

5-2004

Structuring a Wayfinder's Dynamic and Uncertain Environment

Michael D. Hendricks

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STRUCTURING A WAYFINDER'S DYNAMIC AND UNCERTAIN ENVIRONMENT

By

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B.S. University of Delaware, 1986

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A THESIS

Submitted in Partial Fulfillment of the

Requirements for the Degree of

Doctor of Philosophy

(in Spatial Information Science and Engineering)

The Graduate School

The University of Maine

May, 2004

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An Abstract of the Thesis Presented
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Wayfinders typically travel in dynamic environments where barriers and requirements change over time. In many cases, uncertainty exists about the future state of this changing environment. Current geographic information systems lack tools to assist wayfinders in understanding the travel possibilities and path selection options in these dynamic and uncertain settings. The goal of this research is a better understanding of the impact of dynamic and uncertain environments on wayfinding travel possibilities. An integrated spatio-temporal framework, populated with barriers and requirements, models wayfinding scenarios by generating four travel possibility partitions based on the wayfinder's maximum travel speed. Using these partitions, wayfinders select paths to meet scenario requirements. When uncertainty exists, wayfinders often cannot discern the future state of barriers and requirements. The model to address indiscernibility employs a three-valued logic to indicate accessible space, inaccessible space, and possibly inaccessible space. Uncertain scenarios generate up to fifteen distinct travel possibility categories. These fifteen categories generalize into three-valued travel possible partitions based on where travel can occur and where travel is successful. Path selection in these often-complex environments is explored through a specific uncertain scenario that includes a well-defined initial requirement and the possibility of an additional requirement somewhere beforehand. Observations from initial path selection tests with this scenario provide the motivation for the hypothesis that paths arriving as soon as possible to well-defined requirements also maximize the probability of success in

meeting possible additional requirements. The hypothesis evaluation occurs within a prototype Travel Possibility Calculator application that employs a set of metrics to test path accessibility in various linear and planar scenarios. The results did not support the hypothesis, but showed instead that path accessibility to possible additional requirements is greatly influenced by the spatio-temporal characteristics of the scenario's barriers.

ACKNOWLEDGMENTS

I must begin by acknowledging a grateful appreciation to have had the opportunity to live and study in the safety and comfort of the University of Maine. Many others throughout the world are not as fortunate. My wife Charlene, and our three daughters, Kristen, Ellen, and Erin, have each added to my good fortune and created an atmosphere of support and love for which I am thankful to God.

I would like to thank Max Egenhofer, my advisor, who provided an astute balance of freedom and guidance, allowing me to navigate the various snares and pitfalls toward my goal of furthering my knowledge of Geographic Information Science and specifically the completion of this thesis. I would also like to thank the remainder of my committee, Kate Beard, Mike Worboys, and Bob Franzosa, for all their efforts, and particularly Kathleen Hornsby who provided many hours of help over the years on numerous topics related to this thesis.

I would like to thank a number of people who were helpful in the early stages of my research. Discussions with William Mackaness during his brief stay in Orono helped develop a number of initial concepts. I would also like to thank Professors Paul Bauschatz and Alan Stubbs for their interest in various components of my work. There are numerous colleagues whom I shared insightful conversations with over the last three years and am thankful for their friendship; Carlton Brown, Jorge Campos, Steve Cole, Kristen Eickhorst, Jim Farrugia, Chris Frank, Connie Holden, Lars Kulik, Inga Mau, Tom Noonan, Paul Schroeder, Isolde Schlaisich, and Vijay Venkataraman.

None of this would be possible without the leadership of the United States Military Academy selecting me to attend graduate school. I am thankful for their trust and the opportunity they have provided. I would specifically like to thank Dr. John Brockhaus for his many years of friendship and support, and COL(R) Scott Loomer who has provided a continual example of excellence. My hope is that the three years invested in me to study at the University of Maine can be paid back

with many years of service educating and inspiring future leaders at the United States Military Academy.

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CHAPTER 1

INTRODUCTION

Wayfinding is the fundamental human task of selecting and following paths through the environment (Golledge 1999b). The constraint-based and goal-oriented process of wayfinding (Montello in press) typically occurs within a structured space populated with barriers, affordances, and requirements. Examples include a river (as a barrier), a bridge (as an affordance to cross a river), and an appointment at a particular location and time (as a requirement). Geographic information systems (GISs) typically provide decision support to wayfinders with maps and routing algorithms, with the assumption of a static and well-defined environment.

Wayfinders often travel in dynamic environments, however, where barriers and requirements change over time. Changing environmental conditions make path selection more difficult. In addition, wayfinders are often unsure about the future state of barriers and requirements. This uncertainty often manifests itself as indiscernibility between the elements in a set of future possible outcomes. A wayfinder, for example, may not yet know whether a requirement is at 10:00 am or 11:00 am, and must consider both possibilities when selecting paths. Current GISs lack tools to assist wayfinders in understanding the travel possibilities and path selection options in these dynamic and uncertain settings.

The goal of this research is a better understanding of the impact of dynamic and uncertain environments on wayfinding travel possibilities. To model these environments, an integrated spatio-temporal framework is presented, which is populated with barriers and requirements, and partitioned by the wayfinder's maximum travel speed into travel possibility categories. The specific question of path selection with uncertainties is addressed by considering a special uncertain scenario, *the repairmen scenario*, where a wayfinder has a well defined initial requirement, but is unsure whether a second will be added somewhere beforehand. It is

hypothesized that paths arriving as soon as possible to the well-defined requirement will maximize the probability of success in also meeting the possible additional requirement. The evaluation of the hypothesis occurs within a prototype application that implements the presented conceptual framework and models travel possibilities resulting from dynamic and uncertain wayfinding scenarios.

1.1 Wayfinding in Static Environments

Researchers have studied the various aspects of path selection in wayfinding scenarios for decades (Gatty 1979; Hutchins 1983; Bovy and Stern 1990; Golledge 1995; Golledge 1999a; Hunt and Waller 1999; Kirasic 2000). Wayfinding scenarios occur within a geographic space populated with affordances and barriers, along with a set of requirements defining the scenario's goal. Given a start point and movement capability, wayfinders select paths to meet requirements.

Wayfinders can plan travel paths through the environment without navigational aids (Golledge 1995; Golledge 1999b; Loomis *et al.* 1999), but often employ some form of assistance. For hundreds of years maps, compasses, and other tools have helped wayfinders select successful paths (Robinson *et al.* 1995). Topographic maps are particularly effective at supporting wayfinders (Kals 1983) by indicating where travel is possible and where barriers will be encountered. Visualizations are an important tool in the early stages of problem solving (Blaser *et al.* 2000), and viewing the spatial distribution of future travel possibilities when planning paths is no exception.

Analysis and visualization tools now included with GISs and related spatial technologies provide even more detailed terrain descriptions (Goodchild and Longley 1999). Determining the spatial distribution of travel restriction categories is a typical method of supporting wayfinders. For example, categorizing the terrain into unrestricted, restricted, and severely restricted regions is a classification scheme used in military mobility analysis (Department of the Army 1990; Graff

1997). Other spatial analysis tools, such as visibility plots, provide additional support to wayfinders when formulating travel plans.

Least-cost routing algorithms provide further assistance by automatically considering environmental constraints and suggesting optimal paths based on some criteria. Research into path selection tools for in-car navigation systems have long been studied (Mark 1986) and are a critical component of transportation modeling (Pallottino and Scutellà 1998; Mitchell 2000). The sub-discipline of *GIS for Transportation* (GIS-T) is a focal point of path based GIS research related to wayfinding (Waters 1999).

Numerous criteria can be minimized or maximized when calculating paths. Minimizing distance is the classic example (Lee and Preparata 1984; Hershergery and Suri 1999), though many other permutations exist. For instance, minimizing visibility (Lee and Stucky 1998) or determining the path with the least number of turns (Duckham and Kulik 2003) are strategies tailored to particular tasks.

Minimizing travel time introduces the complexities of temporal processes. The majority of this complexity, however, is often generalized away by considering travel time as an attribute no different from elevation or distance. The wayfinder's location becomes a function of time without considering the possibility of temporal change in other aspects of the wayfinding scenario. This is not a realistic assumption since objects in the environment usually change over time.

1.2 Wayfinding in Dynamic Environments

Wayfinders typically travel in dynamic environments. Barriers change over time and requirements are temporally dependent. Combined with the movement of the wayfinder over time, dynamic barriers and requirements create complex travel possibilities through space and time.

Routing algorithms are beginning to support path selection in dynamic environments (Pallottino and Scutellà 1998). Application areas concerned with dynamic path selections include

providing accident information and routing instructions to car drivers (Ozbay and Kachroo 1999) and aircraft path adjustments to avoid changing weather patterns (Myers 2000). These algorithms themselves, however, fail to provide information concerning travel possibilities beyond proposing individual paths.

Maps are the traditional method of communicating complex spatial information (Bertin 1983; MacEachren 1995). Traditional maps, however, have difficulty representing change, though considerable effort is underway to develop appropriate spatio-temporal visualization techniques (Unwin 1996; MacEachren *et al.* 1999; Andrienko *et al.* 2000; Kraak 2000; MacEachren and Kraak 2001). Improved computational capability is allowing the wayfinder to explore changes occurring through space and time with animated cartography (Peterson 1995; Kraak *et al.* 1997; Blok 2000) and other geovisualization tools (MacEachren and Kraak 2001). The capability of displaying spatio-temporal change is expected to continue to increase (MacEachren and Kraak 2001).

When making path decisions in dynamic environments, wayfinders require additional information beyond when changes occur. They must understand the impact of these changes on travel possibilities. A typical strategy used by people in day-to-day dynamic decisions is to project possible future states from known patterns and trends (Barlow 1998). When crossing the street, people predict their own locations, as well as the locations of any cars, to ensure that they do not intersect in space and time. This process of perceiving and understanding elements in a dynamic spatio-temporal environment and projecting their status into the future is referred to as *situational awareness* (Endsley 1988). In the professional world, success of air-traffic controllers (Andre *et al.* 1998; Durso *et al.* 1999; Azuma *et al.* 2000), pilots (Endsley 1995; Zhang and Hill 2000), and military personnel (National Research Council, 1997; Ellis and Johnston 1999) relies heavily on situational awareness in order to make effective decisions in dynamic environments.

What is lacking in most wayfinder support systems is an integrated space-time approach that provides information related to the changing travel possibilities that result from dynamic

environments. To be more effective, these systems must provide information over space and time about changing conditions and the impact of these changes on travel possibilities. To create systems such as these, an integrated space-time approach is required, though there is a lack of space-time integration within GIS as a whole (Langran 1992; Peuquet 2000; Peuquet 2002).

Time geography (Hägerstrand 1967) is a conceptualization of space-time that holds promise in supporting the exploration of travel possibilities and path selection in dynamic environments. Time is integrated orthogonally to space, creating a space-time cube (Pred 1977). Paths through this cube are represented as geospatial lifelines (Hornsby and Egenhofer 2002). By using the wayfinder's maximum travel speed, and considering barriers and requirements, *space-time prisms* are created indicating partitions of travel possibilities (Miller 1991; Forer 1998; Miller and Wu 2000). Using time-geography concepts, space-time can be partitioned based on the constraints of the wayfinding scenario to indicate various travel possibility categories. The significant limitation of this approach is the assumption that future states are known with certainty.

1.3 Wayfinding in Uncertain Environments

Every approach up to this point, including time geography, assumes perfect knowledge of the environment's future state. This is a simplistic and unrealistic assumption. To be more effective at supporting wayfinders in realistic scenarios, uncertainty must be addressed. Uncertainty is realized in many forms (Worboys 1998), but this thesis concentrates on the uncertainty related to the inability to discern between possible outcomes. Uncertainty results when future environmental characteristics are ill defined (Zhang and Goodchild 2002). When planning a path, for instance, uncertainty arises when the wayfinder does not have enough information to know the requirement's exact location or time constraints.

As with both the environmental characteristics and the wayfinder's location, uncertainty can be a function of time. Uncertainty will decrease as additional information becomes available to the wayfinder. The combined result of changing uncertainty, together with the dynamic nature of

the environment and the wayfinder's varying location, create a complex time-dependent system defining travel possibilities through space and time (Figure 1-1).

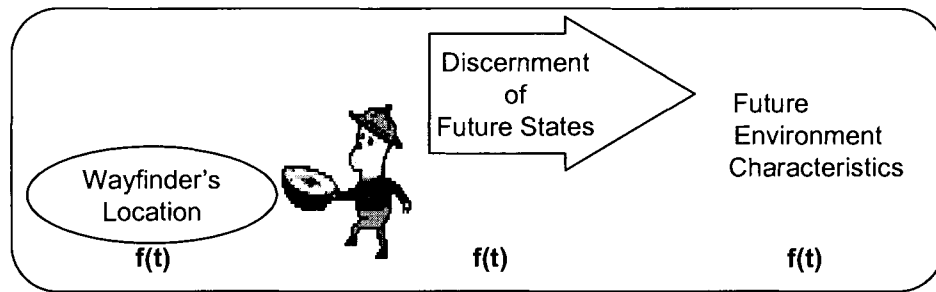


Figure 1-1: Temporally varying elements in uncertain and dynamic wayfinding scenarios.

When uncertainty exists in the characteristics of a wayfinding environment, the locations and times where travel is possible also become uncertain. No longer is space-time crisply categorized into where travel is possible and where it is not. The question of indeterminate boundaries in the spatial realm has been a topic of interest for some time (Burrough and Frank 1996). One method to approach this problem is to introduce a broad boundary (Clementini and Di Felice 1996) between crisply defined accessible and inaccessible space. Using this approach to define travel possibilities results in space-time partitioned into three categories: (1) accessible, (2) not accessible, and (3) the broad boundary space of possibly not accessible.

If characteristics of wayfinding scenarios are well defined, travel possibility spaces provide the wayfinder with a decision tool to select paths. When travel possibility spaces include broad boundaries, paths can intersect partitions indicating possibly invalid or possibly unsuccessful space. In these cases, wayfinders are unsure whether a path will meet the scenario's requirements. To assist wayfinders in these dynamic and uncertain scenarios, additional tools will have to be developed.

One example of an interesting uncertain scenario that demonstrates the complexity involved is termed the *repairmen scenario*. Consider a repairman in the shop planning the day's travels.

Initially the only requirement on the schedule is at Joe's Coffee Shop at 11:00 am. However, the repairman must be ready to go to an additional service call (a requirement) anytime or place beforehand. Initially when selecting a path, the repairman cannot discern whether there will be an additional requirement, or where and when this possible requirement will occur. Assuming that the repairman wants to make money, he will want to travel in such a way to maximize accessibility to the largest percentage of additional requirement possibilities.

To support the repairman, typical GIS tools can display maps indicating the affordances and barriers within the region of interest. The repairman's current location can be determined with the global positioning system (GPS) and the 11:00 am service call's location can be geo-coded and highlighted on the map. In addition, it is becoming possible to provide the repairman with various spatio-temporal visualizations to explore the changing travel conditions inherent at this time of day. From this information, the repairman can select a path between the shop and the 11:00 am service call.

Instead of selecting a path himself, the wayfinder can employ a routing algorithm to minimize values such as distance, gas costs, or time. This algorithm, if sophisticated enough, can account for changing travel conditions when calculating paths. Typically, these algorithms return shortest travel time paths, suggesting to the repairman paths that arrive as early as possible to the requirement or depart as late as possible from the start point.

Current GIS tools, however, offer little assistance in providing a broad understanding of travel possibilities in these scenarios. Where and when should the repairman travel to maximize the probability of reacting to an additional service call? Does staying at the shop, or immediately traveling to the first requirement, maintain accessibility to the maximum number of possible requirements? Current GISs do not answer these questions, and wayfinders, such as the repairman, must formulate travel plans by approximation and rules of thumb.

GIS technology lacks an integrated space-time framework, where changing conditions and uncertainties can be modeled in such a way that travel possibility partitions can be created and

displayed to the wayfinder. In addition, methods to select optimal paths in regard to maximizing accessibility to requirement possibilities in uncertain scenarios is a major shortfall of current systems. What is required is a system that accounts for the wayfinder's location, the changing environment, and the wayfinder's discernment of the environment over time (Figure 1-2). By accounting for these factors, information systems can partition space-time according to travel possibilities, display this information to wayfinders, and suggest paths through space-time that maximize accessibility to requirement possibilities.

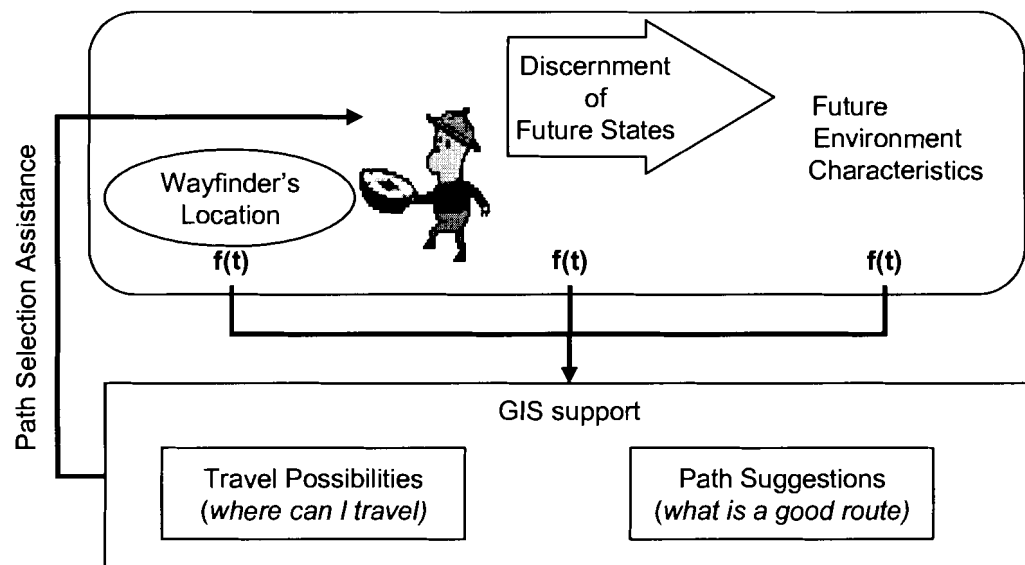


Figure 1-2: Possible GIS support to wayfinders in dynamic and uncertain scenarios.

1.4 Goals and Hypothesis

The goal of this research is a better understanding of the impact of dynamic and uncertain environments on wayfinding travel possibilities. Increased knowledge of the impact of these complex processes will assist those developing next generation travel planning aids. To accomplish this goal, a conceptual model of wayfinding is presented consisting of an integrated

spatio-temporal framework, populated with barriers and requirements. Based on a wayfinding scenario's configuration, space-time is partitioned in a manner that defines the associated certain and uncertain travel possibilities.

To answer questions related to the impact of uncertainty on path selection while wayfinding, the *repairmen scenario* is explored, where a wayfinder has a well defined initial requirement, but is unsure whether a second will be added somewhere beforehand. It can be expected that when planning travel in these instances, wayfinders often employ path selection algorithms that return travel time minimization paths. The question is, "How effective are paths returned from these algorithms in scenarios with uncertainties?" To explore this question and provide additional insight into the impact of uncertainties on wayfinding in general, the following hypothesis is put forward:

"Paths that minimize arrival time to requirements also maximize accessibility to possible additional requirements."

Section 8.3 tests this hypothesis by comparing the arrival minimization path against an accessibility maximization path in various linear and planar wayfinding scenarios. A prototype application, the Travel Possibility Calculator (TPC), serves as the hypothesis testing mechanism, and at the same time provides additional insight into the impact on travel possibilities of change and uncertainty in barriers and requirements.

1.5 Scope

This thesis focuses on wayfinding in dynamic and uncertain environments in the following setting. The spatial domain of this study is movement along the Earth's surface and limits its scope to the 2-dimensional spatial domain. Three-dimensional movement, such as by aircraft or by submersible watercraft, is not addressed. Elevation considerations, however, can be incorporated into this approach as an attribute of the 2-D spatial environment (Maune 2001).

Individual wayfinding is the focus of this work and this wayfinder is modeled as a point object, which is a typical abstraction. In addition, acceleration and deceleration are not considered in this thesis and the related assumption holds that change in the wayfinder's speed immediately takes into effect.

A wayfinder may encounter numerous obstacles in the environment. This thesis models obstacles as absolute blockages. As a result, barriers fully exist and completely block travel or do not exist. The notion of space that can negatively affect the wayfinder is not considered. Wayfinding scenario requirements can take many forms, but this thesis only considers requirements modeled as points in space. This simplification is also a typical abstraction in many GIS applications (Miller and Wu 2000), though in future work more complex requirement realizations would be desirable.

This thesis does not consider uncertainty related to inaccuracy in the sense that the wayfinder is wrong about some element of the wayfinding scenario, nor does it address inherent vagueness. The goal of this work is not to develop a robust statistical approach to path selection, and as a result assumes that all possible outcomes have an equal likelihood of occurring. This assumption results in a uniform probability distribution for each element in a possibility set. This simplification allows the development of conceptual models that can be extended with increased probability refinement in the future. In addition, it also assumes that uncertainty through time remains the same or decreases and does not address the generation of uncertainty during the course of a wayfinding scenario.

The prototype serves as a demonstration and testing tool and does not attempt to develop optimal algorithms or data structures, but instead proposes valid algorithms and possible data structures. As a result, calculations with large spatio-temporal data sets are not considered in this thesis.

1.6 Major Results

To understand better the impacts of change and uncertainty on wayfinding travel possibilities, this thesis develops an integrated model of wayfinding and tests time minimization paths within a prototype application. The specific results are as follows:

- It was demonstrated that an integrated space-time approach to modeling dynamic wayfinding scenarios is a plausible method of representing the impact of changing conditions on travel possibilities. The novel technique of sequentially partitioning space-time into four travel possibility categories, and the use of modal verbs to describe these categories, provides a cognitively straightforward method to represent future travel possibilities to wayfinders selecting paths.
- An extension with uncertainty considerations leads to a three-valued broad boundary technique that represents a wayfinder's indiscernibility of future possibilities. It was shown that sequentially partitioning with uncertainty could result in up to fifteen travel possibility categories. As a result, two generalization schemes were introduced to provide information to the wayfinder about where travel is possible (*valid-space*) and where travel can be successful (*successful-space*).
- Uncertainty in the future states of barriers and requirements can create scenarios where the only valid paths are those that may not succeed in meeting requirements. As a result, this thesis developed a set of metrics to measure a path's accessibility to requirement possibilities over time and a metric of overall path accessibility. With these metrics, wayfinders compare various paths to select the one with the greatest probability of being successful in meeting all requirements.
- Testing of paths in various realizations of the *repairmen scenario* showed that arrival minimization paths do not maximize accessibility to possible requirements in all cases, but instead are impacted greatly by barriers. It was found that the changing

spatio-temporal characteristics of barriers greatly influence the selection of paths that maximize accessibility to possible requirements.

- The development of a Travel Possibility Calculator (TPC) prototype application demonstrated the feasibility of the presented concepts. By integrating space and time in a discrete voxel-based spatio-temporal data structure, wayfinding speed options were shown to be limited when using traditional raster- and voxel-based path algorithms. As a result, a method of accounting for various travel speeds when creating paths in voxel-based spatio-temporal data structures is presented.

1.7 Intended Audience

This thesis will be of interest to engineers and scientists interested in better representing time and uncertainty into wayfinding movement models. Those interested in visualization of spatio-temporal processes will find the travel possibility partitioning technique a viable option for representing complex wayfinding processes. System designers can employ the path-requirement intersection models in future applications, while geographers and transportation engineers will find the results interesting in that they open a whole set of accessibility-based questions related to both time and uncertainty.

1.8 Organization of Remaining Chapters

The remainder of this thesis is organized into the following eight chapters. Chapter 2 reviews the relevant literature to this research, including wayfinding, GIS support in static and spatio-temporal settings, and uncertainty.

Chapter 3 introduces the conceptual framework of a wayfinder's dynamic environment. The framework integrates space and time together into a multidimensional coordinate system, similar to the time-geography approach. Objects exist within this framework, including a point object representing the wayfinder. Based on this framework, the chapter identifies four wayfinding

primitives employed to define wayfinding scenarios: (1) maximum travel speed, (2) start point, (3) barriers, and (4) requirements. Particular attention is placed on the intersection possibilities between the wayfinder's space-time path and requirement objects.

Given a spatio-temporal framework and set of wayfinding primitives, Chapter 4 develops the concept of partitioning space-time into accessibility-based travel possibility categories. The maximum travel speed combines with each of the remaining three primitives to partition space-time according to the accessibility afforded by each primitive. In addition, the chapter highlights the special considerations resulting from combining multiple requirements. Sequentially partitioning individual accessibility partitions generates four travel possibility categories. The chapter ends with a mapping of the accessibility spaces onto modal verbs to describe verbally, travel possibility partitions.

Chapter 5 introduces uncertainty in the barriers and requirements of a wayfinding scenario. This uncertainty is modeled with a three-valued logic where objects or effects exist, do not exist, or possibly do not exist. The possible category is treated as a broad-boundary condition between the two certain cases. The chapter analyzes existence uncertainty of barriers and requirements as well as uncertainty in the spatial and temporal characteristics of point objects and objects with a temporal extent. The impact of these uncertainties on travel possibilities is explored in detail. The extension of the sequential partition approach to include the three-valued partitions resulting from uncertainty, results in a set of fifteen separate partitions of travel possibilities that simplify into two generalized spaces, *valid-space* and *successful-space*. Wayfinders know that paths included in *successful-space* can meet the scenario's requirements.

The special case of uncertainty in requirement combinations is the topic of Chapter 6. It starts with an analysis of the influence of individual requirement uncertainty on the remaining elements of a combined requirement. It is shown that uncertainty in one requirement can create uncertainty in the other requirements of a combination. Three categories of uncertainty in the requirement operation can also exist: (1) uncertainty in the combined requirement's existence, (2) uncertainty

in whether a subsequent requirement will be added, and (3) uncertainty in whether a subsequent requirement will replace an initial requirement. Two special uncertainty scenarios are described: the *repairmen scenario* and the *police officer scenario*. The *repairmen scenario* considers a well-defined requirement with the possibility that a subsequent one will be added somewhere beforehand. The *police officer scenario* also includes a well-defined requirement, but in this case the possibility exists where a subsequent requirement replaces the initial requirement.

Chapter 7 completes the circle by linking travel possibility partition concepts with the initial concept of path selection. The chapter outlines the characteristics of space-time paths, in particular with respect to the conceptual wayfinding model, and introduces the basic characteristic of space-time paths. Two unique paths are (1) an *early bird path* that minimizes arrival time to requirements, and (2) a *procrastinator path* that maximizes the departure time from the start point. The concept of *valid* and *successful paths* follow from these definitions. It is shown that in some uncertain scenarios, path selection is limited to *possibly unsuccessful* paths. In these cases, wayfinders are not sure of success, but can select paths to maximize the possibility of reacting to future possibilities. With this in mind, metrics are introduced to measure accessibility to future requirement possibilities. The chapter ends by demonstrating the use of these metrics with a *repairmen scenario*.

Chapter 8 describes the development of the Travel Possibility Calculator prototype to explore the travel possibilities of dynamic wayfinding scenarios. Four critical components are addressed in detail: (1) a voxel-based space-time data model, (2) an accessibility algorithm, (3) a travel possibility-partitioning algorithm, (4) and path generation algorithms. The hypothesis is evaluated within this prototype, and the evaluation tests various linear and planar wayfinding scenarios with and without temporary barriers. It is found that the hypothesis is not supported in all cases, but instead the *early bird* path's ability to maximize accessibility to future possibilities is heavily dependent on the spatial and temporal configuration of barriers and requirements.

Chapter 9 draws conclusions and indicates future work.

CHAPTER 2

THE WAYFINDER'S DYNAMIC AND UNCERTAIN ENVIRONMENT

This chapter reviews relevant research topics and literature concerned with wayfinding in dynamic and uncertain environments. This research crosses a number of disciplinary boundaries from such diverse fields as the social sciences, computational geometry, computer sciences, and spatial information sciences. This review begins by exploring the issues and related work associated with wayfinding (Section 2.1). The chapter continues by considering the support offered wayfinders by GIS and related spatial technology using static representations of the environment (Section 2.2), followed by extensions to account for spatio-temporal aspects of wayfinding (Section 2.3). The chapter ends by considering tools to address uncertainty in wayfinding (Section 2.4).

2.1 Wayfinding

Traversing the environment to accomplish a task is a central human endeavor (Golledge 1999b). Determining and traversing a path or route from one location to another is referred to as *wayfinding* (Golledge 1999a), *pathfinding* (Bovy and Stern 1990), or *navigating* (Kuipers 1978; Trullier *et al.* 1997). Montello (in press) defines wayfinding as "... the goal-directed and planned movement of one's body around an environment in an efficient way."

The wayfinding process can be decomposed into a planning and execution phase (Timpf 1992; Timpf 2002), and Gärling (1986) describes this division as the formation of the travel plan and the execution of the travel plan. The focus of this thesis is on the formation of travel plans in dynamic and uncertain spatio-temporal environments and the remainder of this literature review reflects this fact by ignoring the extensive body of literature on actually moving through the

environment (Golledge 1995; Allen 1997; Golledge 1999b; Loomis *et al.* 1999; Schneider and Taylor 1999; Kirasic 2000).

One of the fundamental decisions when formulating travel plans is where to move. The trace of movement through the environment is referred to as either a *route* or a *path* (Golledge 1995). Wayfinders select *paths* with a *path selection strategy*. Achieving a goal is the primary driver of a *path selection strategy* (Montello in press), and is often achieved by minimizing or maximizing some cost, such as effort, distance, or time (Golledge 1999a). Features in the environment and the wayfinder's capabilities both play an important role in path selection and are each described in the next two sections.

2.1.1 The Wayfinding Environment

Wayfinder's must have knowledge related to the features in the environment to select paths that meet desired goals (Passini 1992). Wayfinders often classify features in regards to their impact on travel (Janzen *et al.* 2000). One simple classification divides space into barriers and barrier free zones, creating a maze-like environment. Researchers have extensively studied how people choose paths and travel through mazes (Lee and Preparata 1984; Janzen *et al.* 2000). Epstein (1997) identifies four types of maze-like structures: random, and three non-random types: warehouse, furnished room, and office.

In her work on artificial intelligence wayfinding, Epstein (1997) classified environmental features into facilitators and obstructers. Facilitators support efficient travel while obstructers make it more difficult. Three kinds of facilitators are identified: gate, base, and corridors. Gates provide access to new quadrants within maze like environments; a base is a key location from a successful wayfinding event; and a corridor is a narrow space. Four obstructions are identified: dead-end corridors, chambers, bottles, and barriers.

The barrier free spaces encountered by wayfinders typically possess variable effects on movement. One popular method to model variations is with friction surfaces (Douglas 1994).

Another method is to partition space into trafficability categories (Department of the Army 1990; Donlon and Forbus 1999).

Influences on where a wayfinder can travel may physically exist or be human constructs. Smith (1995) described physically existing objects, such as a river or bridge, as *bona fide* objects, while objects that exist as a result of human decision or political decree, such as a out-of-bounds area, or the extent of a city, as *fiat* objects. Bittner (2000) further classifies *fiat* boundaries as to whether they are marked and unmarked. For example, he describes the lines on a parking lot as *marked fiat boundaries* that do not physically bar movement, but nonetheless restrict travel. On the other hand, the entrances to the parking spots are *unmarked fiat boundaries* that afford crossing. This distinction creates an organizational structure where *bona fide boundaries* are barriers, and *fiat boundaries* may be barriers or non-barriers.

Whether *bona fide* or *fiat*, the objects in the environment can change over time. Cole and King (1968) described three classes of changing spatial objects: static, slowly (nearly imperceptibly) changing, and dynamic. This classification scheme is a matter of temporal scale. The width of a river, for instance, normally does not change over the course of a day, but is a highly dynamic system over years and decades (Hamblin 1992). Dynamic spatial objects can change their position, size and shape (Galton 1997). For example, a wayfinder's position in space can change, the size of a traffic jam might increase, and the extent of a fire may change shape.

Wang and Cheng (2001) describe spatio-temporal behavior of objects in the environment as either continuous, discrete, or stepwise. Continuously changing objects are in a constant state of change, such as a weather system or flowing water. Discretely changing objects are static, except for instantaneous changes. For example, the opening of a drawbridge is typically modeled as an instantaneous change resulting in an immediate travel restriction. Objects changing in a stepwise fashion alternate between being static and changing. For example, a car will continuously move through the environment, stop and remain static for some time, and then begin moving again. Modeling these various changes is a challenge for information systems.

Change can be temporary or permanent (Egenhofer 1993b). A drawbridge opening is a temporary change that blocks travel, while the destruction of the bridge on the other hand is a permanent change at the scale of most wayfinding scenarios. In addition, an object's identity may change over time (Hornsby and Egenhofer 2000). A travel restriction zone, for instance, can be created, eliminated, and then reincarnated to restrict travel once again.

The environment encountered by a wayfinder is a complex set of physical and human induced objects influencing travel possibilities. These objects do not remain static, but change over time. The next section considers the characteristics of the wayfinder and how he or she formulates travel plans in this dynamic environment.

2.1.2 The Wayfinder

Wayfinders come in many forms: animals (Schone 1984; Trullier *et al.* 1997), robots (Miura and Shirai 1997; Saffiotti 1997; Muller *et al.* 2000), or human. Wayfinding differences exist in humans by gender (Kwan 1998), age (Golledge 1995; Kirasic 2000), and skill (Kirasic 2000; Seidman and Cleveland 2001). Regardless of these differences, successful wayfinders formulate effective travel plans, through spatial knowledge of their environment (Raubal and Egenhofer 1998). Three levels of spatial knowledge are generally assumed (Siegel and White 1975): (1) *landmark knowledge* referring to salient reference points in the environment, (2) *route knowledge* putting landmarks in sequences, and (3) *survey knowledge*, allowing landmarks and routes to be understood in a general frame of reference.

