form of contrasting case instruction, as a reminder of the affordances of synthesis and how students could learn the GMK. However, I do not discuss the problem contexts, as they are not consequential to the analysis,
and refer the reader back to Chapter 2. Next, I discuss the methods of analysis, and then provide the analysis of two focal students, Max and Lynn. Following this is a discussion of the key findings developed from the analyses. Finally, I end the chapter with a brief conclusion summarizing the contributions.

**Prior Research on Contrasting Case Instruction**

Contrasts can help learners build knowledge needed for reasoning and problem solving (Schwartz & Bransford, 1998; Rittle-Johnson & Star, 2007). Within instruction, a number of cases or instances, called contrasting cases, can be used to indicate key knowledge in the form of a commonality that is present across a range of contexts (e.g., Schwartz & Martin, 2004). For instance, Kuo and Wieman used contrasting cases instruction to increase college students’ knowledge of Faraday’s law (2016). These studies exemplify much of the research on contrasting cases instruction dedicated to comparing the instructional effectiveness of contrasting cases instruction to other modes of instruction.

There are also a class of studies that focus on evaluating ways to improve on contrasting cases instruction, such as the types of cases and scaffolds (Holmes, Day, Park, Bonn, and Roll, 2014; Roll, Aleven, & Koedinger, 2011), while other studies focus on task directives for processing the cases and how they influence learning outcomes. For example, Chin *et al.* (2016) compared an “inventing” directive (*i.e.*, finding a single general explanation) to a “compare and contrast” directive with the set of contrasting cases in the context of range in projectile motion. They found “inventing” led to greater learning of range than “compare and contrast” with middle school students. Similarly, Shemwell *et al.* (2015) showed that, with multiple cases of current induction with a magnet, greater learning occurred with college students using instructions to seek a general explanation.
Many of these studies are quantitative studies that do not investigate students’ processes during instruction. Thus, comparatively fewer have focused on understanding the processes by which students build knowledge within instruction using contrasting cases. Therefore, the purpose of this chapter is to understand how students can use contrast and invariance, two affordances of contrasting cases instruction, to build knowledge. I use an individual think-aloud protocol, which provides data about students’ cognitive processes as they learn using contrasting case instruction (Ericsson & Simon, 1987).

**Methods**

**Learning context: Synthesis**

A task called synthesis was created to build student general metacognitive knowledge (GMK), as described in Chapter 2. Broadly, the GMK targeted an understanding of metacognitive reflection, or thinking about one’s thinking. Synthesis is a form of contrasting cases instruction which involved six cases placed in a $3 \times 2$ matrix (Figure 5.1). Each case was a vignette describing the approach of a hypothetical student to a situation. Each row represented a contrast pair, which consisted of two vignettes of hypothetical students considering the same situation, with one student using the targeted GMK and the other not using the GMK. Taking the rows together, there was contrast across the columns and the same underlying idea, or invariant, within a column. The task directive was to compare the approaches of the hypothetical students in one column to those of students in the other and to state the defining characteristic of the approach in the metacognitive column.

To illustrate how learning can occur in synthesis, consider Figure 5.1. Students saw the $3 \times 2$ matrix in the dashed box. In this matrix, within both
columns, the cases shared the underlying idea, or invariant. In column B, the hypothetical students demonstrated the targeted GMK, while those in column A did not. However, column A had less structure in relation to column B. Students may learn the GMK by searching for the invariant across the three cases in different contexts within column B. The relevant portion of the vignette is underlined in the figure. Across these cases, students could induce and abstract the GMK. Andy, Dana, and Fred all thought about how and why certain features in a situation impacted thinking. In contrast to column B, the hypothetical students in column A were not metacognitive. There was an absence of the targeted GMK in the approaches of the hypothetical students in column A, which were in the same varied contexts as in column B.

The structure of the matrix provided students with two affordances to build understanding of the GMK: invariance and contrast. There was invariance within each column, although A was defined in relation to B through contrast both locally within each row and more globally across each column. The combination of both contrast and invariance could help students differentiate the approaches and understand the defining characteristic of the metacognitive approach in column B.
Participants

Participants were a subset of those who participated in the experimental described in Chapter 2. Thirty-seven students engaged in the synthesis conditions. The two focal students were in the synthesis with an everyday context, which included a contrast pair in an everyday context.

Source of Data

The primary source of data was audio recordings of students’ think-aloud interviews while completing synthesis.

Procedure

I was the sole experimenter and conducted one-on-one think-aloud interviews with students as they processed the $3 \times 2$ matrix shown in Figure 5.2. To familiarize students with the synthesis process, students were asked to complete a practice $3 \times 2$ synthesis matrix, which was in an everyday context (Appendix A). Part of this practice also instructed students to verbalize their general procedure prior to arriving at their responses. When students finished, I provided a brief explanation of the practice synthesis task and a sample procedure. Then I gave students the $3 \times 2$ matrix for synthesis (see Fig. 5.2). On average, students took 5 minutes to complete this task.

Methods of Analysis

Sampling. The purpose of the study was to generate insights into how students can productively build the GMK through contrast and invariance. Thus, I was interested in the students who (1) used both affordances and (2) demonstrated a growth in their ideas. To reduce the sample for in-depth analysis, I coded the transcripts for these two characteristics. The first characteristic was
affordance use. I coded whether students used contrast and invariance separately. Contrast use occurred when students searched for a difference across columns. The contrast may be between cases, between students’ ideas for invariants A and B, or between students’ idea for A/B and a single B/A case. Invariance use occurred when students searched for a commonality between two or more cases within the same column.

The second characteristic concerned whether student’s ideas of the GMK progressed from beginning to end. In particular, the focus was on whether a student’s ideas for the invariant moved closer to the targeted GMK. Students’ responses were coded as either having no change or no relevant change of ideas, or having a relevant change in ideas. To determine whether a relevant change occurred, I first identified all of the student’s generalized statements about two or
Table 5.1: Affordance use and relevant change in ideas during synthesis

<table>
<thead>
<tr>
<th>Affordance use</th>
<th>Relevant change of ideas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
</tr>
<tr>
<td>one</td>
<td>21</td>
</tr>
<tr>
<td>both</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>26</td>
</tr>
</tbody>
</table>

more cases within the same column. A generalized statement was defined as phrases that described what the cases represented and reflected a single idea. If there was only one generalized statement for each column, I interpreted this as no change in definition. If there were multiple generalized statements and each statement gained greater definition than the previous statement, I interpreted this as a change in relevant idea. Two common relevant changes were when: (1) students’ subsequent statements moved toward the GMK via increasing focus on reflective thinking, and students’ subsequent statements become less vague.

Table 5.1 summarizes the results. The analysis revealed that all students generated ideas of invariants for A and for B and contrasted these ideas as their final statement. Thus, this was considered as one affordance use. The results show that almost three-fourth of the students used invariance when stating their ideas for A and B but only contrasted in the end to compare those ideas. For example, a typical student statement, relevant to the GMK, was, “Column B put more thought into their answers, and the people in column A put in less thought in their answers.” Almost the same number of students used both affordances but did not have a change in idea, or had a change in idea but used only one affordance. Just over one-eighth of students used both affordances and showed growth toward the GMK. These five students were the sample for in-depth analysis.
**Choosing the focal students.** I did initial analysis on all five students, which is provided in detail the following subsection. To give a broad overview of the outcome of my analysis, I choose only two focal students, Max and Lynn, and present in-depth analysis for both. I chose to focus primarily on Max because he provided the richest data, as he demonstrated the most varied ways in his use of contrast and invariance and well articulated his ideas. Furthermore, Max also had the greatest growth in his ideas, meaning that that he revised his ideas more often than other students. I chose the second student, Lynn, as a contrast to Max. She demonstrated different uses of the affordances in order to build her understanding. The other three students offered less rich data for a variety of reasons. For instance, one student did not express his ideas clearly and his thoughts often trailed off, which made it difficult to infer how his idea was built. Another student arrived at her ideas quickly with far less use of affordances compared to Max and Lynn. The last student was well-articulated, but his approach was similar to Max and did not add any new use of the affordances that was not already demonstrated by Max and Lynn. In addition, across all three students, Max and Lynn spanned all possible uses of affordances across the five students.

**Initial analysis to choose focal students.** Broadly, I examined students’ transcripts to identify when students’ ideas were changing with their use of the affordances. To do this, for each individual student, I first segmented the student’s transcript into what I refer to as moves. A move was defined as an action that a student undertook with the cases, specifically the use of contrast, invariance, or both. For example, Lynn said, “Like Andy at first, Dana was using a mathematical approach to solving the physics problem, but she doesn’t change her mind like Andy did.” This was a move because the student used invariance by looking for a commonality between two cases in the same column. Specifically, here the commonality of “mathematical approach” between two B cases, Andy and Dana.
Immediately after this statement, the Lynn said, “She [Dana] just kind of thought, it’s kind of the same thing because they thought, well, why, the why behind it.” This was a new move. Even though Lynn used invariance between the same two cases (Andy and Dana) again, she generated a new idea for commonality between.

Next, for each move, I created a schematic using the general layout of the $3 \times 2$ matrix as a basis. As seen in Figure 5.4 for Max, a schematic shows three features of interest. First, it shows which affordances the students used. Contrast is represented with the green rectangle that encompasses cells across the columns. Contrast is also represented with a thunderbolt to indicate what was contrasted (e.g., between cases, between ideas, or between a case and an idea). Invariance is represented with the red rectangle that encompasses cells within a column. Second, the schematic also shows which cases students used, the non-transparent cells. Last, the schematic shows when a student’s idea of the invariant was generated, revised, or refined, as indicated by the letter in the column header. By combining the moves for each student in sequential order, I had a broad overview of the student’s entire process. These schematic maps showed students’ interactions with the cases and when their ideas emerged.

Looking across all these maps together, Max’s process revealed the most diverse affordance use with multiple ideas being generated and revised. Next, Lynn’s process had the greatest contrast to Max’s process. While Max started with an idea for the invariance, Lynn considered individual contrasts by row. The other three students did not offer uses of the affordances different from those offered by Max and Lynn. Student X’s approach and moves were similar to Max but did not have as many varied uses of the affordances, while students Y and Z had limited use of the affordances and of cases to generate their ideas.

**In-depth analysis.** The schematic maps provided information on student uses of contrast and invariance and when their ideas for the invariant were
Figure 5.3: Schematic map of moves for the five focal students’ affordance use
Figure 5.4: Overview of Max’s synthesis process

(a) Max’s ideas of the knowledge as he progresses

(b) Max’s ideas by moves

PHASE 1

MOVES 1-3
Blair “more confident”
Andy “no confidence in his mathematical approach”
Blair does not think μ is valuable and sees μ as a pass
Corey
Andy thinks μ is valuable and uses it

PHASE 2

MOVE 4
Andy uses mathematical approach and changes mind
Dana uses mathematical approach and does not change mind

PHASE 3

MOVE 5

MOVE 6

A
“going with their first thought”
Ellen not misconception

A’
“went with what they thought would work or believed in”

MOVE 7

B
thinking about the “the deepening meaning of the why”

B’
“thinking maybe it’s because it’s adjacent

MOVE 8

Fred
“thinking maybe it’s because it’s adjacent

Dana
“why the steep slope is non-increasing”

Andy
“why he decided to use the value for μ”
(a) Lynn’s ideas of the knowledge she progress

![Diagram showing Lynn's synthesis process]

(b) Lynn’s idea by moves

**PHASE 1**

<table>
<thead>
<tr>
<th>MOVE 1-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blair: &quot;more confident&quot;</td>
</tr>
<tr>
<td>Blair: &quot;does not think μ is valuable and does not pass it&quot;</td>
</tr>
<tr>
<td>Andy: &quot;more confident in his mathematical approach&quot;</td>
</tr>
<tr>
<td>Andy: &quot;thinks μ is valuable and uses it&quot;</td>
</tr>
<tr>
<td>Dana: &quot;goes with what they think would work or believed in&quot;</td>
</tr>
</tbody>
</table>

**PHASE 2**

<table>
<thead>
<tr>
<th>MOVE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andy: uses mathematical approach and changes mind</td>
</tr>
<tr>
<td>Dana: uses mathematical approach and does not change mind</td>
</tr>
<tr>
<td>B: thinking about &quot;the why behind it&quot;</td>
</tr>
</tbody>
</table>

**PHASE 3**

<table>
<thead>
<tr>
<th>MOVE 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: &quot;goes with their first thought&quot;</td>
</tr>
<tr>
<td>While not misconception</td>
</tr>
<tr>
<td>A': &quot;went with what they thought would work or believed in&quot;</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MOVE 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>B: thinking about &quot;the deepening meaning of the why&quot;</td>
</tr>
<tr>
<td>Thinking about &quot;the why behind it&quot;</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MOVE 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fred: &quot;thinking maybe it's because its associate&quot;</td>
</tr>
<tr>
<td>Dana: &quot;why the steeper slope is more tempting&quot;</td>
</tr>
<tr>
<td>Andy: &quot;why be undecided, use the value for μ&quot;</td>
</tr>
</tbody>
</table>
generated and revised. However, what students’ ideas were for the cases and for
the invariants are missing from the schematic map. Thus, I created two additional
representations of different grain sizes for closer analysis of students’ ideas and
affordance use. One representation is a finer-grained representation of each move,
which shows the interaction of the affordance and students’ ideas (Figures 5.4b
and 5.5b). The other representation is a coarser grain size, which shows the
proximity of students’ ideas for the invariants to the GMK (Figures 5.4a and 5.5a).

For in-depth analysis of Max and Lynn move by move, together with these three
representations for each focal student, I returned to the student’s transcript, which
was segmented by moves, to describe and understand their processes.

As further explanations for these representations, the finer-grained
representation of students’ ideas and affordance use are shown Figures 5.4b and
5.5b for Max and Lynn, respectively. To illustrate what these representations
represent, consider Figure 5.4b, Max’s ideas. Max’s idea for a single case is
represented by a solid rounded rectangle. Inside is his abbreviated idea, grounded
in his own words, although sometimes his words are used, which are indicated in
quotation marks. Max’s idea of an invariant is represented by a larger solid
rectangle. Similarly, inside is his abbreviated idea. Whenever Max used contrast, a
curved arrow is shown between what ideas he contrasted. Whenever Max used
invariance, a straight arrow is used. The dashed boxes indicate an implicit idea,
which was not explicitly stated, for a single case, cases, or column.

The coarser grain size of students’ ideas for the invariant is shown in Figure
5.4a and 5.5a. To illustrate, consider Figure 5.4a for Max. The steps represent his
ideas of the invariant, which moved toward the GMK located at the center, highest
level. Thus, the steps indicate the proximity of the ideas to the invariant GMK.
The difference between the steps is not necessarily equidistant and not of
consequence for analysis. The figure also shows that the targeted GMK can be
built from the ideas for A and B, as they are related through contrast. The left steps represent ideas for A, while the right steps represent ideas for B. Note that ideas for A and B do not have to occur simultaneously. And by design of the synthesis task, moving up the steps may require the use of invariance, contrast, or a combination of both.

Analysis

In this section, I present my analyses of the focal students, Max and Lynn, in detail. For each student, I first give an overview of the data and analysis. Each student’s process is broken down into phases. After the overview, I present each phase separately, starting with the description of the data followed by the analysis. The analysis focused on how the students used contrast and invariance to build understanding of the GMK.

Overview of Max

Max started with a vague idea of the invariant for the students in column A, describing them as using a “direct approach.” Building from his idea for A, Max arrived at a well-defined idea of the B, or the invariant for the students in column B, as “trying to think about what [they’re] trying to do.” This was Max’s peak, when his idea was the closest to the Although he ended with stating that A had a “direct approach.” and B as “thinking in different ways,” his ideas had greater meaning than their outward expressions suggested.