When planning paths, wayfinders refer to their internal spatial knowledge of the environment. This knowledge is often modeled as a cognitive map (Nelson 1996; Golledge 1999a; Kitchin and Freundschuh 2000; Barkowsky 2001) or cognitive collage (Tversky 1993). Alternately, wayfinders can consult a wayfinding aid such as a map (Freksa 1999; Casakin *et al.* 2000).

Regardless of whether a wayfinder is employing wayfinding aids or not, an important component of successful wayfinding is *orientation*. Orientation is an awareness of the space we

occupy in the environment along with the important objects in that environment (Howard and Templeton 1966; Hunt and Waller 1999). To be *geographically oriented* means that the wayfinder maintains a sense of where they are relative to the goal, where barriers and other relevant features are located (Montello in press).

Orientation considerations typically focus on the spatial component. Maintaining orientation in dynamic environments, however, is an important and difficult aspect of wayfinding. Maintaining orientation over time can be considered a form of *situational awareness*, a concept that Endsley (1988) defines as “the perception of the elements in the environment within a volume of time and space, the comprehension of their meaning, and the projection of their status in the near future”. Endsley (1995) developed a three level model of situation awareness comprised of: (1) perception, (2) comprehension, and (3) projection. The first level of situation awareness, perception of the elements in the environment, refers “to perceive the status, attributes, and dynamics of relevant elements in the environment” (Endsley 1995). For example, a search and rescue team member may perceive the marks left by a hiker. The second level refers to the comprehension of the current situation. At this stage, an understanding of the significance of perceived elements concerning the goal is achieved. In this example, the searcher may recognize the mark as an imprint of a lost hiker and realize that they are on the right track. The final level of situation awareness is the projection of future status. This projection is normally not too far into the future. To continue the example, the searcher can project how far the lost hiker could move and narrow down the search radius.

This ability to predict future events from current system states is a critical skill. Barlow (1998) states, “Prediction is a matter of identifying a spatio-temporal pattern at an early stage and assuming it will run to completion.” He also identified the following requirements: (1) knowledge of commonly occurring spatio-temporal patterns, (2) ability to classify and discriminate patterns, and (3) skill in quickly identifying that a pattern has started.

Effective representations of future states distinguish experts from those less skilled. Durso and his colleagues (1999) showed that good chess players had a better comprehension of the current state of the game than novice players, but an understanding of the future states is what distinguished experts. Kerstholt and Raaijmakers (1997) also showed that people tend to have difficulty projecting into the future when making decisions, and place proportionally higher value on current system states when evaluating options.

To formulate effective travel plans in dynamic environments, wayfinders must; (1) know where and when changes occur in the environment, and (2) project the impact of these changes on travel possibilities through time. With this information, the wayfinder determines a small *choice set* of paths from all possible options (Thill and Horowitz 1997a; Thill and Horowitz 1997b). From this *choice set*, the wayfinder makes a path selection. Without effectively projecting travel possibilities into the future based on the dynamic environment, this *choice set* will not be optimal and may in fact be faulty. Therefore, it is critical for the wayfinder to understand the travel possibilities resulting from the dynamic environment.

2.2 Supporting Wayfinders in Static Environments

The paper map has been the primary tool of wayfinders when selecting paths for hundreds of years (Musham 1944; Robinson *et al.* 1995). Maps provide information on the distribution of geographic features (Bertin 1983), which wayfinders can use to assess travel possibilities and develop travel plans. More sophisticated maps classify the features based on their impact on travel possibilities (Wilson and Gallant 2000). At an even more detailed level of support, preferred paths can be highlighted on maps, as has been done for years by automobile clubs.

2.2.1 Models of Space

The growth of GIS and related technologies expands the level of support available to wayfinders. To provide this support, the wayfinding environment must be modeled within the computer. The

abstraction of space chosen to model the environment fundamentally influences travel possibility calculations. Three primary models exist; vector space, network space, and tessellated (raster, voxel) space (Peuquet 2002). A vector representation associates travel cost with polygons, and paths can be generated between points. Developing efficient algorithms in this space is the focus of computational geometry (Lee and Preparata 1984; Choi *et al.* 1994; Hershergery and Suri 1999; Sellen *et al.* 2000). Supporting wayfinders moving along transportation networks typically employs a graph data structure (Bovy and Stern 1990; Waters 1999; Miller and Shaw 2001; McQueen *et al.* 2002). Travel costs are assigned to edges or nodes and a rich body of knowledge is available to generate least cost paths through the graph. Modeling cross-country movement is less common, but typically employs a tessellated raster space (Benton *et al.* 1996) or voxels when considering 3-dimensional movement (Scott 1994). Travel costs in tessellated spaces are associated with a friction surface (Douglas 1994) and spread functions generate paths (Demers 1997). Regardless of the data model chosen, the representation of the environment in the computer allows wayfinding aids to be generated to include visualizations, categorizations, and the generations of paths.

2.2.2 Displaying the Wayfinder's Static Environment

The classic method of supporting path planning with GIS is through display and query of the features in the environment. Automated cartography and GIS now allow the wayfinder to move beyond viewing a standard map (Robinson *et al.* 1995) to data exploration (Andrienko and Andrienko 1999; Kraak and MacEachren 1999). With the goal to provide exploratory visualization tools, the field of *geovisualization* is becoming popular (MacEachren and Kraak 2001; Slocum *et al.* 2001; Crampton 2002).

Early adopters of geovisualization tools for wayfinders have been the aircraft community. The goal is typically geared towards maximizing pilot *situational awareness* (Endsley 1994; Endsley and Garland 2000; Olmos *et al.* 2000; Zhang and Hill 2000). Studies have indicated the

importance of color coding (Derefeldt *et al.* 1999) and 2-D versus 3-D displays (Olmos *et al.* 2000). A key factor addressed in much of this work is the importance of decluttering displays to allow optimal understanding of the critical features (Yeh and Wickens 2000). Wayfinders operate in complex environments, and these studies highlight the importance of representing environmental features in an effective and simple manner.

2.2.3 Categorizing the Wayfinder's Static Environment

Spatial analysis tools included within GIS can provide more sophisticated and tailored support to wayfinders. The classification of geographic features based on their attribute or location is a standard tool of GIS (Goodchild 1992). Features can be extracted, for instance, from remotely sensed imagery (Jensen 1996), and classified according to the impact they have on travel. Terrain classification can be as simple as identifying where barriers are located, or as detailed as an analysis of the travel impacts of soils, vegetation, or elevation (Dorey *et al.* 1999). The NATO reference mobility model II (NRMII), is an example of a comprehensive analysis tool to provide terrain effects information (Ahlvin and Haley 1992).

2.2.4 Generating Paths in the Wayfinder's Static Environment

Path generating algorithms in support of wayfinding typically minimize or maximize some attribute, such as effort, distance, or time. These algorithms find paths from an origin to either a single destination, or all possible destinations (Mitchell 2000). The abstraction of space employed influences path generation techniques, and as a result, path considerations in each spatial model are discussed.

2.2.4.1 Generating Paths in Geometric Space

The basic form of the path selection problem in geometric space is; given a set of obstacles, find a Euclidean shortest obstacle-avoiding path between two points (Mitchell 2000). Two examples of

this rich field include; Hershergery and Suri's work with movement along the plane (1999), and Lee and Preparata's work with rectilinear barriers (1984). In addition, paths through 3-dimensional and higher geometric space can be calculated (Choi *et al.* 1994; Sellen *et al.* 2000).

A novel method of addressing path generation in geometric space is based on *symbolic projections* (Chang and Jungert 1986; Chang 1987; Chang and Jungert 1996), and partitions the traveler's 2-dimensional space around barriers, enabling qualitative spatial reasoning about paths and travel possibilities (Holmes and Jungert 1992; Jungert 1992). These symbolic projections record objects' extents along the horizontal and vertical axes (Figure 2-1), resulting in strings of object locations associated with each axis (Chang and Jungert 1986; Jungert 1988). When specifically applied to wayfinding, the focus of the symbolic projection approach is the partitioning of space by projecting the extents of barriers. This method holds promise to provide a simple representation of travel possibilities to wayfinders in dynamic and uncertain environments.

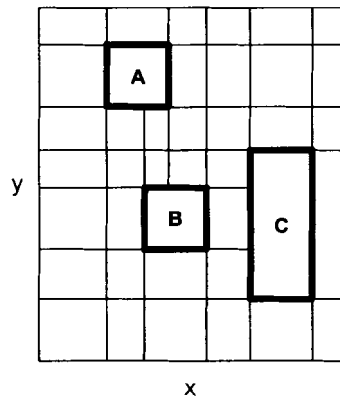


Figure 2-1: Structuring the space of a wayfinder through symbolic projection techniques; barriers A, B, and C are projected onto each axis, creating a grid representing travel possibilities.

2.2.4.2 Generating Paths in Network Space

If the space in the wayfinding scenario is represented as a directed graph, as with most transportation problems, a wide range of least cost path algorithms can be employed. The problem, as concerned with real world wayfinding, involves a simple directed graph $G = (N, A)$,

where N is the set of all nodes, and A is the set of arcs between nodes with associated positive values indicating the travel costs. The goal is to develop a data structure that allows one to ask what is the path length from some start point to all other locations. Length can be measured by travel cost, least visible, least number of turns (Duckham and Kulik 2003), or in time. This results in a shortest path tree (SPT) problem, where the structure is a spanning tree, T , such that each unique path in T represents the shortest path in G , from a source point. The SPT's spanning characteristic provides an opportunity for the wayfinder to query on travel possibilities based on time limits.

The most famous, and still often employed, algorithm to solve the SPT problem is that by Dijkstra (1959). There are numerous extensions of this algorithm, but the basic approach still holds. Zhan (1997) and later with his colleague Noon (1998) compared 15 shortest path algorithms using real transportation networks. They highlight that real transportation networks have unique characteristics in regards to all possible network configurations, namely they are sparsely connected. This fact makes certain algorithms more effective than others. They proposed the following algorithms to be most effective: Dijkstra Approximate Bucket, Dijkstra Double Bucket, and Graph Growth by Pallottino.

2.2.4.3 Generating Paths in Raster Space

When the space of the wayfinder is modeled as a raster representation, calculation of travel possibilities is accomplished with a spread algorithm (Tomlin 1990; Stefanakis and Kavouras 1995; Stefanakis and Kavouras 2002). The general approach is to; (1) generate a friction surface from features in the environment that represent the cost of travel, (2) generate an accumulated cost surface from some start location, that represents the cost of travel from each raster cell to the given start location, and (3) determination of the optimal path from this accumulated cost surface. One difficulty with paths through raster space is which cells should be connected and the associated discretation problems of diagonals (Goodchild 1977).

Collischonn and Pilar (2000) consider the special case of anisotropic travel costs. Anisotropic travel costs are those whose values depend on travel direction. They contend that travel along roads and canals over changing elevations is best modeled anisotropically. Their algorithm finds the path that minimizes uphill travel.

Xu and Lathrop (1995) identified the need for non-adjacent connectivity in raster-based GIS to better model anisotropic spreading of fires and other phenomena. Using their techniques, they increased accuracy from 60 percent to above 95 percent. De Vasconcelos and his colleges also addressed the spread of fire (de Vasconcelos *et al.* 2002).

Lee and Stucky (1998) argue that when calculating least cost paths over digital elevation models, viewshed information can be integrated into the least cost path calculations. Using visibility information, they create four categories of *viewpaths* by creating friction surfaces associated with each. These friction surfaces are hidden path, scenic path, strategic path, and withdrawn path.

Benton and his colleges (Benton *et al.* 1996) focus on implementation issues for what they call tactical route planning, focusing on large raster spaces with trillions of points. They describe a hierarchical route-planning algorithm that allows for faster processing.

There has been limited research on least cost paths in 3-D tessellated spatial structures. Stefanakis and Kavouras (1995; 2002) and Scott are exceptions (1994). Scott's procedure begins with an *origin volume* of binary values, where the origin is assigned a value of one, and all other voxels are set to zero. He populates a second volume, the *friction volume*, with cost values representing the effort to traverse the cell. These two volumes are inputs to a cost volume generation algorithm. The resulting *cost volume* is analogous to an accumulated cost surface. The final input is the *destination volume*, which is similarly structured to the origin volume, except that the destination is given a value of one. The cost volume and destination volume are inputs to the path-finding algorithm.

The GIS support offered to wayfinders in static environments, allows wayfinders to formulate more effective travel plans. The environment most often encountered, however, is dynamic. The next section considers how spatio-temporal GIS can support wayfinders reason about travel possibilities in changing environments.

2.3 Supporting Wayfinders in Dynamic Environments

Tools to support wayfinders in dynamic environments must consider both space and time. The integration of time into GIS has received increasing attention (Peuquet 2001). The first substantive work on the topic was by Langran (1989; 1992) and has since seen an explosion of effort.

2.3.1 Models of Space and Time

Spatial data is conceptualized as either discrete or continuous: often referred to as objects or fields; Temporal data includes this same distinction (Couclelis 1992). Discrete temporal conceptualizations bind analysis to rigid progressions through time, but nonetheless are the most often employed. Typical examples of this technique used in GIS are space-time cubes, sequence snapshots, or base state with amendments (Langran 1992). Continuous conceptualizations of time on the other hand allow a more general treatment of change, but are more difficult to represent and less often employed.

The dimensionality of space and time is also of interest. Disregarding numerous philosophical and relativistic issues, space consists of three-dimensions, while time is one-dimensional. Certain temporal models such as branching time (Frank 1998) can, however, result in a form of multi-dimensionality. This is often the case when using simulations to conduct spatio-temporal data analysis. Time is also directional, while space is not bound in this manner (Galton 1997).

2.3.2 Time Geography

A fruitful research area related to the integration of time and space is Time Geography. Time geography provides a foundation for recognizing paths through space and time (Wachowicz 1999) and measuring accessibility (Miller 1991; Miller 1999). Time geography is a constraint-based approach that does not attempt to predict exact behavior, but instead indicates travel possibilities (Pred 1977) and uncovers structural patterns. Hägerstrand (1967) developed the concept and techniques of time geography to model spatio-temporal behavior of individuals, while Pred (1973; 1977) is responsible for translating much of his work and introducing many of the concepts to English speaking researchers.

The foundation of the theory considers an n -dimensional space and one orthogonal dimension of time. The case of two spatial dimensions (x-y plane) and an orthogonal dimension for time (ascending z-axis) yields a three-dimensional space-time cube. As time progresses, a point object traces a space-time path (Miller 1991) from the origin to the destination, also modeled as a geospatial lifeline thread (Hornsby and Egenhofer 2002). Immobile objects trace vertical lines in the space-time cube, whereas an object moving at a constant speed creates a sloped space-time path, with flatter lines representing faster travel and steeper lines standing for slower travel. Projecting such a space-time path onto the x-y plane creates the *route* traveled through space (Miller 1991).

Most often, the exact path through space-time is unknown. One method of handling this uncertainty is to determine the set of all possible locations that an object can possibly travel to between time intervals. With a given start point and a constant maximum speed, a half cone is created in space-time, which represents the set of all possible locations the traveler can reach, (Figure 2-2a). If a destination point is added to this scenario, a second half cone extending back in time is created and whose intersection with the first half cone creates a potential path space (Miller 1991) or lifeline bead (Figure 2-2b, Hornsby and Egenhofer 2002). This lifeline bead

represents the set of all possible space-time points that the object may have occupied between the origin and the destination, based on heuristics about the object's maximum travel speed. The aggregate of simply connected beads, Figure 2-2c, forms a lifeline necklace (Hornsby and Egenhofer 2002).

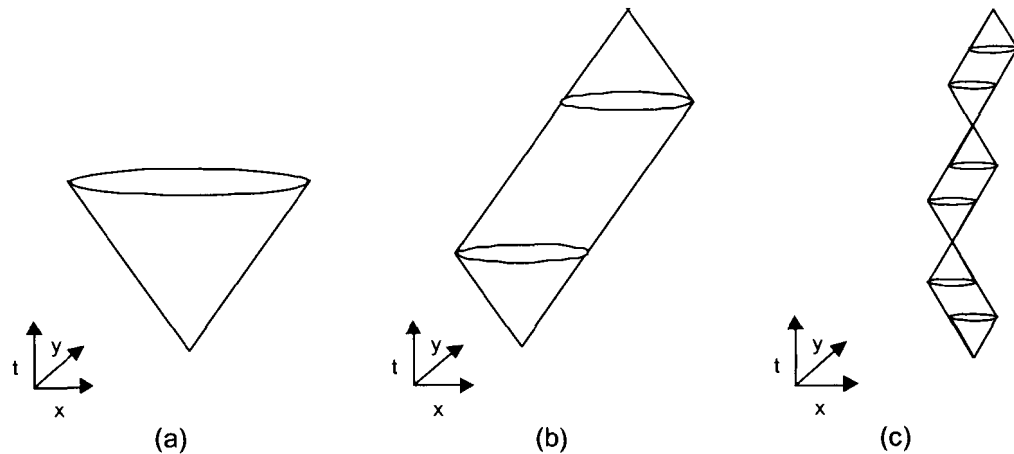


Figure 2-2: Possible travel locations in a space-time volume; (a) a cone, (b) potential path space or lifeline bead, and (c) a lifeline necklace

The lifeline necklace and its associated beads are projected onto the spatial surface to represent accessibility, a qualitative spatial measure used by travelers (Weibull 1980; Miller 1991; Kwan 1998; Miller 1999). A related approach uses isochrones to model accessibility (O'Sullivan *et al.* 2000). While lifeline necklaces support the analysis as to whether two or more individuals could have met, they say little about space-time inside or outside the necklaces and beads.

Time geography considered three classes of constraints that restrict individual movement and shape space-time prisms (Pred 1977). *Capability constraints* restrict movement based on physiological needs, such as sleeping or eating, and the speed limitations of available transportation. *Coupling constraints* limit travel by the requirements to meet other people or

objects in space and time. *Authority constraints* restrict travel as a result of certain activities being only available at certain times.

There has been limited implementation of time-geography concepts within GIS. Miller (1991; 1999; 2000) has explored the integration of time geography from a vector GIS perspective to solve transportation accessibility issues. Forer (1998) argues instead for a raster implementation for the creation of a space-time volume.

Miller's (1991) original work implemented time-geography concepts to generate potential path areas (PPAs) over transportation networks. Potential path areas are projections onto the spatial dimension; this approach does not directly create a 3-D space-time cube. Miller and Wu (2000) highlight the importance of space-time accessibility measures (STAM) and developed a GIS-linked software system to provide decision support for transportation analysis and planning. Their prototype demonstrates the feasibility of calculating individual STAMs, but leaves for future work visualization and statistical measures of multiple individuals.

Forer's (1998) approach chooses to represent time geography in GIS from a raster perspective. He argues that advances in 3-D data structures and computer graphics allow the creation of space-time volumes with voxels, while the creation of space-time volume surfaces and intersections with continuous geometry is too complex. Chen (2000) also argues that discrete volumetric data models, as opposed to surface modeling, better represents volumetric objects and processes.

Forer introduces the term *taxel* to represent a volumetric cell in space-time to highlight the space-time nature of this data structure. He identifies four types of objects that occupy space-time: (1) lifelines, (2) static facilities, (3) mobile facilities, and 4) action volumes, such as a prism indicating potential. These objects can be continuous, in that a contiguous set of taxels span the entire time span of study, or intermittent. Certain *taxels* will be neither of these four and will be empty or inaccessible.

Each object comprises an individual matrix mask, stored as a binary 3-D array. These masks can then easily be compared and queried. He identifies five classes of masks:

- Void masks: represents all inaccessible space-time.
- Discrete structures (N matrices): objects such as buildings, which form columns completely through space-time.
- Dynamic Structures (M matrices): objects that vary over time.
- Actors (K matrices): individuals moving or stationary.
- Action spaces (L matrices): The possibility space for certain actors.

In implementing this approach, he uses ESRI's GRID module to create 2-D raster grids and a custom application to assemble these into a 3-D array. He identifies the immense storage requirements. For example, to represent a 10-meter spatial grid and 5-minute temporal interval for a built up area of 180 km², 1100 million taxels is required. He suggests however, that efficient storage volumetric data, such as octrees (Samet 1990), make this approach plausible.

2.3.3 Generating Paths in Space-Time

Research into shortest path problems in dynamic environments has received limited effort. This work is primarily in transportation planning, and the rise of Intelligent Transportation Systems has increased the need for these tools. Orda and Rom (1990) modeled changing travel costs along edges according to arbitrary continuous functions. The question of traveling in the presence of moving obstacles has also been addressed (Reif and Sharir 1994).

Pallottino and Scutellà (1998) modeled change with a discrete model that holds promise. They refer to this as the *Minimum Cost Dynamic Path Problem*. They introduce a data model, called a Space-Time Network. A Space-Time Network graph (R) is comprised of the nodes of the original graph (G) along with copies of these nodes for each discrete time interval. For example, consider a 4-node graph over 10 time increments (Figure 2-3) The connectivity between nodes indicates time variations of travel. For example, a connection between the same nodes, one time

increment in the future indicates waiting. A link between two separate nodes one time increment apart is twice as fast as the same nodes connected over two time increments. They propose an algorithmic paradigm they call *chrono-SPT* to solve the *Minimum Cost Dynamic Path Problem* with this space-time network. For an example of the options associated with movement from node 1 to node 4 see Figure 2-4. An assumption that they make is that the travel times between nodes is divisible by the time increment in their model. They show how their approach can answer the following classes of problems: 1) minimum travel time to all other nodes with a given departure time, 2) minimum travel time to all other nodes for any departure time, and 3) questions related to time windows, or arrivals and departures within time intervals.

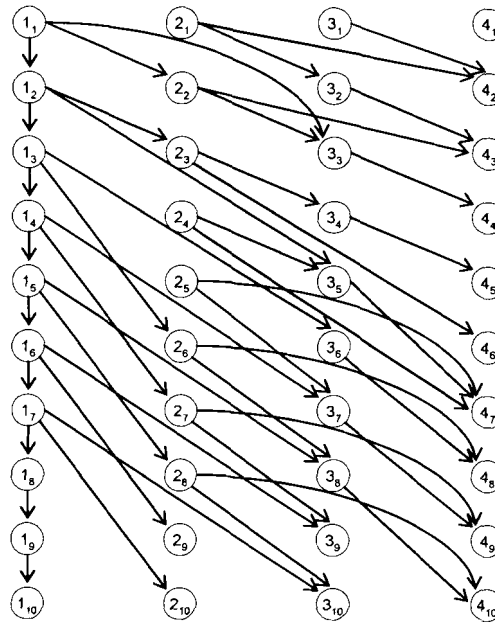


Figure 2-3: Space-Time Network (R) between four nodes over 10 time increments, from (Pallottino and Scutellà 1998).

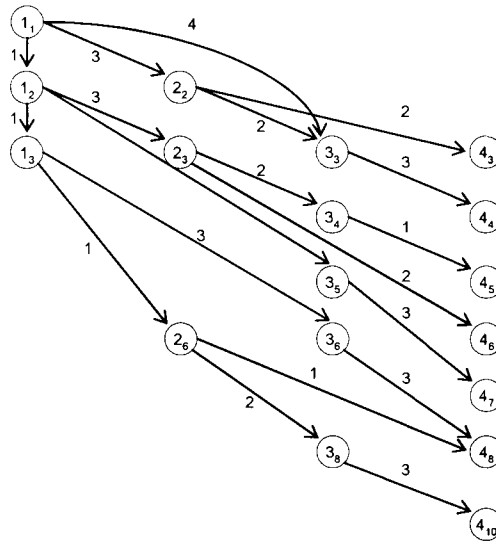


Figure 2-4: Space-time non-redundant portion of R for travel from node 1 to node 4 from (Pallottino and Scutellà 1998).

From this review of techniques of supporting wayfinders in a dynamic setting, it is found that limited support is available. Time geography concepts and Pallotino and Scutella's work on the dynamic path problem, though, hold promise. Complicating this process is the uncertainty often encountered in wayfinding scenarios, which is the topic of the next section.

2.4 Supporting Wayfinders in Uncertain Environments

Most wayfinding models assume a well defined environment and a wayfinder with perfect knowledge of that environment (Duckham *et al.* 2003). This is typically not the case. Wayfinders can be wrong about their location or just not sure where they are located. Features can be placed incorrectly on maps, or details may be lacking. Even if a wayfinder's knowledge of its location and other features of the environment are correct and well defined, wayfinders may be mistaken about the destination or unsure what time to arrive. Imperfections and uncertainties exist in these scenarios that influence the wayfinding process.

These examples also highlight two orthogonal and distinctive categories of imperfection; error and imprecision (Duckham *et al.* 2001). Error relates to the difference between reality and

an observation, often termed inaccuracy. Imprecision, on the other hand, describes a lack of specificity in an observation (Worboys 1998). A wayfinder located in Bangor Maine stating, “I’m located at 21° 18’ north latitude and 157° 50’ west longitude,” is making a precise, yet inaccurate statement, since the coordinates place the wayfinder in Honolulu Hawaii. On the other hand, the statement “I’m located somewhere in Maine” is accurate, but imprecise.

This thesis is particularly interested in imprecision: positional and other forms of error—a central feature in surveying (Leick 2004) and more frequently addressed in other geographic information science fields (Ehlschlaeger and Goodchild 1994)—is ignored. A special form of imprecision, also not within the scope of this work, is vagueness. Vagueness results from borderline cases (Duckham *et al.* 2001). For example, the requirement, “go to the mountains”, is vague, since the wayfinder is uncertain what is meant by mountains, or where the mountains might start. The classification and mapping of vegetation is a vague process (Brown 1998).

A closely related term to imprecision is granularity. Granularity considers the grains or clumps of information. Elements in a grain cannot be discerned (Duckham *et al.* 2001). Course grains provide less detail, while fine grains provide greater amounts of information. For instance, a campus map typically provides information at the granularity of roads, paths, and buildings, but does not include the fine details required to provide direction directly to a particular office. Granularity issues have been explored in relation to both spatial data (Worboys 1998) and spatio-temporal data (Stell 2003). Hornsby and Egenhofer explored moving objects with multiple granulations (Hornsby and Egenhofer 2002) and granularity issues in robotic navigation have been considered (Miura and Shirai 1997). Semantic granularity is also of interest (Fonseca *et al.* 2002).

Classic Boolean logic approaches fail to effectively model imprecision (Duckham *et al.* 2001). Extending the Boolean truth values to include values between 0 or 1, can address imprecision uncertainty and leads to fuzzy set theory (Zadeh 1965). Rather than considering the

probability of belonging to a set, fuzzy representations relate to the degree of membership into a set (Peuquet 2002). The membership function is a critical component of fuzzy sets (Fischer 2000), and truth values indicate the degree of membership. Fuzzy sets are popular in the mobile robot and AI communities (Saffiotti 1997) and are gaining popularity in the spatial information science community (Goodchild *et al.* 1994; Openshaw 1997; Tao *et al.* 1997; Brown 1998; Ratsiatou and Stefanakis 2001; Zhang and Stuart 2001). A major shortfall of fuzzy sets, however, is the difficulty in assigning membership values (Keefe and Smith 1996).

A related approach to modeling imprecision is rough sets. The theory of rough sets was developed by Pawlak (1982; 1991) and motivated by the practical needs to interpret, characterize, represent, and process indiscernibility of individuals (Jeng *et al.* 1998). The key idea in rough sets is an upper and lower approximation, and a set of three truth values; true, maybe, and false (Duckham *et al.* 2001). Consider for example, three locations: location A is certainly a requirement, location B is certainly not a requirement, and location C may be a requirement. It can be said that the upper approximation set of requirements includes both A and C, while the lower approximation includes only A. Location B is not in either set, and is identified as certainly not a requirement.

Rough sets have received limited treatment in the GIScience community (Ahlqvist *et al.* 2000). Rough classification of geographic features (Ahlqvist *et al.* 2000) and the use of rough sets to model imprecise observations of the environment while traveling (Raubal and Worboys 1999) are exceptions.

The difference between an upper and lower approximation is a boundary region (Zhang and Goodchild 2002). Boundary regions are addressed in work on indeterminate boundaries of spatial objects (Burrough and Frank 1996). Cohn and Gotts (1996) extend 'RCC theory' (Cohn and Hazarika 2001) to include representations of indeterminate boundaries. A parallel approach is Clementini and Di Felice's (1996) extension of the 9-intersection model (Egenhofer and Franzosa 1991) where relations between objects are modeled with broad boundaries.

2.5 Summary

This chapter described relevant research related to wayfinding in dynamic and uncertain environments. It was shown that wayfinding is a goal-directed activity where path selection is a major component. The environment encountered by the wayfinder is populated with changing features and wayfinders are often uncertain about the features in the environment and other aspects of the wayfinding process. Traditional wayfinder support assumes a well-defined static environment, and typically provides visualization, categorizations, and path suggestions. The development of GIS and related spatial technologies provides added tools to the wayfinder. Support is lacking in dynamic environments beyond advances in geovisualization. The theoretical foundations of time geography hold promise in providing a conceptual approach to account for changes through time. Support for uncertainty in GIS is also low, particularly with imprecision. Fuzzy and rough set approaches to account for imprecision were considered.

CHAPTER 3

DESCRIBING THE WAYFINDER'S DYNAMIC ENVIRONMENT

Modeling the travel possibilities associated with dynamic environments requires an integrated space-time framework, similar to that used in Time Geography (Miller 1991). This framework combines the spatial dimension(s) with an orthogonal temporal dimension to produce an integrated space-time reference system capable of modeling the changing spatial characteristics of wayfinding scenarios. Objects representing entities and processes exist within this framework, including a point object representing the wayfinder. Over time, the wayfinder point object traces a space-time path indicating the wayfinder's location while moving.

A set of four primitives describes specific wayfinding scenarios. The wayfinder's maximum travel speed restricts the wayfinder's rate of movement and, when combined with the remaining primitives, determines travel possibilities. A start point defines the location and time where the wayfinder begins and two additional primitives, barriers and requirements, complete the wayfinding scenario's description.

This chapter continues by first describing the integrated space-time framework consisting of a container of space-time populated with objects to include a point object representing the wayfinder (Section 3.1). This is followed by a description of the four wayfinding primitives (Section 3.2).

3.1 Wayfinding Framework

Modeling the changing travel possibilities associated with wayfinding in dynamic environments requires a framework that integrates both space and time. When there are spatial and temporal bounds to this environment a closed container of space-time results. Objects to include the

wayfinder exist within this container and represent the framework used to model wayfinders in dynamic settings. Each component of this framework is described in the follow three sections.

3.1.1 Space-Time Container

The most fundamental component of this modeling framework is that space and time are integrated in a manner where time is represented as a dimension orthogonal to space. In addition, space and time are bound, creating an $n+1$ dimensional space-time *container*, denoted by $ST_{(n+1)}$, where n represents the spatial dimension. When two spatial dimensions are represented (i.e., ST_3), the triplet (x,y,t) identifies a distinct space time point, whereas in a linear spatial representation (ST_2), the pair (x,t) identifies a space-time point. While this approach also generalizes to higher dimensions, within this thesis only scenarios in ST_2 and ST_3 are considered. In addition, to simplify a graphic presentation in a planar medium, examples are typically linear ST_2 scenarios, such as a road or rail segment.

The container of space-time creates a closed world of the wayfinder's dynamic environment with the following assumptions:

- The wayfinder experiences time continuously and unidirectionally (Boroditsky 2000).
- An *ego-moving* versus *time-moving* metaphor for temporal change is adopted (Lakoff and Johnson 1980).
- All objects related to the wayfinder's task must be contained in ST.

3.1.2 Objects within the Space-Time Container

A stationary spatial point feature is represented as a linear space-time object orthogonal to the spatial dimension(s). A permanent feature exists unchanged throughout the wayfinding scenario and is modeled with a linear object extending from the top to the bottom of the container (Figure 3-1a). A temporary feature's existence changes over the course of the wayfinding scenario by

either being destroyed or created (Hornsby and Egenhofer 1997), and the objects representing these two cases do not extend the entire temporal length of the container (Figure 3-1b and 3-1 c). A moving point feature creates a non-vertical, monotonic line with respect to the time axis and is referred to as a space-time path, p (Figure 3-1d). The slope of this line represents the travel speed of this object. This thesis does not consider acceleration or deceleration of moving objects, and as a result, space-time paths are a collection of straight segments, where each segment has a constant travel speed (Figure 3-1e). Linear and polygonal objects exist and move in space-time in an analogous manner.

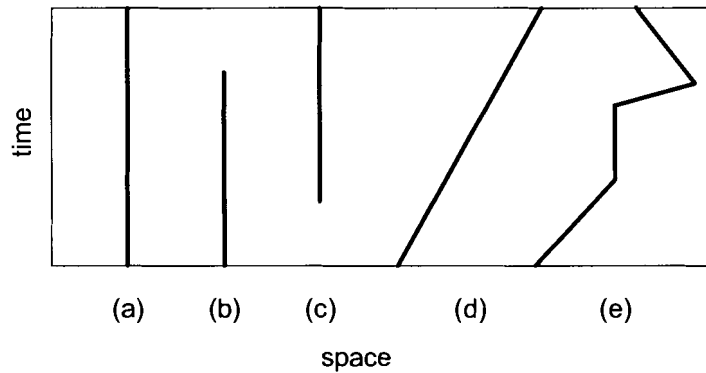


Figure 3-1: Spatial point objects within a linear space-time container, ST_2 : (a) a permanent stationary object, (b) a stationary object ceasing to exist, (c) a stationary object created after some time, (d) an object moving at a constant speed, and (e) a moving object changing speeds.

3.1.3 The Wayfinder

This model of wayfinding generalizes a wayfinder as a point in space, whose location over time results in a linear path in space-time. Whether a wayfinder fits within some space is not addressed. A maximum of one wayfinder object can exist, and this wayfinder object must exist throughout the wayfinding scenario. As a result, the location is the wayfinder's only temporally varying characteristic. Though wayfinders do not always strive to maximize utility (Golledge 1995), this model assumes the wayfinder strives to meet all requirements.

3.2 Wayfinding Scenario Primitives

Within the wayfinding framework's bound container of space-time, the following primitives structure the wayfinder's space by adding a set of constraints:

- the maximum speed limitation of the wayfinder (L);
- space-time information of the start point (O);
- space-time information about barriers that the wayfinder *must not* travel through ($\neg M$); and
- space-time information of requirements through which the wayfinder *must* pass (M).

The values of these primitives define a wayfinding scenario, and adjusting these values allows a wayfinder to model and compare various scenarios.

3.2.1 Maximum Travel Speed

The maximum speed limitation constrains the wayfinder's rate of travel through the environment. This speed limitation provides a method of projecting through space-time the impacts of the remaining three primitives and allows the wayfinder to reason about space-time decision points and their impact on future possibilities.

The variable P denotes accessible space, that is, the set of all paths through some space-time point given a maximum travel speed. The set P is defined both forward (P^+) and backward (P^-) in time. When considering the positive direction, P^+ includes all possible paths from a given space-time point into the future (Figure 3-2a). In the negative time direction, P^- includes all possible paths backwards in time from the given space-time point (Figure 3-2b). Thus, the portion of space-time accessible by all possible paths through some space-time point is the union of P^+ and P^- (Figure 3-2c).

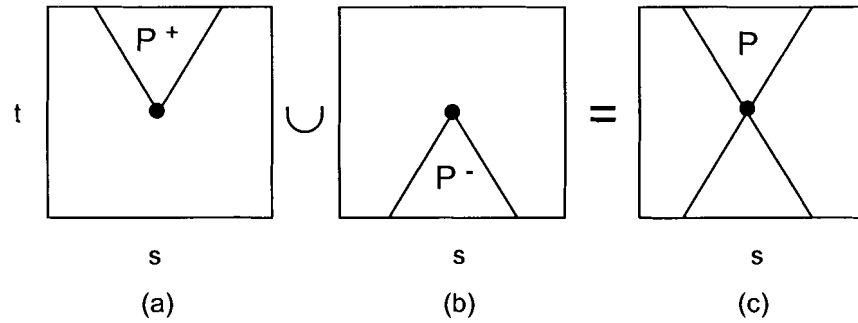


Figure 3-2: The set of all possible paths from some point based on a maximum travel speed: (a) forward in time, (b) backward in time, and (c) the union of the two representing the space-time occupied by the set of all paths through some point.

3.2.2 The Start Point

The second primitive, the wayfinder's start point (O), establishes the scenario's origin in space and time, thereby constraining future travel possibilities. This start point is a special space-time requirement, since the path of the wayfinder must begin at that location and time. The start point intersects the earliest (t is minimum) face or edge of the wayfinding scenario's space-time *container* (Figure 3-3). As a result, possible paths are only available in the positive time dimension and the set P^- is the empty set. The volume of space-time contained in P^+ represents the constrained space-time of a wayfinding scenario introduced by the start point.

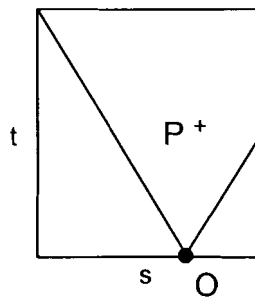


Figure 3-3: The set of all paths P^+ forward in time from the wayfinding scenario's start point, O.

3.2.3 Barriers

Barriers constrain the wayfinder by preventing travel through a portion of space-time. These spaces define where and when the wayfinder may not travel and are referred to as $\neg M$ -spaces. Barriers are classified into those that partition space and those that do not. For example, a river running entirely through the wayfinder's space-time container blocks travel from one side to the other.

Barriers vary in size and shape. The following list gives examples:

- A typical temporary barrier has a spatial extent that begins and ends within the wayfinder's space-time environment, for example, closing a section of road for two hours ($\neg M_1$ in Figure 3-4).
- Partitioning barriers may themselves have a minimal area, but by partitioning space, they may constrain movement over a large spatial extent. For example, closing a bridge creates a point barrier that constrains movement to one side of the river ($\neg M_2$ in Figure 3-4).
- Barriers may also change size over time. For example, a forest fire modeled in this way would appear, increase in size, and then decrease until it goes out ($\neg M_3$ in Figure 3-4).

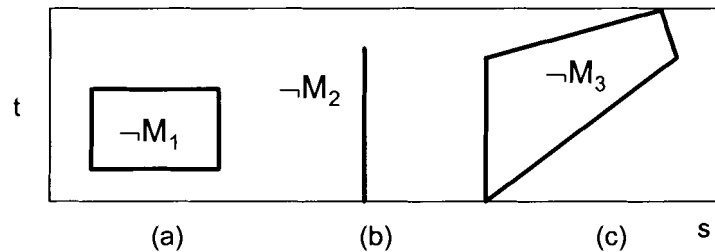


Figure 3-4: Various barrier shapes: (a) a barrier with a spatial extent beginning and ending within the wayfinder's space-time environment ($\neg M_1$), (b) a spatial point barrier that lasts until a specified time ($\neg M_2$), and (c) a barrier appearing, increasing in size, and then decreasing until disappearing ($\neg M_3$).

3.2.4 Wayfinding Requirements

A requirement represents the space-time task necessities of the wayfinder, or the where and when the wayfinder must be in the future. The—possibly empty—set of objects representing the wayfinding scenario’s requirements, in combination with the start point, is referred to as *M-space* (*M*). *M-space* and $\neg M\text{-space}$ do not themselves partition space-time in that there are points in the wayfinder’s space-time *container* that are neither requirements nor barriers.

Simple requirements, such as “meet me at noon at my house,” are represented with a space-time point object. To meet this requirement, the wayfinder’s space-time path must intersect this point. This thesis considers only requirements modeled as spatial point objects and leaves requirements over a spatial extent to future work.

Requirements often extend over a length of time. To model these cases, objects representing these requirements must also extend through time. As a result, a spatial point object becomes a line segment in space-time orthogonal to the spatial dimension(s). The characteristics of requirements at an instance of time (Section 3.2.4.1) and requirements over a temporal interval (Section 3.2.4.2) differ significantly.

3.2.4.1 Requirements at an Instance of Time

Point objects model requirements occurring at a specific location and time, such as “meet at Joe’s Coffee House at 11:00 am.” Successful wayfinder paths intersect this point. Since the wayfinder is also modeled as a point object, at the time of the requirement only two interactions are possible: (1) the wayfinder’s path intersects the requirement object, or (2) the wayfinder’s path does not intersect the requirement (Figure 3-5a).

Identifying the structure imposed on space-time by a requirement helps identify possible interactions. At the time of the requirement, space is divided into the point where the requirement

is located (M) and the remainder of space outside the requirement (M^-). In addition, one can identify the portion of space-time before and after the requirement (Figure 3-5b).

These four space-time components are further generalized into a wayfinder-requirement interaction graph (G) indicating the intersection possibilities between the wayfinder's space-time path and a point requirement (Figure 3-5c). The graph consists of four nodes: (1) a source node representing the wayfinders location before the requirement, (2) a sink node representing the wayfinder's location after the requirement, (3) a node representing the requirement itself, and (4) a node representing the remainder of space at the time of the requirement.

Two traversals of this graph exist representing the travel possibilities related to this requirement: (1) the wayfinder goes through the requirement (p_1 in Figure 3-5a) or (2) the wayfinder misses the requirement (p_2 in Figure 3-5a). The only path resulting in successful wayfinding is p_1 .

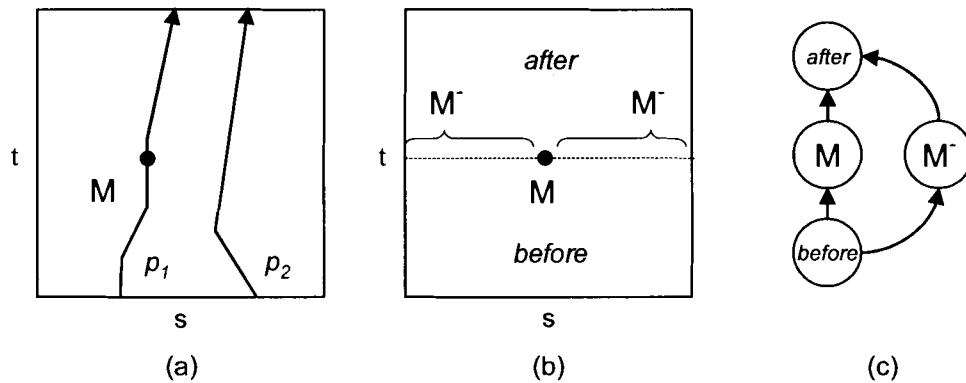


Figure 3-5 Interaction between a point requirement and a wayfinder's path: (a) two realizations of paths through this graph, (b) generalized space-time components of a point requirement, and (c) wayfinder-requirement interaction graph (G) indicating possible wayfinder path interactions with a point requirement.

3.2.4.2 Requirements over a Temporal Interval

A requirement at a point in space occurring over a temporal interval is modeled as a vertical line, orthogonal to the spatial dimension(s). An example of this type of requirement is, "you must be at

work from 9 am to 5 pm.” Unlike point requirements, interval requirements expose additional semantics: does the requirement apply to the entire duration, or would it be satisfied if occupied for only a portion of the entire interval. For example, there is a distinct difference between being at a requirement the entire time versus being there sometime. Additional temporal distinctions are possible, such as “be there at the start” or “you can miss the start.” In addition, often a wayfinder is compelled to be at the requirement for a portion of the requirement’s total temporal extent, for example, “you must spend an hour at work between 8 am and 12 pm.”

To model these distinctions, three temporal components of a requirement are defined: (1) when the requirement starts, referred to as the *start boundary* ($\partial_S M$), (2) when the requirement ends, referred to as the *end boundary* ($\partial_E M$), and (3) during the requirement, referred to as the *interior* (M°) (Figure 3-6a). The wayfinder-requirement interaction graph (G) now consist of six nodes, three of which are parts of the requirement (the start boundary, interior, and the end boundary). Ten directed edges connect the nodes to include a cycle between the interior and exterior resulting from the requirement’s temporal extent (Figure 3-6b).

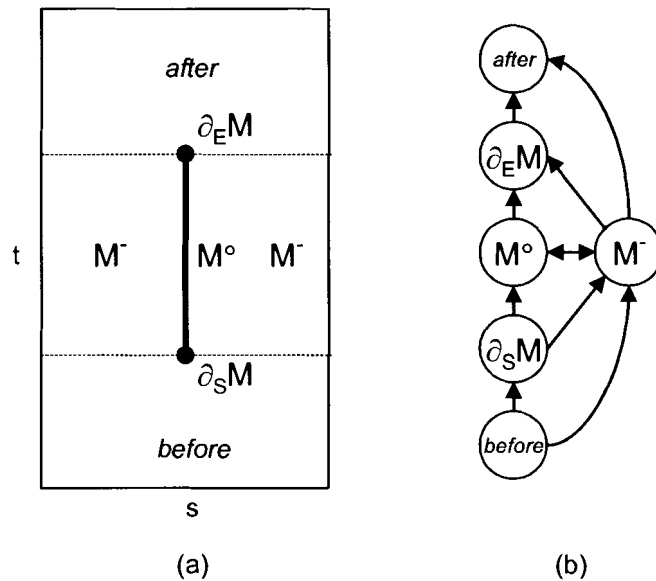


Figure 3-6: A spatial point object existing over a temporal extent: (a) the components of this requirement, and (b) the wayfinder-requirement interaction graph (G) indicating the possible intersection of a wayfinder’s space-time path with the components of this requirement.

There are fourteen traversals with distinct edges through this directed graph, thirteen of which intersect some portion of the requirement. Each case represents a different manner in which the wayfinder meets the requirement. The most restrictive case intersects the requirement the *entire time*. A lattice organizes the fourteen traversals from the most restrictive at the top and least restrictive at the bottom (Figure 3-7).

With this lattice, a wayfinder identifies which intersections fulfill a requirement. Only one case meets the requirement the *entire time* (Figure 3-8a). Four traversals fulfill a *beginning or end* requirement (Figure 3-8b and c). Nine categories meet *start* and *end* requirements (Figure 3-8d and f), whereas ten categories meet the *during* requirement (Figure 3-8e). The least restrictive case allows the wayfinder to meet the requirement *sometime* (Figure 3-8f).

The temporal length a wayfinder must occupy a requirement is not addressed in this lattice. The variable D is used to indicate this time. The value of D must be less than or equal to the total length of the requirement. For requirements that compel the wayfinder to occupy the requirement the entire time, the value of D equals the total temporal length of the requirement.

3.3 Summary

This chapter described the framework and primitives of a wayfinder's space-time environment. The framework consists of a container of integrated space-time. Within this container, geometric objects exist, the critical one being a point object representing the wayfinder. Four primitives define a wayfinding scenario: the wayfinder's maximum travel speed, a start point and time, barriers, and requirements. Wayfinders are successful when their space-time path intersects the scenario's requirements in the correct manner. As a result, wayfinder path-requirement interactions were described.

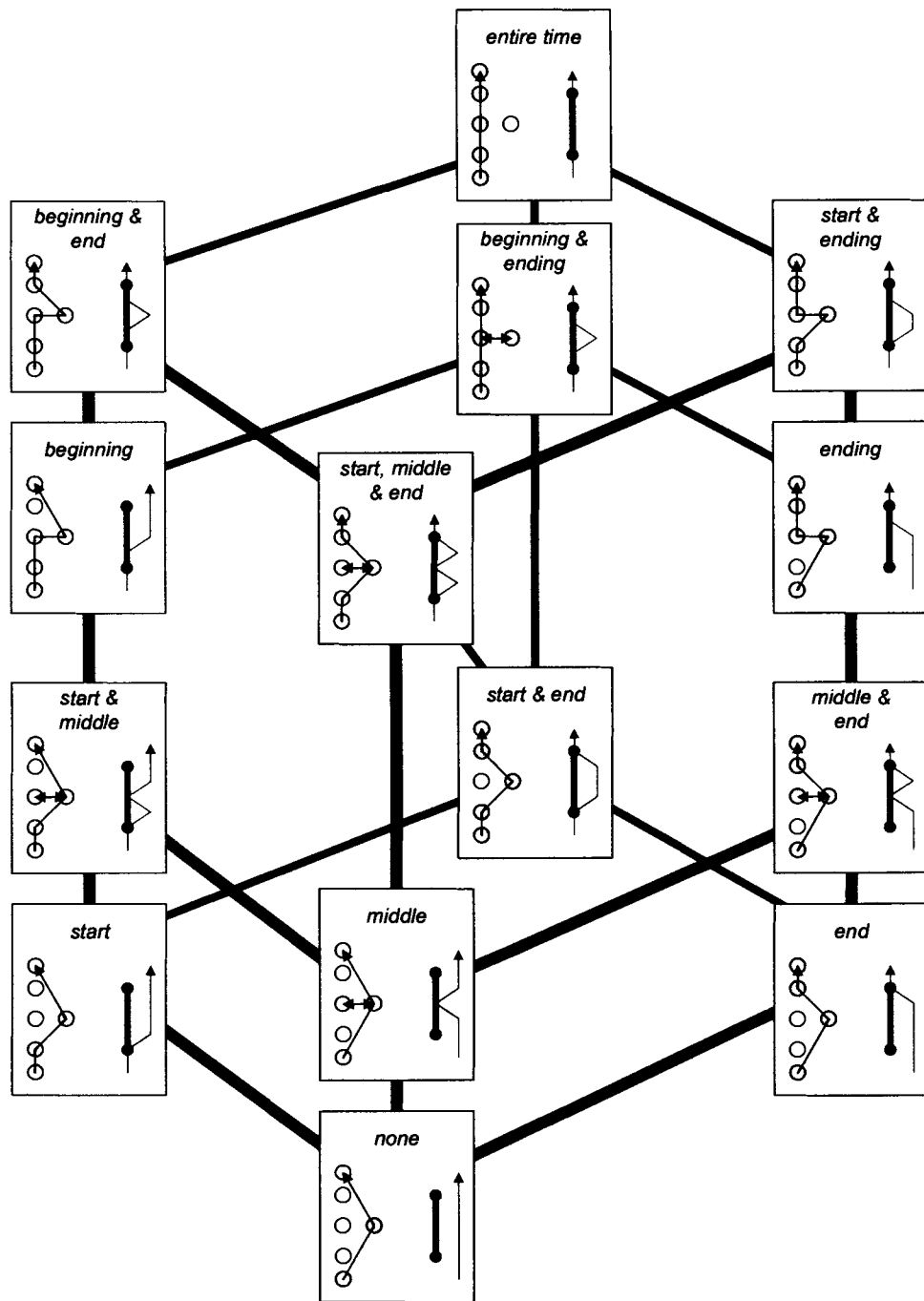


Figure 3-7: Fourteen possible unique edge traversals of the wayfinder-requirement interaction graph (G) organized to indicate the increasing restrictiveness as one goes up the lattice.

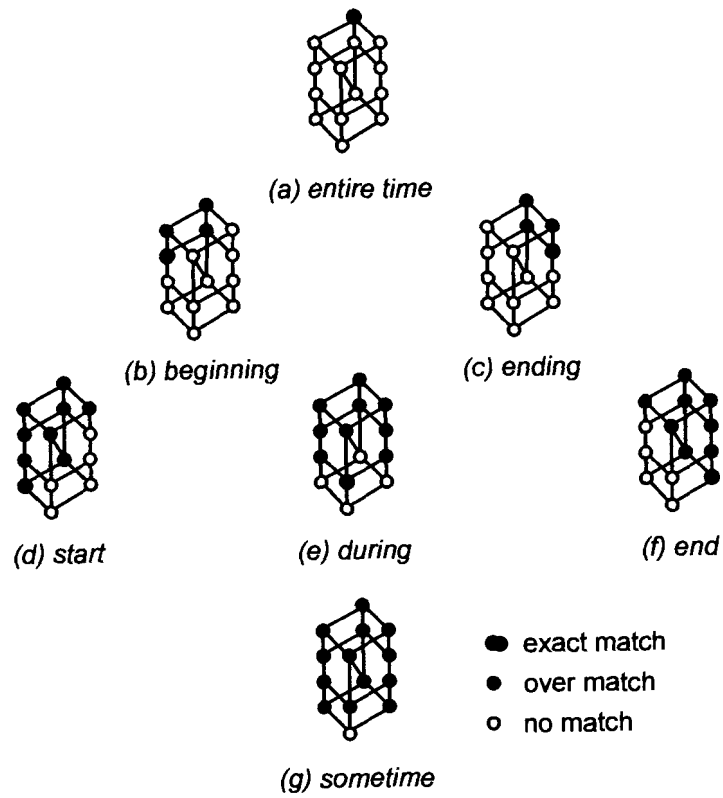


Figure 3-8: The intersection types that meet requirement types.

CHAPTER 4

STRUCTURING THE WAYFINDER'S DYNAMIC ENVIRONMENT

To provide a richer and more complete structure to the dynamic environment of the wayfinder, this thesis presents a systematic approach that builds on the set of wayfinding primitives and partitions space-time into categories of travel possibilities. Partitions are created by calculating accessibility based on the maximum travel speed from; the start point, barriers and requirements. This procedure is similar to that used in the symbolic projection's partitioning of a wayfinder's space (Jungert 1988; Holmes and Jungert 1992). The partitions created from these three combinations are assigned a symbol and its complement to indicate the accessible and inaccessible spaces created from each combination (Figure 4-1). This section continues by describing in more detail three such accessible and inaccessible spaces: *C-space* (Section 4.1), *Y-space* (Section 4.2), and *H-space* (Section 4.3).

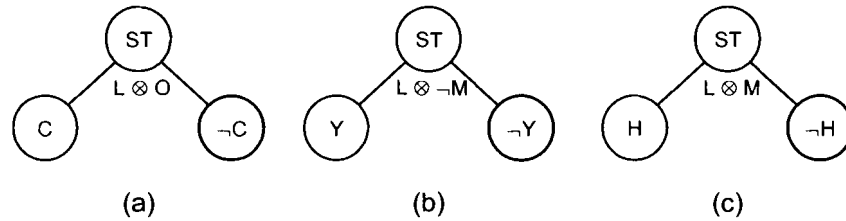


Figure 4-1: Three accessibility partitions of space-time (ST) by wayfinding primitive combinations: (a) speed limitation (L) and start point (O) create *C-space* and *¬C-space*, (b) speed limitation (L) and barriers ($\neg M$) create *Y-space* and *¬Y-space*, and (c) speed limitation (L) and requirements (M) create *H-space* and *¬H-space*.

4.1. The *C-space* of Wayfinding

Combining a wayfinder's maximum travel speed and start point partitions the space-time *container* into two spaces: *C-space* as the set of locations that can be accessed from the start point, and $\neg C$ -space as the inaccessible space. *C-space* is created without consideration of temporary barriers, yielding the most optimistic accessibility given a start point and maximum travel speed.

C-space and $\neg C$ -space are simply connected; the separation of $\neg C$ -space into two separate areas (Figure 4-2) is only a side effect of the planar graphical presentation and would occur only if the scenario were imbedded in a 1-dimensional space. Since the start point intersects space where time is a minimum, movement is only possible in the forward time dimension. As a result, accessibility is calculated only in the forward temporal dimension. The following observations are made about this structure:

- Any valid space-time path must be fully contained within *C-space*.
- If the wayfinder's maximum speed increases, *C-space* also grows.
- The reverse holds true as well, that is, decreasing the maximum travel speed will shrink the *C-space*.

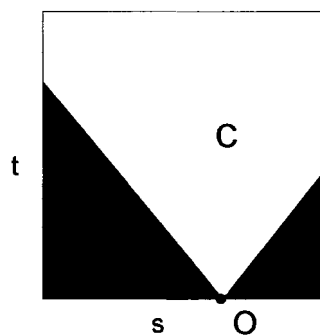


Figure 4-2: Partitioning of the space-time container into *C-space* and $\neg C$ -space as an implication of the wayfinder's start.

4.2 The Y -space of Wayfinding

A wayfinder's maximum travel speed and a set of barriers partition space-time into additional accessible and inaccessible spaces. The inaccessible spaces, referred to as $\neg Y$ -spaces, result from the absolute travel restriction of these barriers. The accessible spaces that remain are referred to as Y -spaces.

The maximum travel speed projections through space-time create a $\neg Y$ -space before and after each temporary barrier with a spatial extent (Figure 4-3a). The $\neg Y$ -space before each barrier is a danger area where, a wayfinder will certainly encounter the barrier. For example, if an explosion occurs when the wayfinder is in this region then injury will occur.

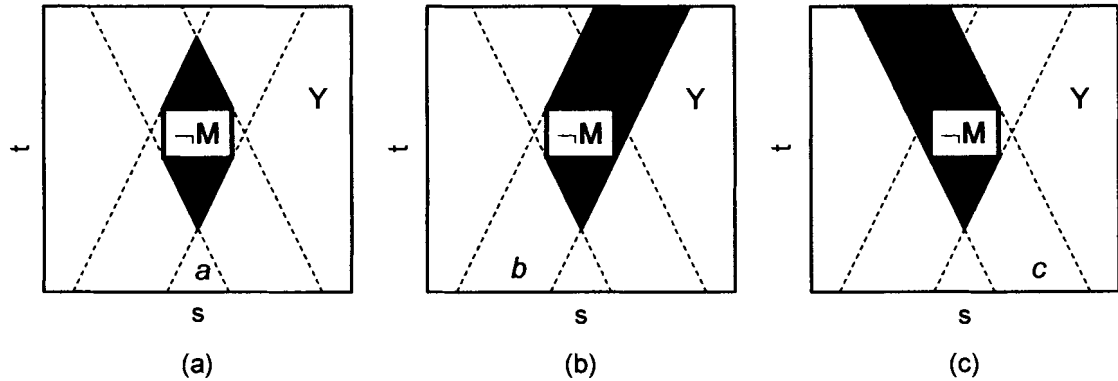


Figure 4-3: A barrier combined with the maximum travel speed limitation partitions space-time: (a) when the wayfinder is in *region a*, $\neg Y$ -space is created before and after the barrier, (b) when the wayfinder is in *region b*, additional $\neg Y$ -space is created after the barrier, and (c) when the wayfinder is in *region c* a different $\neg Y$ -space is created.

Barriers that partition space create additional $\neg Y$ -spaces that are dependent on the location of the wayfinder. For example, in Figure 4-3b a wayfinder in *region b* finds that the barrier creates a *space-time shadow* of $\neg Y$ -space. This *shadow* is different if the wayfinder is located in *region c* (Figure 4-3c). An example of this type of barrier is a closed bridge over a space-partitioning river. The inaccessible space resulting from the barrier, the blocked bridge, is dependent on what side

of the barrier the wayfinder is located. Unlike the $\neg C$ -space, the partitioning nature of the barrier creates $\neg Y$ -spaces that are not simply connected and in fact represent disconnected spaces.

4.3 The H -space of Wayfinding

The existence of space-time requirements compels the wayfinder to be at some location and time. As a result, space-time is partitioned into those locations and times that the wayfinder should travel through to still meet the requirement, and those inaccessible locations and times where they should not travel through or they will not meet the requirement. The inaccessibility in this case, as opposed to barriers, is not a matter of absolute physical inaccessibility, but relative inaccessibility as a result of the desire to meet the requirement. This thesis assumes that the wayfinder strives to meet all requirements.

The combination of maximum travel speed and the set of requirements create a third partition of the space-time into H -space and $\neg H$ -space. The H -space represents where and when the wayfinder should travel while still meeting the requirements. Therefore, H -space is the set of all points in space-time meeting this condition. The remainder of the space-time environment that is inaccessible as a result of meeting these requirements is $\neg H$ -space. This section continues by describing the accessibility resulting from different requirement shapes.

4.3.1 Point Requirement H -space

A point is the simplest requirement representation. To partition space-time based on a wayfinder's maximum speed and a point requirement, one determines the set of all possible paths forward and backward in time through the point requirement. The set of all possible paths forward in time from a point based on a maximum travel speed is P^+ , whereas the set of all possible paths backward in time from the point requirement is P^- (Figure 4-4). The union of these two sets, along with the point requirement, M , is the H -space of this requirement (Equation 4.1).

$$H = P^+ \cup P^- \cup M \quad (4.1)$$

The remainder of the space-time container is $\neg H$ -space.

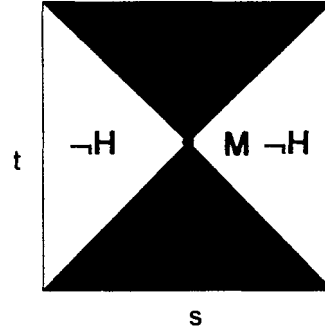


Figure 4-4: Partitioning space-time into H -space and $\neg H$ -space as a result of the wayfinder's maximum travel speed and space-time point requirement, M .

4.3.2 Temporal Extent Requirement H -space

In the previous case, the start and end of the requirement occurred simultaneously. As a result, the wayfinder's latest arrival time and earliest departure time are equal and coincide with the requirement. When requirements occur over a temporal interval, however, this is not necessarily the case. For example, when a requirement is for the *entire time*, the latest arrival time for the wayfinder is at the start of the requirement ($\partial_s M$), while the earliest departure time from the requirement is the end ($\partial_e M$). However, when a requirement is for *sometime*, the wayfinder must only intersect the requirement for an instance of time and, therefore, may arrive at the start and depart immediately (earliest departure time), or may wait until the end to arrive (latest arrival time).

The value of the latest arrival time (LA) and earliest departure time (ED) are determined by considering the duration (D) the wayfinder must occupy the requirement's interior (M°). The latest arrival time takes the value of the requirement's ending time minus its duration (Equation 4.2), while the earliest departure time is the requirement's start time plus the duration (Equation 4.3).

$$LA = \partial_E M - D \quad (4.2)$$

$$ED = \partial_S M + D \quad (4.3)$$

For a requirement occurring at an instance of time, the start and end of the requirement are equal, resulting in equal values of LA and ED. When a temporal interval requirement is for the entire time, the value of D is equal to $\partial_E M$ minus $\partial_S M$, which results in the earliest departure time to be the end of the requirement and the latest arrival time equaling the start. If, on the other hand, the wayfinder is only required to be at the requirement *sometime*, D equals zero, and the latest arrival time is the end and the earliest departure is the start. The value of D will also equal zero when a requirement is not met, but in this case, the values of LA and ED will be empty, since a requirement does not exist.

A wayfinder may also be compelled to be at the start or end of a requirement. In these cases, the earliest departure and latest arrival times are constrained. For example, if the requirement is for the *start* of a requirement, the latest arrival time and earliest departure time are also at the start. If, on the other hand, the requirement is for the *start* and *end*, the wayfinder's actions are constrained in a more complex manner. In this case, there exist two values for ED and LA. The wayfinder must arrive no later than the start of the requirement, can then depart, but must arrive again at the requirement no later than the end, at which time the wayfinder can then depart once again.

In the same way, requirements may combine start and end components with duration components. Consider the requirement to be at the start and any 20 minutes of a one-hour requirement starting at noon. The wayfinder must arrive no later than noon and has the option to depart immediately. The value of D is 20 minutes and yields values for ED of 12:20 pm and LA of 12:40 pm. The wayfinder must arrive at the requirement at noon, and has the option of (1) staying until 12:20, (2) traveling to other locations until 12:40, (3) staying until the end, or (4) some intermediate case between these options.

Given these values for the earliest departure time(s) and latest arrival time(s), the *H-space* of the requirement is constructed. The P^+ *space* projects from the earliest departure time and the P^- *space* projects from the latest arrival time. Again, the union of these two spaces along with the requirement itself is the requirement's overall *H-space*. The *H-spaces* resulting from the fourteen intersections of the wayfinder's space-time path with the requirement, where the value of *D* is 1/3 the total time of the requirement, is shown in Figure 4-5.

4.4 Combining Requirements

Often a wayfinding scenario consists of more than one requirement. To model these scenarios, individual requirements are combined with the operators; *and*, *or*, *after*, and *before*. When two requirements are combined with the *and* operator, both requirements must be met, but can be met in any order. Combining two requirements with the *or* operator, on the other hand, constrains the wayfinder to visit only one requirement. The exclusive or (xor) is not considered in this case, so both requirements can be visited if desired. The final two operators, *before* and *after*, constrain the wayfinder to visit the requirements in a particular order. This approach does not restrict the wayfinder from visiting the second requirement earlier in the wayfinding scenario if desired.

A wayfinder can visit two requirements, M_1 along with M_2 , in the following ways: (1) only visit the first requirement, (2) only visit the second requirement, (3) visit the first requirement then the second, or (4) visit the second requirement followed by the first. Each of the four requirement operators—*and*, *or*, *before*, and *after*—allow a different set of these possibilities to fulfill the combined requirement (Table 4-1). The *or* operator allows the wayfinder to visit the two requirements in any way and, therefore, is the least restrictive combination of the four choices. The two ordering operators, *before* and *after*, are the most restrictive combination, allowing only one possibility. In addition, it can be seen that the *and* operator is the combination or union of both ordering relations.

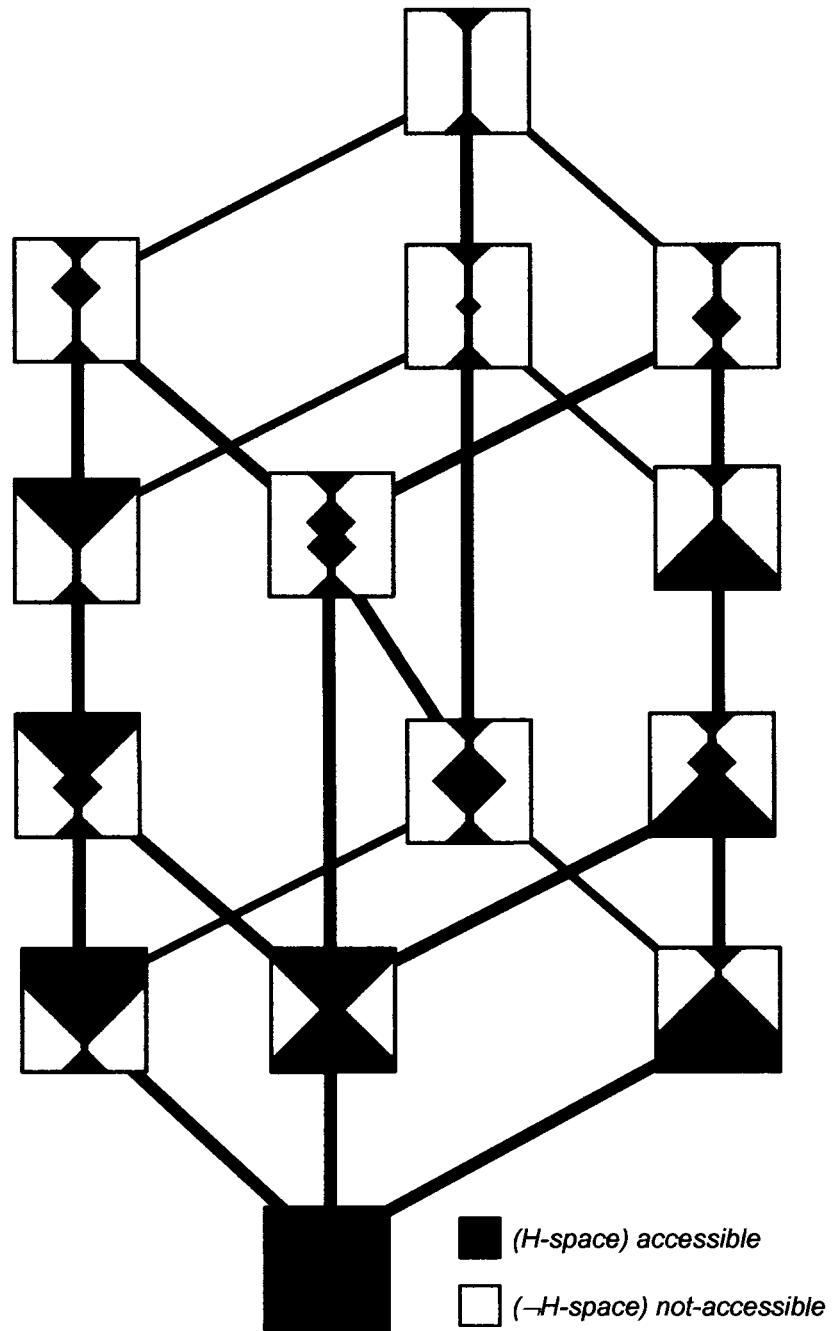


Figure 4-5: The *H-spaces* resulting from the fourteen realizations of the wayfinder's space-time path with the requirement (Figure 3-7) where the value of *D* is 1/3 the total time of the requirement.