Figure 5.4a shows Max’s key moves in building his understanding from the start of his process to the peak. Max was at the first step when he generated his initial idea of the invariant for A as a “direct approach” [A]. He gained a firm foothold at this step by explaining how each of the A cases was an example of his idea (move 1). In his next three moves, Max moved up two steps and made
significant progress. First, he formed an implicit general idea between two B cases as thinking about “why” \[b\], which prompted him to revise his idea for A as “doing something but . . . not really thinking about it” \[A’\] (move 3). Max then elaborated on this idea as he contrasted it to his first idea for B as “trying to find new ways to do things” \[A”\] (move 4). At his peak of his progress towards building the GMK, Max refined his idea for B through Andy who was “thinking about what he was trying to do” \[B’\] and extended it to all cases, thus moving closer to the targeted GMK (move 5). He gained further clarity on his idea by explaining his idea for B for both A and B cases (move 6-8). Not shown on the figure is his final summary statement of his ideas, which contained more meaning than their outward expressions.

Max’s process is organized and presented in three chronological phases. These phases are centered on the generation of his ideas, and they are marked on the right side of Figure 3. These phases are as follows:

- Phase I: Generating the initial idea of the invariant for A (move 1)
- Phase II: Revising the idea of the invariant for A, while generating one for B (moves 2-5)
- Phase III: Refining the idea for the invariant B (moves 6-8)

More generally, as seen in Figure 5.4a, in the first phase, Max generated an initial idea of the invariant for A and applied it to each A case, thereby instantiating his idea. In the next phase, Max took the opposite of his idea for A and projected it onto a B case. He formed an implicit latent idea between two cases in column B, leading him to consider and revise his idea for A. Finally, in the last phase, when Max instantiated his idea for B on one of the B cases, he elaborated on his idea and extended the idea to include the other B cases. He proceeded to instantiated
his idea on last two B cases, as well as its opposite on an A case. He ended by stating his final ideas for A and B.

**Phase I: Generating the idea of the invariant for A**

**Description**

Move 1: After reading all of the cases by columns, Max generated an idea of the invariant in column A as using a “direct approach.” He then proceeded to instantiate “direct approach” upon the cases. For each case, he recapped the situation, and then described how each supported his idea:

- Blair “doesn’t want to alter her way of thinking. She sees that someone else got a different answer and she thinks that they made a mistake. So she doesn’t want to alter her way of thinking.”

- Corey “gets the wrong answer and realizes what he did wrong, but then he decides you’d always have to do that [use energy conservation], which is probably not the best choice. You shouldn’t always do something one way.”

- For Ellen, “if that analogy was used in, like, a problem sense, she’s searching for an answer that she already knows, kind of, and – but she sees something else but puts it back down and finds the right answer.”

**Analysis**

**Using invariance.** Max’s first move illustrates the broad process of generalization and abstraction, as discussed in Chapter 4. Max was able to abstract an idea, or generalization, from all three cases. As he was reading, Max searched for the invariant within the cases of column A, and he generated his first
Table 5.2: Transcript of Max’s process during synthesis segmented by moves

<table>
<thead>
<tr>
<th>Move</th>
<th>Schematic</th>
<th>Description</th>
<th>Transcript</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="#" alt="Schematic" /></td>
<td>Generating A generalization</td>
<td>OK. Let's see. So... um, the students in column A have, they, they have a direct, like, approach to the problem that they're doing.</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td><img src="#" alt="B" /></td>
<td>Instantiating generalization on A1-A3</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td><img src="#" alt="B" /></td>
<td>Using A generalization to project a contrast onto B1</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td><img src="#" alt="B1" /></td>
<td>Finding commonality (implicitly) between B2 and B3</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>b</td>
<td>Revising A generalization by contrasting to b</td>
</tr>
<tr>
<td>4</td>
<td>A''</td>
<td><img src="#" alt="B" /></td>
<td>Generating B generalization through contrast to A''</td>
</tr>
<tr>
<td>5</td>
<td>A''</td>
<td><img src="#" alt="B" /></td>
<td>Using a contrast to generate an invariant</td>
</tr>
<tr>
<td>6</td>
<td>A''</td>
<td><img src="#" alt="B" /></td>
<td>Instantiating an invariant onto a contrast pair</td>
</tr>
<tr>
<td>7</td>
<td>A''</td>
<td><img src="#" alt="B" /></td>
<td>Further instantiating B' on B3</td>
</tr>
</tbody>
</table>
idea of the invariant for A as a “direct approach.” He returned to the cases to instantiate his idea, as shown in Figure 5.4 move 1. In doing so, he was evaluating whether his working idea applied to the cases and thus began to build a definition of what he meant by “direct.”

The data indicate that “direct” took on three different meanings around a central idea. First, a direct approach corresponded to not changing one’s thinking, as evidenced by Blair, who did not “alter her way of thinking.” A direct approach also referred to deciding to always use one method to solve a problem, and Max judged Corey’s decision as not being a good one. Finally, “direct” meant keeping focus and ignoring distractions in order to arrive at the “right” answer as in the case of Ellen. Although Max appeared satisfied with his instantiations on the A cases, as suggested by restating his generalization in his next move, his descriptions hinted otherwise. At this point, his three meanings for “direct” clustered around a central idea, although not one that is complete or one that he is able articulate at this point. Together, they suggested something more than “direct” and perhaps hinted toward a latent idea of a less thoughtful approach. Although his generalization took on various meanings, Max gained enough traction so that he could move forward.

Phase II: Revising the idea for A while generating one for B

Description

Move 2: After instantiating “direct approach” upon the cases in column A, Max restated that column A had a more direct approach to the students’ problems. Then he began to visit the cases in column B to contrast it with a “direct approach,” but only got to Andy, who he described as “reconsider[ing] his answer” and, implicitly, as the opposite of Blair, who did not “alter her way of thinking.”
Move 3: Max continued on to the last two B cases, Dana and Fred, and he implicitly found a local commonality between them as thinking about “why.” He said that Dana was thinking “about why the steeper slope could have caused . . . someone to think that could have caused a faster speed,” and Fred realized “why he wanted them [Fruity O’s], and then decides not to get them.”

Move 4: In contrast to his idea for Dana and Fred, Max returned to column A and revised his initial idea of A from “direct approach” to say that, “The students in A are just kind of – they’re doing something, but they’re not really thinking about it.”

Move 5: Max elaborated on his revised idea for A while he contrasted it to his idea of the invariant in B, thus generating his first idea for B, the GMK. He said, “Well, they are thinking about it, but they have a very, like, straight line of thinking where the students in column B are trying to find new ways to do things.”

Analysis

Instantiating an invariant as an opposite. At the start of phase II, Max took his idea for the invariant of A to interpret Andy’s case [B1]. Max described Andy’s action as the opposite of his general idea of a “direct approach” and, more specifically, to Blair’s “direct” approach. Max described Andy who reconsidered his idea as the opposite of Blair who did not change her mind. As shown in Figure 3 for move 2, Max maintained his general idea of a “direct approach” together with his instantiation of Blair when he contrasted Andy’s case by indicating that it was the opposite of both. Maintaining the general idea with an instantiation when contrasting involves a coordination of invariance and contrast. This coordination was also a type of instantiation; Max instantiated the opposite of his general idea for one column onto a case in the other column. Thus,
Max increased his definition of a “direct approach” but only to one of his three meanings that he articulated in move 1. His contrast of Andy had a looser relation to the other two A cases (i.e., using one method and keeping focus) while having a clearer relation to Blair.

**Using invariance when contrast fails.** At the start of his visitation of the B cases, Max had begun by using contrast with the first case (move 2), but the data suggest that contrast failed for the next two cases (move 3). Unlike with Andy, Max’s descriptions of Dana [B2] and Fred [B3] were not clear contrasts to their counterparts, Corey [A2] and Ellen [A3], or to his general idea of a direct approach. Instead, Max transitioned to using invariance, meaning that he searched for a commonality between Corey [A2] and Ellen [A3]. The shift from contrast to invariance could be due the fact that contrasting Dana and Fred as employing the opposite of a direct approach did not hold as well as with Andy. Although he stayed close to the provided descriptions within the cases, Max extracted similar statements from Dana and Fred. As shown in Figure 2, Max described both Dana and Fred as thinking about “why”: Dana thinks about why other students would think steeper implied faster, and Fred thinks about why he wanted the Fruity O’s. With these two cases, Max implicitly abstracted a local invariant between Dana and Fred [b], shown in the dashed box in Figure 5.4b, moves 3-4. This was further evident in move 4, when Max returned to his initial idea of A. He revised his initial idea of a “direct approach” to “doing something but they’re not really thinking about it” [A’], which was the absence of thinking “why,” something he implicitly found similar across Dana and Fred. As shown in Figure 5.4a, this move was his first major step toward the GMK. Max moved from three different meanings of a “direct approach” to possibly one meaning. At this point, how the A cases exemplify this remained unclear, as Max did not revisit the A cases to instantiate
this revised idea. However, what is clear is that Max now saw A in a new light, which was brought about in contrast to the two B cases.

Reflecting further on moves 2-4 in Figure 5.4b, Max shifted or toggled back and forth between using contrast and invariance. This toggling had two consequences. The first was that when contrast did not work, Max switched to invariance as a way to avoid a roadblock and as a means to continue his progress. The second was that it helped Max gain further clarity about his idea for the invariant for A. In his toggling, Max formed an implicit local invariant across two B cases, furthering his progress in two distinct ways. As discussed earlier, it led Max to revise his initial idea of A to “not really thinking about it.” It is possible that his latent idea of a less thoughtful approach, implied from move 1, was taking form. Contrast brought out greater precision to his idea for A, which could not be accessed through the use of invariance alone. Thus, the use of invariance and the use of contrast are complementary processes. As a result, Max could articulate what he could not articulate earlier. Furthermore, Max may have generated a latent idea of the invariant for B. Because he knew that his idea for A and B were linked through contrast, Max’s idea of A as “not really thinking about it” suggested that a latent idea for B might be the presence of “thinking about it.” This is supported by his descriptions of Dana and Fred as thinking as “why” in move 3. Thus, by toggling between invariance and contrast, Max made progress in building the GMK by building greater definition for A, as well as by generating an important latent idea for B.

**Generating an invariant for B.** During phase 2, Max leveraged his ideas for A multiple times (moves 4-5) to generate his first idea of the invariant for B (move 6). Max sequentially used invariance and contrast to arrive at his idea. As mentioned already, in move 4, Max implicitly abstracted a local invariant between Dana and Fred but not one that appeared to accommodate Andy. Yet, this was
sufficient enough to initiate Max to use contrast and revise a “direct approach” [A] to “not really thinking about it” [A’] (move 3). Then, in move 5, Max elaborated to say that “they are thinking about it, but they have a very . . . straight line of thinking” [A’’] as a contrast to his idea of the invariant for B as “trying to find new ways to do things” [B] (move 5). This was his first idea for B. His abstraction for B was likely informed and extended from his idea of the local invariant between Dana and Fred, who were thinking “why”, as well as the contrast to his idea of A as a “straight line of thinking” [A’’]. Thus, both invariance and contrast played complementary roles in helping Max generate his first idea for B. Furthermore, Max arrived at his first idea for B by building his ideas for A. However, at this point, his idea for B was vague, especially since Max did not say how the B cases illustrated “finding new ways.” In spite of this ambiguity, his prior moves suggest that there was more to his idea than what he generally stated. His idea for B was another step forward toward building the targeted GMK.

**Phase III: Refining the idea for the invariant of B**

**Description**

Move 6: Max revisited the cases in row-wise fashion. Starting with the first row, he contrasted Blair and Andy. He said, “Blair gets the correct answer and . . . thinks that Andy got the wrong answer, and then Andy realizes what he got wrong and thinks about what he was trying to do.” He then extended the idea from Andy to apply to all of the cases in B, saying “that kind of goes for all of them.”

Move 7: Moving on to the second row, Max instantiated the opposite of Andy’s approach to Corey’s approach, pointing out that Corey got the wrong answer but “he doesn’t think about why he got it wrong.” Then, for Dana, he instantiated Andy’s approach on Dana as well as contrasted it
to Corey, noting she was correct but yet she still thought “about what the other students could have done to get the wrong answer.”

Move 8: Arriving at the last row, Max did not talk about Ellen but further instantiated Andy’s approach on Fred, who “thinks about why he wants them [Fruity O’s]”

Final statement: Andy ended with his final statement of his ideas for A and B. He said, “The main difference is that the students in column B are thinking in different ways as opposed to students in column A who have more of a direct approach.”

Analysis

Elaborating on an idea from instantiating. At the start of phase 3, Max had already generated an idea for the GMK as “finding new ways to do things.” However, revisiting the cases suggests that he was tentative about his idea and what he did next suggested that he knew more about his idea when he described Andy’s action as metacognitive, thinking about his intentions [B1]. As seen in Figure 2 move 6, Max began with the first row cases. He differentiated how Andy “thinks about what he was trying to do” from the approach of Blair, touching upon metacognitive thinking for the first time, and he extended the idea to the other two B cases. The data indicated that Max saw the same general idea exemplified by Andy across the other two cases, Dana and Fred, when he said “that kinda goes for all of them” [B’]. However, Max did not explicitly state the more general idea. It is unclear whether Max was instantiating his idea for B of “finding new ways” on Andy’s case, but his seamless extension of Andy’s approach to the entire column suggests that he may already have had in mind what the commonality could be. Perhaps contrasting Blair’s and Andy’s cases brought his latent idea for B to surface, an idea that appeared in line with his prior articulated ideas in moves 3-4. That is, Max’s description of Andy resonated with his implicit idea spanning Dana and Fred as thinking about “why” and noticeably contrasted
with his prior idea of A as ‘not really thinking about it.” Although Max was not explicit in what “goes for all of them” in the B cases, Max nonetheless gained greater clarity for his idea for B in move 6 through Andy. Max’s idea of B became less vague as he did not previously have a specific instance for his idea.

In move 6, Max coordinated his use of contrast and invariance. Max contrasted Blair and Andy to see that Andy was thinking about what he was trying to do. He immediately extended his idea for Andy to the other cases. That is, rather than treating the contrast within a row independently from the invariance within a column, Max was also able to consider the invariant at the same time.

**Instantiating an idea.** After Max extended his idea for Andy to the other B cases (move 6), he continued to the next row and instantiated his idea for Andy on both cases (move 7). In these moves, Max gained further definition in his idea for B. Figure 2 move 6 shows that Max described Corey as someone who “doesn’t think about why he got it wrong,” while Dana is someone who “thinks about what the other students could have done to get the wrong answer.” This was another example of Max coordinating contrast and invariance. Here, Max maintained his idea of the invariant, as exemplified by Andy, when instantiating its absence and presence across a contrast pair. Specifically, Max pointed out, in respect to Andy’s approach, that there was an absence of thinking with Corey and a presence of thinking with Dana. Max furthered instantiated on Fred, the last case in B, saying that Fred “thinks about why he wants them” (move 8). With each instantiation, Max gained greater clarity about his idea of the invariant because he spanned a greater number of possible contexts that accommodated his idea.

In addition to seeking the invariant within B, Max attended to correctness across the cases, which varied within each column. In move 7, Max also pointed out that Blair was correct and Andy was incorrect. In his next move, Max also
noted correctness across Corey and Dana. Max seemed to emphasize that Corey was in column A and was incorrect, while Dana was correct. Although correctness was irrelevant to the invariant, noticing this contextual difference was important because it addressed a conditional aspect of his idea for B. Max came to realize that his idea applied whether one arrived at the correct reasoning or not within a given situation. Thus, by attending to correctness and instantiating its presence and absence across the rows, Max also gained further clarity about his idea for B.

**Final summary statement of ideas.** Max ended with stating his final ideas for A as a “direct approach” and B as “thinking in different ways.” Neither of his outward expressions captures the full essence of his idea. Although his final expressed idea for A is identical to his initial idea, a “direct approach” has greater definition and meaning as developed through his multiple instantiations and contrasts to his idea for B. For his idea for B, he ends with “thinking in different ways,” which has greater precision than his initial idea of “trying to find new ways to do things.” However, it still lacked the precision he attained at the peak, when he elaborated his idea for B in the context of Andy as thinking about his intention (move 6).