Combination operator	M1	M2	M1 then M2	M2 then M1
$M_1 \text{ or } M_2$	yes	yes	yes	yes
$M_1 \text{ and } M_2$			yes	yes
$M_1 \text{ before } M_2$			yes	
$M_1 \text{ after } M_2$				yes

Table 4-1: Combination possibilities of requirements.

4.4.1 The *H-space* of the *Before/After* Operators

Two ordered requirements place travel restrictions on each other. For example, the ordered combination of, “go to the Post Office *after* work is over at 6 pm,” implies the wayfinder cannot arrive at the Post Office until 6 pm plus the travel time from work to the Post Office. Assuming that the Post Office closes at 7 pm, the wayfinder cannot depart work later than 7 pm, minus the travel time between the two requirements. In the first case, the initial requirement places a travel restriction on the subsequent requirement, and in the second case, the subsequent requirement places a travel restriction on the first.

Ordered combination restrictions are modeled by considering the intersection of one requirement with the other’s accessible space (P^+ and P^-). The first constraint results from the restriction that a wayfinder cannot arrive at a subsequent requirement until first visiting the initial requirement. An accessibility space (P^+_{M1}) forward from the earliest depart time (ED) models this constraint. The space-time contained in this accessibility space represents the travel restriction of the initial requirement.

Subsequent requirements intersect the initial requirement’s accessibility space in three ways. The first possibility is that the subsequent requirement is entirely contained in P^+_{M1} (Figure 4-6a) and the ordering operator places no special constraints on the subsequent requirement. Alternately, the subsequent requirement can be disjoint from the accessibility space (Figure 4-6c). This situation results in an invalid combination since it is impossible to visit the two requirements

in the specified order. A third case results when the subsequent requirement is partially contained in $P^+_{M_1}$ (Figure 4-6b). If the subsequent requirement is for the *entire time*, both its start and end points must be fully contained in P^+ . On the other hand, if the subsequent requirement is for *sometime*, then only some portion of the subsequent requirement must intersect P^+ .

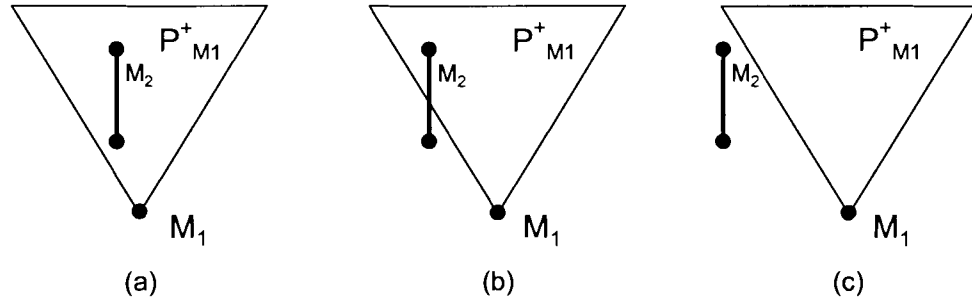


Figure 4-6: Possible intersection of an earlier requirement, M_1 and a second requirement M_2 .

If a valid ordered combination is possible, but the subsequent requirement is only partially contained in P^+ , the subsequent requirement is transformed to represent the restriction of having to first visit the initial requirement. This transformation is the intersection of the subsequent requirement with the initial requirement's accessibility space (Equation 4.4). Figure 4-7 shows three possible transformed requirements resulting from the travel constraints of M_1

$$M_2^* = M_2 \cap P^+_{M_1} \quad (4.4)$$

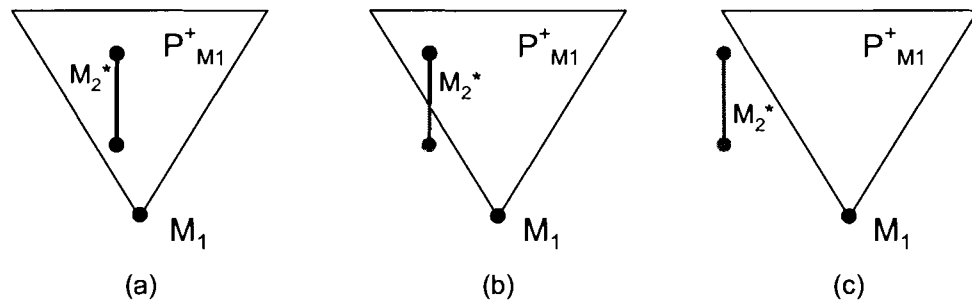


Figure 4-7: Transformation of a requirement by a previous requirement's projected travel constraint.

To fulfill the second constraint of an ordered combination, the wayfinder must depart the initial requirement in time to reach the subsequent requirement. The procedure is analogous to calculating the initial requirement's constraint on the subsequent one, except the accessibility space ($P_{M_2}^-$) is oriented backward in time from the latest time the wayfinder can arrive (LA) at the subsequent requirement. The intersection of M_1 with $P_{M_2}^-$ transforms the first requirement to represent the restriction of having to get to the subsequent requirement before it ends (Equation 4.5).

$$M_1^* = M_1 \cap P_{M_2}^- \quad (4.5)$$

Each transformed requirement generates an individual *H-space*. The individual *H-spaces* combine into an overall *H-space* through an intersection operation.

$$H_{(M_1 \text{ before } M_2)} = H_{M_1^*} \cap H_{M_2^*} \quad (4.6)$$

To demonstrate these concepts, consider the two requirements, “go to the Post Office” (M_1), along with “go to the Hardware Store” (M_2). Assume that both are open between 9 am and 5 pm. The combined requirement, “go to the Post Office *before* the Hardware Store”, (M_1 *before* M_2 , Figure 4-8a), requires a transformation of each requirement to represent the ordered constraints (Figure 4-8b and c). The resulting transformed requirements, M_1^* and M_2^* , create individual *H-spaces* (Figure 4-8d and e). The combination of these individual *H-spaces* yields the overall *H-space* resulting from the combined requirement (Figure 4-8f).

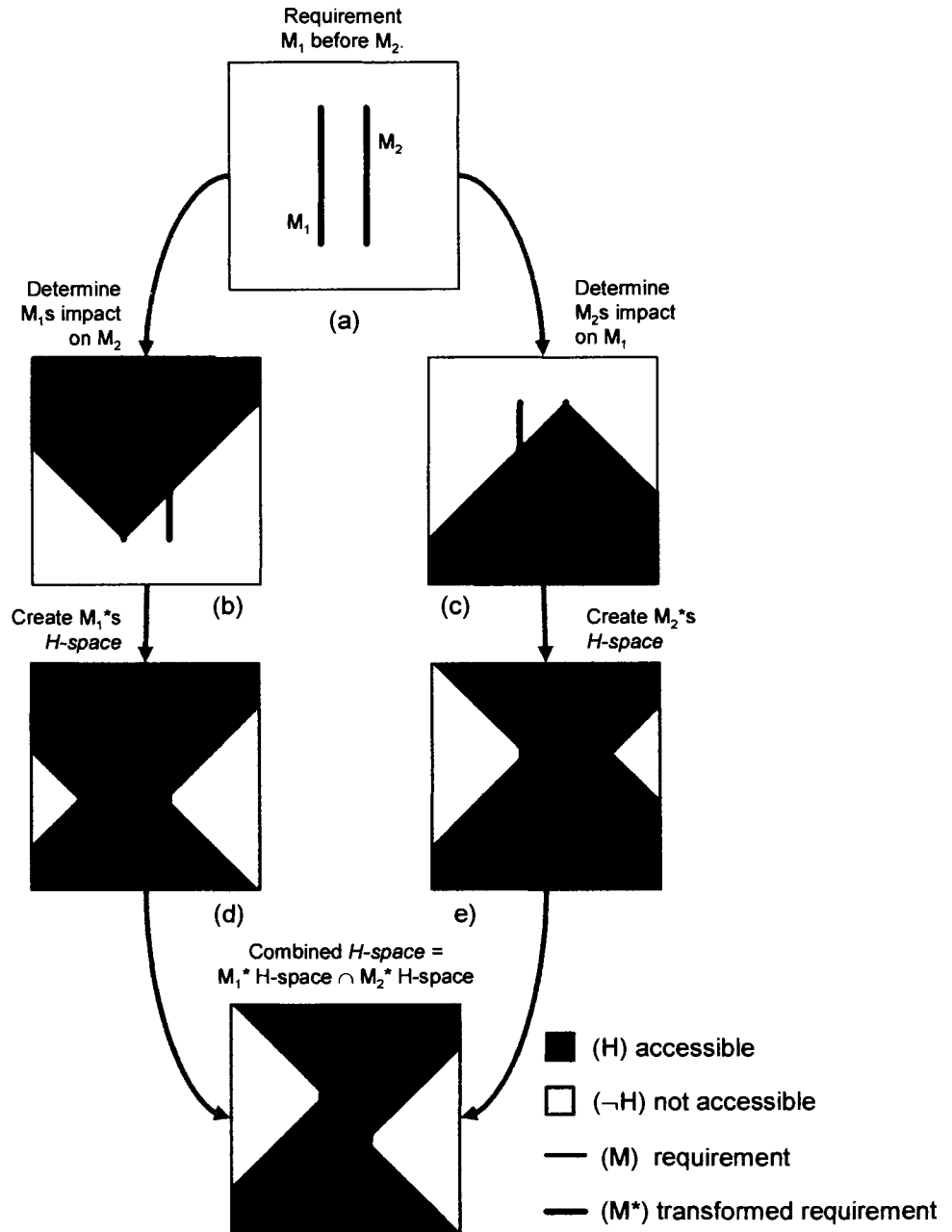


Figure 4-8: Creation of H -space for an ordered requirement combination: (a) “go to the Post Office (M_1) before going to the Hardware Store (M_2),” (b and c) both requirements constrain the other, (d and e) each requirement is transformed based on the other’s constraints, and (f) intersecting the two transformed requirements, M_1^* and M_2^* , results in the overall constrained H -space.

4.4.2 The *H-space* of the *and* Operator

The *and* operator allows the wayfinder to visit two requirements in any order; M_1 *before* M_2 or M_1 *after* M_2 . As a result, the *and* operator's *H-space* is the union of the individual *H-spaces* from both ordered combinations, *before* and *after* (Equation 4.7):

$$H_{(M_1 \text{ and } M_2)} = H_{(M_1 \text{ before } M_2)} \cup H_{(M_1 \text{ after } M_2)} \quad (4.7)$$

In addition, the transformed requirement space (M^*) for each requirement is the union of the transformed space in both ordered instances (Equations 4.8 and 4.9)

$$M_1^*_{(M_1 \text{ and } M_2)} = M_1^*_{(M_1 \text{ before } M_2)} \cup M_1^*_{(M_1 \text{ after } M_2)} \quad (4.8)$$

$$M_2^*_{(M_1 \text{ and } M_2)} = M_2^*_{(M_1 \text{ before } M_2)} \cup M_2^*_{(M_1 \text{ after } M_2)} \quad (4.9)$$

4.4.3 The *H-space* of the *or* Operator

Combining requirements with the *or* operator is the least restrictive combination since it allows any single requirement or combination to meet the combined requirement. As a result, the combined *H-space* is the union of the *H-spaces* of all four possibilities (Equation 4.10).

$$H_{(M_1 \text{ or } M_2)} = H_{M_1} \cup H_{M_2} \cup H_{M_1 \text{ after } M_2} \cup H_{M_1 \text{ before } M_2} \quad (4.10)$$

Since each *H-space* created from an ordered combination is derived from the intersection of the two requirement's individual *H-spaces*, they are subset of the union of each individual *H-space* (Equation 4.11)

$$H_{M_1 \text{ before } M_2} \cup H_{M_1 \text{ after } M_2} \subset H_{M_1} \cup H_{M_2} \quad (4.11)$$

The union of a set with a subset of that union is equal to the set, therefore, the ordered *H-spaces* can be removed and the *H-space* of two requirements combined with the *or* operator is the union of each individual requirement's *H-space* (Equation 4.12)

$$H_{(M_1 \text{ or } M_2)} = H_{M_1} \cup H_{M_2} \quad (4.12)$$

To illustrate the difference between the four requirement combination operators, consider a scenario with two requirements (M_1 and M_2) each for sometime (Figure 4-9a). The ordered combination, M_1 *before* M_2 , (Figure 4-9b), partitions space-time differently than M_1 *after* M_2 (Figure 4-9c). Combining these two requirements with the *and* operator (M_1 *and* M_2) allows the wayfinder to visit the requirements in any order (M_1 *before* M_2 or M_1 *after* M_2) and creates an overall *H-space* with more possible travel space, since it is union of the two ordered operators (Figure 4-9d). The final operator, *or*, allows the greatest travel flexibility Figure 4-9e).

4.5 Sequential Partitioning the Wayfinder's Dynamic Environment

To model the overall travel constraints of a wayfinding scenario the wayfinding primitives can sequentially partition each other in a hierarchical manner. The first partition of space-time is created by combining the maximum speed limitation and the start point, yielding *C-space* and $\neg C$ -space. The *C-space* is then partitioned by a set of barriers ($\neg M$) into *Y-space* and $\neg Y$ -space. The final step partitions *Y-space* with the requirements (M) into *H-space* and $\neg H$ -space. This sequential partitioning of space-time leads to the hierarchy shown in Figure 4-10. The leaf nodes of this graph indicate the four basic travel possibility partitions of a wayfinding scenario: $\neg C$ -space, $\neg Y$ -space, $\neg H$ -space, and *H-space*.

In addition to the order of sequential partitioning used in this approach (start point—barriers—requirements), there exists five alternate orderings. Each separate ordering can categorize inaccessible space ($\neg C$, $\neg Y$, and $\neg H$) differently, however, the size and shape of the accessible space (*H-space* in this approach) is independent of the chosen order.

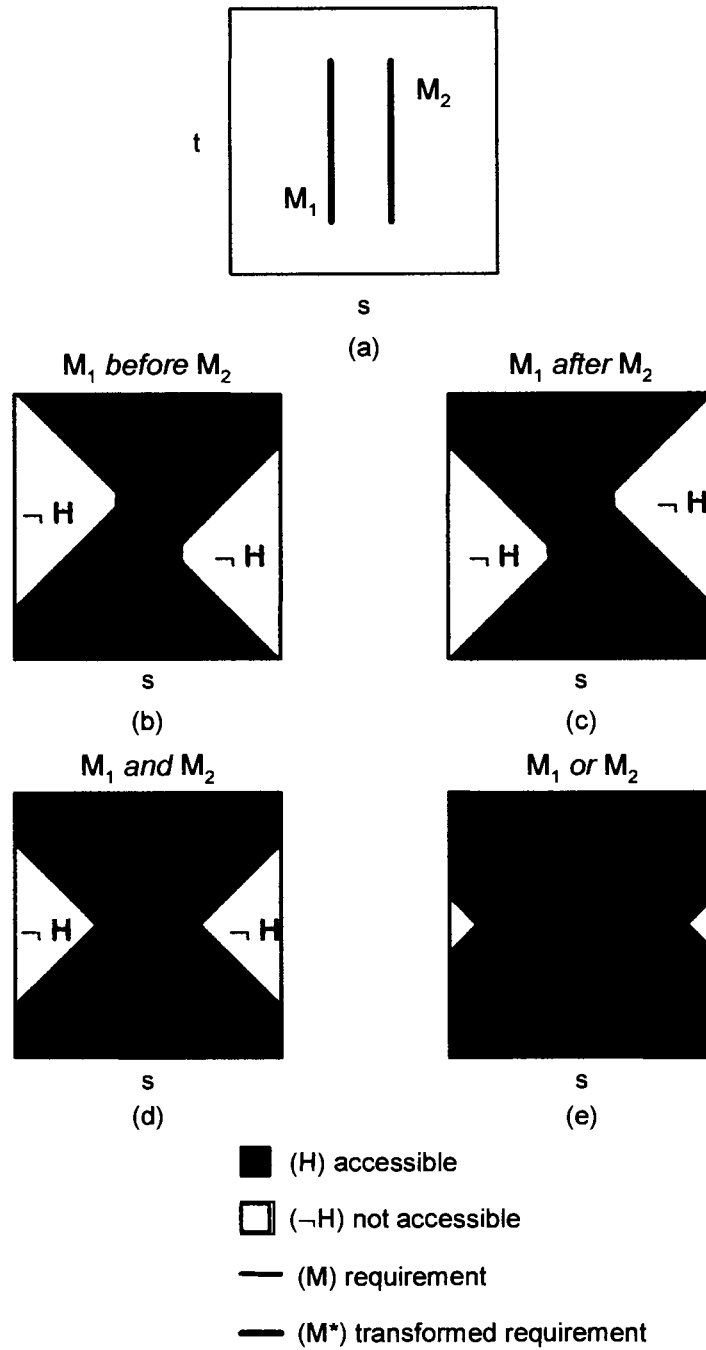


Figure 4-9: Accessible space of (a) the combination of two sometime requirements results in, (b) M_1 before M_2 , (c) M_1 after M_2 , (d) M_1 and M_2 , and (e) M_1 or M_2 .

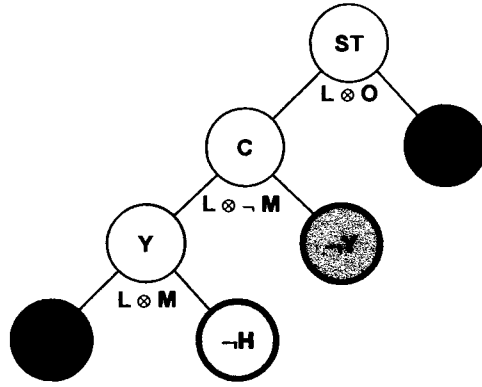


Figure 4-10: Sequential partitioning of space-time into the four basic travel possibility partitions.

A wayfinder who sequentially partitions space-time in this manner can quickly identify, either by query or visualization, those locations in space and time where: (a) travel is impossible due to travel speed capability ($\neg C$), (b) travel is impossible as a result of temporary barriers ($\neg Y$), (c) travel is possible, but requirements will not be met ($\neg H$), and (d) those portions of space-time where the wayfinder can travel and meet the scenario's requirements (H).

To illustrate the sequential partitioning technique, consider a wayfinding environment consisting of a maximum travel speed, start point O , a requirement M , and two barriers, $\neg M_1$ and $\neg M_2$ (Figure 4-11a). The maximum travel speed, combined with the wayfinder's start point, partition space-time into C -space and $\neg C$ -space (Figure 4-11b). The C -space is then partitioned by combining the barriers with the maximum travel speed to yield Y -space and $\neg Y$ -space (Figure 4-11c). In addition, spaces influencing potential $\neg Y$ -spaces are delineated by dashed lines. Finally, Y -space is partitioned by combining the wayfinder's requirement with the travel speed limitation creating H -space and $\neg H$ -space (Figure 4-11d).

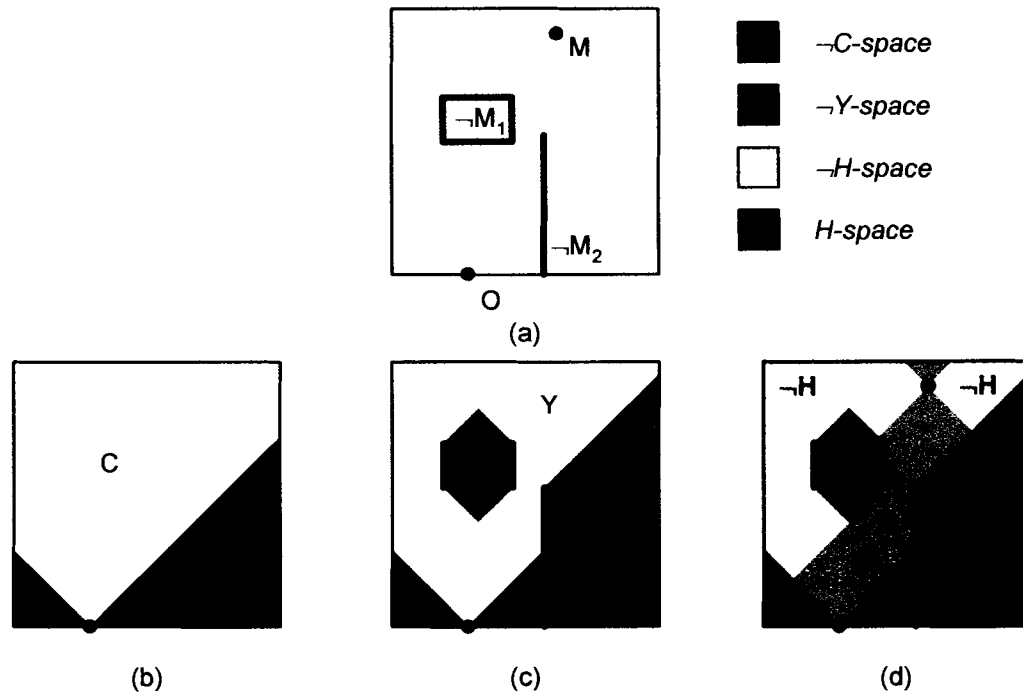


Figure 4-11: The structuring of a wayfinder's space-time environment by sequential partitioning: (a) combining the speed limitation and start point to partition the space time container into *C-space* and *¬C-space*, (b) partitioning *C-space* by combining the barriers and maximum travel speed into *Y space* and *¬Y-space*, and (c) partitioning *Y-space* by the requirement into *H-space* and *¬H-space*.

4.6 Describing with Modal Verbs

Until this point, space-time partitions have been presented in primarily a graphical manner. A complementary method of employing the modal verbs *can*, *may*, *must*, and *should* provides a plausible verbal description of the partition primitives of a wayfinding scenario. Modal verbs indicate whether things, events, or relations are actual, possible, or necessary (Johnson 1987). Sweetser (1990) argues that the meaning of modal verbs as used in the physical or social realm are similarly used for argument and reasoning. In an inherently physical act such as wayfinding, describing spatial and temporal constraints modally provides insights for argument and reasoning of the wayfinding task.

Though each modal verb is used in various ways during normal conversations, as employed in this approach *can* is related to a positive ability (capability); *must* denotes obligation or compelling force; and *may* is roughly associated with permission or lack of a potential barrier (Sweetser 1990). Johnson (1987) relates each of these modal verbs to various image schemata, which structure knowledge through abstract high-level experiential gestalts of common situations. For example, *must* is a requirement; *may* is the removal of restraint; and *can* is enablement. In addition, we include one other modal verb to describe a wayfinder's space-time structure, *should*, indicating a weaker form of *must*. Defining a wayfinder's requirements (*must*), capabilities (*can*), and permissions (*may*) while traveling through a volume of space-time provides a concise, yet simple description of a wayfinding scenario.

Modal verbs are associated with the various wayfinding primitives and the partitions of space-time. We begin by assigning the modal verb *must* (M) to requirements and *must not* (\neg M) to barriers. The modal verb *can* (C) effectively describes the accessible partition of space-time created by the combination of the start point and maximum travel speed, or the space that the wayfinder's capability allows travel to. Its complement, *cannot* (\neg C), describes the inaccessible spaces of this partition. The modal verb *may* (Y) effectively describes the partition of *Y-space* that the wayfinder has access to based on temporary barriers in space-time. Again, its complement, *may not* (\neg Y), is employed to describe the spaces made inaccessible as a result of barriers in space-time. Finally, the modal verbs *should* and *should not* are employed to describe *H-space* and \neg *H-space*, respectively. A summary of these modal verb assignments, along with example usages, is shown in Table 4.2.

Space	Modal Verb	Example Usage
M	Must	"You <i>must</i> be at work by 9:00."
¬M	Must not	"You <i>must not</i> cross the river."
C	Can	"I <i>can</i> be at Joe's Diner at 7:00."
¬C	Cannot	"I <i>cannot</i> get to Joe's Diner at 6:30."
Y	May	"You <i>may</i> cross the river when the drawbridge is down."
¬Y	May not	"You <i>may not</i> get to Joe's Diner at 6:45 because the drawbridge is open."
H	Should	"To get to the Joe's Diner by 7:00, I <i>should</i> cross the drawbridge before it opens."
¬H	Should not	"To get to the post office by 7:00, I <i>should not</i> go first to the grocery store."

Table 4-2: Assignment of modal verbs to the primitives and partition spaces of the wayfinder's space-time environment.

4.7 Summary

This chapter introduced an approach to structure the travel possibilities of a wayfinder's dynamic environment with the wayfinding primitives: the maximum travel speed, the starting point, barriers, and requirements. This structure was created with distinctive partitions of space-time resulting from combining the speed limitation with each of the remaining three primitives. Sequentially partitioning space-time with these primitives generates four travel possibility categories. Wayfinder's can view and query these partition spaces to understand better travel options and select successful paths. Describing partitions with modal verbs provide the wayfinders the added capability to query and describe travel possibilities with natural language expressions.

CHAPTER 5

UNCERTAINTY IN BARRIERS AND REQUIREMENTS

Wayfinding scenarios often include uncertainty in the existential, spatial, and temporal characteristics of barriers and requirements. A wayfinder, for example, may not know whether a drawbridge's scheduled opening will occur, or may be unsure at what time to meet a friend for lunch. When there is uncertainty in a scenario, travel possibility partitions are also uncertain.

The term uncertainty is not well defined throughout the GIScience literature (Worboys 1998). In this thesis, uncertainty results when the wayfinder cannot determine the spatio-temporal characteristics of wayfinding primitives. It does not consider the related concept of inaccuracy in the sense that the wayfinder is wrong about some element of the wayfinding scenario nor does it address inherent vagueness. For example, the scenario where a requirement is at 11 am, but the wayfinder believes it to be at 10 am, or a vague spatial expression such as, "go to the mountains" are not considered.

When a wayfinder cannot determine the exact spatio-temporal characteristics of a scenario's wayfinding primitives, the travel possibilities resulting from these primitives are also indeterminate. For example, a wayfinder that cannot determine a requirement's location from among a set of two possibilities can also not determine the future travel possibilities through space and time. In this case the wayfinder selects a path that retains accessibility to either location for as long as possible in the hope that additional information becomes available that allows the the requirement's location to be determined.

This thesis assumes that all possible events have an equal likelihood of occurring, resulting in a uniform probability distribution. Given n independent possible events, the probability of any individual event is equal to $1/n$ in the discrete case, and similarly in the continuous case. This approach also assumes that the correct values are always within the set of possibilities. A scenario

would not occur in which the wayfinder is uncertain whether a requirement is at 10 am or 11 am, when in fact it is at 8 am.

The possibility of more than one realization in the value of a wayfinding primitive creates a more complex partitioning of travel possibilities than in those scenarios where the wayfinder is certain about these values. To account for uncertainty, this thesis extends the binary partitioning of space-time approach into a three-valued partitioning consisting of *accessible*, *inaccessible*, and *possibly inaccessible*, a broad boundary much like the one used for spatial relations (Clementini and Di Felice 1996).

To explore these wayfinding uncertainties and their influence on travel possibilities, this chapter continues by distinguishing four categories of uncertainty encountered in dynamic wayfinding scenarios (Section 5.1). A description of the impact of these uncertainties on travel possibility partitions follows (Section 5.2). The chapter continues by considering the sequential combination of uncertain possibility spaces (Section 5.3).

5.1 Categories of Uncertainty in Wayfinding Scenarios

Uncertainty in the characteristics of barriers and requirements, or their impact on travel, is treated with a three-valued logic consisting of: *existing* (X), *not existing* ($\neg X$), and *possibly existing* ($\Diamond X$). A barrier or requirement must exist in all possible outcomes to be classified as *existing*. Likewise, to classify barriers and requirements as *not existing* they must not exist in every possible outcome. When barriers and requirements exist in some cases, but not others, they are classified as *possibly existing*.

5.1.1 Existence Uncertainty

Of the four wayfinding primitives, the maximum speed and the start point must exist in all cases. Uncertainty in the existence of barriers and requirements, though, is common. Consider the scenario, “if the North drawbridge opens, it will open at 8 am,” or “you may or may not have to

go to work Saturday.” In these cases, the existence of the barrier ($\neg M$), or requirement (M), is uncertain and the object representing the primitive only possibly exists, represented by $\Diamond\neg M$ and $\Diamond M$, respectively. Therefore, there are two outcomes with this uncertainty. In one case, the barrier or requirement exists, whereas in the other case it does not. Because of the uniform probability limitation used in this thesis, there is a 50% likelihood of either possible outcome.

5.1.2 Uncertainty in the Spatio-Temporal Components of a Point Object

Even if a wayfinder is certain that a barrier or requirement exists, she may be uncertain about its position in space and time. This uncertainty may be between specific discrete possible outcomes, such as “meet me at 9 am, 10 am, or 12 pm,” or between a continuous range of possible outcomes, as in the case “meet me sometime between 9 am and 12 pm.” In the first example, three discrete times can occur, each with a $1/3$ probability of occurring (Figure 5-1a). In the second example, the requirement may be anytime within the given continuous range (Figure 5-1b). Assuming a resolution of one hour, four possibilities exist—8 am, 9 am, 10 am, and 11 am—each with a 25% likelihood of occurring.

In addition to temporal uncertainty, the spatial location may be uncertain. Again, this uncertainty may be discrete, as in “meet me either at Joe’s Coffee House, Bobs Café, or the Post Office” (Figure 5-1c), or continuous as in, “meet me somewhere in town” (Figure 5-1d).

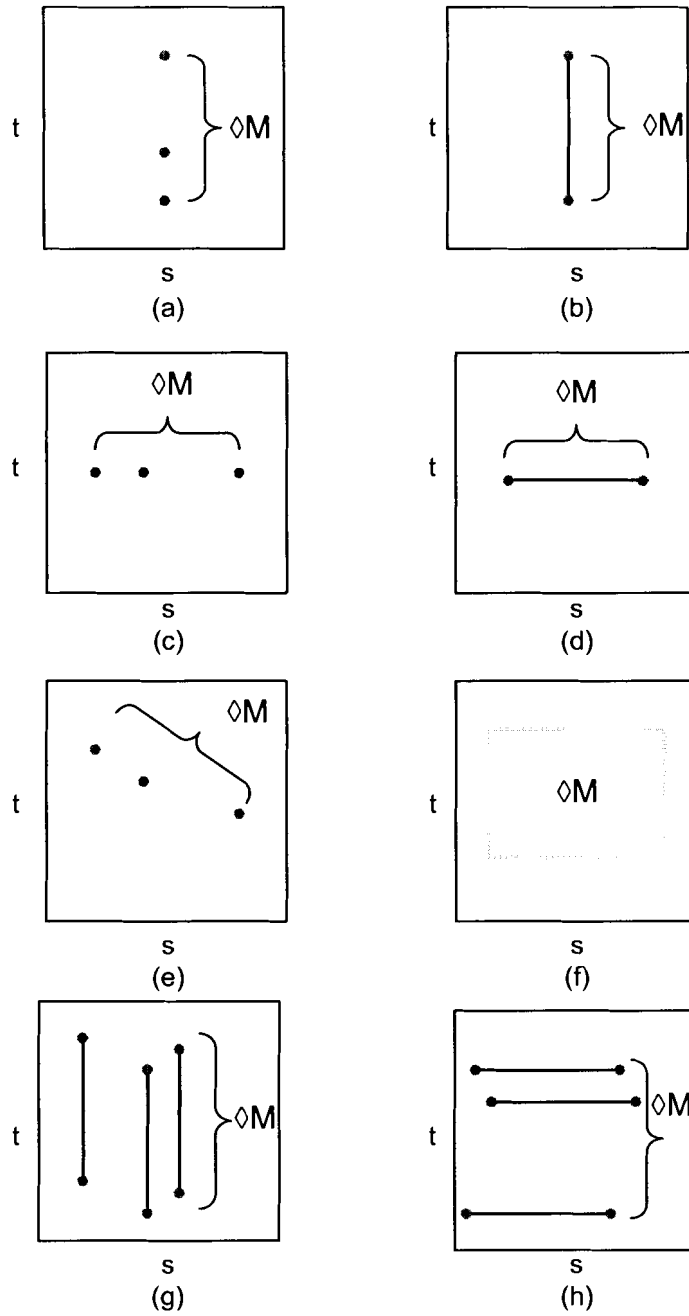


Figure 5-1: Possible space-time point uncertainty: (a) discrete temporal, (b) continuous temporal, (c) discrete spatial, (d) continuous spatial, (e) discrete temporal and spatial, (f) continuous temporal and spatial, (g) discrete spatial, continuous temporal, and (h) continuous spatial, discrete temporal.