**Overview of Lynn**

Compared to Max, Lynn generated her ideas of the invariant more gradually and had less varied uses of contrast and invariance. Nevertheless, her piece-meal approach provided additional insight into the processes by which students built the GMK. Her first essential idea for B was generated by finding a commonality two B cases (Figure 5.5) as thinking about “why,” which was later elaborated on and extended to the B column as “thinking about the deepening meaning of why.” This was the peak of her process, the closest she came to the
Table 5.3: Transcript of Lynn’s process during synthesis segmented by moves

<table>
<thead>
<tr>
<th>Move</th>
<th>Schematic</th>
<th>Description</th>
<th>Transcript</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>![A B](A1 B1)</td>
<td>Generating a contrast between A1 and B1</td>
<td>Column A, the first column, the first row. Blair ignores the values for $\mu$. She correctly applies Newton’s 2nd law. She checks her answer with Andy who got a different answer. She thinks that Andy made a mistake by using $\mu$. Andy uses $f = \mu N$ to calculate static friction. He checks his answer with Blair and realizes he should have used Newton’s 2nd Law instead. So already it’s obvious that Blair is more confident with her work than Andy. He checks his answer with Blair and realizes he should have used Newton’s 2nd Law instead. He wonders why he used $\mu$. He -- like I said, lack of confidence in his answer and then, so he has no confidence in his mathematical approach. He decides that having a number for $\mu$ made him think he needed to use it.</td>
</tr>
<tr>
<td>2</td>
<td>![A B](A1 B1)</td>
<td>Generating another contrast between A1 and B1</td>
<td>So he thinks he was influenced by the inclusion, or the fact that $\mu$ was included when it didn't need to be, and Blair ignores the value for, for $\mu$. So that, it's two totally different think -- like, ways of thinking. She was able to look past what she thought was not valuable in the problem, but Andy used it because he thought since it was included in mu must be valuable in problem.</td>
</tr>
<tr>
<td>3</td>
<td>![A B](A1 B1)</td>
<td>Attempting to generate a contrast between A2-B2</td>
<td>[reads Corey’s case then Dana’s] So again in this situation, Corey, Corey was wrong and he realizes that energy conservation gives the right answer, so he decides to always use energy conservation, which isn't a bad approach because energy is always conserved, but Dana uses energy conservation and get the right answer. A lot of students answered incorrectly because they thought that steeper implies faster.</td>
</tr>
<tr>
<td>4</td>
<td>![A b](A1 B1)</td>
<td>Generating an local idea b between B1-B2</td>
<td>So like Andy [B1] at first, Dana [B2] was using a mathematical approach to solving the physics problem, but she doesn't change her mind like Andy did.</td>
</tr>
<tr>
<td>5</td>
<td>![A b'](A1 B1)</td>
<td>Generating essential local idea b' between B1-B2</td>
<td>She [B2] just kind of thought, it's kind of the same thing because they thought, well, why, the why behind it.</td>
</tr>
<tr>
<td>6</td>
<td>![A' b'](A1 B1)</td>
<td>Generating idea of invariant for A, instantiating on A3 revises</td>
<td>[reads Ellen] So all three students in column A, they go with their first thought and their, and like what they, what they thought which might, or most of their cases were misconceptions besides Ellen and, Oh, I want Honey Nut O's, but they went with what they thought would work, or what they believed in.</td>
</tr>
<tr>
<td>7</td>
<td>![A' B](A1 B1)</td>
<td>Generating an idea for B</td>
<td>[reads Fred] So Fred [B3], he was going for what he believed in and then the box of Fruity O's jump out at him, so he was tempted to use them but then he thought... like student, like, basically the students in column B all end up in the end thinking, in thinking, is, like, the deepening meaning of the why.</td>
</tr>
<tr>
<td>8</td>
<td>![A' B](A1 B1)</td>
<td>Instantiating the invariant on the B cases</td>
<td>So he was, he was thinking maybe it's because it's adjectives like &quot;naturally flavored&quot; and &quot;fruity&quot; make the cereal seem healthy, where Dana [B2] was thinking why the steeper slope is more tempting and then Andy [B1] was thinking, he was thinking, he was wondering why he decided to use the value for $\mu$.</td>
</tr>
</tbody>
</table>
GMK, as she ended with B having a “systematic” approach. Like Andy, her final statement may not capture the essence of her idea.

As shown in Figure 5.5b move 1, Lynn did not have an immediately idea for the invariant for either column. Starting with the first row, Lynn generated two contrasts. Her first two moves are not shown in Figure 5.5a, as she did not yet have an idea for the invariant. She was on the ground level, but she had potential contrasts from which to build. Lynn moved on to the second row, but ran into difficulty. Similar to Max when his use of contrast failed (move 3), she switched to using invariance. Lynn found two commonalities between two B cases, with one having the essential idea as thinking “why,” which marked her first step toward the GMK (move 4). Moving on to the last case in the A column, she generated her first idea for A as “going with their first thought” [A], but found an exception in Ellen’s case, and revised her idea to accommodate Ellen to say that A “went with what they thought would work or what they believed in” [A’] (moves 5-6). In her last move, she moved to the last case in the B column. When she described the case, she generated her idea for B, in line with her prior idea for B, as thinking about “the deepening meaning of the why” [B], which she then applied to all three B cases (move 7-8), touching upon reflective thinking related to the GMK.

Her process can be segmented out into three parts as follows:

- Phase I: Contrasting individual paired contrasts (moves 1-2)
- Phase II: Generating an idea of a local invariant for B cases (moves 3-5)
- Phase III: Generating ideas of the invariants (moves 6-8)

More generally, in the first phase, Lynn started by generating ideas through individual contrasts, which were neither consequential to the GMK. In the next phase, after a failed attempt use of contrast between two cases, she generated ideas
for the local B invariant, of which one was a consequential latent idea for B. In the final phase, she generated ideas for the invariants for both A and B. After generating her idea for A, she instantiated it on an A case, which did not fit, and returned to revise it. While considering the last B case, an idea for B emerged from her latent idea from the prior phase. She then proceeded to instantiate it on the other two B cases. She ended by stating her final ideas for A and B.

**Phase I: Generating potential ideas of the invariant through single contrasts**

**Description**
Move 1: To start, Lynn read the first row. When she finished with Blair’s case, she started on Andy’s case but stopped after the second bullet point to comment. She noted, “So already it’s obvious that Blair is more confident with her work than Andy.” She continued to read, stopping once more after the third bullet point to confirm her prior observation: “Like I said, lack of confidence in his answer . . . so he has no confidence in his mathematical approach.”

Move 2: Lynn finished reading Andy’s case and contrasted Blair and Andy again. She saw two different ways of thinking, in which Blair “was able to look past what she thought was not valuable in the problem, but Andy used it because he thought since it was included it must be valuable in problem.”

**Analysis**
Unlike Max who read the cases by columns and generated an initial idea of the invariant from the start, Lynn read the cases by rows, an approach that lent itself to contrast. For the cases in the first row, Blair and Andy, she described two contrasts. The first contrast was a presence/absence of confidence in an approach, which appeared to be the most obvious for Lynn. The second contrast was a
presence/absence of the ability to look past an irrelevant problem feature. Although these contrasts were context-specific and not yet generalized to the other cases, Lynn produced some potential ideas to work with as she moved onto the next two rows. At the end of this phase, Lynn had not generated an idea of the invariant. Despite not yet generated an idea, she progressed as she laid groundwork for her potential ideas of the invariant.

**Part II: Generating local ideas of the invariant for B**

**Description**

Move 3: Lynn moved on to the second row and read Corey’s and Dana’s cases in their entirety, then recapped Corey’s approach but reread the first two bullet points of Dana’s case.

Move 4: She then shifted her focus to Andy and Dana in the B column, finding a commonality and a difference: “So like Andy at first, Dana was using a mathematical approach to solving the physics problem, but she doesn’t change her mind like Andy did.”

Move 5: Lynn stated the essential relationship between the Andy and Dana, finding that Dana “just kind of thought it’s kind of the same thing because they thought, well, why, the why behind it.”

**Analysis**

At the start of phase 2, Lynn continued the use of contrast with the next row but had difficulty. Previously, Lynn’s use of contrast between Blair and Andy was successful as she generated two distinct contrasts. However, the data suggest that Lynn’s use of contrast between Corey and Dana was challenging, and she did not generate any clear contrasts (move 3). After reading both cases, Lynn summarized Corey’s approach, pointing out that he was incorrect and that he did not use energy conservation. Then, she attempted to contrast Dana, but instead,
she fell back on re-reading Dana’s case. In response to her difficulty, she shifted from contrast to invariance, and in her next move, Lynn found a commonality and difference between Andy and Dana in column B (move 4). The data indicate that she may have used one of her individual contrasts from the prior row as the basis for a commonality. She noticed that Dana was like Andy in using a mathematical approach but also noticed that Dana differed from Andy in the presence of confidence in her answer. The notion of confidence appeared to be a potential idea for the invariant, as this was the first thing that she noticed in move 1, in which she made two specific comments concerning Andy’s confidence. Seeing this difference, Lynn may have been unsatisfied with this commonality, because consequently, she continued to search for another one between Andy and Dana (move 5). She saw that they thought the “same thing,” and thought about “the why behind it.” This was a key move for Lynn, as she generated an essential idea for B, as shown in Figure 3. At the end of phase 2, Lynn has achieved a step in her progress with an idea that is related to reflective thinking.

Phase III: Generating the ideas of the invariants

Description

Move 6: Arriving at the last row, Lynn read Ellen’s case in column A. Then, she stated her first idea of the invariant for A as “going with their first thought.” She continued to point out that these thoughts were “misconceptions” for everyone in the column except for Ellen, who thought, “Oh, I want Honey Nut O’s.” She then modified her idea to say, “They went with what they thought would work or what they believed in.”

Move 7: Lynn reached the last case, Fred, in column B. After she read the case, she began to recap the situation but paused slightly as she attempted to
describe what Fred thought, and she saw a connection to her prior latent idea for B as thinking “why.” She connected Fred to this by stating her first idea of the invariant for B as “all end up in the end thinking . . . the deepening meaning of the why.”

Move 8: Lynn instantiated her idea on each B case, starting with Fred and ending with Andy:

- Fred “was thinking maybe it’s because it’s adjectives like ‘naturally flavored’ and ‘fruity’ make the cereal seem healthy.”
- “Dana was thinking why the steeper slope is more tempting.”
- “Andy was thinking . . . he was wondering why he decided to use the value for μ.” Final: Lynn ends by stating her final ideas of the invariants: “The students in column B, just to recap what I’ve been saying, they use a more systematic approach than using instinct to make their decisions.”

Analysis

Revising an idea after instantiating. In this last phase, Lynn appeared to use invariance only. She generated an idea for A and for B without explicitly relating them to one another, illustrating the processes of abstraction and generalization as discussed in Chapter 4. At the beginning of this phase, Lynn started with column A. She had arrived at the last row, and after she finished reading Ellen’s case, she generated an idea for A. However, she quickly modified her idea. Lynn first described column A as “going with their first thought,” pointing out that A went with their thoughts that may be “misconceptions” but recognized that Ellen was an exception. The data suggest that Lynn instantiated her idea on Ellen, as seen in Figure 5.5b, move 4. Perhaps noticing that her idea
was too specific and did not account for Ellen, she revised her idea to include Ellen. Her revised idea for A was that “they went with what they thought what would work or believed in,” which excluded correctness. As shown in Figure 5.5a, her idea for A was a step toward the right direction, but it was vague. Her idea hinted at more, perhaps that A was not thoughtful, but this was left unexamined. Lynn did not continue to describe how the cases represented her idea. Rather, she appeared satisfied, as implied when she moved on to the last case in column B, that of Fred. Nonetheless, Lynn generated an idea for A, which may help her later differentiate an idea for B.

**Instantiating an idea.** In her last two moves, Lynn considered column B, generated an idea for it, and instantiated her idea onto the cases. In move 7, Lynn read and recapped the final case, Fred, but slightly paused. Then she said, “The students in column B all end up in the end thinking . . . the deepening meaning of the why.” Recall that this idea is similar to the commonality that she articulated between Andy and Dana in her prior move as “the why behind it” (move 5). Her pause may indicate that she saw how Fred’s approach not only was connected to her latent idea of thinking “why” (move 5) but also something more, as suggested with “deepening.” She then proceeded to instantiate her idea on each case in column B. As shown in Figure 3, this marked her final step toward progress to the GMK. These instantiations provided greater definition for her idea. But unlike Andy, Lynn did not explicitly use contrast to refine her idea for B using the A cases. It is unclear to what extent she used contrast between the cases or ideas of the invariants across columns to help her differentiate and build her idea for either B or A.

**Final summary statement of ideas.** In her final statement of her ideas, Lynn described A as “using instinct” and for B as “systematic.” Like Andy’s final statement, the outward expressions were both vague. On the final statements
alone, her idea for B did not capture the essence of her idea formed in this phase. Nevertheless, the process by which she arrived at her idea for B (Fig. 5.5b, moves 6-7) suggests that more lie below the surface of “systematic.”

**Discussion**

The purpose of this study was to generate insights about productive processes that may support students in building general metacognitive knowledge (GMK) via synthesis and beyond. I analyzed, in detail, the ways in which two students, Max and Lynn, engaged in synthesis, and I illustrated how students could abstract and generalize their understanding of the underlying idea across the cases. Furthermore, the analyses provide insight on how these two students’ ideas interacted with their uses of the cases. Drawing upon my analyses, I developed three important insights about productive synthesis processes. The first concerns the evaluation of their ideas by instantiating them on the cases. The last two are more general to synthesis. These insights are supporting students to acquire language for their ideas and helping students to self-regulate by utilizing the built-in affordances of synthesis.

**Instantiating to evaluate.** When the two students abstracted and generalized an idea of the invariant across the cases, their initial ideas tended to be vague. A common strategy that students used to evaluate those ideas was to instantiate them onto the cases from which the ideas were drawn, or the constituent cases. Instantiation was the process of explaining how a specific case was an example of the idea. Instantiations provided students with the criteria to judge whether their ideas were adequate and, consequently, to revise and to refine those ideas. Furthermore, these instantiations provided more explicit descriptions of a student’s idea, which gave it greater clarity and definition. For example,
Max’s initial idea of A was vague but gained greater meaning across the different contexts when instantiated on the A cases (Fig. 5.4, move 1). Most commonly, instantiations immediately followed students’ generation of an idea for a column. For instance, after Max generated an idea of A as having a direct approach, his next move was to explain how each of the A cases demonstrated a direct approach (Fig. 5.4, move 1). Later on when he generated his idea for B, Max immediately went through each of the B cases to instantiate the idea (Fig. 5.4, moves 6-8). Similarly, once Lynn generated an idea for B, she immediately proceeded to explain how each case supported her idea (Fig. 5.5, move 7).

Not all instantiations were on the constituent cases. One unique approach was to take the opposite of the idea for one column and project it onto a case in the other column. Max demonstrated this when he explained how a case in one column was the opposite of his idea for the other column. For example, Max explained how Andy “reconsidered his idea,” which was the opposite of “direct approach” and the opposite of Blair who did not “alter her way of thinking” (Fig. 5.4, move 2). He did this once more when he instantiated his idea for B, exemplified through Andy as “thinking about what he’s trying to do,” to explain its absence in Corey’s case and its presence in Dana’s case (Fig. 5.4, move 7).

Also, these two students did not always instantiate their ideas on the cases. For instance, when Max revised his idea for A (Fig. 5.4, moves 4-5), he did not revisit the A cases. Similarly, Lynn formed local ideas between two cases without instantiating on either of the cases (Fig. 5.5, moves 2-3). Other times, students instantiated their idea on selected cases, as seen with Lynn instantiated her idea for A on one case only (Fig. 5.5, move 5). When these students did not instantiate or instantiate only on some cases, they were insufficiently checking their ideas, limiting their ability to detect potential problems with their ideas.
When Max and Lynn instantiated, they were either successful or unsuccessful. Successful instantiations occurred when all the cases supported the idea. Thus, when one or more of the cases did not support the idea, instantiations were unsuccessful. In some instances of successful instantiations, Max and Lynn explicitly elaborated on their ideas of the invariant for A or B within a specific context. The explanations provided greater precision to the idea and led to the refinement of their idea. For example, when Max instantiated his idea for B on Andy’s case, he started with “finding new ways to do things” and ended with Andy who “thinks about what he was trying to do,” which led to an extension to the other B cases (Fig. 5.5, move 6). By contrast, sometimes successful instantiations led to elaborations that were not explicit but implicit. In implicit elaborations, the two also elaborated on their ideas, but their outward expression for their idea did not change. For example, after Lynn generated her idea for B (“thinking about the deepening meaning of why”), which was vague, she instantiated her idea on each of the B cases (Fig. 5.5, move 7). Her idea had meaning within each specific context that extended beyond her summary statement to include the concept of reflection. Thus, her idea gained further clarity and definition. Nonetheless, her outward expression of her idea did not change.

In an unsuccessful instantiation, one or more cases were not an instance of an idea. When this occurred, the data showed that students had two different approaches. One approach was to reconsider and revise their ideas to accommodate such cases by reevaluating the contextual details of the cases that did not fit. For example, Lynn generated an idea for A, found an exception with an A case, and revised her idea to include the exception (Fig. 5.5, move 6). Here, Lynn used an unsuccessful instantiation to self-regulate through evaluating and improving her idea.
Another approach to an unsuccessful instantiation was glossing over whether the cases actually supported the idea. Neither Max nor Lynn fully demonstrated this approach, but Max came close at one point (see Fig. 5.4 move 1). When Max took his idea of “direct approach” for A and applied it to each of the A cases, he described three divergent meanings for “direct.” Although his idea gained definition, it lacked cohesion. Had Max stopped here, he would not have likely progressed as far in building his understanding toward the GMK. But he was likely aware that “a direct approach” was inadequate since he later returned to revise this idea to “not really thinking about it” (Fig. 5.4 move 4).

The primary learning mechanism of contrasting case instruction, such as synthesis, is suggested to be schema induction, which involves extracting the relevant information from multiple cases to induce the shared abstract structure, or underlying idea (e.g., Kellman, Massey, and Son, 2010). Less emphasized is the importance of instantiation. However, the data show that instantiation is an essential process in building the knowledge. Thus, students’ approaches to instantiation can help inform the design of inductive tasks, such as synthesis, as well as inform pedagogy.

Instantiation helps students in two important ways: as the means for elaboration and as a feedback mechanism. Students’ ideas are often vague to start. Through instantiation, the idea gains greater definition when students elaborate on their idea, providing increasingly detailed information about how their idea is exemplified within each specific context. Students’ instantiations can proceed in two ways. One way is to instantiate onto the cases immediately after the idea is generated. Another way is to instantiate the idea onto a contrasting case. Instantiation onto the cases that constitute the idea is important, but an instantiation of the opposite of an idea onto a contrasting case is also beneficial.
because it contributes to a better differentiate idea and more rigorous evaluation of the idea.

Most importantly, instantiation is also a feedback mechanism, which leads to the revision and refinement of ideas. When students instantiated their idea on the cases, students may be evaluating whether their cases fit their ideas. Often, after generating an idea, students immediately instantiated their idea on the constituent cases immediately. Given the very short temporal duration between generating an idea and instantiating, students may be prone to confirmation bias, which is the tendency to confirm or justify a drawn conclusion. In this context, the drawn conclusion is students’ ideas. To help students to impartially evaluate whether the cases fit their idea, perhaps delaying students from instantiating immediately could mitigate confirmation bias effect. But a potentially more rigorous approach to check an idea is to instantiate the opposite of an idea onto a contrasting case. Because a student. A future study could determine scaffolding students to use these ways of instantiation can improve the quality of students’ ideas.

**The need for language.** The students made considerable progress on building their understanding of the GMK, although they did not quite reach a precise and accurate expression for the invariant. Consequently, students expressions for their ideas were generally lengthy (comparable to their initial or final ideas) and often vague. This was an expected outcome that is characteristic with this mode of instruction. Students may require a more formal definition of the GMK to complete their understanding (Schwartz and Martin, 2004).

A possible explanation for students’ struggle is that they did not have the language with which to articulate their ideas, and thus build precise concepts of them. Lengthy expressions, generally arising from the instantiations, were not economical, but they tended to be more precise. Vague expressions did not provide
enough information about the idea, but they tended to be more economical. Students struggled with generating an outward expression for their ideas that was both economical and precise, or succinct. Often, they sacrificed precision for more economical expressions. This was reasonable because maintaining a precise but lengthy idea for A and B while considering the cases simultaneously may be too taxing on working memory. Thus, vague and less lengthy outward expressions for their idea may be easier to maintain. However, the consequence is that their ideas lose some information. Perhaps if students had language that was succinct, students’ ideas may have progressed further.

To illustrate, with Max, his idea for A started with a vague idea for A (“a direct approach”) but gained greater precision while taking the form of lengthier expressions (“not really thinking about it” to “straight line of thinking”). His idea for B fared better, which gained some precision from start (“trying to find new ways to do things”) to finish (“thinking in different ways”); however, neither captured the essence of the invariant as well as his more lengthy but precise phrasing in between which described the individuals in B as thinking about their intentions (“trying to think what [they’re] trying to do”). Perhaps if Max had language at this moment, at the peak of his progress, he would not have fallen back onto less precise expression for his idea. With a succinct outward expression for his idea, his idea could retain greater meaning.

By contrast, Lynn had greater success than Max for her idea for A. She started with a lengthy expression “going with their first thought”), revised to an even lengthier one (“they went with what they thought would work, or what they believed in”), and ended with a succinct outward expression for A (“using instinct”). Similarly, her idea for B started vague but economical, becoming precise but lengthy, and ended vague but economical (“thinking why” to “thinking about the deepening meaning of why,” and ending on “systematic”). In between her last
two ideas for B, if Lynn had succinct language, she may have maintained the essence of reflective thinking, which is lost with her final idea for B as “systematic.”

How can students acquire language to better articulate their idea with greater precision and accuracy? One approach would be to provide students with the language before they engage in synthesis. For instance, beforehand, students could learn the definition of metacognition as thinking about one’s own thinking. However, I would argue that this approach might short-circuit the generative process of synthesis. To a much greater extent, the task would instead involve recognizing and applying the knowledge, which does engage students in engaging with the deep structure of idea. Another option is to introduce the definition of metacognition after inductive instruction, such as synthesis, as advocated by other researchers who use this type of instruction (e.g., Kapur, 2011; Schwartz and Bransford, 1998). In fact, in the larger quantitative study (Chapter 2), I used this approach. Immediately after synthesis, students received direct instruction via a scripted lecture, which defined the GMK and provided students language connecting to their more limited expressions so that they can make sense of their idea. One disadvantage of this approach is that it may be too abrupt, especially for the students who did not progress as far as Max and Lynn in building and refining their ideas. Thus, the gap between their ideas and the GMK may be too large for students to reconcile in a direct instruction format. Given the observations of this present study, a better approach would be to insert an activity between synthesis and direct instruction that supported students to continue to revise and refine their ideas. One way would be for students to receive sample ideas with more succinct language. Another way would be to teacher-led discussion of students’ ideas, which would provide students access to language that is varied in length and precision so that they could improve their idea.
Commitment to improving an idea. A signature feature of Max, more so than of Lynn, was his demonstrated need to improve his idea, which likely contributed to his success in building his understanding. He went from a vague notion of column A to a well-defined idea for B. Along the way, his ideas were frequently modified and refined, moving closer toward the idea of metacognitive thinking. During this process, he built upon his initial idea for A to generate an idea for B. Once he generated an idea for B, he could have stopped as the task goal was accomplished; yet, he persisted. This distinguished him from Lynn who stopped once she generated an idea for B. Most likely, Max was unsatisfied with his idea for B and recognized that it was insufficient. His continued efforts to improve his idea may be an indicator that he had ability to gauge when his ideas were inadequate – *i.e.*, his ability to self-regulate. Max took full advantage of the affordances of the synthesis, using both contrast and invariance to build his idea while also using the built-in criteria in the cases to evaluate.

Not all students may be able to self-regulate like Max. There may be many reasons why students may struggle to self-regulate during synthesis. First, students do not understand the goal of synthesis. That is, students are not aware that their ideas can change. College students typically engage in classroom assignments in which their ideas are not expected to change, so they may view the synthesis task in the same light. Thus, supporting students to make their ideas visible by externalizing in a representation might facilitate the synthesis process better because students can see what their ideas are and whether their ideas are changing. For instance, a graphic organizer with a double staircase diagram, such as in Figures 5.5 and 5.4, could guide students through the process and highlight the need for ideas for both A and B and for the change ideas to occur. This more productive framing of the synthesis might shift students’ learning orientations and encourage students to become more engaged ([Elby & Hammer](#)).
Second, students may not know how to effectively use instantiation to elaborate and evaluate their ideas. Thus, explicitly prompting students to instantiate their ideas onto cases may help students move in the right direction. Moreover, for further support, especially for the unsuccessful instantiations in which students gloss over whether or not their ideas fit all of the cases, students could be provided with another row of contrasting cases, in reserve, that contradicts their ideas and could therefore encourage for students to revise their idea.

Finally, the synthesis task places a high demand on working memory (Sweller, 1994). As seen with Max, an exception, he was able to hold multiple cases and ideas in working memory while coordinating his use of contrast and invariance (Fig. 5.4 moves 2, 6, 7). To compare, Lynn primarily used contrast and invariance independently, and she considered individual contrasts and searched for commonalities between two cases rather than all three simultaneously (Fig. 5.5 moves 1-4), which was perhaps her strategy to manage the cognitive load. Nevertheless, off-loading students’ working memory could be beneficial to the synthesis process. If students used representations to externalize their ideas, such as the kind of graphic organizer as discussed above, this would free up working memory for more productive student engagement.

**Conclusion**

In this chapter, I analyzed the processes of two students, Max and Lynn, while they engaged in synthesis. Max and Lynn utilized both affordances of synthesis, contrast and invariance, to build their ideas. Analyses of their process revealed three main findings. First, instantiation is an essential process during synthesis, which students used to elaborate and assess their ideas. Second,
students struggled with balancing generating outward expressions for their ideas that were both economical and precise. There is a need for language to help students build more succinct ideas, so that their ideas do not lose information. Finally, persistence and commitment to improving one’s idea during synthesis is important. These findings are valuable as they can inform the design of contrasting cases and instruction on supporting students learning during synthesis.

In the larger context of this dissertation, the findings articulated in this chapter complement those from Chapter 4. The findings provide valuable insight into how synthesis helped students learn the GMK. In particular, they also shed light on how a change in relevant ideas may have helped build more robust, metacognitive knowledge.
The prior chapters described an in-depth investigation of understanding how students can build SDF-related GMK with synthesis. The findings point toward promising classroom applications using synthesis as an instructional approach to build GMK. However, these findings were drawn from an interview-based study under controlled conditions. The same instructional approach in a real-world classroom may not result in similar learning outcomes. Thus, as a preliminary step toward applying the findings to authentic classroom instruction, I created a stand-alone assignment aimed at building students’ SDF-related GMK using synthesis. I drew upon my prior findings to guide and inform the design. A goal of the study was to support and optimize the synthesis process in order to improve student learning of the GMK.

This chapter is organized into three sections. The first section describes the overview of design and implementation, including the instructional materials and procedures. Next, the results are reported. Following this, I discuss the findings and potential future work.

Methods

Design and Instructional Sequence

The broad goal was to support the synthesis process in order to improve student building of the targeted GMK. The study was designed and administered using Qualtrics, a web-based platform, for easy distribution and access for a large-enrollment course. The design of the prior study was the foundation, and modifications were made in light of findings from that study.
Starting point: Prior design. To recap, the prior study consisted of four primary phases: (1) processing contrast pairs individually, (2) synthesizing the same contrast pairs, (3) listening to a scripted lecture containing an overview of the GMK, (4) completing assessment items on the learning of the GMK. In this study, the four phases remained largely intact and served mostly the same purposes. It is important to note, that in the original study, phase 1 supported the intervention in multiple of ways. In this study, phase 1 used the same pairs as the synthesis task, which may help reduce student cognitive load during synthesis. Thus, phase 1 allowed students to more fully engage in the synthesis process without being additionally burdened with processing the content of the pairs. However, the online study diverged from the in-person interviews in a number of significant ways.

Departure 1: Decoupling SDFs from the GMK. Previously, when learning about the GMK, students also learned about SDFs. More specifically, in synthesis, along the way to abstracting and generalizing the GMK, students also needed to abstract and generalize the idea of an SDF and its impact. Explicit instruction on SDFs occurred with the GMK in the scripted lecture (phase 3). In the prior study, students did not appear to struggle with understanding SDFs. This was evident in the pre-synthesis phase, when processing the contrast pairs separately. Approximately three fourths of students expressed the general idea of SDFs. After the scripted lecture, although not explicitly prompted, nearly all of the students (93%) identified the SDF with its associated cued reasoning for the given situation at least once across the four post-test items (see Tables 2.9 and 2.7 for items). This suggests that students may have learned more about SDFs than about the GMK from the scripted lecture. Learning about the SDF and GMK together demands a higher cognitive load than learning about each separately. Thus, decoupling the learning of SDFs from the learning of the GMK during
synthesis could further reduce students’ cognitive load, and consequently, students could focus on abstracting and generalizing the GMK. Therefore, explicit instruction of SDFs was added into the existing design as the leading phase. In this new phase, students were instructed about SDFs and their impact. SDF instruction consisted of students answering a physics question containing an SDF, reading discussions of both a correct response to the question based on the relevant problem feature(s) and an incorrect response based on the SDF, reading an overview of SDFs and their impact, and completing a practice question in which they were asked to identify the SDF and its associated reasoning.

Departure 2: Using task prompts to focus on SDF-related thinking. With a new leading phase focused on SDF instruction, all subsequent task prompts were targeted specifically to SDF-related thinking. Previously, the prompts were general and did not explicitly specify the focus of student thinking. Targeting SDF-related thinking in the prompts could help students focus their search for commonalities among the cases to build definitions closer to the targeted GMK.

Departure 3: Reducing the number of contrast pairs and post-test assessment items. Another departure from the original design concerned the number of contrast pairs and assessment items. There were no time limits imposed on the tasks, but as a practical consideration, the goal was to keep the assignment relatively short and to keep completion time under 30 minutes, which could more easily, in the future, be adopted into standard classroom practice. Thus, the number of contrast pairs in synthesis was reduced from three to two. Students considered two contrast pairs sequentially, and they considered the same two contrast pairs in a $2 \times 2$ matrix for synthesis. Similarly, the number of post-test items was reduced from four to three. These were not the same items
as in the prior study; however, two items from the prior study were adapted for this study in order to have a more sensitive measure of student learning.

**Departure 4: Omitting everyday context from the contrast pairs.**
Because prior results indicated that there was no difference between using contrast pairs exclusively in physics contexts or including an everyday context in students’ abstraction and generalization of the GMK, all paired contrasts were within the physics domain.

**Departure 5: Varying the contrast highlighted in synthesis.** Prior findings suggested students could not differentiate between the levels of metacognitive thinking related to the SDF-cued action. Many students conflated the targeted GMK, the reflection on the thinking that led to the action, with the reflection on the incorrect nature of the SDF. One potential reason for this conflation may be that the contrast between the cases did not adequately highlight these nuanced differences. Instead of an absence/presence contrast of the targeted GMK, an alternative is juxtaposing the two levels of reflection on the SDF-cued action *i.e.*, why the SDF is incorrect vs. why the SDF is tempting) across the columns of the matrix for synthesis. This is a “near miss” contrast (Gick & Paterson, 1992), in which the contrast differs only in one dimension. Both involved reflective thinking about an SDF-cued action but differ in which aspect of the SDF-cued action are evaluated. Such a contrast may help students differentiate the GMK from other ways of reflective thinking, which may bring greater definition to student understanding of the GMK. Additionally, with this contrast, students may also abstract and generalize a related complementary approach to the GMK. That is, students have the potential to abstract two distinct, complementary aspects of reflective thinking about the SDF, while the absence/presence contrast, by design, offers only one.
**Two conditions: Varying contrast.** To investigate whether varying the contrast would improve students’ learning of the GMK during synthesis, students were split into two conditions. Everything was identical for the conditions except for the contrast used in synthesis. The types of contrast designed to support building understanding for the GMK were different in the two conditions. Across both conditions, in the synthesis matrices, the second columns were identical, all having cases with the same underlying structure of the targeted GMK. The hypothetical students think about why the SDF impacted their thinking. However, the first columns were different across the two conditions. One condition, called the absence-presence (AP) condition, used a contrast of absence/presence of the GMK, as in the prior investigation. Thus, in the left column, there was an absence of the targeted GMK. In this contrast, column 1 does not have a clear definition without column 2, the GMK. The other condition is called the two-way-reflection (TR) condition. Unlike the AP condition, column 1 has a clear underlying idea, or invariant, in contrast to column 2. In this condition, both columns in the matrix were similar because they involved reflective thinking about an SDF-cued action. However, the nature of the reflection was different, as the targeted GMK was reflection on why an SDF was tempting while the other column illustrated reflection on why an SDF was incorrect.