In many cases, there is uncertainty in both the temporal and spatial components of a point object. Considering both discrete and continuous uncertainties, four outcomes exist. First, there may be discrete uncertainty in both the temporal and spatial components of an object. This results in a set of discrete space-time points representing the possible outcomes for the object (Figure 5-1e). In the second case, uncertainty is continuous in both space and time, creating a polygon of uncertainty (a volume in ST_{2D} scenarios) representing where in space and time the point object may be located (Figure 5-1f). The last two cases arise when one component has discrete uncertainty whereas the other has continuous uncertainty (Figure 5-1g and h). These eight types are not exhaustive, since there are numerous combinations of these uncertainties, such as “meet me either at Joe’s Coffee House between 8 am and 9 am, or somewhere in town at 10 am.”

5.1.3 Uncertainty in Objects with a Temporal Extent

Requirements often arise over a temporal interval. An object representing a requirement with a temporal interval has three components: a start, a duration, and an end. Any combination of two defines the temporal interval. For example, the scenario, “the drawbridge will be open from 8 am until 9 am” describes the same time interval as, “the drawbridge will be open at 8 am for 1 hour.” To define a temporal interval with certainty, the wayfinder must know the value of any two components. If the wayfinder only knows one component with certainty, the object’s time of existence is uncertain.

Consider the situation where the wayfinder knows the start time with certainty, but not the duration and end. If there is complete uncertainty in these two values, the end of the object is possibly anytime after the start (Figure 5-2a). In many cases, however, the wayfinder has information to limit uncertainty. Again, this uncertainty can be either discrete or continuous. For example, “starting at 8 am, the draw bridge will be open for 1 or 2 hours” (Figure 5-2b), as opposed to, “starting at 8 am, the drawbridge will be open between 1 and 2 hours” (Figure 5-2c).

If instead, the wayfinder knows the end with certainty, similar possible intervals result, but the alignment is in the opposite temporal direction (Figure 5-2d-f).

A different space-time object results when the wayfinder only knows the duration with certainty. As with the previous cases, there may be complete uncertainty in the start and end, for instance “sometime in the future the drawbridge will open for an hour” (Figure 5-2g). Again, there may also be discrete uncertainty information with either the start or end; for example, “the drawbridge will open for an hour either at 8 am or 9 am” (Figure 5-2h). The continuous case also occurs, “the drawbridge will open for an hour sometime between 8 am and 10 am” (Figure 5-2i).

5.1.4 Uncertainty in Earliest Departure and Latest Arrival Times

Requirements with temporal uncertainty have uncertain earliest departure (ED) and latest arrival (LA) times. For example, when there is uncertainty in whether a point requirement is at 8 am or 10 am, the wayfinder does not know how soon to depart or how late to arrive (Figure 5-3a). In one case, the earliest departure time is 8 am, whereas in the other it is 10 am. The accessible space-time forward in time is different from these two points. The first case, an 8 am departure, is optimistic and identifies points in space-time that possibly will not be accessible if the time of the requirement turns out to be 10 am. The accessibility forward in time from 10 am, on the other hand, identifies points that are accessible in either case (Figure 5-3b). As a result, the requirement has a certain earliest departure time (ED) of 10 am and a possible earliest departure time (\Diamond ED) of 8 am.

In a likewise manner, it is possible in one case to arrive as late as 10 am, but in the other case to arrive at 8 am. The accessible spaces backwards in time indicate that only the case of 8 am properly describes accessibility in both cases (Figure 5-3c). As a result, 8 am is the certain latest arrival time (LA) and 10 am is the possible latest arrival time (\Diamond LA).

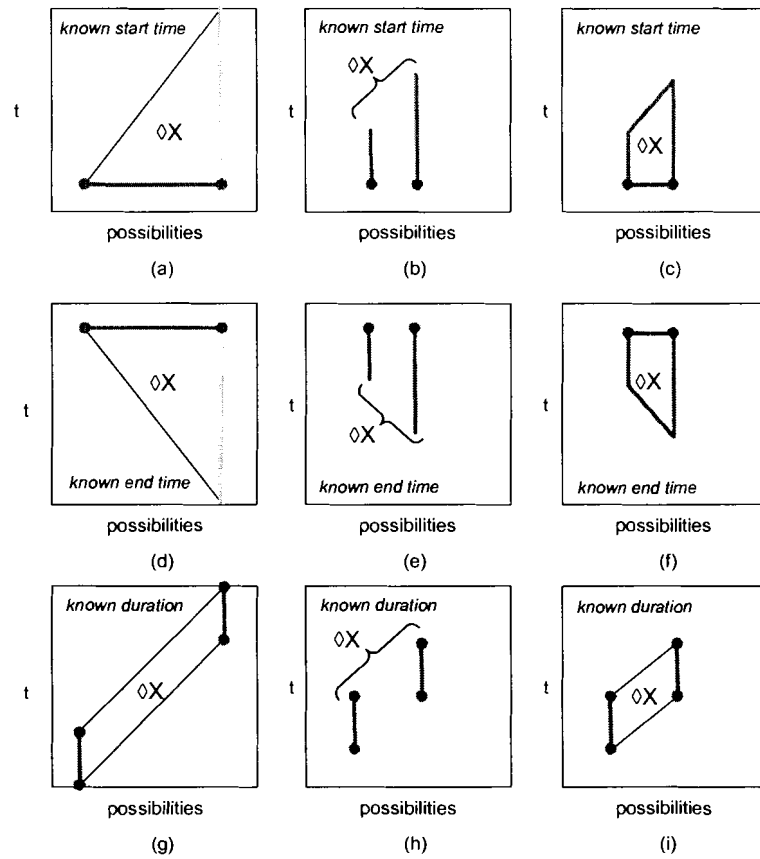


Figure 5-2: Uncertainty in objects with a temporal interval: (a) known start continuous uncertain end, (b) known start discrete end uncertainty, (c) known start continuous end uncertainty, (d-f) known end, (g) known duration uncertain start and end, (h) known duration discrete uncertain start and end, and (i) known duration continuous range uncertainty in start and end.

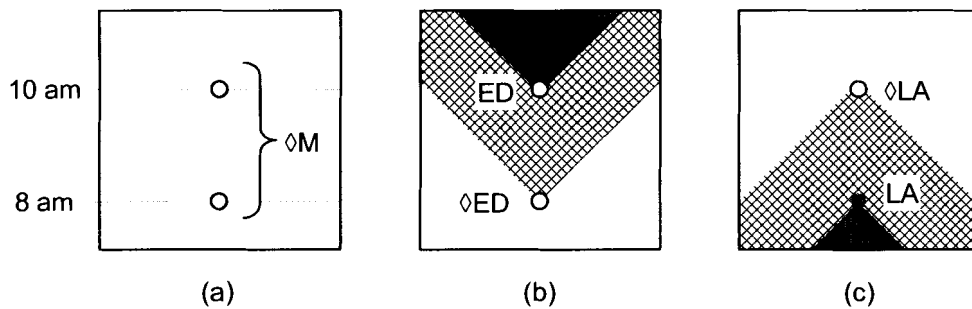


Figure 5-3: Uncertainty in earliest departure time (ED) and latest arrival time (LA) resulting from (a) a discrete temporal uncertainty, creating two possible accessibilities (b) forward and (c) backward in time.

5.2 The Impact of Uncertainty on Travel Possibilities

Uncertain primitives create uncertain travel possibility partitions. If a space-time point is accessible in all possible cases, it is classified as *accessible*. If a space-time point is not accessible in all possible cases, it is classified as *not accessible*. Points in space-time accessible in some cases, but not others, are classified as *possibly not accessible*. This creates a broad boundary condition between *accessible* and *inaccessible* accounting for the uncertainty in wayfinding primitives. A combination matrix (Table 5-1) shows the rule to create the overall travel possibility space from the accessibility of individual possible outcomes. The next three sections explore the broad boundary partitions created from different primitive uncertainties.

<i>Accessibility (A)</i>		<i>Possible outcome 1</i>	
		A	$\neg A$
<i>Possible outcome 2</i>	A	A	$\Diamond \neg A$
	$\neg A$	$\Diamond \neg A$	$\neg A$

Table 5-1: Combination matrix for the overall accessibility space resulting from two possible outcomes.

5.2.1 The Impact of Existence Uncertainty

To explore the impact of existence uncertainty, consider an example where a barrier possibly exists ($\Diamond \neg M$). Two possible cases arise, each with equal probability: (1) the barrier exists (Figure 5-4a), and (2) the barrier does not exist (Figure 5-4b). In the first case, the barrier creates inaccessible space, *may not* space ($\neg Y$) (Figure 5-4c), while in the second it does not (Figure 5-4d). The partition a wayfinder *may* travel through in all possible cases is identified as *Y-space*, the partition the wayfinder *may not* travel through in all possible cases is identified as $\neg Y$ -space. The remainder of space-time where travel is uncertain is *possible* $\neg Y$ -space ($\Diamond \neg Y$). The possible existence of this barrier creates a small diamond of $\Diamond \neg Y$ -space and the remainder of space-time is *Y-space* (Figure 5-4e). Uncertainty in the existence of requirements is handled similarly.

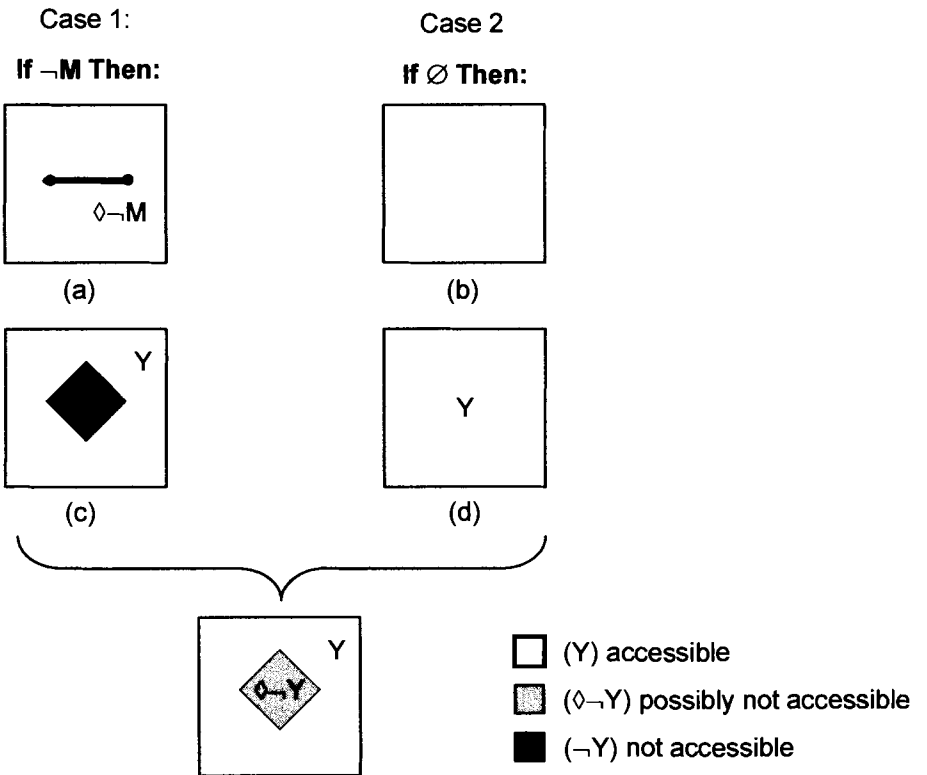


Figure 5-4: The impact of barrier existence uncertainty: (a) in one case the barrier exists, (b) in the second it does not exist, (c) in the first case inaccessible space ($\neg Y$) results, (d) in the second case there are no barrier constraints (Y), and (e) the overall travel possibility space yielding a partition of possibly not accessible space.

5.2.2 The Impact of Point Object Uncertainty

Uncertainty in the spatial or temporal characteristics of a point object results in numerous possible outcomes. The wayfinder can calculate accessibility from each discrete case, or from the extremes of a continuous uncertainty. The eight point uncertainty types (Section 5.1.1) create three-valued travel possibility partitions (Figure 5-5). Continuous *Y-space* is maintained only in the case where there is no uncertainty in the spatial location of the point requirement, in all other cases a wayfinder is forced to create paths through *possibly should not space*. This characteristic is addressed further in chapter seven.

5.2.3 The Impact of Uncertainty in Objects with a Temporal Extent

Uncertainty in a requirement with a temporal extent (Section 5.1.5) creates uncertainty in the earliest departure and latest arrival. Departure and arrival uncertainties define the uncertain travel possibility spaces of a requirement. Consider, for example, the following uncertain requirement, “Be at home to meet the plumber at 9 am and stay there until he is done.” This requirement starts at 9 am and ends sometime between 9 am (the plumber immediately fixed the problem) and the scenario’s maximum time, assume 5 pm.

In one extreme case, the requirement begins and ends at 9 am (Figure 5-6a). In the other case, the requirement begins at 9 am, but does not end until 5 pm (Figure 5-6b). Equations 4.2 and 4.3 calculate earliest departure (ED) and latest arrival times (LA). The first case yields values for ED and LA both equaling 9 am, while in the second case, ED equals 5 pm and LA equals 9 am. With these values, accessibility from each case is calculated (Figure 5-6c and d). By combining the two cases, the wayfinder creates a three-valued partition of space-time indicating the overall travel possibilities of this uncertain scenario (Figure 5-6e).

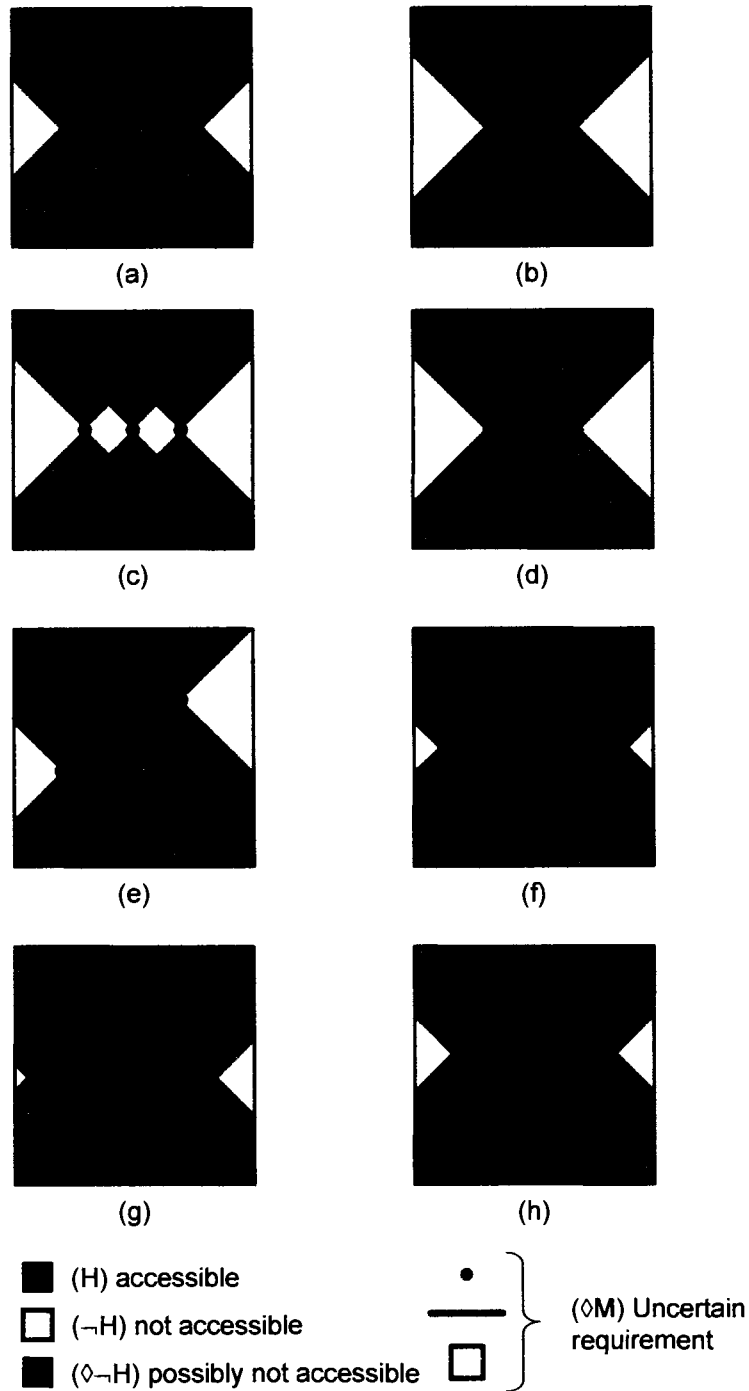


Figure 5-5: Three-valued possibility partitions for various point uncertainties: (a) discrete temporal, (b) continuous temporal, (c) discrete spatial, (d) continuous spatial, (e) discrete temporal and spatial, (f) continuous temporal and spatial, (g) discrete spatial, continuous temporal, and (h) continuous spatial, discrete temporal.

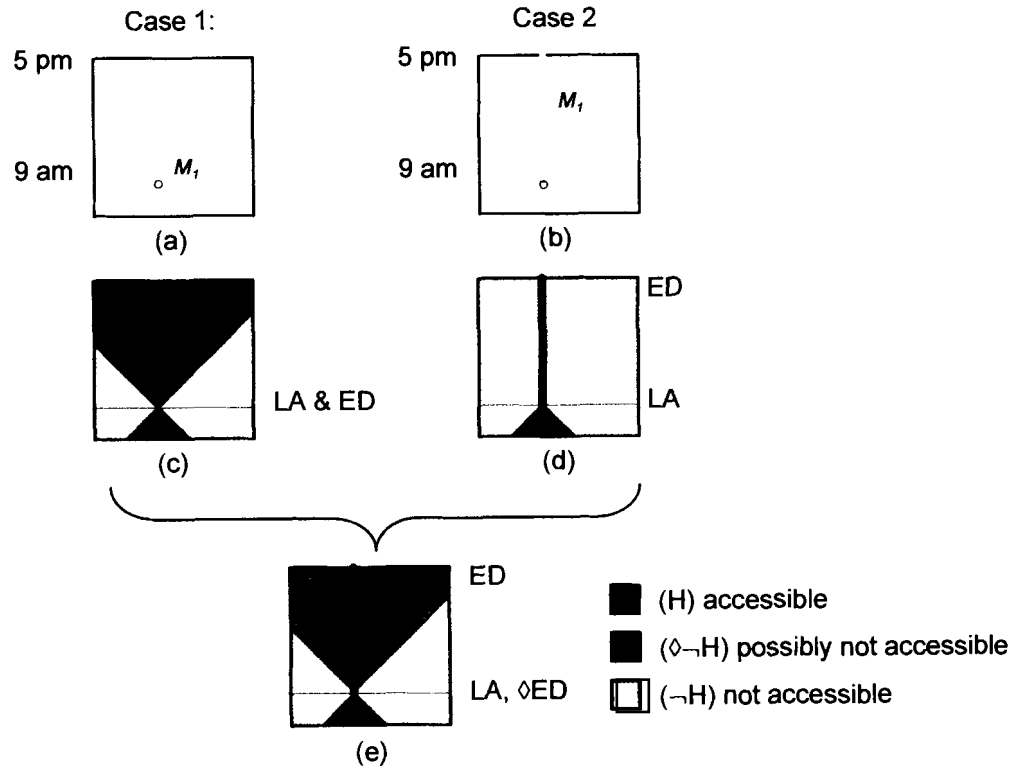


Figure 5-6: The travel possibility space resulting from a requirement with uncertainty in the temporal extent: (a) the wayfinder is only compelled to be at the requirement for an instance at 9 am, (b) the wayfinder must occupy the requirement the entire time starting at 9 am, (c) accessibility for case 1, (d) accessibility for case 2, and (e) combined accessibility to yield a three-valued accessibility partition of space-time.

5.3 Sequentially Partitioning Uncertainty Spaces

Wayfinding scenarios with uncertainty create three-valued partitions in one or more of the basic travel possibility spaces: *C-space*, *Y-space*, and *H-space*. Sequentially partitioning three-valued partitions is more complex than binary partitioning. Sequentially partitioning binary travel possibility spaces results in a tree graph structure with four leaf nodes, where each leaf represents a particular travel possibility (Section 4.5). Sequentially partitioning three-valued travel possibility spaces results in a tree graph with fifteen leaf nodes (Figure 5-7).

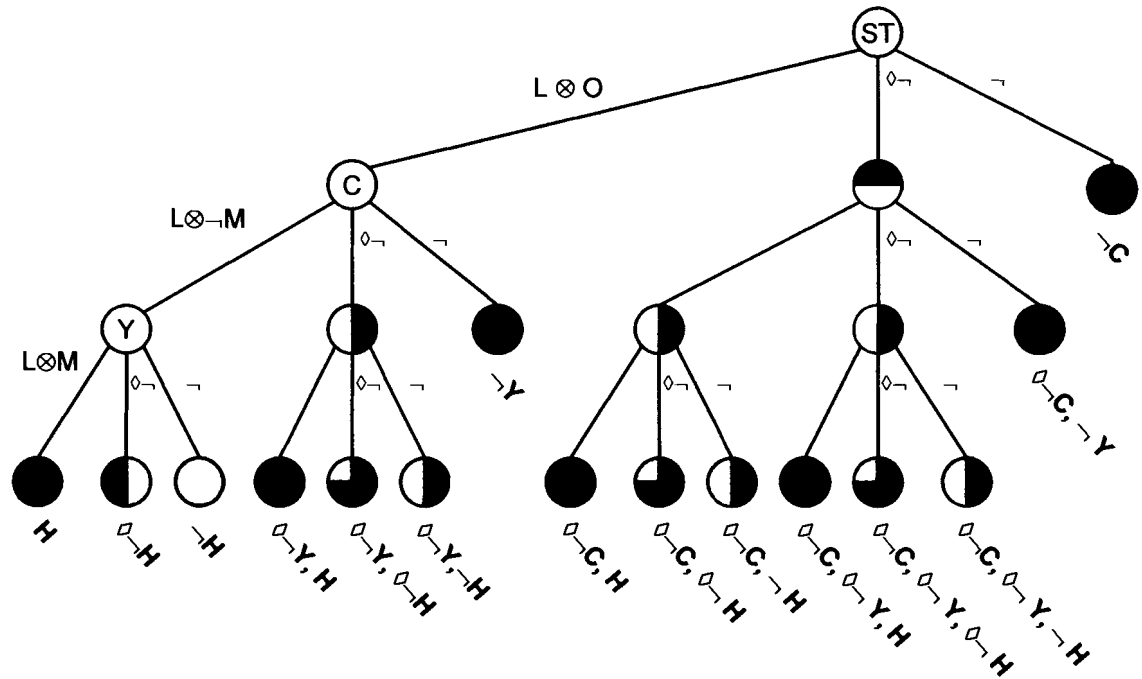


Figure 5-7: Sequentially partitioning three-valued travel possibility spaces yields fifteen categories of travel possibilities.

These fifteen partitions describe the travel possibilities of uncertain wayfinding scenarios, however, reasoning with this number of classifications is challenging. Wayfinders can generalize these fifteen partitions into single three-valued partitions based on some criteria. One

generalization is based on whether travel is possible, or *valid* (V), while a second is based on whether travel can be *successful* (S) (Figure 5-8).

The *valid* partition (V) indicates, regardless of requirements, where travel is possible, grouping three leaf node partitions into one. *Invalid* partitions ($\neg V$) also group three leaf nodes, indicating where travel is impossible. The remaining nine leaf nodes are grouped into one generalized partition, *possibly invalid* ($\diamond \neg V$), that indicates where the wayfinder is uncertain about the possibility of travel. This generalization of fifteen partitions into three allows the wayfinder to identify quickly those spaces and times where travel paths can be planned.

Adding the consideration of requirements creates a generalization into *successful* partition space. Success is defined as meeting the requirements of a wayfinding scenario. Only one leaf node partition, *H-space*, is certainly *successful* (S). Seven of the fifteen leaf nodes are classified as *unsuccessful* ($\neg S$), since even with all the other potential uncertainties the wayfinder is sure that the scenario's requirements will not be met if traveling in this space. The remaining seven leaf node partitions each have uncertainty in whether requirements will be met, and are grouped into a single broad boundary space, defined as *possibly not successful* ($\diamond \neg S$). Generalizing the fifteen leaf nodes resulting from a sequential partitioning of the space-time of an uncertain wayfinding scenario, provides wayfinders a much simpler schema to explore travel possibilities.

Consider a wayfinding scenario with uncertainty (Figure 5-9a). The wayfinder is unsure about the existence of a temporal barrier, and is not certain about the time of the requirement. Sequentially partitioning space-time results in a travel possibility space (*P-space*) with six partitions, three of which have uncertainty (Figure 5-9b). This space is generalized into *valid-space* (*V-space*) to indicate where valid paths can be created (Figure 5-9c). In addition, *success-space* (*S-space*) is generated, indicating where paths that meet the scenario's requirements can be created (Figure 5-9d).

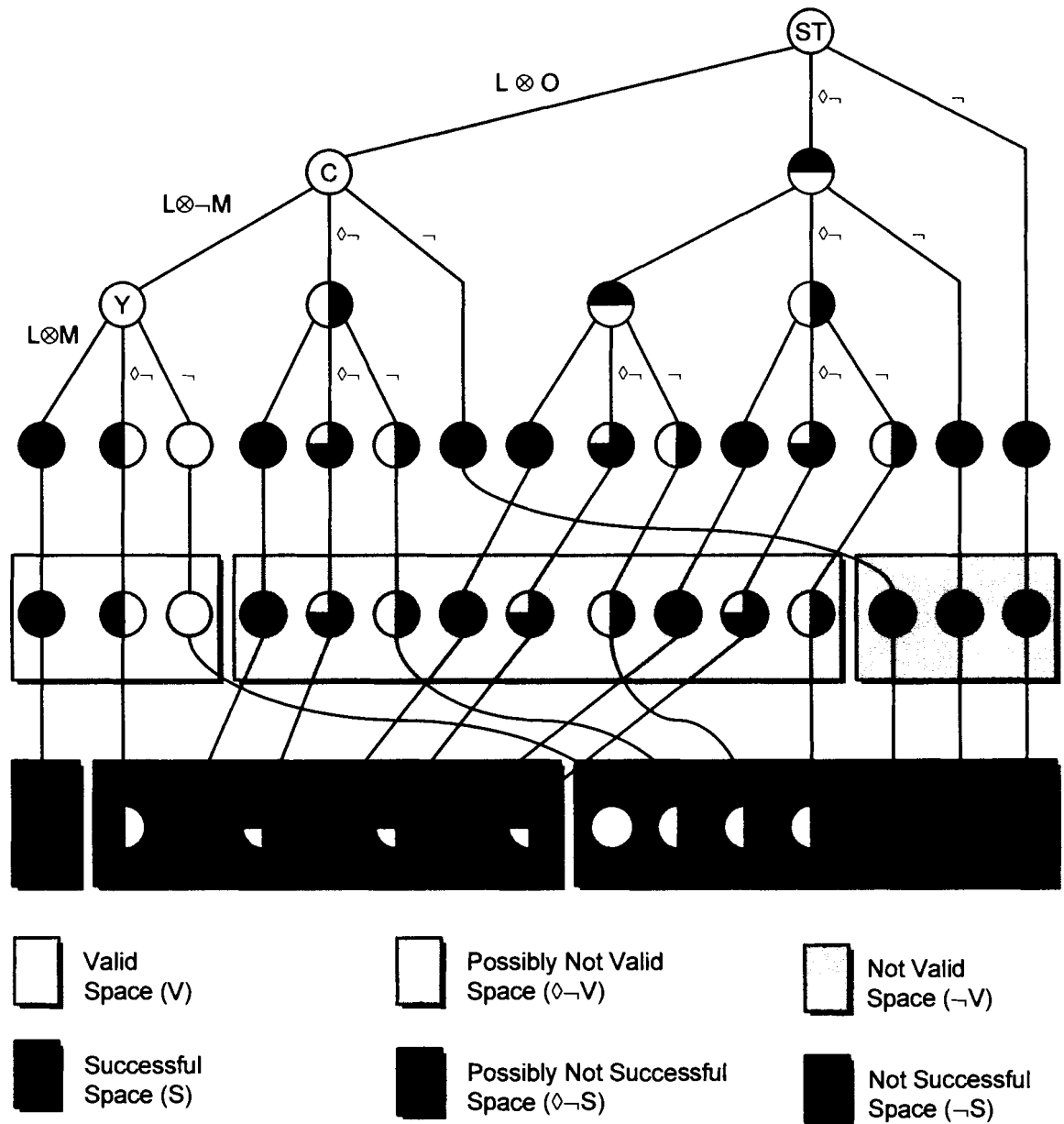


Figure 5-8: Mappings of the fifteen travel possibility partitions into valid space and successful space generalizations.

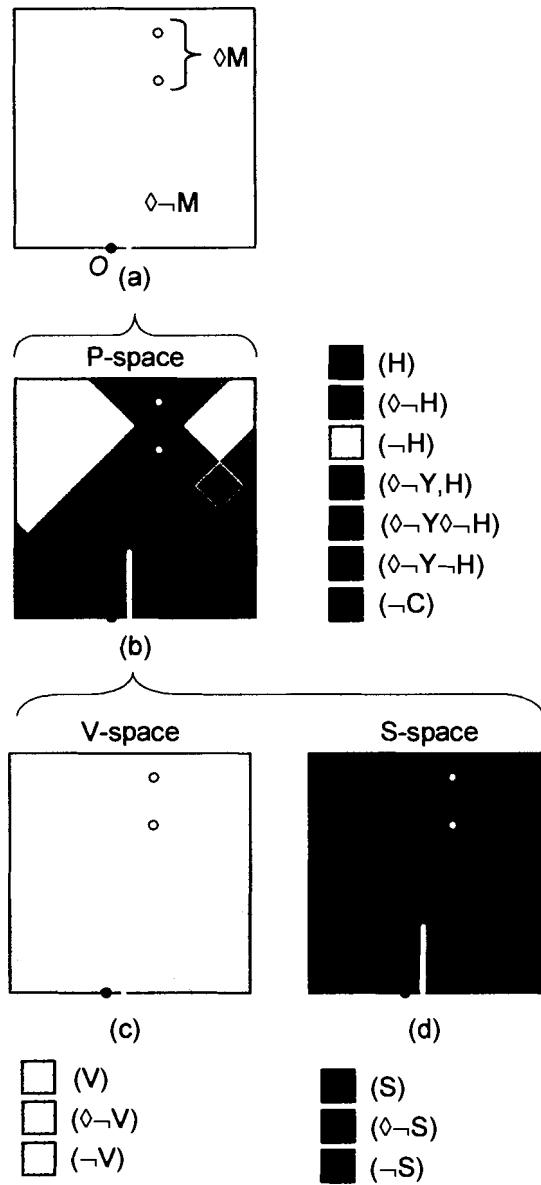


Figure 5-9: Partitioning of a wayfinding scenario (a) into: (b) travel possibility space, *P-space*, (c) travel validity space, *V-space*, and (d) travel success space, *S-space*.

5.4 Summary

This chapter considered the impact of uncertainty arising from the indiscernibility of possible values that barriers and requirements could take in a wayfinding scenario. Three categories of uncertainty were introduced: (1) existential uncertainty, (2) spatial and temporal uncertainty of a point object, and (3) uncertainty in objects with a temporal extent along with the related issue of earliest arrival and latest departure times. Sequentially partitioning 3-valued spaces creates up to 15 travel possibility categories. These categories can be generalized into either *valid-space* or *successful-space*.

CHAPTER 6

UNCERTAINTY IN REQUIREMENT COMBINATIONS

Wayfinding scenarios often include requirement combinations that contain uncertainty. This uncertainty can take one of two forms: (1) there may be spatial or temporal uncertainty in the individual requirements of the combination, or (2) there may be uncertainty in the overall combination. In the scenario, “mail a package at the post office before getting groceries,” uncertainty in when the post office opens is of the first form, while uncertainty in whether to get groceries before or after stopping at the post office is an example of the second form. Wayfinders must understand and account for both forms of uncertainty when partitioning space-time into travel possibility spaces.

Uncertainty in one requirement can propagate to every requirement in a combination, greatly influencing travel constraints. When a wayfinder is unsure about a requirement’s earliest departure or latest arrival times, he or she cannot be sure how this requirement constrains others. The procedure for modeling travel restrictions between the components of a requirement combination (Section 4.4.1) must be adjusted to account for individual requirement uncertainty.

Even when wayfinders are sure about the characteristics of individual requirements, there may be uncertainty in the overall combination operation. Though numerous uncertainties arise with requirement combinations, three are explored in detail: (1) uncertainty in the existence of the entire combination, (2) uncertainty in whether a second requirement exists, and (3) uncertainty in whether a second requirement will replace an initial requirement. Each category of combination uncertainty creates a unique partitioning of space-time, constraining the wayfinder’s travel possibilities.

Particularly complex cases arise when there is uncertainty in both the individual requirements and the overall combination operation. There are two particularly interesting and relevant realizations of this situation: (1) the *repairman scenario* and (2) the *police officer scenario*.

In the *repairmen scenario*, the wayfinder has a requirement, but is unsure whether a second requirement at some unknown location and time will be added to the scenario. Since the initial requirement must still be met, valid locations for this additional requirement are within the initial requirement's *should space* (*H-space*). The wayfinder, however, cannot be sure when and where within this space the requirement will occur, or if it will exist at all.