**Final instructional sequence.** As shown in Figure C, the final instructional sequence had five phases: (1) learning about SDFs, (2) processing paired contrasts individually, (3) synthesizing paired contrasts, (4) watching a video explanation on the GMK, and (5) completing three assessment items. In the new initial phase, students were instructed about SDFs and their impact, as discussed earlier. In phase 2, students processed two contrast pairs separately. For each pair, they were prompted to describe how the SDF-related thinking of the hypothetical student on the right differed from that of the hypothetical student on
the left. In the next phase, the same contrast pairs were combined into a $2 \times 2$ matrix for synthesis. The task directive was to describe how the SDF-related thinking of the hypothetical students in the right column differed those in the left column. Following synthesis, students watched a 2-minute video explaining the GMK in the context of the synthesis task. Finally, in the last phase, students completed three post-test items on their learning of the GMK.

**Context and Participants**

The study took place at a mid-size public university in the northeastern part of the United States in Spring 2016. Participants were 127 undergraduate students enrolled in a calculus-based introductory electricity and magnetism course, which was the second semester of a two-semester introductory course sequence. Generally, students were engineering majors in their first year. Students received a small amount of extra credit for their participation.

**Procedure**

To elicit student participation, I made an announcement at the beginning of the class and provided students with a short overview describing what they would do in the study. I directed students to a link that was posted on the course
webpage and active for two weeks near the end of the spring semester. Upon clicking on the link, a welcome message informed students that there were four parts to the study, which would have an approximate average time for completion of 25-30 minutes. In addition, the message requested that students complete the entire study in one session. Once they started, students progressed through the phases as shown in Figure 2. Students were randomly assigned to a condition, with half processing the absence/presence (AP) contrast (e.g., absence versus presence of the GMK) and the other half processed the two-way-reflection (TR) contrast (e.g., thinking why an SDF is correct versus the GMK).

The first phase of the study was learning about SDFs, which consisted of primarily of three parts. These parts involved students (1) answering a physics question containing an SDF and receiving feedback on their responses, (2) reading an explanation about SDFs in the context of the prior question, and (3) practicing identifying an SDF and the cued reasoning in a physics question. In the first part, students answered the friction problem, which was a question used in the prior study (see Fig. 2.2). Later, this question is also used in a contrast pair. In brief, the question presented a situation of two identical boxes on different surfaces. The boxes have the same applied 30 N horizontal forces, and they remain at rest. Students were asked to compare the friction forces between the boxes and surfaces. The correct response requires that students recognize the two relevant problem features, identical applied forces and the fact that the boxes remain at rest. The most common incorrect response is based on the difference in the coefficients of static friction, which is the SDF. The question was multiple-choice, and students selected their answers. They were also asked explain the reasoning behind their answers, which they typed into the provided textbox.

In the next page, a follow-up question asked students to choose which feature(s) of the problem they used to arrive at their answer. The options were:
coefficients of static friction, boxes remain at rest, masses of the boxes, and horizontal applied forces. After submission, students received feedback on their responses and read an explanation for the correct answer, which pointed out the relevant features of the problem. Following this, for students who were incorrect (e.g., used the SDF to arrive at their answer), the next page informed these students that their responses corresponded to the most common incorrect response based on the coefficients of static friction. An explanation about the incorrect answer was given, which emphasized why the coefficients of static friction were incorrect to use. However, for the students who were correct, they were given the most common incorrect answer and asked to choose the feature(s) of the problem they thought a student who arrived at this answer might have used. They were then given an explanation of the incorrect answer, which was the same explanation that was given to the incorrect students. Finally, using this question as a springboard, ”alluring distracting features” were defined.

After reading about SDFs, students completed one sample problem in which they identified the SDF and the associated cued reasoning. The practice problem is shown in Figure 6.2. In this question, three playground slides of the same height but with varying steepness were shown. The question asked which slide would give a girl the greatest possible speed at the bottom of the slide. For this question, the primary SDF is the relative incline of the slides, which may lead to the incorrect conclusion that a steeper slide will produce greater speed. The correct response is that all slides would give the girl the same speed. Students were not expected to answer the question, and the correct answer was provided as well as a link to the explanation. The practice problem was multiple-choice, and students were asked to first select the most likely alluring feature, and then select the incorrect conclusion that would be drawn on the basis of the alluring feature. Students were also asked to provide a brief explanation in the textbox. After
submitting their responses, students read an explanation of the SDF, which discussed why the SDF was incorrect to use in this situation.

In the second phase, students processed two contrast pairs sequentially. The pairs presented a physics question and two hypothetical students’ responses to the question along with the vignettes. The physics questions were the same as the prior study. The first was the friction question that students answered in the prior phase. The second question was in the kinematics context. (See Fig. 2.2) As a reminder, in this question, students are shown a position versus time graph for two cars and asked when, if ever, the two cars had the same speed.

Each contrast pair was shown as a PowerPoint presentation and embedded into the webpage. To help students process the cases, students needed to advance through the slides. Each slide revealed an additional bullet point for both vignettes simultaneously to facilitate side-by-side comparisons. The prompt was to explain how the hypothetical student’s thinking related to the SDF on the right differed from the left hypothetical student. Students typed in their responses in the textbox provided beneath the embedded PowerPoint.
Students processed the contrast pairs in their assigned condition. The vignettes for each contrast pair are shown in Figures 6.3 and 6.4 for the AP and TR conditions, respectively. For both conditions, the right case for each pair was always the same, while the left case differed depending on the contrast. That is, in the AP condition, the contrast is the absence versus presence of the GMK, while in the TR condition, the contrast is thinking about why the SDF is incorrect versus the GMK. To illustrate, in the first pair, Andy and Blair considered the friction problem (Figure 5, a), which is shown in the first row of Figure 6.3. In both conditions, Blair thinks about why she was drawn to the coefficients of static friction, while in the AP condition, Andy does not. However, in the TR condition, Andy instead thinks about why the coefficients were not required for the correct response. Similarly, in the second pair containing the kinematics graph problem (Fig. 2.2), Dave thinks about why he was drawn to the intersection point, while in the AP condition, Cathy does not. In the TR condition, Cathy thinks about why the intersection was not the answer. The same contrasts are therefore highlighted across both pairs in the AP condition and both pairs in the TR condition.

In phase 3, students considered the same two contrast pairs as in the prior phase together in synthesis. To familiarize students with synthesis process, students completed a $2 \times 2$ practice matrix in an everyday context. The prompt was to compare how column 2 differed from column 1. A textbox under the matrix was provided for students to type in their responses. After submission, students read an explanation of the matrix and general approach procedure.

Next, students saw the $2 \times 2$ synthesis matrix, which contained the same two contrast pairs as the prior phase. Thus, there was invariance within the columns and contrast across the columns. Students in the AP condition considered the $2 \times 2$ matrix in Figure 6.3, while students in the TR considered the $2 \times 2$ matrix in Figure 6.4. Column 2 was the same in both cases, which demonstrated

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1 Absence of the GMK
Not thinking about why an SDF is tempting

- Andy is tempted to use the values for $\mu$.
- However, he remembers Newton’s 2nd law and correctly applies it.
- He checks his answer with Blair who got a different answer.
- He thinks Blair made a mistake by using $\mu$.

2 GMK
Thinking about why an SDF is tempting

- Blair uses $f = \mu N$ to calculate static friction.
- After she checks her answer with Andy and realizes she should have used Newton’s 2nd Law, she gets the right answer.
- She wonders why she was tempted to use $\mu$, which gave the maximum static friction forces.
- She decides that having numbers for $\mu$ made her think she needed to use them.

Irrelevant differences

- correct/incorrect
- does not use/uses SDF

Cathy thinks the intersection is the answer.
- She checks her answer with the key, and she finds that she is wrong.
- She then remembers that slope corresponds to the velocity and gets the right answer.

Dave wants to choose the intersection but decides that this point is where the positions are the same.
- He then recalls that the velocities are the same when the slopes are the same.
- He thinks about why he initially focused on the intersection.
- He realizes that he was drawn to the intersection because it was an obvious place where something was the same.

Figure 6.3: $3 \times 2$ matrix for synthesis with absence-presence contrast

1 Reflecting thinking about the SDF
Thinking about why an SDF is incorrect

- Andy is tempted to use the values for $\mu$, but he remembers Newton’s 2nd law and correctly applies it.
- He checks his answer with Blair who got a different answer.
- He wonders why $\mu$ is not required for this problem.
- He decides that the problem did not ask for the maximum friction force, given by $\mu$.

2 GMK
Thinking about why an SDF is tempting

- Blair uses $f = \mu N$ to calculate static friction.
- After she checks her answer with Andy and realizes she should have used Newton’s 2nd Law, she gets the right answer.
- She wonders why she was tempted to use $\mu$, which gave the maximum static friction forces.
- She decides that having numbers for $\mu$ made her think she needed to use them.

- Dave wants to choose the intersection but decides that this point is where the positions are the same.
- He then recalls that the velocities are the same when the slopes are the same.
- He thinks about why he initially focused on the intersection.
- He realizes that he was drawn to the intersection because it was an obvious place where something was the same.

- Cathy notices the intersection point.
- She thinks the intersection point is the answer, but she checks with a posted solution and is wrong.
- She then recalls that slope corresponds to the velocity and gets the right answer.
- She thinks about why the intersection is not the answer.
- She realizes that the intersection point indicates the same position but not the same speed.

Figure 6.4: $3 \times 2$ matrix for synthesis for the two-way-reflecting contrast
the GMK. The $2 \times 2$ matrix fit within the web browser when in full-screen, such that no scrolling was required. For reference, a link to the physics questions, which opened in a new window, was provided. Students typed in their answers in the textbox directly below the matrix. The synthesis prompt was the following: "Considering the situations in column 2 together, how does the thinking related to the alluring distracting feature of the students in column 2 differs from that of the students in column 1?" However, because the contrast varied across conditions, students’ ideas for column 1 were also of interest. Thus, in the next page, there was a follow-up synthesis prompt. The $2 \times 2$ matrix was provided again and the prompt focused on column 1 only. This prompt focused on column 1, the contrast of the GMK, stating: "Considering the situations in column 1 together, characterize the thinking related to the alluring distracting feature of the students in column 1."

Students were then asked to watch a short scripted video explanation of the GMK. The video, which directly followed synthesis, explained the $2 \times 2$ matrix. The purpose of the video was to potentially enhance student learning about the GMK from synthesis. The video consisted of PowerPoint slides with a voice-over narration. The slides showed the $2 \times 2$ matrix and highlighted key parts of the vignettes that corresponded to the narration of the explanation. In the video, after giving an overview of the purpose, the narrator pointed out the SDFs for each contrast pair and explained how the SDF affected each hypothetical student in their thinking. Then, the narrator discussed the difference between the two columns of the matrix by, pointing out the GMK in column 2, modeling an ideal student response. Finally, the narrator described the benefits of the GMK.

The video was identical in structure for the two conditions, but the discussions of the differences between the two columns necessarily differed. The description of column 2, the GMK, was the same, but the description of column 1
was based on the relevant contrast, either the absence of thinking about why SDFs impact thinking (absence-presence condition) or thinking about why SDFs are incorrect (two-way-reflection condition). For both conditions, the video lasted a little over 2 minutes. The video script and slides are provided in Appendix C.

Measures

There were a total of five measures to determine the effectiveness of the synthesis phase in each condition, as well as the differences in the learning between the two conditions. Two were process measures and collected in synthesis. One measured the extent to which students generalized and abstracted the GMK (column 2). The other measured students’ ideas of the contrast to the GMK (column 1). The last three measures were post-test items, which assessed the learning of the GMK.

Synthesis. Student ideas of the GMK (column 2) were coded according to how well students abstracted and generalized the GMK using the same three-level scale of students’ articulation of the GMK, which was used in the prior study and is described in Chapter 2. Sample responses are provided in Table 6.1. Responses were scored as level 2 if they articulated the GMK as reflecting on the thinking processes that led to the SDF-cued action. For example, a level 2 response could describe thinking about why a person would be drawn to the distracting features. Level 1 responses described reflecting on an SDF-cued action. Such a response typically discussed reflecting on why an SDF-cued response was incorrect. All other responses not related to reflective thinking about an SDF were scored as 0.

Student ideas for the contrast of the GMK (column 1) were also coded. By design, the invariant in column 1 did not represent the targeted GMK, and thus students could only give level 0 or level 1 responses for column 1 in either
Table 6.1: Sample student responses from synthesis coded

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Synthesis</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Column 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Column 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Post-test</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>General</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>No elements</th>
<th>Some elements of the GMK</th>
<th>Full elements of the GMK</th>
</tr>
</thead>
<tbody>
<tr>
<td>No reflective thinking related to a choice</td>
<td>The students in column 2 both believe that the alluring distracting feature is included in order to trip them up during the problem and bring them to the wrong conclusion.</td>
<td>The students in column 2 went back and thought about why they were incorrect, the students in column 1 just realized they were wrong or didn't think back on what the source was.</td>
<td>The students in column 2 recognize why a student would be drawn to the distracting features given to them in the problem.</td>
</tr>
<tr>
<td>Reflecting about a choice</td>
<td>The students in column 2 recognize why a student would be drawn to the distracting features given to them in the problem.</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>Reflecting about the thinking leading to a choice</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
</tbody>
</table>

Column 1 thinking understands that extra information is given, but it isn't always needed. The thinking is that why is the alluring feature not needed and then coming to an understanding of why it is not used. The alluring distracting features should be thought about in why the person was drawn to this feature even if it does not impact the problem and why it is not an important aspect of the problem.

Post-test items. Table 6.2 summarizes the three post-test items of a different type. The first item was an open-ended response question that explicitly asked students to describe important and productive ways to think about SDFs. The purpose of this question was to assess what students had learned about thinking related to SDFs. In addition, this type of item would be the most sensitive of the three in detecting any learning of the GMK. The item was coded using the same three-level scale, shown in Table 2.4. A level 2 response articulated thinking about why a person is drawn to an SDF, while a level 1 response articulated thinking about why an SDF-cued response is incorrect. A level 0
response did not articulate reflecting thinking about the SDF and typically focused on content understanding and strategies to avoid the SDF.

The last two items included an application item (physics) and a recognition item (everyday), which were adapted from the prior study. The purpose of these items was to determine the extent to which students were able to differentiate the GMK from other ways of reflective thinking about the SDF. For the application item, the scenario presented the approach of hypothetical student, Jay, on a qualitative physics question. Jay used the SDF in the physics question to arrive at an incorrect response, and then he thinks about why the SDF was incorrect for the situation. Students were asked to explain another way that Jay could think about the SDF. Student responses on the application item were coded either as articulating the GMK or not. Response coded as articulating the GMK had to include descriptions of thinking about why the difference in masses was tempting. Other responses typically described further thinking about the content, such as thinking how the masses affected the motion of the pucks.

The final item was the recognition item, which was also a transfer item in an everyday context of purchasing headphones and not within the physics domain in which the learning of the GMK occurred. The recognition item presented four approaches of hypothetical people to their purchase decisions. Only one, Cathy, used the GMK, and thought about why she was tempted to buy one pair of headphones over the other (level 2). Another person, Ally, thought about her SDF-cued action. She was attracted to one set of headphones due to the sleek design, and later thought about why these headphones would not be a good choice (level 1). The question was multiple-choice, and students were asked to choose the hypothetical person who was reflecting on the role of alluring features. If students choose Ally, their response was coded as level 1, and if students choose Cathy, their response was coded as level 2. All other student responses were coded as level 0.
Table 6.2: Post-test items measuring student learning of the GMK

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>What are important and productive ways in which students can think about alluring distracting features?</td>
</tr>
<tr>
<td>Application</td>
<td>The following clicker question is posed in physics lecture.</td>
</tr>
</tbody>
</table>

The diagram depicts two pucks on a frictionless table. Puck II is four times as massive as puck I. Starting from rest, the pucks are pushed across the table by two equal forces.

Which puck will reach the finish line with greater kinetic energy?
A. Puck I  
B. Puck II  
C. Both will have the same.  
D. There is not enough information to decide.

Correct response: C. Both will have the same.  
Click here for the explanation based on the work-energy theorem.

Jay incorrectly answers that puck I would have the greater kinetic energy because of the difference in the masses. He reasons that a smaller mass would cross the finish line with a greater velocity and therefore more kinetic energy. After he is shown the correct answer, Jay correctly applies the work-energy theorem.

The difference in masses is an alluring distracting feature. Jay thinks about why this difference in masses does not lead to different final kinetic energies. How else could Jay productively think about this alluring distracting feature (the difference in masses)? Please explain.

Recognition | Four people are buying new wireless headphones. They are deciding between two headphones that have roughly the same price. The approaches the individuals use to select their headphones are provided.

*Audiophile On-Ear Bluetooth*  
*Urban Bass 2 Wireless*

The reasoning the individuals use to select their headphones is described below.

A. Ally is tempted to buy the Urban Bass headphones because of their sleek look. However, she remembers the Audiophiles were more comfortable, and she purchases them. Later she thinks about why the Urban Bass were not a good choice.

B. Brian finds the Audiophile headphones more comfortable and lighter than the Urban Bass headphones. After carefully thinking about the choices, he purchases the Audiophiles.

C. Cathy is inclined to buy the Urban Bass headphones but prefers the Audiophiles’ sound quality. She purchases the Audiophiles. She thinks that she wanted Urban Bass because they are popular with her friends.

D. Danny likes the sleek design of the Urban Bass, but the Audiophiles were the only ones available in the store. He purchases the Audiophiles, and he later regrets his decision.

Which student is thinking is about how an alluring distracting feature impacts his or her reasoning?
Results

Student Responses From Synthesis

Table 6.3 shows the distribution of student responses in synthesis. The first row reports student responses for the GMK (column 2 of the contrast matrix). Overall, approximately 39% of all students were able articulate some or full elements of the GMK. The table also shows the distribution between conditions. For column 2, approximately the same percentage of students in both conditions articulated the full GMK for column 2. However, fewer students in the two-way-reflection condition (TR) articulated some partial elements of the GMK (i.e., thinking about why an SDF is incorrect) than those in the than the absence-presence (AP) condition.

The next row of Table 6.3 shows student responses for the contrast of the GMK (column 1 of the contrast matrix). No AP students articulated column 1 as having some elements of the GMK, or reflective thinking (i.e., thinking about why an SDF is incorrect) compared to 27% of TR students. There was a statistically significant difference between the two treatments in student responses of the contrast to the GMK (column 1), Fisher’s exact test $p < 0.001$. In the TR condition, 13% articulated the full GMK for column 2 of the contrast matrix (Figure X) and articulated reflective thinking for column 1. For those who missed the targeted GMK in column 2 (scored 0), about 18% of these students articulated the reflective thinking in column 1. In summary, between conditions, there was no difference in the number of students generating the full GMK. Nevertheless, in the TR condition, 13% students differentiated between two ways of reflecting on the SDFs (i.e. they articulated the full GMK for column 2 of the contrast matrix as well as some elements of the GMK for column 1).
Table 6.3: Distribution of student responses on all items

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMK (column 2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>absence-presence contrast</td>
<td>61% (78)</td>
<td>21% (26)</td>
<td>18% (23)</td>
</tr>
<tr>
<td>two-way-reflecting contrast</td>
<td>35% (56)</td>
<td>29% (18)</td>
<td>16% (10)</td>
</tr>
<tr>
<td>Contrast to the GMK (column 1)</td>
<td>87% (110)</td>
<td>13% (17)</td>
<td>----</td>
</tr>
<tr>
<td>absence-presence contrast</td>
<td>0 (63)</td>
<td>0</td>
<td>----</td>
</tr>
<tr>
<td>two-way-reflecting contrast</td>
<td>73% (47)</td>
<td>27% (17)</td>
<td>----</td>
</tr>
<tr>
<td>Post-test</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>General</td>
<td>66% (84)</td>
<td>19% (24)</td>
<td>15% (19)</td>
</tr>
<tr>
<td>Application</td>
<td>94% (120)</td>
<td>---</td>
<td>6% (7)</td>
</tr>
<tr>
<td>Recognition</td>
<td>16% (20)</td>
<td>50% (64)</td>
<td>34% (43)</td>
</tr>
</tbody>
</table>

Post-Test Items

Table 6.3 also shows the students responses for the post-test items. These results are not shown by condition for simplicity, as there was no difference between the conditions on three post-test items. For the general item, students were asked to describe important and productive ways to think about the SDF. Approximately one third of students articulated some aspect of reflecting thinking about the SDF, with 15% expressing the full GMK and 19% with some elements, or thinking about why an SDF was incorrect. For the application item, there was a floor effect, with only 6% of student responses offering the GMK as another way to think about an SDF in the physics situation. For the recognition item, 34% of students were able to differentiate the GMK from other ways of reflective thinking related to the SDF in an everyday context, with nearly half of students confounding the GMK with reflecting why an SDF-choice was incorrect.
Table 6.4: Correlations between post-test items and student responses from synthesis

<table>
<thead>
<tr>
<th></th>
<th>General</th>
<th>Application</th>
<th>Recognition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synthesis</td>
<td>0.157</td>
<td>0.046</td>
<td>0.088</td>
</tr>
<tr>
<td>General</td>
<td>--</td>
<td>0.481**</td>
<td>0.022</td>
</tr>
<tr>
<td>Application</td>
<td>--</td>
<td>--</td>
<td>0.037</td>
</tr>
</tbody>
</table>

** Correlation is significant at the 0.01 level (2-tailed).

Table 6.4 shows the correlation between post-test items and student responses of the GMK from synthesis. Student responses after synthesis did not correlate with any post-test item. Among post-test items, the recognition item did not correlate with any of the other items, while the general and application item had statistically significant correlation. Taken together, this indicates that the post-test items may be unreliable as a measure of students' understanding of the GMK.

**Discussion**

Broadly, a goal of this study was to examine the viability of supporting student building of the GMK via synthesis within the context of authentic classroom instruction using a web-based platform. The overall results were encouraging and indicate that learning of the GMK occurred. Within synthesis, approximately 40% of students abstracted and generalized aspects of the GMK, with 21% articulating some elements and 19% articulating the full GMK. Across the three post-test items, student performance varied but nonetheless indicated some learning, although there are concerns regarding the reliability of the items. As a practical goal, the study is a modest first step toward moving the medium of
instruction from interview-based to web-based and toward developing a stand-alone activity that can be flexibly incorporated into both small- and large-enrollment introductory physics courses.

The modifications to the instructional sequence appeared to support students in abstracting and generalizing well-differentiated and precise ideas of metacognitive thinking within synthesis. In students’ responses articulating the GMK, their expressions successfully used the language of SDFs. Although many modifications were made, the most likely contributor was the explicit SDF instruction, which may have helped students acquire language to express their idea of the GMK with greater precision. Another contributor may be the directed prompts, which expressly focused students on SDF-related thinking within the cases across which they would see contrast and commonalities. Furthermore, both SDF instruction and directed prompts may have also reduced students’ cognitive load, so that students could better focus on the key feature of the GMK.

The study showed an impact on student learning of the GMK, but there are concerns. One concern is the very weak correlation between student responses in synthesis to their responses on any of the three post-test items. As discussed in Chapter 4, these should show some correlation if synthesis were the cause for student learning of the GMK, as seen in the prior study. But an important finding from the quantitative analysis of the prior study was that changes in students’ relevant ideas of the GMK during the synthesis process was also related to greater learning of the GMK, as measured on the post-test. In light of this, perhaps the 39% of students who generated some understanding of the GMK did not have a change in relevant ideas during synthesis. With the change in medium (think-aloud interview to online interface) for this study and with its data limited to students’ final responses to synthesis, how students arrived at their responses is unknown. In contrast, substantial process data were available from the in-person interviews.
Other possible explanations may stem from the modifications made for the online version. For instance, the change in medium may be a factor. Students completed the study on their own, so they may have been distracted while completing the task. Another possibility may be that the SDF instruction and reduced contrast pairs reduced student cognitive load. In particular, with the SDF instruction prior to synthesis, students may not have had to grapple with the idea of the SDF and GMK simultaneously, so they could primarily focus on the GMK. Think-aloud interviews with the target population, introductory college students, can examine students’ process as they complete the synthesis. Other studies could manipulate the modifications to isolate the impact of a modification on student learning.

Together, the post-test items did not have strong inter-relatedness, and therefore not reliable in measuring student learning of the GMK. Individually, there are separate explanations for student performance on each item. First, for the general item, only about one-sixth of students articulated the GMK as an important and productive way to think about SDFs, which was originally expected to be the most sensitive measure. The particular phrasing of the item, which asked for students to identify important and productive ways of thinking, likely underestimates the number of students who may have learned the GMK because they did not perceive the utility value, or the usefulness and relevance, of the GMK beyond the learning context (Hulleman, Durik, Schweigert, & Harackiewicz, 2008). Instead, many students articulated cognitive strategies for avoiding SDFs in physics questions, such as better understanding the physics concepts, rereading the question, and anticipating “trick” questions.

Next, in the application item, only 6% of students offered the GMK as an approach to think about an SDF in a physics situation. This result indicates that there was a floor effect, and the item was difficult for students. However, one
interesting finding was that there was a strong correlation between student responses on the general item and the application item. Although the frequencies were small and both items were difficult, nearly all (but one) of the students who transferred the GMK to this situation also described the GMK as an important and productive approach in the general response item. This suggests that the utility value of the GMK may play a part in its transfer to the application item. Researchers argue that the transfer of knowledge may be promoted by increasing the utility value of that knowledge (Engle, Nguyen, & Mendelson, 2011; Hulleman et al., 2008). A potential way to increase the utility value of the GMK is to use an expansive framing, such as connecting the GMK to contexts outside of the physics domain (Engle et al., 2010). For instance, including an everyday context may help students see the applicability of the GMK beyond the physics classroom. In the prior study in this dissertation, including an everyday context did not play a role in improving student abstraction and generalization of the GMK. Perhaps an everyday context can play a role in increasing the utility value of the GMK during the intervention.

A future study could embed questions throughout the intervention to track students’ perceptions of the utility value of the GMK as they complete the associated tasks. Finally, student responses on the recognition item, which was in an everyday context, were not related to any of their responses on other post-test items or from synthesis. One-third of students (many of whom did not arrive at the GMK during synthesis) were able to transfer the GMK to an everyday situation and differentiate the GMK from the other reflective approaches. This may be because the everyday context was familiar, as college students may have more direct, relevant experiences within the context of purchasing headphones that might make the GMK more salient than in a physics context.
The study also investigated whether the type of contrast in synthesis could help students build a more complete understanding of the GMK. Students were given either an absence versus presence (AP) contrast of the GMK or reflective thinking (TR) contrast. Results show that there was also no difference in the frequencies of responses articulating the full GMK in either contrast condition. In other words, the type of contrast did not affect student generation of the GMK. In addition, both conditions performed similarly on post-test items. One potential explanation is that the modifications to the study design were sufficient to help students to discriminate and abstract the full GMK within the absence-presence condition. As mentioned earlier, the explicit SDF instruction before synthesis and the direct prompts may have helped students notice the key features of the GMK.

Although the type of contrast did not affect student learning of the GMK, one can imagine that both kinds of reflection related to SDFs (thinking about why SDFs are tempting and thinking about why they are incorrect) may be beneficial to learn together, as in the two-way-reflection contrast (TR) condition. Learning both simultaneously within synthesis may highlight the importance of reflecting on SDFs in general and the different ways to do so. In addition, one may speculate that if students learned these two approaches together in relation to one another, students may associate these approaches. Thus, if one approach were given, then perhaps the other approach would be cued. However, the application item, which was designed to measure this possibility, was a difficult item and did not detect a difference between conditions. A future study would use more sensitive items to detect whether there is a difference in student learning associated with the two types of contrast, and more broadly, explore how to leverage the learning of both types of reflection within and beyond synthesis.

It is not clear if the modifications negatively impacted students’ productive engagement during synthesis. The data suggest that there was abstraction and
generalization of the GMK, but the manner in which students engaged with their ideas and with the cases is unknown. As mentioned earlier, the prior study showed that students who had a change in ideas during synthesis had greater learning of the GMK. Perhaps some combination of the SDF instruction, direct prompts, reduced number of contrast pairs, and the change in medium of instruction somehow precluded or minimized the productive processes involved in synthesis that were observed in the prior study. Further studies would tease out the effects of each of these modifications and determine the contribution to the learning of the GMK within synthesis.
Chapter 7
CONCLUSION

The purpose of this dissertation was to understand how to build student general metacognitive knowledge (GMK) that was useful for physics learning. In Chapters 3 and 4, the findings show that synthesis, a form of contrasting case instruction, can be an effective method for helping college students build the understanding and awareness of reflecting on how and why certain features, or salient distracting features (SDFs), in physics problems can impact their thinking. In addition, the findings about students who did or did not engage in synthesis suggest that telling students about the GMK is not an effective method. Moreover, an explicit focus on building student GMK is necessary, as students tend not to build GMK on their own. Whereas the primary focus of many studies is on metacognitive regulation, such that metacognitive knowledge is often overlooked, this work shows that students who understand the GMK can apply that knowledge, which may help them later use that knowledge to regulate their thinking.

Broadly, this work also suggests that contrasting case instruction, which has primarily been shown to be effective in building generalizable and transferable conceptual and procedural knowledge across a variety of domains and age groups, may also be extended to metacognitive knowledge. Furthermore, as discussed in Chapter 5, adding to the research on contrasting case instruction, a key finding is that during synthesis, instantiation (i.e., explaining how an idea fits or does not fit a particular case) is an essential process that students can utilize to self-regulate and to build understanding of the GMK. Moreover, instantiation was also used as a way for students to refine their ideas of the GMK.
As a preliminary effort to apply these findings to develop an intervention for use in authentic classroom instruction, Chapter 6 described the design and implementation of a synthesis activity to build student GMK via an online platform in a calculus-based introductory physics course at a midsize public university. The findings indicate that students could generate the GMK during synthesis, although the stability of their learning of the GMK could not be adequately ascertained, as the post-test items were found to be unreliable measures. Nevertheless, this effort is a modest but important step forward in understanding how to create stand-alone synthesis activities that can be flexibly incorporated into both small- and large-enrollment introductory physics courses.

Implications for Teaching

In light of the findings presented in this dissertation, two general recommendations are made for instruction. First, follow-up instruction to synthesis may be necessary for students to further refine their understanding of the GMK, as well as to recognize the value of the GMK for physics learning. These additional efforts, however, may not require restructuring course content. For instance, an instructor could supplement pre-existing physics problems that contain potential SDFs with prompts that direct students to apply the GMK to the problem. Second, it may be valuable for instructors to support students in further differentiating the ways in which students may reflect on the impact of SDFs. Reflecting on the content and content understanding (e.g., why an SDF is incorrect for a given situation) is important, but equally important is reflecting on the associated thinking processes (e.g., why and how an SDF impacts thinking). Both ways of reflection are essential for helping students to “think like a physicist” (Chasteen, Pollock, Pepper, & Perkins, 2012; Redish, 2003).
Recommendations for Future Work

In general, the discussion in Chapters 4-6 offered specific ways to further investigate how students can build the GMK via synthesis. For instance, stemming from the findings in Chapter 4, future studies may compare the effectiveness of synthesis to other common modes of instruction, such as tell and practice, which is used extensively for strategy training, on student learning of the GMK. From Chapter 5, which describes productive processes in synthesis, a possible study could investigate the extent to which instantiation within synthesis can support student learning of the GMK. Finally, from Chapter 6, a study could examine how the types of contrast used in synthesis may support students to differentiate the two ways of reflecting on SDFs.