The *police officer scenario* is similar to the *repairmen scenario*, but instead of the possibility of adding a second requirement, the possibility exists that an additional requirement, at some unknown place and time, will take precedence and replace the initial one. In this scenario, if the possible additional requirement exists, the initial requirement is no longer valid and no longer constrains travel. As a result, valid locations for this possible additional requirement are much more flexible than in the *repairmen scenario*, and occur anywhere within the initial requirement's *may-space* (*Y-space*), which is based only on the start point and maximum travel speed of the wayfinder.

This chapter continues by first analyzing the impact of individual requirement uncertainty on requirement combinations (Section 6.1). An exploration follows of three categories of requirement combination uncertainty and their impact on travel possibility partitions of space-time (Section 6.2). Section 6.3 considers the two identified special cases of complex uncertainty—the *repairmen scenario* and the *police officer scenario*.

6.1 Combining Uncertain Requirements

Creating the *should space* (*H-space*) for a combined requirement with uncertainty, in one or both requirements, is similar to the procedure without uncertainty (Section 4.4.1). This section

describes the ordered combinations of *before* and *after*. Two remaining combinations—*and* along with *or*—extend from these concepts.

In ordered combinations, the earliest departure time of the initial requirement, plus travel time, constrains subsequent requirements. In an analogous manner, the subsequent requirement constrains the initial requirement. Uncertain constraints result when a wayfinder is unsure of the latest departure and earliest arrival times of a requirement. To account for these uncertainties, a wayfinder calculates all possible constraints between requirements in a combination.

Consider the ordered combination, “get a package from Jim (M_1) and go to the Post Office (M_2)”, where the wayfinder is unsure whether to meet Jim at 9 am or 10 am and there is a half hour travel time between the two locations. This situation creates an ordered combination with discrete temporal uncertainty in the initial requirement (Figure 6-1a). The wayfinder determines that the earliest departure time (ED) from Jim’s location is 10 am, but with the possibility (\Diamond ED) it is as early as 9 am. With these values, the wayfinder calculates the constraints on when he can arrive at the Post Office (Figure 6-1b). The Post Office requirement, initially anytime during the scenario, is possibly transformed into a requirement starting at 9:30 am ($\Diamond M_2^*$) and certainly into a requirement starting at 10:30 am (M_2^*) (Figure 6-1c). Scenarios that are more complex can be treated in a similar manner.

After creating each transformed requirement, the next step is to create *should space* (*H-space*) for each transformed requirement. These *H-spaces* are three-valued broad-boundary possibility spaces. The overall *H-space* of the ordered combination results by intersecting the individual *H-spaces* (Table 6-1.)

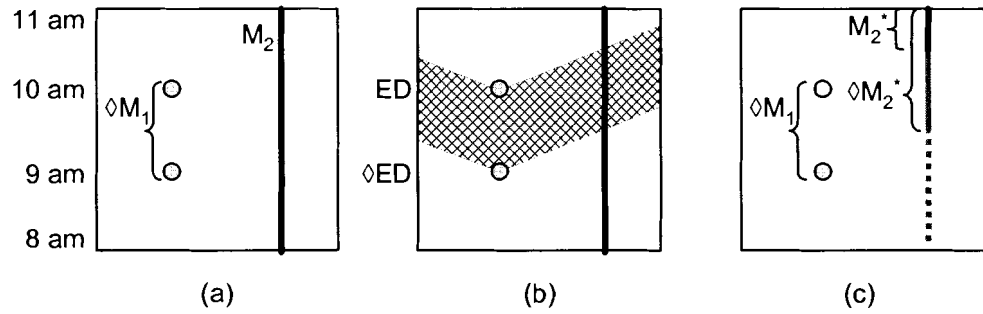


Figure 6-1: An ordered requirement combination (M_1 before M_2) with: (a) discrete temporal uncertainty in the initial requirement generates, (b) certain (gray) and possible (cross-hatched) constraints which result in (c) the transformation of M_2 .

	H	$\diamond\text{--}H$	$\text{--}H$
H	H	$\diamond\text{--}H$	$\text{--}H$
$\diamond\text{--}H$	$\diamond\text{--}H$	$\diamond\text{--}H$	$\text{--}H$
$\text{--}H$	$\text{--}H$	$\text{--}H$	$\text{--}H$

Table 6-1: Combination matrix for two broad boundary H -space partitions.

To summarize this procedure, consider a wayfinding scenario with the ordered combined requirement, “mail a package at the post office before getting groceries”—*sometime M_1 before sometime M_2* —with a continuous uncertainty of M_1 ’s start time (when the post office opens) between 8 am and 9 am (Figure 6-2a). Each requirement transforms the other (Figure 6-2b and c). The subsequent requirement’s transformed space has become uncertain as a result of the initial requirement’s uncertainty. The resulting transformed requirements, both the certain and uncertain components, create individual three-valued partitions (Figure 6-2d and e.) These individual partitions combine to yield the overall H -space resulting from the combined requirement. This partitioning represents the constrained space-time of this ordered combination of uncertain requirements (Figure 6-2f).

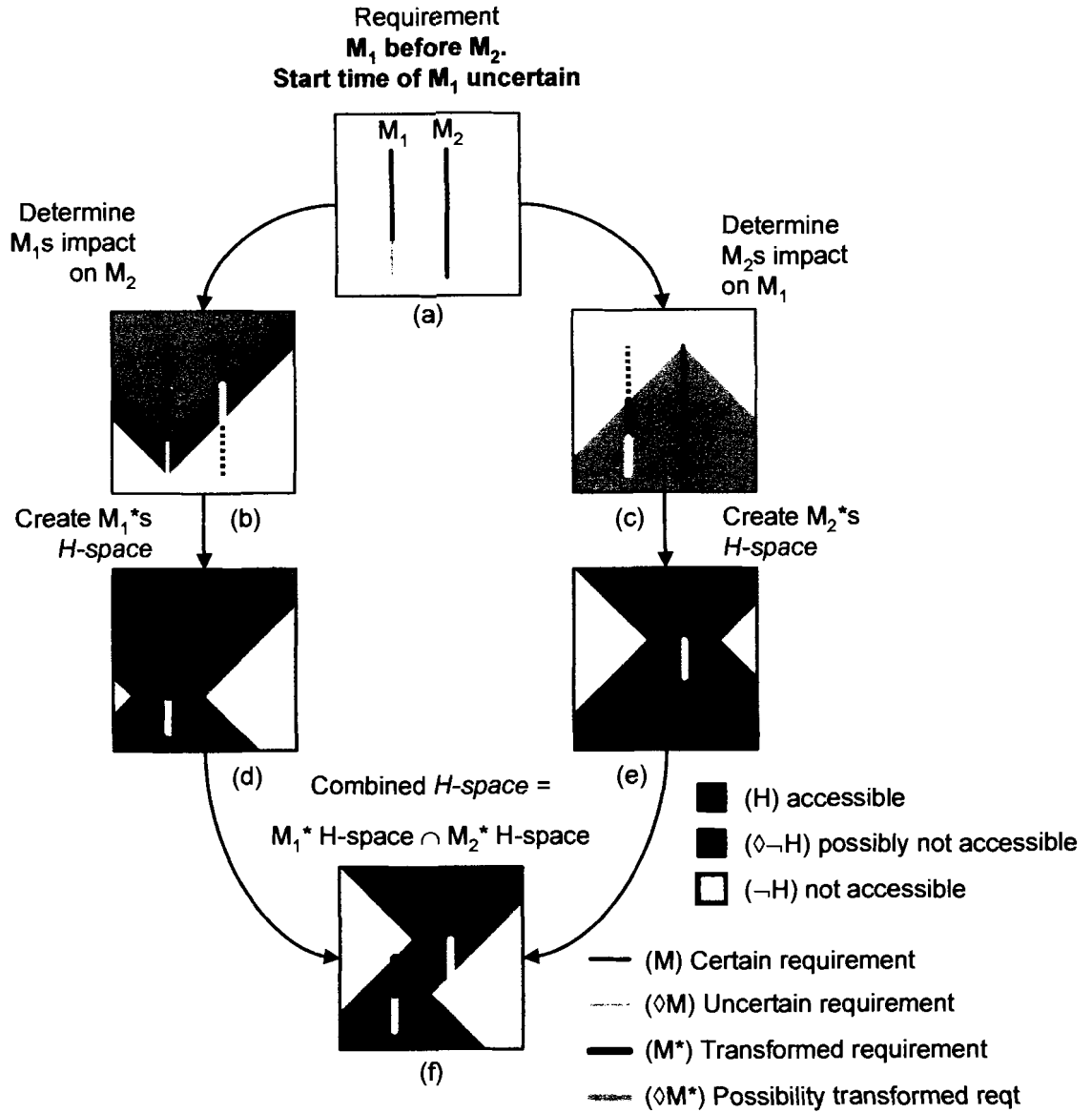


Figure 6-2: Combining requirements with uncertainties: (a) an ordered combined requirement (M_1 before M_2) with uncertainty in M_1 , (b) M_1 certainly and possibly constrains M_2 , transforming M_2 into an uncertain requirement, (c) M_2 constrains M_1 , (d and e) the transformed requirements created three-valued partitions, (f) intersecting the two transformed requirements, M_1^* and M_2^* , resulting in the overall three-valued partitioned space.

6.2 Uncertainty in Requirement Combinations

In addition to combining uncertain requirements, there may be uncertainty in the combination operation. Consider, for example, the following uncertain requirements: (1) “possibly go to both the post office and the grocery store,” (2) “go to the post office and possibly also the grocery store,” and (3) “go to the post office, or possibly the grocery store instead.” The wayfinder is sure when and where the individual requirements are valid, but in each case is unsure about how they are combined. In the first example, the entire combination is uncertain; the wayfinder possibly will have to go to both, or maybe neither. The wayfinder knows for sure, in the second example, that an initial requirement is valid, but is unsure about the addition of a subsequent requirement. In the third example, though it is a combined requirement, the two possibilities are individual requirements: (a) go to the post office or (b) go to the grocery store. This example is similar to the *or* operator; however, the difference is that in this case, one of the requirements is valid while the other is not, and currently the wayfinder does not know which possibility is correct. The next three sections address each of these scenarios.

6.2.1 Uncertainty in Combined Requirement Existence

In the first type of combination uncertainty, the existence of the entire combination as a whole is uncertain. In one case, the combined requirement is valid and constrains the wayfinder’s travels. In the second case, the combined requirement does not exist. Three-valued broad boundary partitions of space-time result, indicating the travel possibilities of this uncertain combination.

Consider the scenario, “possibly go to both the post office (M_1) and the grocery store (M_2).” In the first case, the combined requirement exists and the wayfinder goes to both locations (Figure 6-3a). It is also possible that the requirement will not exist and there will be no constraints on travel (Figure 6-3b). Each possibility creates an *H-space* (Figure 6-3c and d). Combining these

two cases (Table 5-1) creates a three-valued broad boundary partition of space-time, indicating the impact of this uncertain requirement combination (Figure 6-3e).

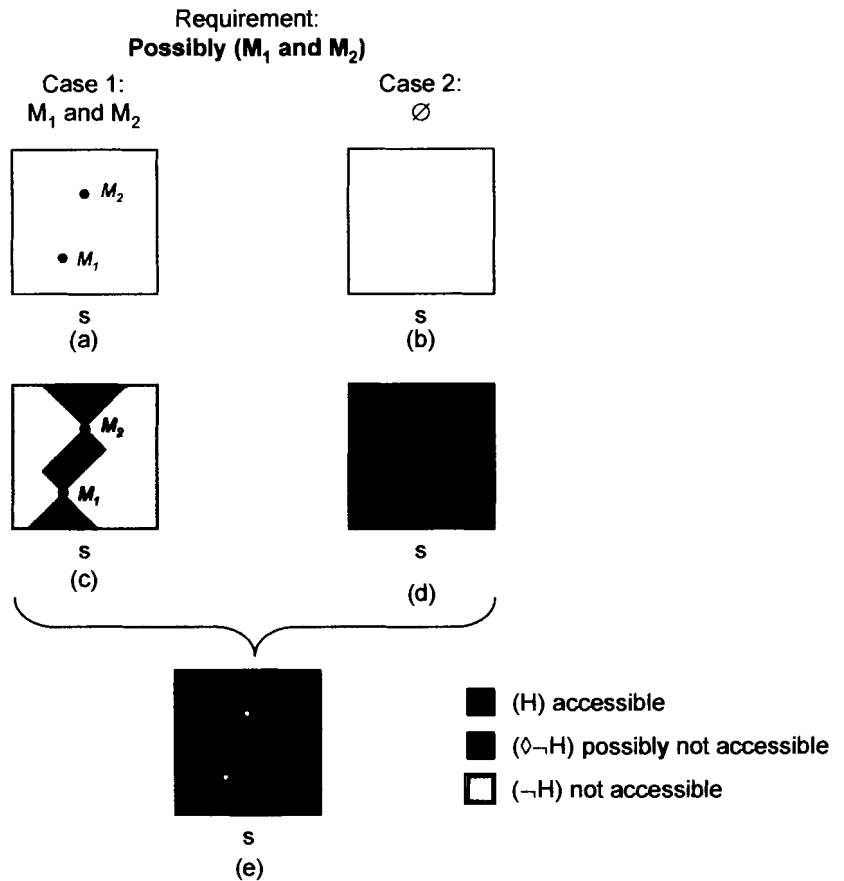


Figure 6-3: Uncertainty in combined requirement existence: (a) in one case the requirement exists, (b) in another case it does not, (c and d) each case partitions space, and (e) combining these possibilities (Table 5-1) creates a three-valued broad boundary partition of space-time.

6.2.2 Uncertainty in Subsequent Requirement Existence

The second type of requirement combination uncertainty considers a possible additional requirement. The initial requirement exists in all cases. The wayfinder, however, is unsure about the addition (*and* operator) of a subsequent requirement, as “travel to the Post Office and possibly also the Grocery Store.” This type of uncertainty is not necessarily ordered, but may incorporate the ordering operators of *before* and *after*, further restricting travel.

Creating the three-valued partitions of space-time begins by considering the case where only the first requirement is valid (Figure 6-4a), followed by the possibility of both requirements being valid (Figure 6-4b). Each possibility partitions space-time (Figure 6-4c and d). Combining the two possibilities (Table 5-1) creates a three-valued broad boundary partition representing the travel constraints of this uncertain combined requirement (Figure 6-4e).

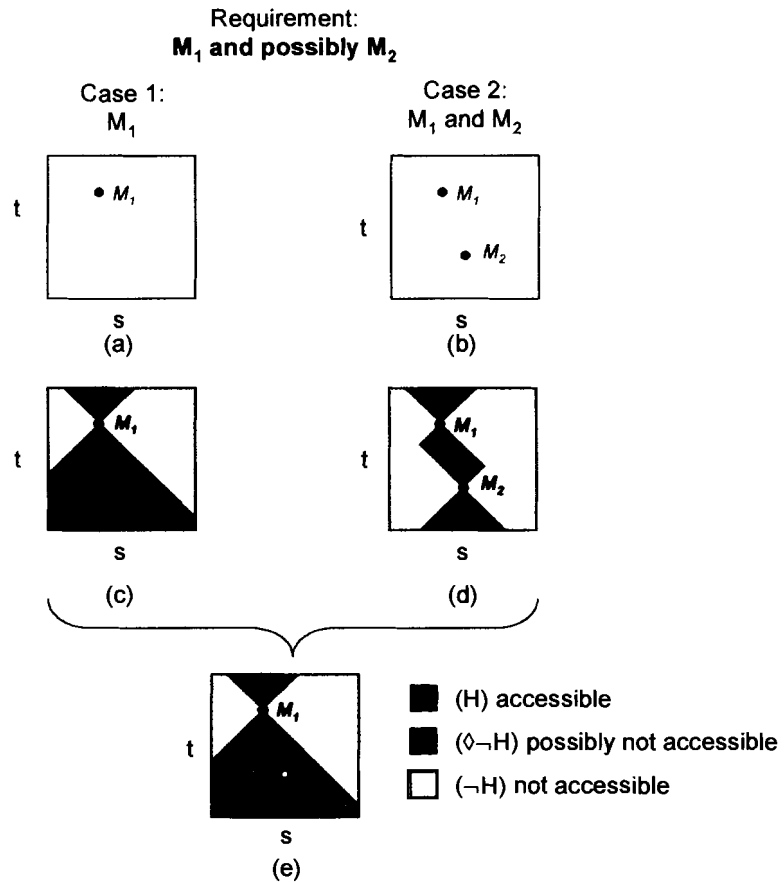


Figure 6-4: Uncertainty in a subsequent requirement's existence: (a) in one possibility only the initial requirement exists, (b) an alternate possibility is that both requirements exist, (c and d) each partitions space-time, and (e) both possibilities combine to yield a three-valued broad boundary partition of travel possibilities.

6.2.3 Uncertainty in Correct Requirement

When a wayfinder is unsure whether a subsequent requirement will replace an initial one, she cannot be sure which requirement is valid. In one case, only the first requirement exists and restricts travel, while in the other case only the subsequent requirement restricts travel. Though there is no restriction that either requirement must be contained in the other's *H-space*, the resulting three-valued partitioning of space is greatly influenced by whether they do or do not. If the requirements are within each other's *H-space*, the combined *H-space* is continuous through time (Figure 6-5). However, if the requirements are not contained in each other's *H-space*, there does not exist a continuous *H-space* through time (Figure 6-6). How the wayfinder should choose paths through the regions of uncertainty is the topic of Chapter Seven.

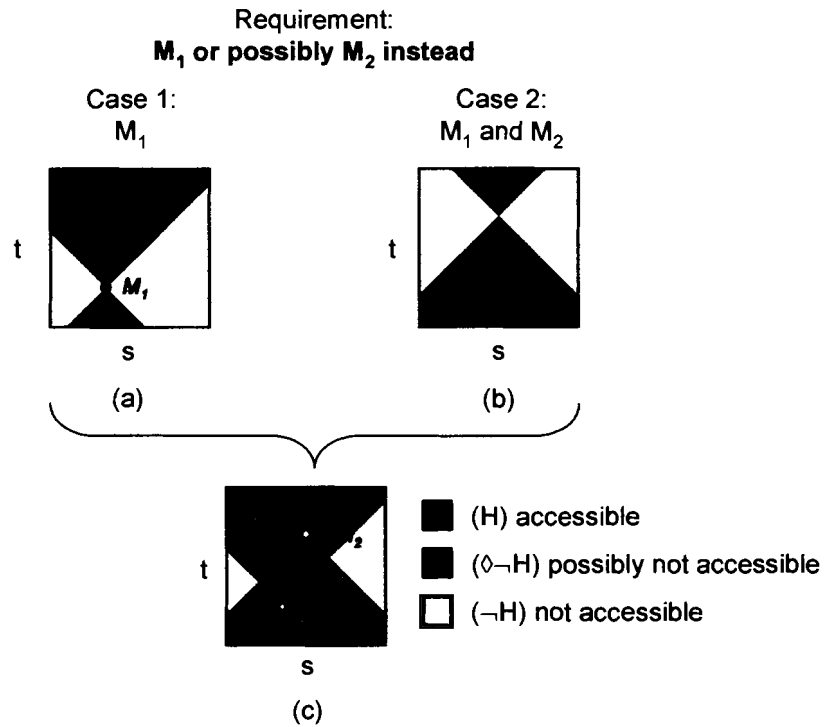


Figure 6-5: Uncertainty in correct requirement where both requirements are within each other's *H-space*.

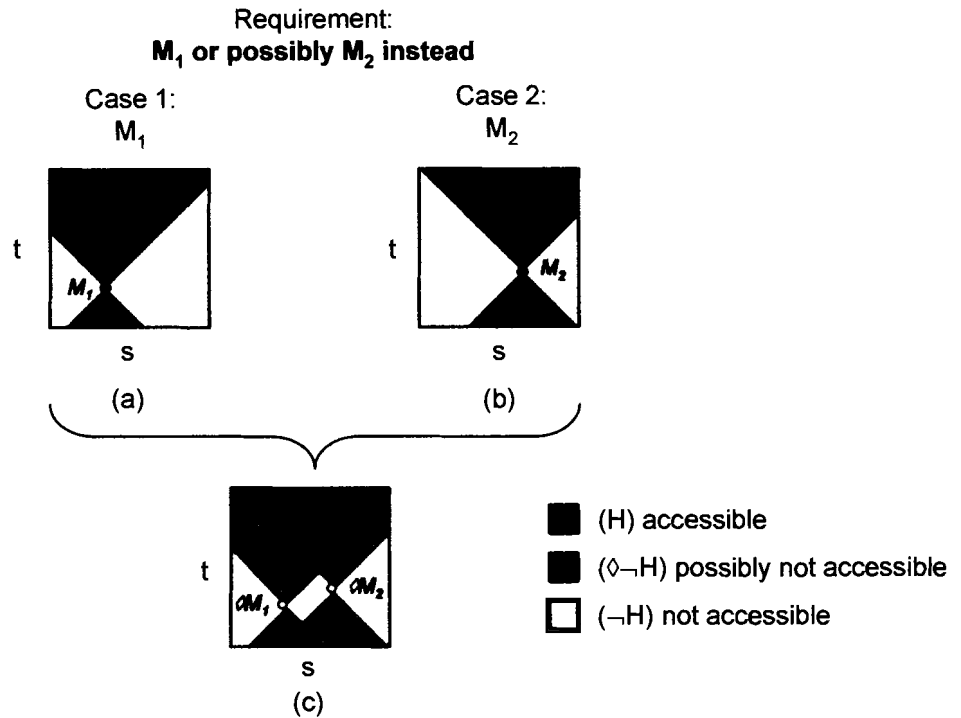


Figure 6-6: Uncertainty in correct requirement where both requirements are not within each other's *H-space*.

6.3 Uncertainty in Both the Requirement and the Combination

Two specific scenarios include uncertainty in both the requirement and the combination itself, dubbed the *repairmen scenario* and the *police officer scenario*. The wayfinder, in both cases, begins with an initial requirement and is unsure if an additional requirement is included. In the *repairmen scenario* this additional requirement is added to the initial one (Section 6.2.2), while in the *police officer scenario* the additional requirement replaces the initial requirement (Section 6.2.3). Both cases produce complex three-valued broad boundary partitions of travel possibilities.

Travel impact calculations related to the possible events associated with the combination of two or more uncertain requirements must include considerations for the broad boundaries representing uncertainty. Table 6-1 extend the combination rules described in Table 5-1 to account for broad boundary travel possibility spaces.

	H	$\Diamond\neg H$	$\neg H$
H	H	$\Diamond\neg H$	$\Diamond\neg H$
$\Diamond\neg H$	$\Diamond\neg H$	$\Diamond\neg H$	$\Diamond\neg H$
$\neg H$	$\Diamond\neg H$	$\Diamond\neg H$	$\neg H$

Table 6-2: Combination matrix for the accessibility space of two possible outcomes with broad boundaries.

Consider a repairman named Ted, and Sue, a police officer, each planning an 11:00 am lunch at Bob’s Coffee House. Ted currently has no repair calls, but the possibility exists that one may arise prior to lunch. Ted, as a result, plans his travels with the uncertain combined requirement, “go to Bob’s Coffee House and sometime prior to this, maybe somewhere else.” Sue, on the other hand, has begun to expect interruptions to her lunch break, therefore operating with a different uncertain requirement combination, “go to Bob’s Coffee House, but be prepared at anytime to go somewhere else instead.” These two uncertain requirement combinations partition space-time differently.

6.3.1 Possible Additional Requirement Somewhere (*Repairman Scenario*)

In the *repairmen scenario*, the wayfinder is certain of an initial requirement, but an additional one is possible. This additional requirement’s location and time is also uncertain, but restricted to space and time that still allows the initial requirement to be met. Ted the repairman is sure that he has a requirement to go to Bob’s Coffee House for an hour, starting at 11 am. The uncertainty arises as a result of combining the additional uncertain requirement “be prepared to go anywhere in town before hand if a customer request comes up.” Since the initial requirement is still valid, the possible space-time partition accessible to him is restricted to the first requirement’s *should-space*.

One potential outcome to consider is that only the initial requirement will exist, and the possible additional requirement never arises (Figure 6-7a). This one requirement creates a *should-space* representing its travel constraints (Figure 6-7c). A second potential outcome exists, where

the second requirement is added to the scenario somewhere within the initial requirement's *should-space*, since the initial requirement must still be met (Figure 6-7b). The two combined requirements create a separate *should-space* (Figure 6-7d). Combining two potential outcomes creates an overall three-valued partition of space-time related to this uncertain scenario (Figure 6-7e).

Continuous *H-space* does not exist, and as a result, Ted is uncertain about success in this scenario. This uncertainty arises from the possibility that the potential additional requirement may occur at a place and time inaccessible to the wayfinder. Determining what path the wayfinder should choose that will maximize the chance of success in such settings is the topic of Chapter Seven.

6.3.2 Possible Substitution of Requirement Somewhere (*Police Officer Scenario*)

The *police Officer scenario* is less restrictive, but eventually creates even larger uncertainties for the wayfinder, because there is the possibility that a second requirement replaces the initial one and that this new requirement occurs in space and time anywhere the wayfinder *may* reach. Sue, the police officer, knows that she has an initial requirement at 11 am at Bob's Coffee House, but must be ready to respond to an emergency *in lieu* of this initial requirement. If this change in the scenario occurs, no longer is she required to go to Bob's Coffee House. As a result, the new requirement is no longer constrained to the initial requirement's *should-space*, but is restricted to the *may-space*.

Requirement:
 M_1 and possibly M_2 also,
 Where M_2 is possibly inside M_1 's H -space

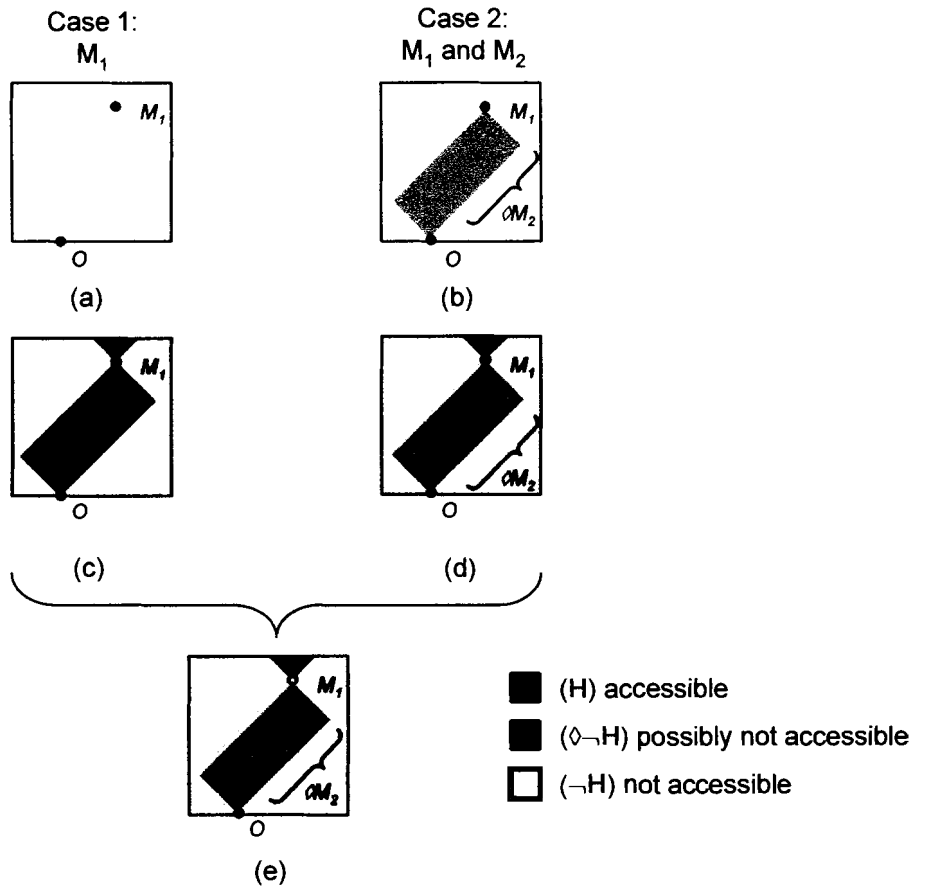


Figure 6-7: The impact of the possible addition of an uncertain requirement ($\Diamond M_2$) to an initial requirement (M_1): (a) in one case only the initial requirement exists, (b) in the second case both requirements exist, (c and d) each possibility creates its own *should-space*, and (e) combination yielding an overall *should-space* representing the travel restrictions of this uncertain scenario.

As with the *repairmen scenario*, one potential outcome is where only the initial requirement exists (Figure 6-8a). In this scenario, the second potential outcome is that a second requirement replaces the initial one. The location of this replacement requirement is uncertain, but exists somewhere within the wayfinder's *may-space* (Figure 6-8b). Each potential outcome creates *should space* (Figure 6-8c and d). The combined uncertain requirement indicates a very uncertain scenario (Figure 6-8e), which is indicative of scenarios where wayfinders are required to react to requirements over large areas, such as emergency response providers.

6.4 Summary

This chapter extended uncertainty concepts to include uncertainty in requirement combinations. It was shown that ordered combined uncertainties project uncertainty between requirements. In addition, uncertainty in the combination itself can exist. The impact of uncertainty in a combined requirement as a whole was demonstrated, along with the impact of uncertainty in the addition of a second requirement. A third uncertain combination, the possibility of replacing the initial requirement with a second, identified the different possibility spaces created based by the relationship between each requirement's accessible space.

Additional complexity arises when uncertainty exists in both the individual requirements of combination, and the combination as a whole. Two common realizations of this situation are described; the *repairmen scenario*, and the *police officer scenario*. In the *repairmen scenario*, uncertainty exist in whether to add a subsequent requirement somewhere. The *police officer scenario*, on the other hand, describes uncertainty in whether a subsequent requirement replaces an initial one. In both scenarios, large portions of the space-time environment become *possible should not space*, success in these scenarios is not assured.

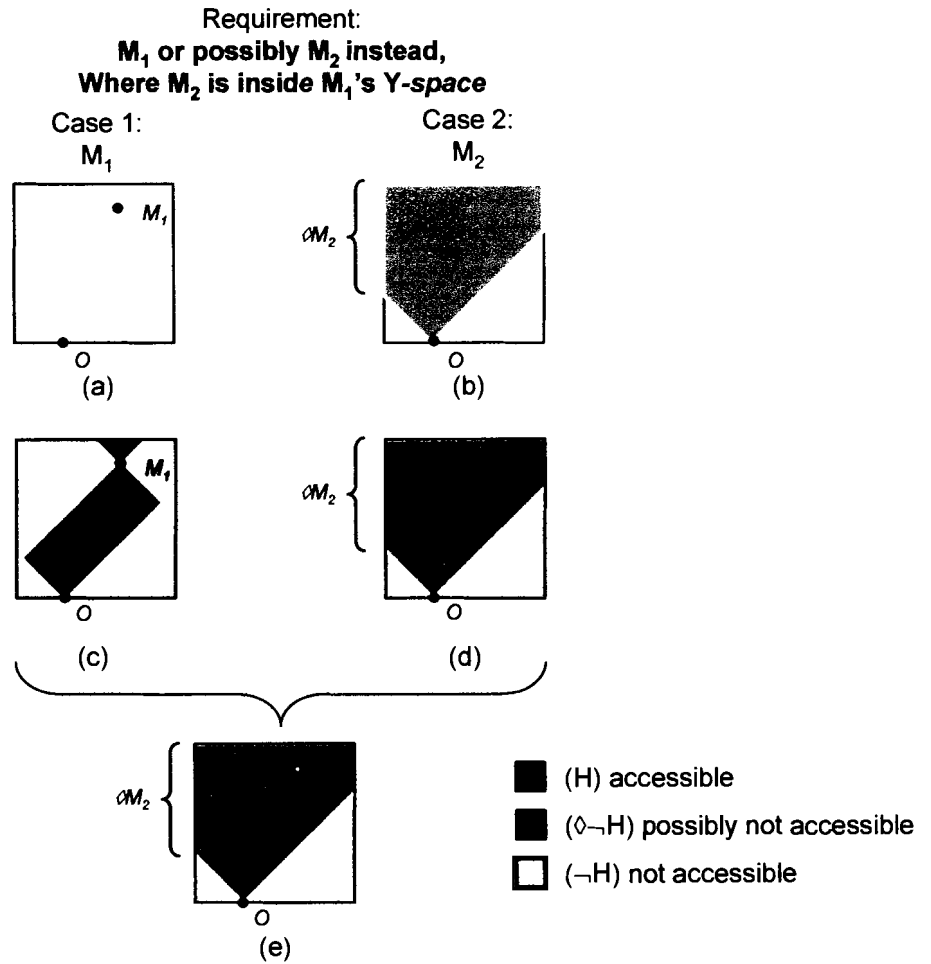


Figure 6-8: The impact of the possible substitution of an initial requirement (M_1) with a new uncertain requirement ($\Diamond M_2$): (a) in one case only the initial requirement exists, (b) in the second case the second requirements exists, (c and d) each possibility creates its own *should-space*, and (e) combination yielding an overall *should-space* representing the travel restrictions of this uncertain scenario.

CHAPTER 7

SELECTING PATHS THROUGH TRAVEL POSSIBILITY PARTITIONS

Paths are an integral part of any wayfinding activity (Hunt and Waller 1999) and effective wayfinders select paths that meet a scenario's requirements. Paths meet requirements, when modeled within an integrated space-time framework, by intersecting the requirement object (Section 3.2.4). Paths that achieve this condition are *successful paths*.