More broadly, although it has been shown that synthesis can be an effective method for building GMK, further studies are needed to understand how educators can support student use of the metacognitive knowledge to monitor their thinking. Part of the motivation for the investigation described in this dissertation stemmed from the finding that students who demonstrate the requisite conceptual understanding on one problem are often unable to successfully apply that understanding on a second problem that contains a salient distracting feature, abandoning correct formal reasoning in favor of a more intuitive line of reasoning cued by the SDF. Thus, it is imperative that future studies investigate the extent to which interventions designed to build GMK through synthesis also improve student performance on physics problems containing SDFs. In addition, research exploring the possible relationship between the development of this kind of metacognitive knowledge and student learning of physics may guide future generations of research-based instructional materials.
REFERENCES


McDermott, L., & Shaffer, P. (n.d.).


## Appendix A

### INSTRUCTIONAL PROTOCOL

<table>
<thead>
<tr>
<th>Opening</th>
<th>[Greet and thank student for participation.]</th>
</tr>
</thead>
</table>
| **IRB** | • First, before we begin, I would like you to read and fill out a consent form.  
• Do you have questions about concerning the consent form?  
• I would like to remind you of your right to withdraw from participation at any time. Should you choose to exercise this right; no repercussions or questions will be asked. Your participation is also confidential. Any answers or discussions from this interview will not associated with your name. |
| **Entrance briefing** | • Before we begin, I’d like to ask you if you’ve heard anything about the study from other students, like what they did or what they learned. |
| **Explaining the think aloud protocol** | • Let’s begin.  
• In this study, I’m interested in how you are thinking when answering physics questions.  
• In order to do this, I am going to ask you to think aloud as you work through the questions are you given. What I mean by “think aloud” is that I want you to say everything you are thinking from the first time you see the question until you reach an answer. I would like you to talk constantly from the time I present each problem until I tell you to stop. I don’t want you to plan what you say or try to explain to me what you are saying or thinking or doing. It is not necessary to use complete sentences. Just act as if you are alone in the room speaking to yourself. It is important that you keep talking. I will occasionally remind you to keep talking if you fall silent for too long.  
• Do you understand what I want you to do? |
| **Think aloud practice** | • We will start with a practice problem to get you used to thinking aloud. Are you ready?  
• Remember to just say everything that goes through your mind while you answer this question.  
  o How many windows are in the house or apartment where you live?  
  o Imagine that your shoes are untied. Please think aloud and describe everything you do when you tie your shoes. |
| **Think aloud follow-up** | Was there anything that you were thinking about that you didn’t say aloud?  
Just some things for you to know.  
1. I won’t be able to answer any questions in you think aloud.  
2. If you have questions, please go ahead and ask them, but I won’t respond until after the think aloud is complete.  
3. If you’re silent for more than 5 seconds, I will ask you continue to think aloud. |
### Considering contrasts pairs separately

| Contrast pair 1: Andy and Blair |  
|---------------------------------|---|
| **First, I have a pair of student responses and a written description about what they are thinking for three different scenarios. This is the first pair.** |  
| **We have Blair and Andy. This is a homework problem for Blair and Andy. They are working on this problem in the Physics Learning Center. You’ll see what each of them wrote down. And you’ll also see in the box what they were thinking.** |  
| **Explain how Andy’s reasoning approach is different from Blair’s reasoning approach. Please remember to think aloud.** |  
| **[When done] Was there anything that you were thinking about that you didn’t say aloud?** |  

| Contrast pair 2: Dana and Corey |  
|---------------------------------|---|
| **This is the second pair.** |  
| **We have Dana and Corey. They are in physics lecture together. This question you see is posed as a clicker question. Again, you’ll see the written work and a description of what Dana and Corey are thinking after the answer is explain.** |  
| **Explain how Dana’s reasoning approach is different from Corey’s reasoning approach. Please remember to think aloud.** |  
| **[When done] Was there anything that you were thinking about that you didn’t say aloud?** |  

| Contrast pair 3: Fred and Ellen |  
|---------------------------------|---|
| **This is the third pair.** |  
| **We have Fred and Ellen. They are on their own and working on this homework problem. Again, you’ll see their written work and a description of what Fred and Ellen are thinking as they answer the question.** |  
| **Explain how Fred’s approach is different from Ellen’s approach. Please remember to think aloud.** |  
| **[When done] Was there anything that you were thinking about that you didn’t say aloud?** |  

| Pre-screening |  
|----------------|---|
| **Please think aloud as you answer this question. [Give pre-screening question]** |  

### Synthesis students only

| Practice contrast matrix |  
|--------------------------|---|
| **We are now going to look at all of the students at once in a 3 x 2 matrix and try to find an explanation that describes all the cases. But before we do that, I have a warm-up to help understand what I want you to do.** |  
| **Please read the directions, and think aloud.** |  
| **Do you have any questions about what I want you to do?** |  
| **Please remember please continue to think aloud.** |  

| Explaining practice contrast matrix |  
|------------------------------------|---|
| **In looking at this table, you noticed that column A was were animals were work animals and column B was animals that were pets. To see this you looked at all of the things in A together and found something in common, and then compared it to column B.** |  
| **The idea of work animal and pets had to work for each row.** |  

| Synthesis |  
|-----------|---|
| **Here is the table of the students’ responses and find one explanation that makes column B different from column A. I want you to keep this idea of having the pattern work for each pair of rows in mind. Please continue to think aloud.** |  

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| What are alluring features | I’d like to talk to you about “thinking about the role of alluring features” in physics questions. You called it _______________.
| | • Let’s begin to talk about what “alluring features” are in physics questions. In all these cases, there is an alluring feature.
| | o An alluring feature is something in the problem that automatically captures our attention.
| | • Can you go through each case and identify what an alluring feature was for each student? |
| Prompts for clarifying the general explanation. Ask students to reevaluate if needed. | • I noticed that you said the difference in column A was …
| | • How does that apply for each student?
| | • I also heard you say that the difference in column B was …
| | • What do you mean by …
| | • Can you tell me more about …
| | • This is what I thought I heard… Did I understand you correctly?
| | • How does ____ different from ____? How is ____ the same as ____? |
| What are alluring features | I’d like to talk to you about “thinking about the role of alluring features” in physics questions. You called it _______________.
| | • Let’s begin to talk about what “alluring features” are in physics questions. In all these cases, there is an alluring feature.
| | o An alluring feature is something in the problem that automatically captures our attention.
| | Can you go through each case and identify what an alluring feature was for each student? |
| Alluring features cue spontaneous reasoning | • The next thing I’d like to you about is about “the role of alluring features” or the “impact of alluring features”
| | • Alluring features cue us on how we reason or think about the question in a certain way and may lead us down an incorrect reasoning path.
| | • For example, you said _______________. For Andy, how did the alluring feature cue him to think?
<p>| | o The coefficients cued Andy to use the equation and even though he knew Newton’s 2nd law, he wasn’t able to think about it because he was focused on the coefficients. |</p>
<table>
<thead>
<tr>
<th>Thinking about alluring features</th>
</tr>
</thead>
<tbody>
<tr>
<td>• It is good to think about “the role of alluring features”.</td>
</tr>
<tr>
<td>• Can you think about a reason why?</td>
</tr>
<tr>
<td>• Because alluring features captures all our attention and gives us tunnel vision. We can’t see anything else, and we’re might not even be aware that it’s happening.</td>
</tr>
<tr>
<td>• Even if it’s not an alluring feature for us in the moment it might be later on and impact how we respond to a different question.</td>
</tr>
<tr>
<td>• Being on the lookout for alluring features and taking some perspective on why someone is attracted by the alluring feature and what type of tempting reasoning it cues, can help us avoid and get out of these unintentional thinking traps.</td>
</tr>
<tr>
<td>• Do you have questions or thoughts on the role of alluring features?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assessment questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>• I’m going to ask you a few more questions.</td>
</tr>
<tr>
<td>• Here is the first question. Think aloud as you answer the question.  [puck]</td>
</tr>
<tr>
<td>• Here is the second question. Think aloud as you answer the question.  [ball]</td>
</tr>
<tr>
<td>• Going back to this question  [puck] think aloud as you answer the question.</td>
</tr>
<tr>
<td>• Here is the third question. Think aloud as you answer the question.  [vacation]</td>
</tr>
<tr>
<td>• Here is the fourth question. Think aloud as you answer the question.  [head]</td>
</tr>
<tr>
<td>• Finally, going back to this question  [vacation] think aloud as you answer the question.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exit briefing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other students may ask you about what we did today. It’s okay for you to say how it went for you or how you felt about it, but please do not reveal details about questions that I asked you or the situations.</td>
</tr>
</tbody>
</table>
Appendix B

INSTRUCTIONAL MATERIALS

Task Directives for Each Contrast Pair

<table>
<thead>
<tr>
<th>Contrast pair 1</th>
<th>Blair and Andy are working in the Physics Learning Center (PLC) together. They are working on a homework problem together. On the sheet, you’ll see their written work. The box below their written work describes what they are thinking.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>How is Andy approaching the situation differently from the way that Blair is approaching it?</td>
</tr>
<tr>
<td>Contrast pair 2</td>
<td>Dana and Corey are in physics lecture. The following clicker question was posed to the class. On the sheet, you’ll see their written work. The box below their written work describes what they are thinking after the class has discussed the question.</td>
</tr>
<tr>
<td></td>
<td>How is Dana approaching the situation differently from the way that Corey is approaching it?</td>
</tr>
<tr>
<td>Contrast pair 3P</td>
<td>Ellen and Fred are working on a homework problem on their own. On the sheet, you’ll see their written work. The box below their written work describes what they are thinking.</td>
</tr>
<tr>
<td></td>
<td>How is Fred approaching the situation differently from the way that Ellen is approaching it?</td>
</tr>
<tr>
<td>Contrast pair 3E</td>
<td>Ellen and Fred are grocery shopping. On the sheet, the box describes a situation.</td>
</tr>
<tr>
<td></td>
<td>How is Fred approaching the situation differently from the way that Ellen is approaching it?</td>
</tr>
</tbody>
</table>

Pre-screening question | Are there any similarities between the approaches used by all the students?
Post-Test Items

Recognition: Physics context

In physics recitation, the following question was posed.

A student throws a ball straight up in the air and times how long it takes the ball to reach its maximum height.

Afterwards, the student throws the same ball with a greater initial speed.

Compared to the first throw, will the amount of time taken for the ball to reach its maximum height be greater, less, or the same?

In the second throw, the time for the ball to reach its maximum height is greater. Because the acceleration is constant, the second ball has a greater change in velocity (from $v_2$ to 0 m/s) and therefore must require more time to reach its maximum height.

Four students, who all eventually arrive at the correct answer, start thinking about the question as described below.

A. Amanda knows that the acceleration due to gravity is constant, so the greater the change in speed the greater the time.

B. Bobbi focuses on the speed and thinks that a greater speed would imply less time to reach the maximum height. She wonders why she focused on the speed before considering the height.

C. Chris thinks the second ball travels higher because it was thrown with a greater initial speed. As a result, it must take longer to reach its maximum height.

D. David is tempted to say that the second throw will take less time to reach its maximum height because the ball has a greater initial speed, but then remembers that acceleration is constant.

Which student (or students) can be described as reflecting on the role of alluring features in impacting one's thinking? Please explain.
Suppose the coefficient of static friction between box A and the floor is 0.4, as shown at right. The coefficient of static friction between box B and a different floor is 0.6, as shown below right. $m_A = m_B = 10$ kg.

A horizontal 30 N force is applied to each box, and both boxes remain at rest.

Is the magnitude of the friction force exerted on box A greater than, less than, or equal to that exerted on box B? Explain. If there is not enough information, please state so explicitly and describe what additional information is required.

Blair

- Blair ignores the values for $\mu$.
- She correctly applies Newton's 2nd Law.
- She checks her answer with Andy who got a different answer.
- She thinks that Andy made a mistake by using $\mu$.

Andy

- Andy uses $f = \mu N$ to calculate static friction.
- He checks his answer with Blair and realizes he should have used Newton's 2nd Law.
- He wonders why he used $f = \mu N$.
- He decides that having a number for $\mu$ made him think he needed to use it.
A young girl wants to select one of the (frictionless) playground slides illustrated below to give her the greatest possible speed when she reaches the bottom of the slide. Which should she choose?

5. It doesn't matter. It would be the same for each.

Corey
- Corey thought that steeper slope means faster; which was wrong.
- He realizes energy conservation gives the right answer.
- He decides to always use energy conservation.

Dana
- Dana uses energy conservation and gets the right answer.
- A lot of students got it wrong because they thought that a steeper slope was faster.
- She thinks about why steeper slope is tempting.
Ellen

The motions of two cars are described by the position vs. time graphs shown above. When, if ever, are the speeds of the cars the same?

Ellen notices the intersection point. She thinks the intersection point could be the answer. She recalls that the slope corresponds to the velocity and gets the right answer.

Fred

The motions of two cars are described by the position vs. time graphs shown above. When, if ever, are the speeds of the cars the same?

Fred's eyes are drawn to the intersection point. He then looks at the slope and realizes the velocity is the same where the slope is the same. He thinks about why he focused on the intersection point. He decides that he chose the first thing that he saw, the intersection.
Fred

- Fred is grocery shopping and looks for Honey Nut O’s.
- A box of Naturally Flavored Fruity O’s jumps out at him and he is tempted to choose it over Honey Nut O’s.
- He thinks maybe it’s because adjective like “naturally flavored” and “fruity” make the cereal seem healthy.

Ellen

- Ellen is in the cereal aisle and searches for Honey Nut O’s.
- She picks up a box of Naturally Flavored Fruity O’s, but puts it back down.
- She puts a box of Honey Nut O’s in her cart.
### 3 × 2 practice synthesis matrix

Warm-up: Compare the two columns below and answer the question.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Border collie herding ducks.</td>
<td>Border collie playing Frisbee.</td>
</tr>
<tr>
<td>Helper monkey feeds person.</td>
<td>Man hangs out with monkey companion.</td>
</tr>
<tr>
<td>Carrier pigeon delivers messages.</td>
<td>Parrot sits on top of its cage.</td>
</tr>
</tbody>
</table>

1. Describe a single overall difference that makes Column B different from A.

2. What did you do to figure out your answer?
In general, how are students in column B approaching the situation differently from the way that students in column A are approaching it?
### Figure B.6: $3 \times 2$ synthesis matrix (with everyday context)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Blair ignores the values for $\mu$.</td>
<td>• Andy uses $f = \mu N$ to calculate static friction.</td>
</tr>
<tr>
<td>• She correctly applies Newton’s 2\textsuperscript{nd} law.</td>
<td>• He checks his answer with Blair and realizes he should have used Newton’s 2\textsuperscript{nd} Law instead.</td>
</tr>
<tr>
<td>• She checks her answer with Andy who got a different answer.</td>
<td>• He wonders why he used $\mu$.</td>
</tr>
<tr>
<td>• She thinks that Andy made a mistake by using $\mu$.</td>
<td>• He decides that having a number for $\mu$ made him think he needed to use it.</td>
</tr>
<tr>
<td>• Corey thought that steeper implies faster, which was wrong.</td>
<td>• Dana uses energy conservation and gets the right answer.</td>
</tr>
<tr>
<td>• He realizes energy conservation gives the right answer.</td>
<td>• A lot of students answered incorrectly because they thought that steeper implies faster.</td>
</tr>
<tr>
<td>• He decides to always use energy conservation.</td>
<td>• She thinks about why the steeper slope is tempting.</td>
</tr>
<tr>
<td>• Ellen is in the cereal aisle and searches for Honey Nut O’s.</td>
<td>• Fred is grocery shopping and looks for Honey Nut O’s.</td>
</tr>
<tr>
<td>• She picks up a box of Naturally Flavored Fruity O’s, but puts it back down.</td>
<td>• A box of Naturally Flavored Fruity O’s jumps out at him, and he is tempted to choose it over Honey Nut O’s.</td>
</tr>
<tr>
<td>• She puts a box of Honey Nut O’s in her cart.</td>
<td>• He thinks maybe it’s because adjectives like “naturally flavored” and “fruity” make the cereal seem healthy.</td>
</tr>
</tbody>
</table>

In general, how are students in column B approaching the situation differently from the way that students in column A are approaching it?
Ally, Brian, Cathy, and Danny are buying new wireless headphones. They are deciding between two headphones that have roughly the same price.