Partitioning a wayfinding scenario into travel possibility categories simplifies the path selection process, because wayfinders know that only paths contained inside *valid space* are possible and paths that meet requirements are contained inside *successful space*; all other paths can be ignored. When wayfinding scenarios include uncertainties, continuous partitions of *valid space* and *successful space* do not always exist. In these instances, wayfinders select paths that intersect *possibly invalid* and *possibly unsuccessful space*, and as a result, they cannot be sure they will meet all their requirements. Wayfinders, however, can increase the probability of success in these situations by selecting paths that maximize accessibility to requirement possibilities.

To assist wayfinders when selecting a path through *possibly unsuccessful space*, a set of metrics measures a path's accessibility to the requirement possibilities. These metrics provide a mechanism to compare various paths within a scenario and assist the wayfinder to select paths with the greatest chance of success. This procedure is followed by an example scenario, and the data suggest that paths minimizing arrival time also maximize accessibility to possible additional requirements, which is the hypothesis put forward in this thesis.

This chapter continues by considering general characteristics of paths through space-time (Section 7.1) and then investigates a number of specific paths that minimize and maximize these characteristics (Section 7.2). This leads to the definition of *valid* and *successful paths*, which are

critical concepts for the assessment of the hypothesis (Section 7.3). Section 7.4 introduces metrics of accessibility to possible requirements for points and paths. With these metrics, path accessibility is measured within a *repairmen scenario* (Section 7.5).

7.1 Characteristics of Paths through Space-Time

Space-time paths that meet requirements begin at a start point, intersect the requirement(s), and continue until time runs out. Requirements divide paths into legs. For example, a scenario with a single requirement has a path with two legs: one before and one after the requirement.

The legs of a space-time path are categorized as follows (Figure 7-1): The *initial leg* begins at the start point and ends at the time of the first requirement; legs between the first and subsequent requirements are *intermediate legs*, with the leg ending at the last requirement being the *terminal leg*. For a path with a single requirement, the initial leg and the terminal leg coincide and no intermediate legs exists. The portion of the path after the last requirement is the *post requirement leg*. During the post requirement leg, wayfinders are no longer constrained to meet requirements.

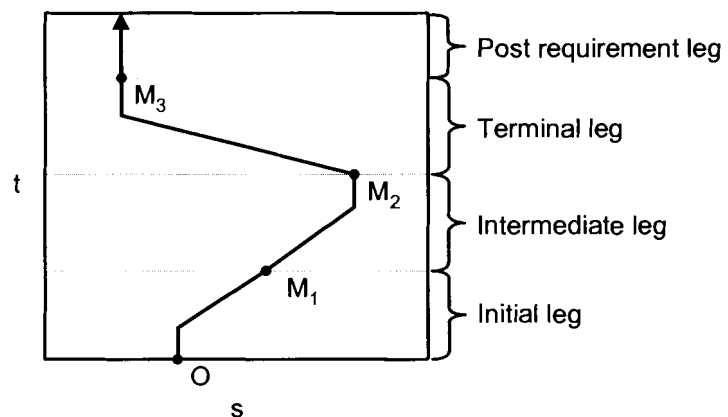


Figure 7-1: Types of legs in a wayfinding path.

Each leg has a number of characteristics: it has a start and an end, each with a spatial and temporal component. Additional measures further describe the legs of a wayfinding path (Figure 7-2):

- *Length*: The spatial distance of the leg as calculated from the projection of the space-time leg onto the spatial dimension(s).
- *Departure Time*: The last moment the path occupies the start point.
- *Arrival Time*: The first moment the path occupies the end point.
- *Greatest Speed Required*: A measure of the fastest speed the wayfinder must use along the leg.

A path's total spatial length is the sum of all leg lengths. The path's departure time is the departure time of the initial leg, whereas the path's completion time is the arrival time of the terminal leg (Figure 7-3).

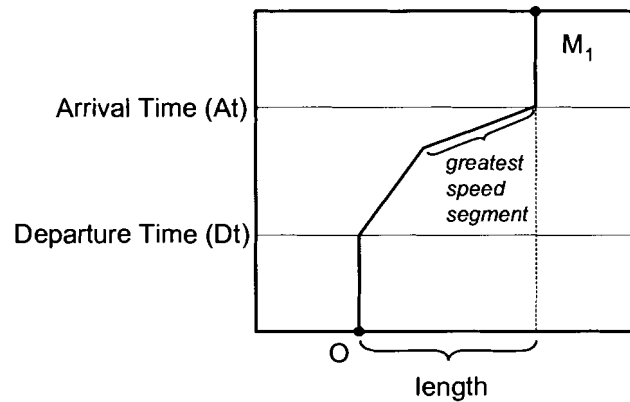


Figure 7-2: Basic characteristics of a space-time leg.

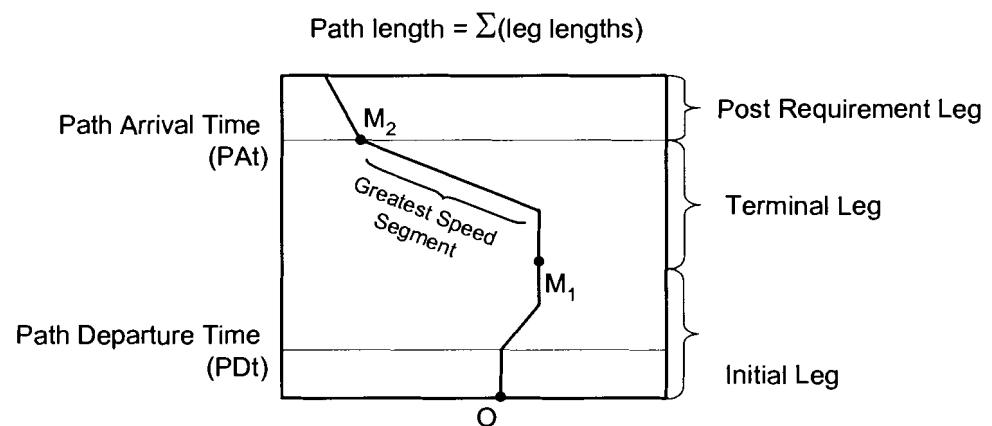


Figure 7-3: Path with multiple requirements.

7.2 Various Minimum and Maximum Legs

Minimizing and maximizing particular metrics creates special paths through space-time. Minimizing length creates a *shortest path*, while minimizing arrival time creates a path of earliest arrival, called the *early bird path* (Figure 7-4a). Maximizing the departure time produces a *procrastinator path* (Figure 7-4b), which allows the wayfinder to remain at the leg's start point for as long as possible. Minimizing the greatest speed required to reach the leg's end point creates a steady state path, called a *tortoise path* (Figure 7-4c).

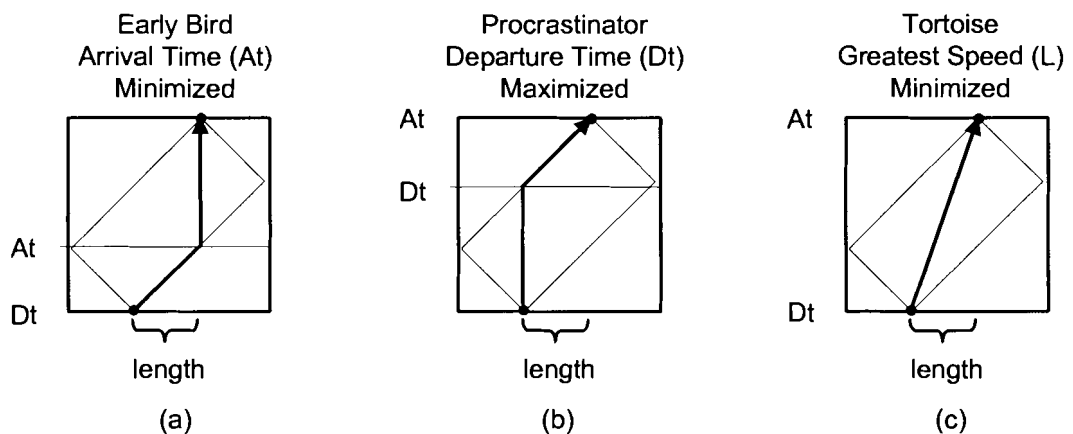


Figure 7-4: Three space-time paths: (a) *early bird path* minimizes the arrival time at the requirement, (b) a *procrastinator path* maximizes the departure time, and (c) a *tortoise path* minimizes the greatest required speed.

In some scenarios the shortest path and arrival minimization path are not equal. In these cases, wayfinders must choose whether distance or time minimization is more important. Consider, for instance, a wayfinder with a requirement and temporary barrier existing at the same location, but at different times (Figure 7-5a). The latest departure time path is unaffected by this barrier (Figure 7-5b). The earliest arrival paths, however, are more complex. One path minimizes distance and arrives as early as possible (Figure 7-5c). The absolute earliest arrival path, however, has a greater length than the shortest path (Figure 7-5d).

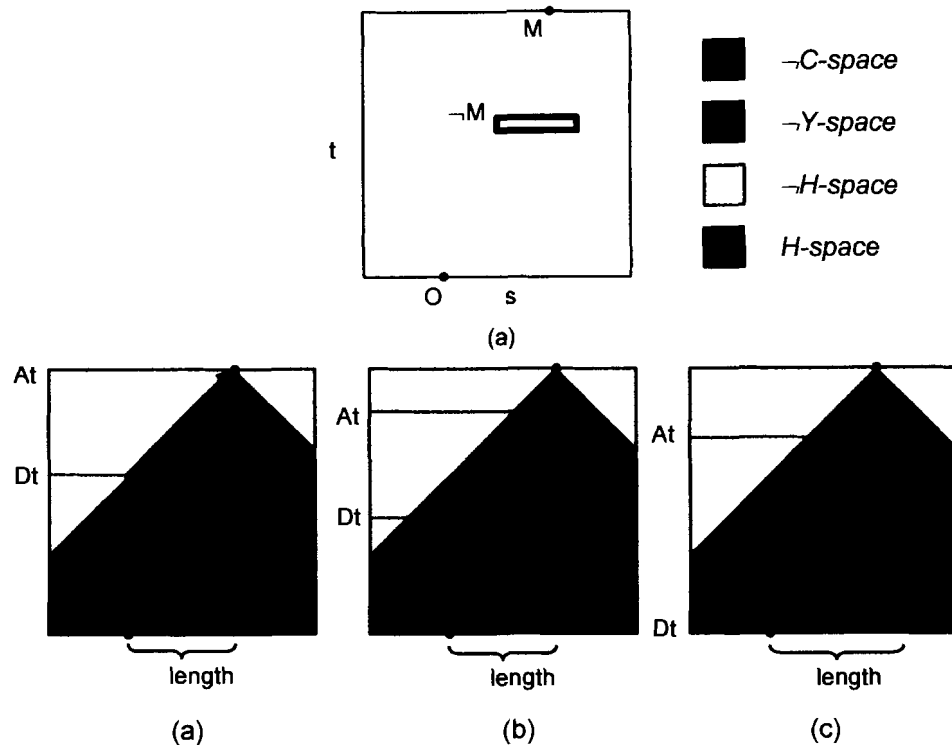


Figure 7-5: The effects of a temporary barrier on earliest arrival and shortest distance paths: (a) a scenario with a temporary barrier at the same location of a later requirement, (b) the latest departure path is unaffected, (c) the shortest distance path arriving as early as possible, and (d) absolute earliest arrival path.

7.3 Valid and Successful Paths

Scenarios with uncertainty produce up to fifteen travel possibility partitions, only three of which are classified as valid: the three sub-partitions of *may-space* (H , $\diamond\neg H$, and $\neg H$). Paths contained within *valid-space* (V) and remaining below the maximum travel speed are *valid paths* (Vp) (Figure 7-6a). On the other hand, *invalid space* ($\neg V$) specifies where travel is impossible. Paths intersecting *invalid-space* are *invalid paths* ($\neg Vp$) (Figure 7-6b). The remaining partitions indicate where travel is uncertain and is classified as *possibly invalid space* ($\diamond\neg V$). Paths intersecting *possibly invalid space* are *possibly invalid paths* ($\diamond\neg Vp$) (Figure 7-6c). The selection of *possibly invalid paths* should be avoided to ensure success, because the wayfinder may not be able to travel along these paths.

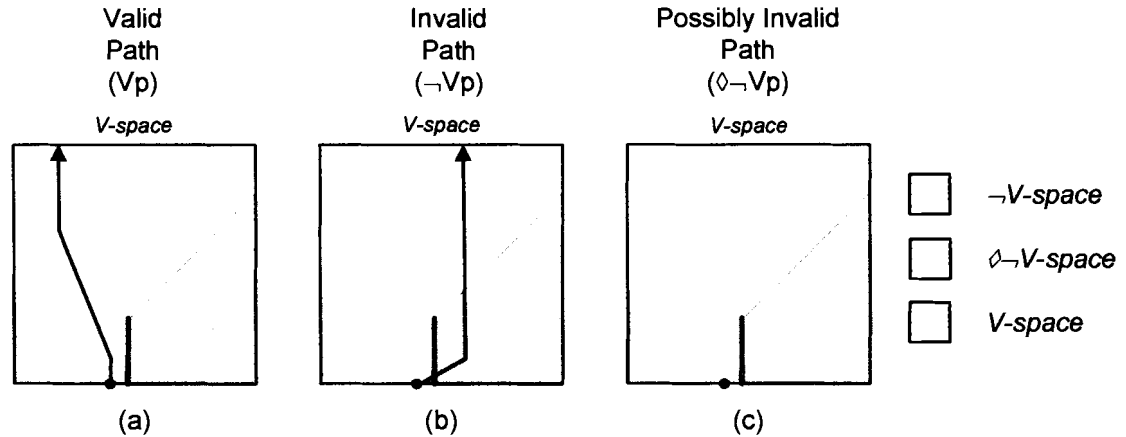


Figure 7-6: Paths through valid space (V -space): (a) valid path (V_p), (b) invalid path ($\neg V_p$), and (c) possibly invalid path ($\diamond \neg V_p$).

The selection of a *valid path* does not ensure a wayfinder can meet a scenario's requirements, it only ensures the wayfinder can follow the selected path. Classifying paths according to their ability to succeed in meeting requirements does, on the other hand, allow the wayfinder to choose successful paths. *Successful paths* (S_p) meet three conditions: (1) the path must be valid, (2) the path must intersect the scenario's requirements, and (3) the path must remain within *successful space* (S) (Figure 7-7a). *Successful space* is the portion of space-time where wayfinders create paths that meet requirements (Section 5.3). Of the three *valid space* partitions, only one—*H-space*—is also *successful space*. Wayfinders are sure that paths intersecting *unsuccessful space* cannot meet the scenario's requirements and are classified as *unsuccessful paths* ($\neg S_p$) and should not be selected (Figure 7-7b). In some scenarios, *possibly unsuccessful space* exists. *Possibly unsuccessful paths* ($\diamond \neg S_p$) intersect this space and wayfinders who select these paths are unsure about whether they will meet the scenario's requirements (Figure 7-7c).

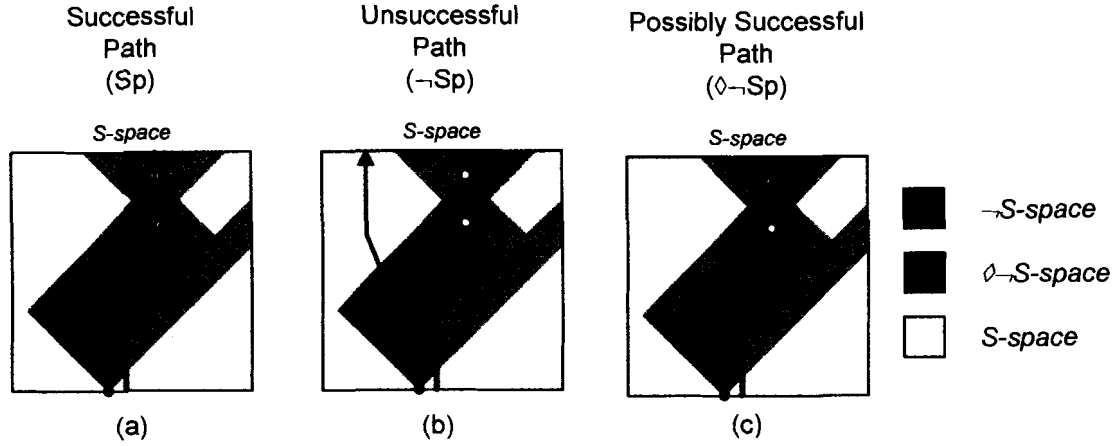


Figure 7-7: Paths through *successful space*: (a) successful path (Sp), (b) unsuccessful path ($\neg Sp$), and (c) possibly unsuccessful path ($\diamond \neg Sp$).

7.4 Measures of Accessibility to Possible Requirements

When continuous successful space does not exist in a scenario, as in the case of the *repairmen scenario* (Section 6.3.1), wayfinders are forced to select *possibly unsuccessful* paths. Unlike *successful* paths, which all have a 100% probability of success, *possibly unsuccessful* paths vary in their accessibility to requirement possibilities. Wayfinders, as a result, must have metrics to measure a path's accessibility to uncertain requirements.

7.4.1 Point Accessibility Metrics

Accessibility to possible requirements from a point in space-time is measured by calculating the intersection of the point's accessible space, $P_{(s,t)}^+$, with the possible requirement, $\diamond M$ (Equation 7.1):

$$a_{\diamond M}(s,t) = P_{(s,t)}^+ \cap \diamond M \quad (7.1)$$

With discrete uncertainty, the units of accessibility are count values. For example, the accessibility shown in Figure 7-8a is 2 discrete possibilities. Accessibility to requirements with

continuous uncertainty, on the other hand, is measured in units of space-time, for example, 4.5 km · min for the scenario in Figure 7-8b.

Dividing accessibility by the total uncertain requirement yields the percentage of requirement possibilities accessible from this point (Equation 7.2):

$$\alpha_{\diamond M}(s,t) = (P_{(s,t)}^+ \cap \diamond M) / \diamond M \quad (7.2)$$

The percentage of accessibility in the scenario shown in Figure 7-8a is 66%, while the continuous uncertainty scenario in Figure 7-8b results in an accessibility percentage of 50%.

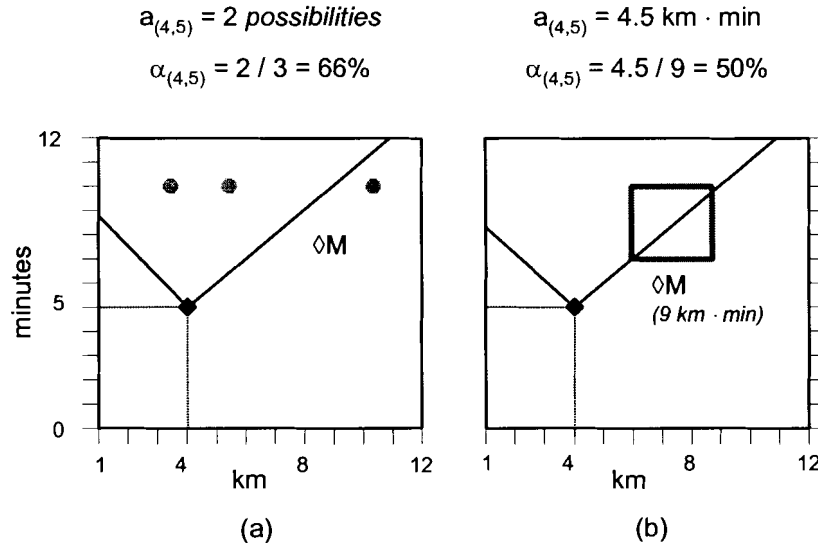


Figure 7-8: Calculation of accessibility to possible requirement locations: (a) discrete case, and (b) continuous case.

Calculating accessibility to a set of possible requirements for all points in space-time generates *accessible space* ($A_{\diamond M}$). Discrete uncertainty partitions space-time into equal values of accessibility (Figure 7-9a). Continuous uncertainty, on the other hand, creates a field of varying accessibility values (Figure 7-9b). These partitions and fields relate to successful space in that *successful space* (S) partitions have a value of 1, *unsuccessful spaces* ($\neg S$) have a value of 0, and *possible unsuccessful spaces* ($\neg S$) have values ranging between 0 and 1.

A path fully contained in *S-space* maintains 100% accessibility to requirement possibilities and ensures scenario success. Paths intersecting *possibly unsuccessful space*, on the other hand, do not maintain full accessibility and may fail in meeting a scenario's requirements.

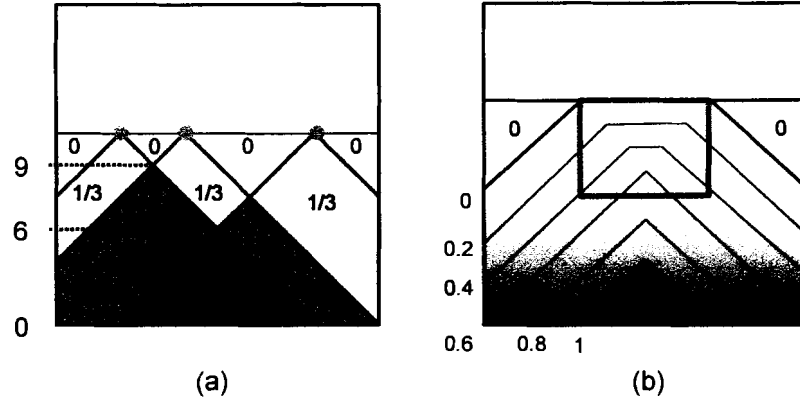


Figure 7-9: Accessibility of space-time to an uncertain requirement (A_{0M}): (a) the discrete case and (b) the continuous case.

7.4.2 Path Accessibility Metrics

For every time, t , there is a maximum accessibility value, $\alpha_{\max \diamond M}(t)$, within accessible space. For example, the scenario shown in Figure 7-9a yields 100% accessibility when $t = 6$ ($\alpha_{\max \diamond M}(6) = 1$) and 66% accessibility when $t = 9$ ($\alpha_{\max \diamond M}(9) = 2/3$). Paths that occupy locations with maximum accessibility throughout time are *accessibility maximization paths* ($p_{\max a_{0M}}$). Accessibility maximization paths are not necessarily unique for a wayfinding scenario. For instance, Figure 7-10 highlights two different accessibility maximization paths for the same scenario.

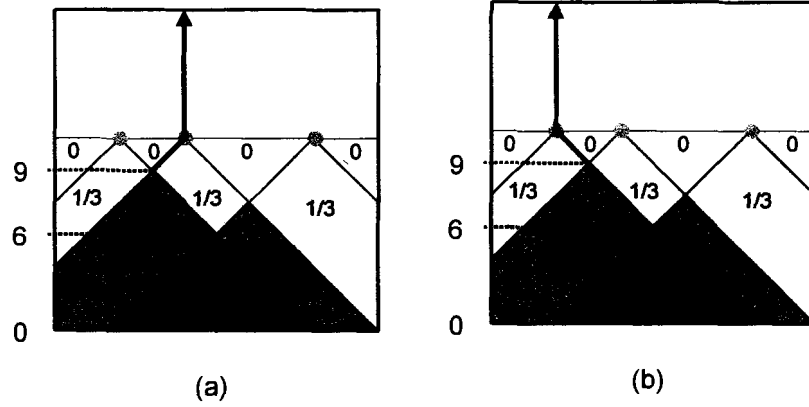


Figure 7-10: Accessibility maximization paths.

The measure of a path's accessibility at a moment of time is equal to the accessibility of the path's location at time t (Equation 7.3).

$$p\alpha_{\phi M}(t) = \alpha_{\phi M}(s, t) \text{ where } s \text{ is the path location at time } t \quad (7.3)$$

The ratio of any path's accessibility to the accessibility maximization path's value measures how closely the path comes to maximizing accessibility at that moment of time (Equation 7.4).

$$p\alpha_{\phi M}(t) / \alpha_{\max}(t) \quad (7.4)$$

7.4.3 Overall Path Accessibility Metrics

In addition to measuring accessibility at moments of time, wayfinders can measure a path's overall accessibility. For discrete uncertainty, the overall accessibility metric equals the sum of the path's accessibility over the time (Equation 7.5).

$$pa_{\phi M} = \sum p\alpha_{\phi M}(t) \quad (7.5)$$

When uncertainty is continuous, an integral yields the overall accessibility value (Equation 7.6).

$$pa_{\phi M} = \int p\alpha_{\phi M}(t) \quad (7.6)$$

The ratio of a path's overall accessibility to the *accessibility maximization path's* overall value is a single metric indicating how close a path, as a whole, comes to maximizing accessibility (Equation 7.7).

$$pa_{\phi M} / p_{max}a_{\phi M} \quad (7.7)$$

7.4.4 Path Accessibility Metrics for a *Repairmen Scenario*

To demonstrate the use of path accessibility metrics the *repairmen scenario* (Section 6.3.1) is analyzed. Other scenarios, including the *police officer scenario*, are handled in likewise manner. The *repairmen scenario* occurs along a road segment 12 kilometers long lasting 12 minutes. The maximum speed is 1 km/minute and the start point is at the 3 km point. There are no barriers and the initial requirement is 9 km down the road in 12 minutes (Figure 7-11a). The *repairmen scenario* includes uncertainty in whether a second requirement is added some time and place before the initial requirement (Section 6.3.1). The primitive values generate a travel possibility space (Figure 7-11b), and this space is generalized into *successful space* (Figure 7-11c).

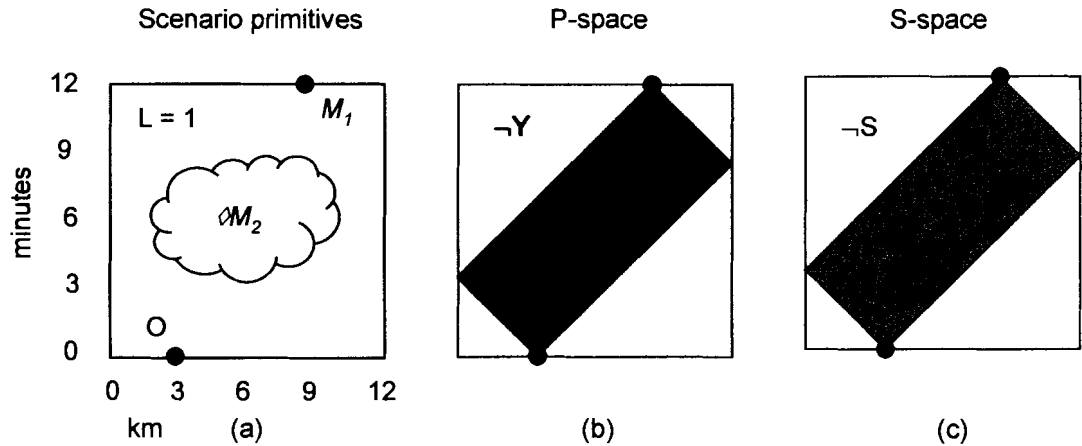


Figure 7-11: A *Repairmen Scenario*: (a) scenario primitives, (b) resulting travel possibility space (*P-space*), and (c) generalized successful space (*S-space*).

Minimizing the arrival time generates an *early bird path*, while maximizing departure time creates a *procrastinator path*. In both cases, these paths are *possibly unsuccessful paths* ($\diamond \rightarrow S$), because they intersect *possibly unsuccessful space* (Figure 7-12).

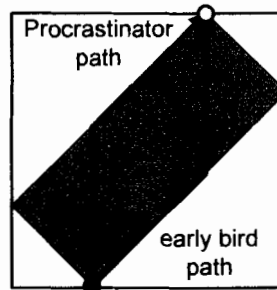


Figure 7-12: Early bird and procrastinator paths for a *repairmen scenario*.

Because of the spatial and temporal uncertainties with the possible additional requirement, the wayfinder is forced to select *possibly unsuccessful paths* and cannot be sure of success. The question to ask becomes, “which path offers the highest probability of success?” To answer this question, an accessibility space (A) is generated for the scenario (Figure 7-13a) and from it, an accessibility maximization path (Figure 7-13b).

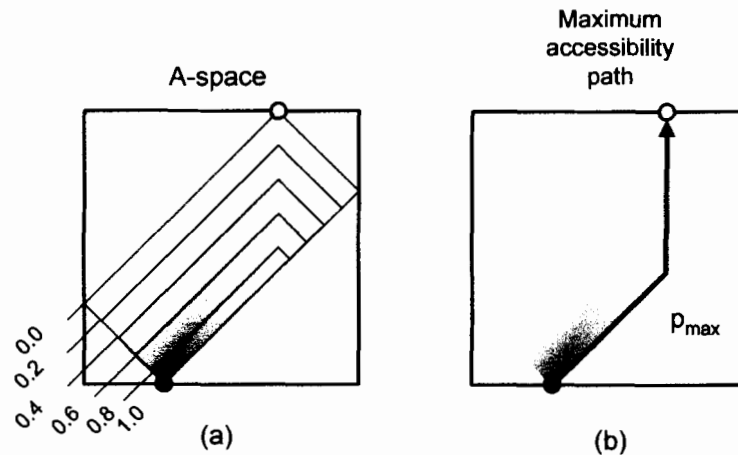


Figure 7-13: Accessibility maximization for the *repairmen scenario*: (a) *accessibility space (A-space)*, and (b) *maximum accessibility path*.

The *early bird* and *procrastinator* paths can now be compared to the accessibility maximization path over time. A plot of accessibility for each path and the overall amount of possible requirements still in the future are shown in Figure 7-14, along with the total amount of possible requirement space (*H-space*). From this plot it can be seen that the *early bird path* equals the accessibility maximization path through time. In addition, the *early bird path* has more accessibility than the *procrastinator path* at all times.

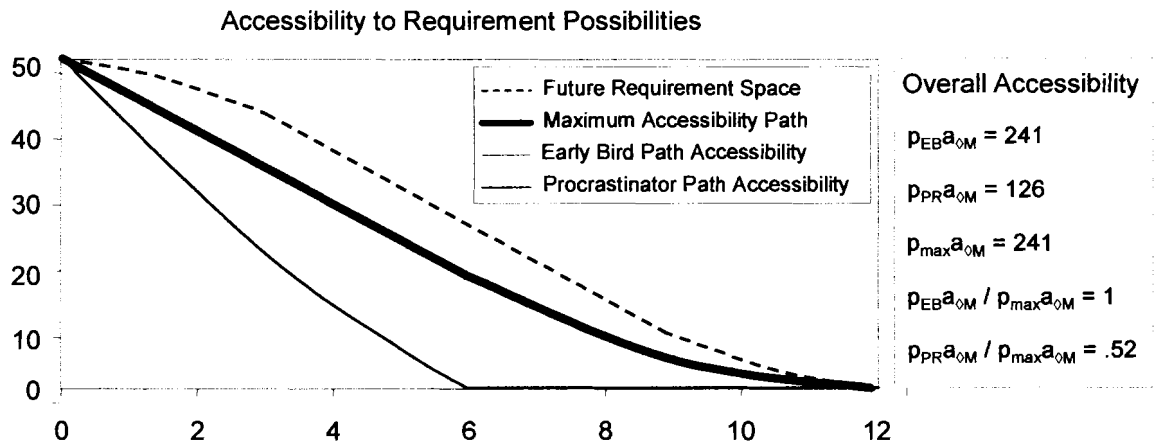


Figure 7-14: Accessibility to requirement possibilities.

The overall path accessibility values yield values for the *early bird path* of 241, the *procrastinator path's* overall accessibility is 126, and the accessibility maximization path is 241. The ratio of the *procrastinator path* to the accessibility maximization path is 52%. The *early bird path's* ratio is 1, indicating that the early bird path maximizes accessibility to possible requirements in this version of the *repairmen scenario*.

These data provide motivation for the thesis hypothesis, “arrival minimization paths (*early bird paths*) also maximize accessibility to possible additional requirements.” In Chapter 8, this hypothesis is tested further in various permutations of the *repairmen scenario*.

7.5 Summary

This chapter described the selection of paths through travel possibility partitions. After first considering space-time path characteristics, three min-max paths were identified, one of which is the focus of the hypothesis: the arrival minimization path, or *early bird path*. The chapter also highlighted the distinction between the two generalized spaces, *valid space* (V) and *successful space* (S). Broad boundary classifications of these spaces, *possibly invalid space* and *possibly not successful space*, result from uncertainties in wayfinding primitives.

To be sure of success, wayfinders select paths in *success space*, however, in many scenarios, continuous *successful space* does not exist, so that *possibly unsuccessful* paths must be selected. In these cases, wayfinders are unsure of success and one needs metrics to select the paths with the greatest probability of success. As a result, a set of accessibility metrics was presented and used to assess paths in a *repairmen scenario*. The data from this one scenario upheld the thesis's hypothesis that arrival minimization paths also maximize accessibility to possible additional requirements.

CHAPTER 8

EVALUATION WITH A PROTOTYPE

The development of a prototype Travel Possibility Calculator (TPC) provides a hypothesis testing mechanism and demonstrates the utility of creating and visualizing travel possibility partitions. The prototype models a generalized 2-dimensional wayfinding space with a discrete 3D voxel-based spatio-temporal data structure. In this way, an integrated space-time framework is achieved that is capable of modeling an assortment of dynamic and uncertain spatial-temporal wayfinding scenarios.

Wayfinding scenarios are established by defining values for the maximum speed, start point, barriers, and requirements. Based on these primitives, an algorithm partitions space-time into four basic travel possibility categories (Section 4.5) and, if uncertainty is present, up to eleven additional broad boundary partitions (Section 5.3). The prototype displays the distribution of travel possibility partitions through space and time with various visualization tools, including time sliced planar maps, time series accessibility graphs, and summary tables. Given a partitioned travel possibility space, separate algorithms create three paths between the start point and a single point requirement: (1) the arrival minimization, *early bird*, path, (2) the departure maximization, *procrastinator*, path, and (3) a random path.