Audiophile On-Ear Bluetooth Headphones

Urban Bass 2 Wireless Headphones

The reasoning the individuals use to select their headphones is described below.

A. Ally, who has purchased Audiophile headphones in the past, is tempted to purchase the Audiophile headphones. However, she really likes the powerful bass from the Urban Bass headphones.

B. Brian finds the Audiophile headphones more comfortable and lighter than the Urban Bass headphones.

C. Cathy is inclined to buy the Urban Bass headphones but prefers the Audiophile sound quality. She thinks that she wanted Urban Bass because they are popular with her friends.

D. Danny likes the sleek design of the Urban Bass headphones. He also thought they felt sturdier than the Audiophile headphones.

Which student (or students) can be described as reflecting on the role of alluring features? Please explain.
Application: Physics context

You are in physics lecture. The following clicker question is posed.

The diagram depicts two pucks on a frictionless table. Puck II is four times as massive as puck I. Starting from rest, the pucks are pushed across the table by two equal forces.

Which puck will reach the finish line with greater kinetic energy?

A. Puck I
B. Puck II
C. Both will have the same.
D. There is not enough information to decide.

The correct answer (C) because of the work-energy theorem (or energy law). Since both pucks have the same force applied over the same distance, it has the same work, and thus the same kinetic energy.

The distribution of student responses is given below. A lot of students wrongly answered that puck I would have the greater kinetic energy.

Unprompted application  What should a student do to make sure he or she fully understands the reasoning involved and learns as much as possible from this question?

Prompted application  Describe how reflecting on the role of alluring features can enhance student understanding of the reasoning involved.
Application: Everyday context

Your friend is planning a week’s vacation for winter break. Your friend plans to spend most of the time outdoors swimming and enjoying the sandy seashore.

Your friend has currently narrowed the options down to two places that are reasonably priced and asks for your advice. The websites give only a limited amount of information about the two options.

Option A
Sunny weather
Nice beaches for swimming
A quality hotel

Option B
Lots of sunshine
A beautiful rocky sea coast
An ultramodern hotel

Unprompted application Your friend is thinking about choosing B, but you suspect that your friend is not thinking clearly about this decision.

Prompted application What would you tell your friend to help your friend make the best decision between these two vacation options
In this assignment, there is a total of four sections. Three sections are learning sections, and the final section asks questions about what you have learned. The assignment will take approximately 25-30 minutes to complete. Complete the assignment on your own. Please try your best.

For your participation, Dr. Clark has offered two extra credit options: your lowest written homework score replaced with a 100%, or 8 points towards your prelim total. You will select your option and enter your name at the end of the assignment.

Let’s begin!

### SECTION I

Suppose the coefficient of static friction between box A and the floor is 0.4, as shown at right. The coefficient of static friction between box B and a different floor is 0.6, as shown below right. $m_A = m_B = 10$ kg.

A horizontal 30 N force is applied to each box, and both boxes remain at rest.

Is the magnitude of the friction force exerted on box A ($f_A$) greater than, less than, or equal to that exerted on box B ($f_B$)?

- $f_A > f_B$
- $f_A < f_B$
- $f_A = f_B$

Explain your reasoning.
Suppose the coefficient of static friction between box A and the floor is 0.4, as shown at right. The coefficient of static friction between box B and a different floor is 0.6, as shown below right.

\[ \mu_A = 0.4, \quad \mu_B = 0.6 \]

A horizontal 30 N force is applied to each box, and both boxes remain at rest.

Is the magnitude of the friction force exerted on box A greater than, less than, or equal to that exerted on box B?

Correct answer: \( f_A = f_B \)

Relevant features: The boxes remain at rest and the horizontal forces must be balanced

Explanation: Because each box remains at rest, the acceleration is zero and thus the net force on the box must be zero. The static friction force and the horizontal force must be equal in magnitude in order to have no net force. Since the same 30 N horizontal force is applied to each box, the magnitudes of the friction forces on both boxes must also be equal.
A common incorrect answer is that \( f_A < f_B \). What feature(s) of the problem do you think a student who arrived at this answer might have used?

- coefficients of static friction
- masses of the boxes
- boxes remain at rest
- horizontal applied forces

**Explanation of the Common Incorrect Answer**

Your response is incorrect.

**Common incorrect answer:** \( f_A < f_B \)

**Feature often used for incorrect answer:** Coefficients of static friction

**Explanation:** A common incorrect answer is \( f_A < f_B \), and the associated reasoning typically focuses on the numerical coefficients of static friction, which differ for the two boxes. The coefficients of static friction may be used to find the maximum static friction force in each case, but often, like in the problem above, the static friction forces are not at their maximum values.
Alluring Distracting Features

Suppose the coefficient of static friction between box A and the floor is 0.4, as shown at right. The coefficient of static friction between box B and a different floor is 0.6, as shown below right.

\[ \mu_A = \mu_B = 10 \text{ kg} \]

A horizontal 30 N force is applied to each box, and both boxes remain at rest.

Is the magnitude of the friction force exerted on box A greater than, less than, or equal to that exerted on box B?

**Alluring distracting features:** Coefficients of static friction

The numerical coefficients of static friction serve as alluring distracting features.

An alluring distracting feature is something in the problem that automatically captures our attention. They may cue how we reason or think about the question and may lead us down an incorrect reasoning path. Being on the lookout for potential alluring distracting features can help us avoid common pitfalls.

Next, you will complete an exercise in which you will be asked to identify the alluring distracting feature associated with a particular physics problem.
Exercise

A young girl wants to select one of the (frictionless) playground slides illustrated below to give her the greatest possible speed when she reaches the bottom of the slide. Which should she choose?

4. Her speed would be the same for each slide.

Correct response: 4. Her final speed would be the same on each slide. Click here for an explanation.

Which of the following is most likely an alluring distracting feature for this problem?

- [ ] the height of each slide
- [ ] the length of each slide
- [ ] the relative incline of each slide

If a student based his or her reasoning on the alluring distracting feature you identified above, the student would incorrectly conclude that the girl would have the greatest possible speed at the bottom of:

- [ ] 1. Slide 1
- [ ] 2. Slide 2
- [ ] 3. Slide 3
- [ ] 4. Her speed would be the same for each slide.

Briefly explain your reasoning.

The alluring distracting feature is explained on next page.
A young girl wants to select one of the (frictionless) playground slides illustrated below to give her the greatest possible speed when she reaches the bottom of the slide. Which should she choose?

4. Her final speed would be the same on each slide.

**Alluring distracting feature:** The relative incline of each slide

**Common incorrect response:** Slide 1

**Explanation:** Many students think that the girl will have the greatest possible speed on the steepest slide, slide 1. It is true that the acceleration is larger on slides with larger inclines. However, the total time spent on the incline must be taken into account in order to determine the speed of the girl when she reaches the bottom of the slide. Although slide 1 has the greatest acceleration, the girl will spend the least amount time on the slide.
In this section, you will examine student approaches on two different problems. The first one is below.

Problem 1: Andy and Blair

Andy and Blair are working together on the same problem that you worked on earlier. Their responses are shown below in the boxes. Click on the PowerPoint to advance the slides.

Suppose the coefficient of static friction between box A and the floor is 0.4, as shown at right. The coefficient of static friction between box B and a different floor is 0.6, as shown below right.

A horizontal 30 N force is applied to each box, and both boxes remain at rest.

Is the magnitude of the friction force exerted on box A greater than, less than, or equal to that exerted on box B?

How does Blair's thinking related to the alluring distracting feature differ from Andy’s thinking related to the alluring distracting feature?
Problem 2: Cathy and Dave

Cathy and Dave are working on the following problem individually. Their responses are shown in the boxes below. Click on the PowerPoint presentation to advance the slides.

The motions of two cars are described by the position vs. time graphs shown at right. When, if ever, are the speeds of the cars the same?

---

CATHY

The cars have the same speeds at point B.

- Cathy thinks the intersection is the answer.
- She checks her answer with the key, but she finds that she is wrong.
- She then remembers that slope corresponds to the velocity and gets the right answer.

DAVE

At A where the cars have equal slopes.

- Dave wants to choose the intersection but decides this point is where the positions are the same.
- He then recalls that the velocities are the same when the slopes are the same.
- He thinks about why he initially focused on the intersection.
- He realizes he was drawn to the intersection because it was an obvious place where something was the same.

---

How does Dave’s thinking related to the alluring distracting feature differ from Cathy’s thinking related to the alluring distracting feature?
In this section, you will examine all four student approaches simultaneously in a 2 x 2 matrix. To help you understand what you will need to do, please complete the warm-up exercise below.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Border collie herding ducks." /></td>
<td><img src="image2" alt="Border collie playing frisbee." /></td>
</tr>
<tr>
<td>Helper monkey feeds person.</td>
<td>Ross with Marcel (pet monkey).</td>
</tr>
</tbody>
</table>

Considering the situations in column 2 together, how do they differ from the situations in column 1?

Check your response on the next page.
<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>work animals</td>
<td>pets</td>
</tr>
<tr>
<td>Boarder collie herding ducks.</td>
<td>Border collie playing frisbee.</td>
</tr>
<tr>
<td>Helper monkey feeds person.</td>
<td>Ross with Marcel (pet monkey).</td>
</tr>
</tbody>
</table>

In column 1, the animals are work animals. In column 2, the animals are pets. To see this, you look at all of the things in column 1 together and find something in common and then compare it to column 2. The distinction between work animal and pet has to apply to each row.
The matrix below shows all four students approaches on the two different problems. Please click here to refer back to the problems.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andy is tempted to use the values for $\mu$, but he remembers Newton’s 2nd law and correctly applies it. He checks his answer with Blair who got a different answer. He wonders why $\mu$ is not required for this problem. He decides that the problem did not ask for the maximum static friction forces, given by $\mu$.</td>
<td>Blair uses $f = \mu N$ to calculate static friction. After she checks her answer with Andy and realizes she should have used Newton’s 2nd Law, she gets the right answer. She wonders why she was tempted to use $\mu$, which gave the maximum static friction forces. She decides that having numbers for $\mu$ made her think she needed to use them.</td>
</tr>
<tr>
<td>Cathy thinks the intersection is the answer, but she checks with the key and finds that she is wrong. She then remembers that the slope corresponds to the velocity and gets the right answer. She thinks about why the intersection is not the answer. She realizes that the intersection indicates the same position but not the same speed.</td>
<td>Dave wants to choose the intersection but decides that this point is where the positions are the same. He then recalls that the velocities are the same when the slopes are the same. He thinks about why he initially focused on the intersection. He realizes that he was drawn to the intersection because it was an obvious place where something was the same.</td>
</tr>
</tbody>
</table>

Considering the situations in column 2 together, how does the thinking related to the alluring distracting feature of the students in column 2 differs from that of the students in column 1?
Andy is tempted to use the values for \( \mu \), but he remembers Newton's 2nd law and correctly applies it. He checks his answer with Blair who got a different answer. He wonders why \( \mu \) is not required for this problem. He decides that the problem did not ask for the maximum static friction forces, given by \( \mu \).

Blair uses \( f = \mu N \) to calculate static friction. After she checks her answer with Andy and realizes she should have used Newton's 2nd Law, she gets the right answer. She wonders why she was tempted to use \( \mu \), which gave the maximum static friction forces. She decides that having numbers for \( \mu \) made her think she needed to use them.

Cathy thinks the intersection is the answer, but she checks with the key and finds that she is wrong. She then recalls that the slope corresponds to the velocity and gets the right answer. She thinks about why the intersection is not the answer. She realizes that the intersection indicates the same position but not the same speed.

Dave wants to choose the intersection but decides that this point is where the positions are the same. He then realizes that the velocities are the same when the slopes are the same. He thinks about why he initially focused on the intersection. He realizes that he was drawn to the intersection because it was an obvious place where something was the same.

Considering the situations in column 1 together, characterize the thinking related to the alluring distracting feature of the students in column 1.

An explanation is provided on the next page.
Play the video below to learn how the thinking related to the alluring distracting features of the students in column 2 differ from that of the students in column 1.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andy is tempted to use the values for $\mu$. However, he remembers Newton's 2nd law and correctly applies it.</td>
<td>Blair uses $f = \mu N$ to calculate static friction. After she checks her answer with Andy and realizes she should have used Newton's 2nd Law, she gets the right answer.</td>
</tr>
<tr>
<td>He checks his answer with Blair who got a different answer. He thinks Blair made a mistake by using $\mu$.</td>
<td>She wonders why she was tempted to use $\mu$, which gave the maximum static friction forces. She decides that having numbers for $\mu$ made her think she needed to use them.</td>
</tr>
<tr>
<td>Cathy thinks the intersection is the answer. She checks her answer with the key and finds that she is wrong. She then remembers that slope corresponds to the velocity and gets the right answer.</td>
<td>Dave wants to choose the intersection but decides that this point is where the positions are the same. He then recalls that the velocities are the same when the slopes are the same. He thinks about why he initially focused on the intersection. He realizes that he was drawn to the intersection because it was an obvious place where something was the same.</td>
</tr>
</tbody>
</table>

Did the video play correctly?

No [ ]
Yes [ ]
SECTION IV

You’re almost done! This is the final section with three questions.

1. What are important and productive ways in which students can think about alluring distracting features?
2. The following clicker question is posed in physics lecture.

The diagram depicts two pucks on a frictionless table. Puck II is four times as massive as puck I. Starting from rest, the pucks are pushed across the table by two equal forces.

Which puck will reach the finish line with greater kinetic energy?

A. Puck I
B. Puck II
C. Both will have the same.
D. There is not enough information to decide.

Correct response: C. Both will have the same.  
Click here for the explanation based on the work-energy theorem.

Jay incorrectly answers that puck I would have the greater kinetic energy because of the difference in the masses. He reasons that a smaller mass would cross the finish line with a greater velocity and therefore more kinetic energy. After he is shown the correct answer, Jay correctly applies the work-energy theorem.

The difference in masses is an alluring distracting feature. Jay thinks about why this difference in masses does not lead to different final kinetic energies. How else could Jay productively think about this alluring distracting feature (the difference in masses)? Please explain.
3. Ally, Brian, Cathy, and Danny are buying new wireless headphones. They are deciding between two headphones that have roughly the same price.

Audiophile On-Ear Bluetooth Headphones

Urban Bass 2 Wireless Headphones

The reasoning the individuals use to select their headphones is described below.

A. Ally is tempted to buy the Urban Bass headphones because of their sleek look. However, she remembers the Audiophiles were more comfortable, and she purchases them. Later she thinks about why the Urban Bass were not a good choice.

B. Brian finds the Audiophile headphones more comfortable and lighter than the Urban Bass headphones. After carefully thinking about the choices, he purchases the Audiophiles.

C. Cathy is inclined to buy the Urban Bass headphones but prefers the Audiophiles’ sound quality. She purchases the Audiophiles. She thinks that she wanted Urban Bass because they are popular with her friends.

D. Danny likes the sleek design of the Urban Bass, but the Audiophiles were the only ones available in the store. He purchases the Audiophiles, and he later regrets his decision.

Which student is thinking is about how an alluring distracting feature impacts his or her reasoning?
The study has been completed! Thank you for your participation.
Enter in your information below to receive your well-deserved extra credit.

Enter your MaineStreet login (e.g., john.smith):

Select your choice for extra credit:

100% on lowest written homework assignment
+8 points towards prelim score

Your participation has been recorded.

Please do not share specific details of the assignment with anyone. Sometimes if people know what the assignment is about, that knowledge will affect their responses even when they don’t mean for it to, and then the data are not valid.

If you have questions, comments, or concerns, or you would like to learn more about the purpose of assignment, please contact Thanh at thanh.le@maine.edu. Your participation is greatly appreciated!
BIography OF THE Author

Thanh Lê was born in Midland, Texas. After graduating from Oak Grove High School in San Jose, California, she attended the University of California, Berkeley where she earned a Bachelor of Arts in Physics in 2001, Single Subject Teaching Credential in 2003, and a Masters of Arts in Education in 2005. She taught high school math and physics in Oakland and Richmond, California before attending the University of Maine. Thanh K. Lê is a candidate for the Doctor of Philosophy degree in Physics from The University of Maine in August 2017.