To answer the hypothesis, “do arrival minimization paths also maximize accessibility to uncertain additional requirements?” the prototype exhaustively calculates, from every point in space-time, accessibility to all possible requirements. These calculations yield an *accessibility space* that can produce maximum accessibility paths. The hypothesis test compares the *early bird path* in various linear and planar scenarios against the maximum accessibility path. If the *early bird path* equals the maximum accessibility path in all scenarios the hypothesis is accepted. If the *early bird path* deviates significantly, the hypothesis is rejected.

This chapter continues with a basic explanation of the prototype (Section 8.1), with detailed descriptions of four critical components: (1) the space-time voxel data structure, (2) the possibility volume creation algorithm, (3) travel possibility partitioning algorithm, and (4) path creation algorithms. This portion is followed by a description of the evaluation procedure that includes psuedo-code of the evaluation algorithm and a description of the testing scenarios (Section 8.2). Section 8.3 presents the results of the evaluation, and Section 8.4 provides a general discussion of results.

8.1 Prototype

Developing a prototype Travel Possibility Calculator (TPC) provides an exploratory tool of the travel possibilities and paths associated with generic wayfinding scenarios, and at the same time creates a mechanism for hypothesis evaluation. The prototype was written as a stand-alone application in Microsoft Visual Basic 6.0 on a Dell Latitude C810 laptop PC computer.

A single scenario session begins by setting the values for the four wayfinding primitives: (1) the wayfinders maximum travel speed, (2) the start location and time, (3) a set of barriers as realized with a friction volume, and (4) a set of requirements. An accessibility algorithm partitions space-time into the four primary travel possibility categories ($\neg C$, $\neg Y$, $\neg H$, and H). With these partitions, the *early bird*, *procrastinator*, and random paths are generated. The user explores the distribution of travel possibility partitions by viewing planar time-slice maps. These maps color code each voxel according to either its travel partition category (Figure 8-1) or voxel accessibility rating (Figure 8-2). To further explore the travel possibilities of the wayfinding scenario, the user can view various time series graphs, indicating partition variability over time (see the upper right-hand corner of Figure 8-1).

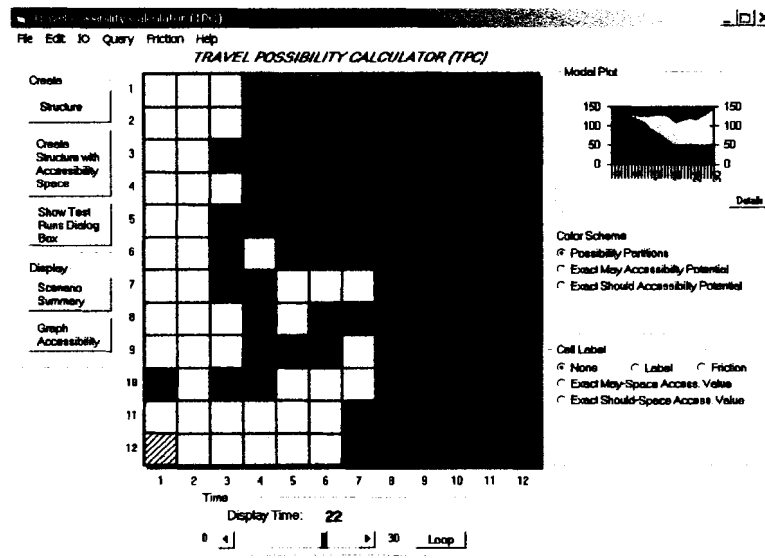


Figure 8-1: Time-slice map with voxels color-coded according to travel possibility partitions.

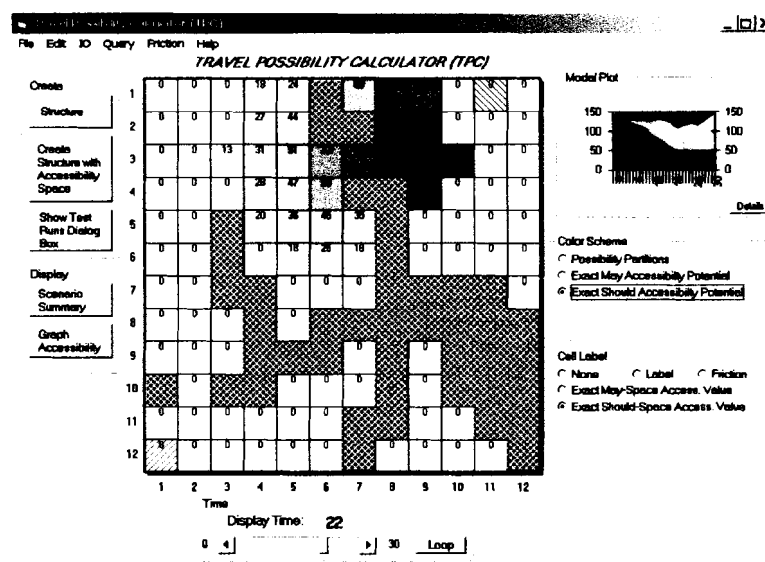


Figure 8-2: Time-slice map with voxels color-coded according to accessibility to *should-space* where darker green represents greater accessibility to possible requirements in *should-space*.

To implement partitioning and path selection concepts into the prototype, four critical concerns are addressed. The first consideration is the selection of an appropriate data structure to model the dynamic space-time of wayfinding scenarios. With a spatio-temporal data structure chosen, the next step is the implementation of a suitable accessibility algorithm, which is called repeatedly by an overall partitioning algorithm. Working together, these two algorithms create travel possibility partitions of individual wayfinding scenarios. From this partition space, path generation algorithms were developed that produce the *early bird* and *procrastinator paths*. Each process is addressed in the next four sections.

8.1.1 Voxel-Based Space-Time Data Model

Voxel-based volumetric data models have been found to be an effective method of representing changes in 3-D space (Samet 1990; Chen *et al.* 2000; Kaufman 2000). Consequently, a discrete voxel-based spatio-temporal data structure functions as the prototype's fundamental framework. This decision is similar to Forer's (1998) use of voxels to represent a space-time cube in his work with time geography. This 3-D voxel representation is composed of a 12 x 12 tessellation of space and an orthogonal dimension of up to 101 discrete time units for a maximum of 14,544 voxels. This small representation of space was determined to be a large enough to demonstrate the concept of space-time travel possibility partitions and allowed faster processing.

Though problems with travel calculations over spatial grids has long been addressed (Goodchild 1977), movement calculations through 3-D voxel space still typically focus on movement from one cell to its immediate 26 neighbors (Scott 1994). In these spatial approaches, the time of travel between an origin voxel and its neighbors is used in various path selection algorithms.

When calculating paths through voxels of space-time, sometimes referred to as taxels (Forer 1998), a number of unique challenges arise. No longer are all 26 adjacent cells connected. Assuming instantaneous travel is not allowed, only nine voxels—one time increment in the

future—and nine voxels backward in time are connected to any cell. Restricting travel to these 18 connections hard wires the wayfinder’s speed into the data structure. For example, if the spatial resolution (i.e., size of voxels in the two spatial directions) is 10 meters and the temporal resolution (voxel size in the temporal direction) is 1 minute, the wayfinder’s speed is set at 10 meters per minute. This is not an acceptable limitation and further methods of representing movement in this space-time volume must be considered.

One alternative permits non-adjacent voxel connections forward (and backward) in time (Figure 8-3a). These additional connections allow slower travel speeds, but the discrete nature of the data structure still restricts movement to a small subset of possible speeds. This restriction is particularly apparent at higher values (Figure 8-3b).

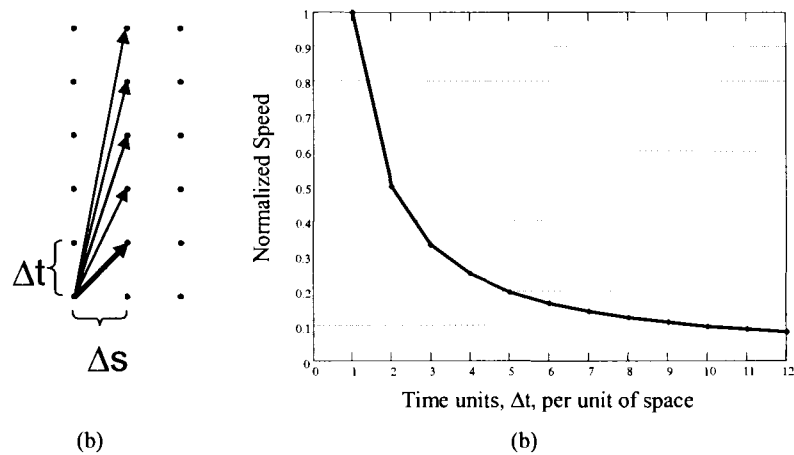


Figure 8-3: Space-time voxel speed limitations: (a) five possible space-time voxel connections and (b) normalized speed associated with various connections.

To work around this restriction, this thesis implements a methodology that maintains a regular spaced voxelization of space-time, but stores with each voxel a *waiting time* value to model a wider range of travel speeds. Calculating a movement vector between origin and destination voxels, and storing the additional time required to reach the destination as a *wait time*, permits the modeling of various travel speeds (Figure 8-4). Subsequent calculations from this voxel must add the *wait time* to the overall travel time to other destination voxels (Figure 8-4).

This extension to the simple voxel representation of space-time accommodates a full range of speeds and yields a more realistic representation of a wayfinder's movement.

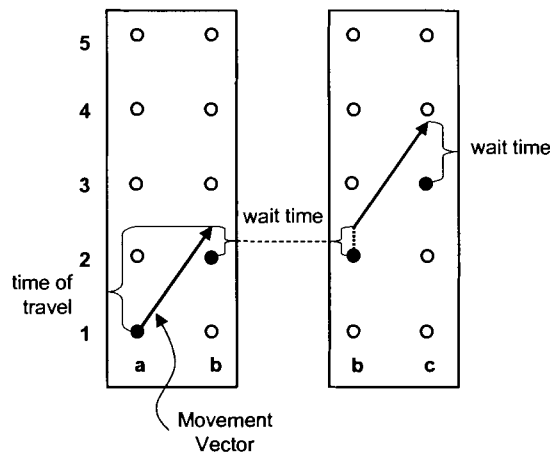


Figure 8-4: The use of *wait times* to model various travel speeds.

8.1.2 Accessibility Algorithm

The algorithm calculating accessibility forwards and backward in time from an origin is a critical component of this prototype. Accessibility is calculated from a voxel or set of voxels with a modified spread function (Xu and Lathrop 1995). The ordered nature of the temporal dimension ensures that a path beginning at time t and ending at $t+2$ must also exist at some location at $t+1$. This characteristic allows the algorithm to divide the potentially large single calculation of accessibility into many small accessibility calculations between each time increment.

The algorithm (Figure 8-5) begins by determining the earliest arrival time from the origin at time t to all possible voxels at $t+1$. The algorithm then determines the earliest arrival times from accessible voxels at $t+1$ to all possible voxels at $t+2$. This process continues until reaching the scenario's time limit. Voxels not reached during this procedure have a null value for earliest arrival time and are classified as inaccessible. The result of this algorithm is the labeling of each voxel in space-time with accessibility values as measured by earliest arrival times.

Algorithm: Basic Accessibility Algorithm

```

for t = startPoint.t to timeLimit do
  for each voxel, fromVoxel, in t do
    for each connected voxel, toVoxel, in t+1 do
      waitTime = timeofTravel(fromVoxel,toVoxel) + fromVoxel.waitTime - 1
      if (waitTime < toVoxel.waitTime and waitTime < 1) then
        toVoxel.waitTime = waitTime
      end if
    next toVoxel
  next fromVoxel
next t

```

Figure 8-5: Basic accessibility algorithm.

8.1.3 Travel Possibility Partitioning Algorithm

Travel possibility partitions are modeled with binary masks populated with the accessibility algorithm (Section 8.1.2) (Figure 8-6). Accessible voxels receive a value of 1, while inaccessible voxels are given a value of 0. The algorithm's first phase begins with the creation of individual accessibility masks, which are the realizations of the three basic travel possibility spaces (*C-space*, *Y-space*, and *H-space*). The *C-Mask* is populated with accessibility from the start point and the assumption that temporary barriers do not exist. In a similar manner, the algorithm creates the *Y-Mask*, except that all temporary barriers block travel as defined in the wayfinding scenario. Each requirement possibility populates a separate *H_r-Mask* by calculating accessibility both backwards and forwards in time from the requirement. These separate *H_r-Masks* combine into an overall *H-Mask* representing the scenario's requirement-based constraints.

The second phase of the algorithm combines individual masks in a manner similar to map algebra (Tomlin 1990) (Figure 8-6). The *C-Mask* and *Y-Mask* combine to model the first two steps in the sequential partitioning process (Section 4.5) and produce a *CY-mask* with three possible values ($\neg C$, $\neg Y$, and Y). The *CY-mask* then combines with the overall *H-Mask*, resulting in the partitioning of space-time into four travel possibility partitions ($\neg C$, $\neg Y$, $\neg H$, and H).

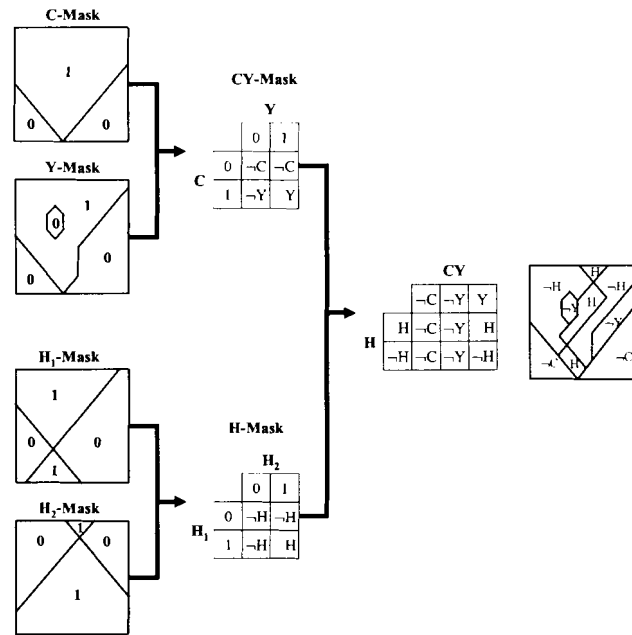


Figure 8-6: Travel possibility partitioning scheme.

8.1.4 Early Bird and Procrastinator Path Generation Algorithms

The *early bird path* minimizes arrival times by traveling to requirements as soon as possible. To model this behavior, the *early bird path* generation algorithm works backwards in time from the destination, and requires the calculation of earliest arrival values for each voxel, from the start point forward in time. The algorithm begins by assuming it is located at the requirement and considers voxel accessibility for the previous time increment, $t-1$. The algorithm first checks whether the occupied location at t is also accessible at $t-1$. If so, the voxel at $t-1$ is the path's next location. This check ensures the *early bird path* arrives as soon as possible to voxels along the path. If the occupied location at $t-1$ is not accessible, the algorithm selects the voxel with the earliest arrival time at $t-1$ as the path's next location. The algorithm continues in this way until reaching the start point, whereupon the path location at each time increment is determined.

```

Algorithm: Create early bird path
//set path location at time of requirement to requirement location
pathLocation(reqtTime) = reqtLocation
//Loop through time
for t = endpoint.t to (startPoint.t-1) step -1 do
    //check same location at t-1
    if A(pathLocation(t),t-1) = accessible then
        pathLocation(t-1) = pathLocation(t)
    else
        //from accessible voxels, find minimum earliest arrival at t-1
        pathLocation(t-1) = minimum wait time of connected voxel at t-1
    end if
next t

```

Figure 8-7: *Early bird path* generation algorithm.

The *procrastinator path* generation algorithm employs the same strategy as the *early bird path* algorithm, but in the temporally opposite direction. This algorithm requires the earliest arrival values from the requirement backwards in time, and begins at the start point. The algorithm attempts to remain at the same spatial location over time, and when forced to move, chooses the voxel with the earliest arrival time. The result of this algorithm is a path that remains where it is as long as possible until forced to travel to the requirement.

8.2 Evaluation Procedure

The procedure for testing the hypothesis compares the *early bird path* to the path maximizing accessibility. The maximum accessibility path is derived through an exhaustive calculation of accessibility from every point in space-time to *should-space (repairmen scenario)*. For comparison, the evaluation also considers the *procrastinator path* and a random path. Comparisons are made in various linear and planar scenario configurations. If the *early bird path* equals the maximum accessibility path in all cases, the hypothesis is accepted. If however, the *early bird path* does not equal the maximum accessibility path, the hypothesis is rejected and alternate explanations must be considered.

8.2.1 Evaluation Algorithm

The hypothesis evaluation process measures *early bird path* accessibility in numerous wayfinding scenarios. For each scenario, the partition algorithm creates travel possibility space and three paths: *early bird path* (p_{eb}), *procrastinator path* (p_{pr}), and a random path (p_{ra}). These paths generate accessibility metrics, which are used to measure the closeness of fit to the maximum accessibility path (Section 7.4). The algorithm used to test scenarios is shown as pseudo-code in Figure 8-8.

Algorithm: Testing Algorithm

```
for each scenario do
  p-space = CreatePspace(scenarioPrimitives)
  for t = start.time to requirement.time
    //total future should voxels
     $T_H(t) = \text{CalcTotalVoxels}(H\text{-space}, t)$ 
  next t
  //create paths
   $p_{EB} = \text{CreateEarlyBirdPath}(pSpace)$ 
   $p_{PR} = \text{CreateProcrastinatorPath}(pSpace)$ 
   $p_{Ra} = \text{CreateRandomPath}(pSpace)$ 
  for each path ( $p_{EB}, p_{PR}, p_{Ra}$ ) Do
    for t = start.time to requirement.time Do
      //create path accessibility
       $pa_H(t) = \text{CalcPathAccessibility}(hSpace, t)$ 
    next t
  next path
  for every point ( $x, y, t$ ) in ST Do
    //create accessibly space
     $A_H(x, y, t) = \text{CalcAccessibility}(hSpace, x, y, t)$ 
  next point
  //create maximum accessibility path
   $p_{max}a_H = \text{CreateAccessibiiltyMaxPath}(MaxA_H)$ 
  For paths ( $p_{EB}, p_{PR}, p_{Ra}, p_{Max_H}, p_{Max_Y}$ ) Do
    //create path overall accessibility
     $pa_H = \text{sumAccessibilityOverTime}(\text{path})$ 
  next path
  for paths ( $p_{EB}, p_{PR}, p_{Ra}$ ) Do
    //create accessibility ratios
     $pa_H / T_H$ 
     $pa_H / p_{max}a_H$ 
  next path
next scenario
```

Figure 8-8: Hypothesis Evaluation algorithm.

8.2.2 Wayfinding Scenarios

The evaluation of path accessibility occurs in a representative set of linear (1-dimension) and planar (2-dimension) wayfinding scenarios. The linear scenarios are modeled with a 1 x 11 spatial grid and the planar scenarios with an 11 x 11 grid. In all cases the start point begins when time = 0 and there exists one point requirement at various locations and times. In some scenarios, temporary barriers exist, while in other scenarios the barriers are static.

8.2.2.1 Linear Scenarios without Barriers

Linear scenarios restrict the wayfinder's movement to a line segment bound at both ends (B_1 and B_2). A central point (C) exists at the line's midpoint when friction is uniform. Considering these three key locations generates five scenarios. In all cases, the scenario is tested with requirement times of $t = \{5, 10, 15, 20, 25, \text{ and } 30\}$.

- **Remain at Center ($C_0 \rightarrow C_t$):** Often ignored as a trivial wayfinding scenario, a requirement at the same location as the start point, but later in time, has the potential to produce interesting results. This first scenario considers remaining at the center (Figure 8-9a).
- **Remain at Boundary ($B_0 \rightarrow B_t$):** As opposed to remaining at the center, in this scenario the wayfinder remains at a boundary (Figure 8-9b).
- **Start at Boundary and Travel to the Center ($B_0 \rightarrow C_t$):** In most cases requirements are not at the same location as the start point. One simple scenario is where the wayfinder begins at a boundary and travels to the center (Figure 8-9c).
- **Start at the Center Travel to Boundary ($C_0 \rightarrow B_t$):** The opposite case also exists where the wayfinder begins at the center and moves to the boundary (Figure 8-9d).

- **Start at One Boundary and Travel to Opposite Boundary ($B1_0 \rightarrow B2_0$):** The last barrier-free linear scenario tests path accessibility when traveling through the center while moving from one boundary to another (Figure 8-9e).

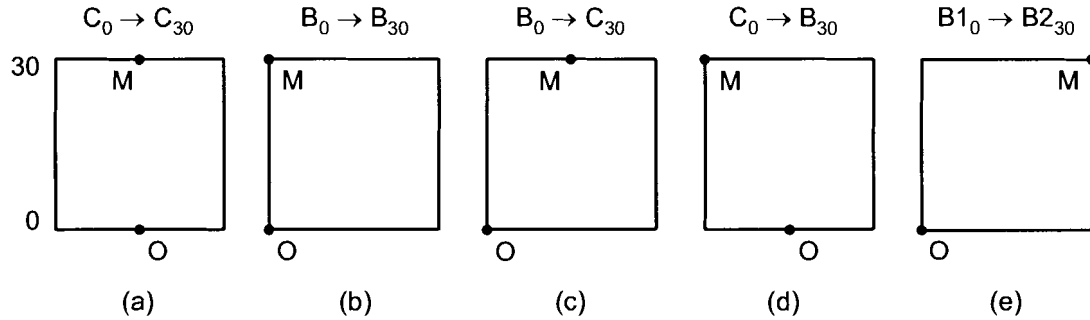


Figure 8-9: Barrier-free linear wayfinding scenarios with a requirement at $t = 30$: (a) remain at the center, (b) remain at a boundary, (c) move from a boundary to the center, (d), move from the center to a boundary, and (e) move from one boundary to another.

8.2.2.2 Linear Scenarios with Temporary Barriers

The next set of linear scenarios introduces temporary barriers at various locations and times. These scenarios assume a start point at one boundary and a requirement at the other with a point barrier existing at either the center, near the start point, or near the requirement. The barriers can begin and end their existence at various times ($B1_0 \rightarrow \neg M_{(x,m,n)} \rightarrow B2_{30}$), where the triplet (x,m,n) indicates a barrier's position, start time and end time. In all cases the time of the requirement is $t = 30$.

- **Barrier Existing Until Some Time ($B1_0 \rightarrow \neg M_{(x,0,n)} \rightarrow B2_{30}$):** The scenario with a barrier until some time is tested with three barrier locations and two ending times, which results in six tests (Figure 8-10).

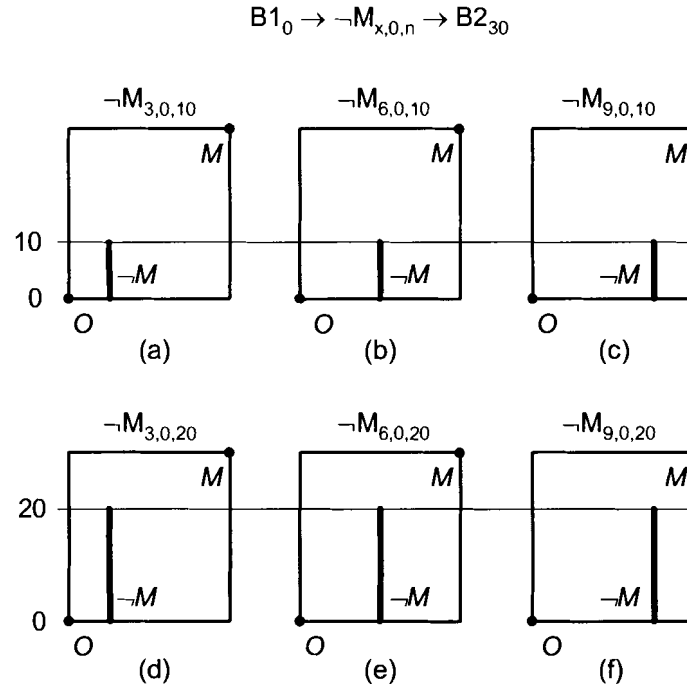


Figure 8-10: Linear scenario with a barrier until some time: (a) barrier near start until $t = 10$, (b) barrier in center until $t = 10$, (c) barrier near requirement until $t = 10$, (d) barrier near start until $t = 20$, (e) barrier in center until $t = 20$, (f) barrier near requirement until $t = 20$.

- **Barrier Existing After Some Time ($B1_0 \rightarrow \neg M_{(x,m,30)} \rightarrow B2_{30}$):** In some cases, barriers initially do not exist, but then become active. This scenario tests the same six spatial-temporal configurations as with barriers existing until some time (Figure 8-11).

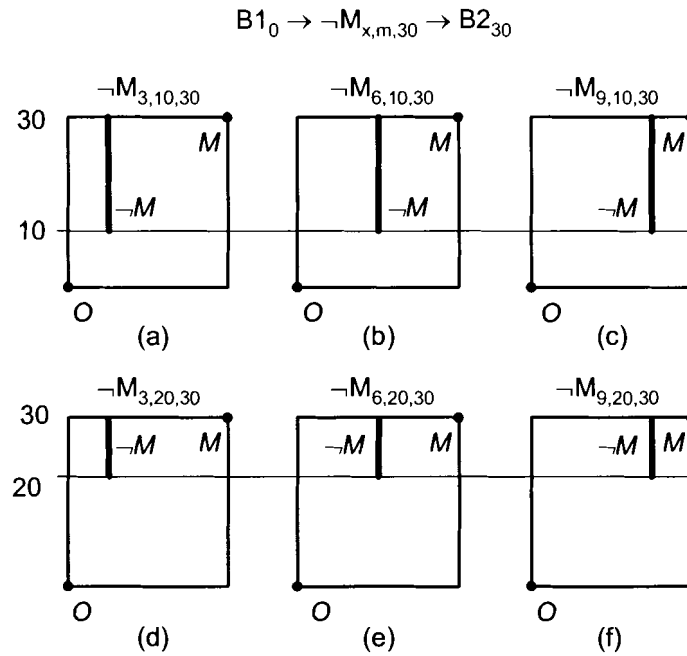


Figure 8-11: Linear scenario with a barrier after some time: (a) barrier near start after $t = 10$, (b) barrier in center after $t = 10$, (c) barrier near requirement after $t = 10$, (d) barrier near start after $t = 20$, (e) barrier in center after $t = 20$, (f) barrier near requirement after $t = 20$.

- Barrier Existing From Some Time until Later (L3c):** An even more complex environment arises with intermittent barriers: barriers that begin and end during the scenario. This scenario includes a barrier located at the central place whose beginning and end are variable. The barrier can begin to exist at $t = 5$ and end as late as $t = 25$. Using increments of five time units, ten temporal intervals result.

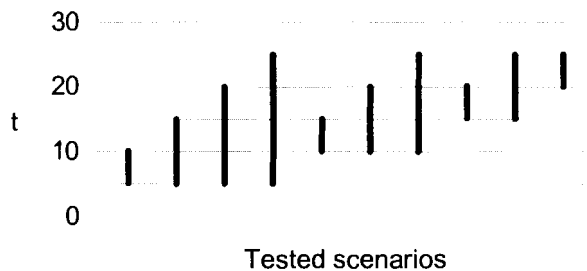


Figure 8-12: Ten tested intermittent barriers.

8.2.2.3 Planar Wayfinding Scenarios

Extending the wayfinder's environment into a second dimension creates more realistic and complicated scenarios. The first set of tests in a planar environment re-evaluates a sample of the linear scenarios to determine if one-dimensional findings scale up to two dimensions.

Barriers in linear environments always block travel to some portion of space. This is not necessarily the case in two-dimensions. A river with a bridge, for example, does not block travel to the other side, but does effect travel times. A linear barrier's impact on travel is dependent on its location and the position of any gap in the barrier.

- **Linear Barrier with Static Gap:** In these scenarios, a linear barrier cuts across a spatial region of interest, dividing space into two regions. A permanent gap exists that allows travel between the two sides. Various configurations of barrier location and gap position test the impact of these changes. In one case, the linear barrier partitions space in half, in the second case it partitions space so one side of the barrier is half the size of the other, and in a third case one side of the partition is a small corridor. The gap in this barrier can be in the center or at one end. For each configuration, six possible locations exist for either the start point or requirement (Figure 8-13). Thirty-six different start point and requirement combinations are possible from these six locations, for example, *remain at a* or *travel from a to e*. The six barrier configurations and these thirty-six combinations create 216 possible scenarios.

2-Dimensional Scenario linear barrier with a gap

$W_m G_n$ where m = barrier position, n = gap position

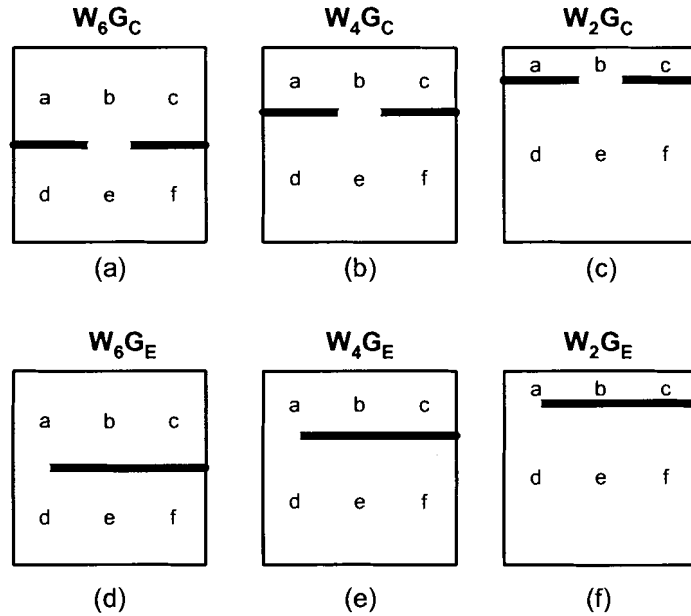


Figure 8-13: Various linear barriers with gap configurations; six possible start and requirement locations are identified in each.

8.3 Hypothesis Evaluation

The hypothesis considers path selection where wayfinders arrive as soon as possible to requirements, and wondering whether this strategy provides the greatest chance of also meeting a second requirement added somewhere beforehand. The hypothesis, as stated in Section 1.6, is as follows:

“Paths that minimize arrival time to requirements also maximize accessibility to possible additional requirements.”

The evaluation focuses on the *repairmen scenario*, where a wayfinder has a known requirement, but may have another one added before hand. The evaluation begins by first conducting tests in a barrier-free linear environment where the location and time of either the start point or the requirement change. The evaluation continues by introducing temporary barriers at

various locations and times. The scope of testing expands into the second dimension by first re-evaluating a subset of the linear scenario configurations. The next set of tests explores various configurations of gaps within linear barriers, where the location of both the linear barrier and gap change in each scenario. The final set of tests consider the impact of changing the time when gaps open and close. All scenarios employ the accessibility metrics introduced in Section 7.4, and if the *early bird path* maximizes accessibility in all cases, the hypothesis is supported.

8.3.1 Linear Scenarios without Barriers

Path accessibility tests for the six linear scenarios without barriers produced mixed results. In general, the *early bird path* maximized accessibility to possible additional requirements only when remaining at or moving towards the center (Table 8-1). A number of specific observations can be made from the data.

Observation 1: When time is limited, path choice is also limited and, as a result, the early bird path maximizes accessibility.

When the requirement time is equal to the earliest arrival time, all paths must depart immediately and travel at maximum speed to the requirement. This situation results in the creation of minimal amounts of *should-space* and a very inflexible scenario. The only additional requirements a wayfinder can meet in these scenarios are those directly on the path to the initial requirement.

Observation 2: Early bird paths fail to maximize accessibility when moving towards or remaining at a boundary.

When moving away from the center, the *early bird path* failed to maximize accessibility. When time is limited ($C_0 \rightarrow B_s$) the *early bird paths* maximized accessibility (*Observation 1*), but as the time of the initial requirement increases the percentage of maximum accessibility afforded by the *early bird path* decreases until becoming steady at 88%.

The *procrastinator path*, by staying longer at the center, outperforms the *early bird path* when traveling from the center to the boundary. The same pattern holds when remaining at a

location: the *early bird path* maximizes accessibility when remaining at the center, but fails to maximize accessibility when remaining at the boundary.

Observation 3: A path's percentage of maximum accessibility often varies over time.

A wayfinder moving through the center from one boundary to the other produces interesting results when viewed over time (Figure 8-14). Initially the *early bird path* maximizes accessibility as it moves towards the center. As the path continues past the center, however, it no longer maximizes accessibility. When the *early bird path* arrives at the requirement (point b in Figure 8-14a), its accessibility equals the *procrastinator path*. The two paths have equal accessibility because both are currently located at a boundary. As time progresses, however, the *procrastinator path's* percentage of maximum accessibility drops precipitously until it rises again as the *procrastinator* begins moving towards the requirement (point d in Figure 8-14a).

Observation 4: Accessibility maximization paths travel as quickly as possible to the center, remain there until possibilities begin to decrease, and then slowly move to the requirement.

When considering the movement from one boundary to another, the accessibility maximization path stops at the center (point a in Figure 8-14a). Maximum accessibility is maintained by remaining at this location, until forced to move again (point c in Figure 8-14a). The departure time is related to accessibility to the latest departure time from the start point: the time the procrastinator begins to move (point d in Figure 8-14a). When the accessibility maximization path does begin moving, it does so at a slow steady rate.

Hypothesis Test Summary						
Linear Scenarios without Barriers		Should-Space Accessibility				
		Total Voxels Summed over Time	Maximum Accessible Voxels Summed over Time	Max Voxels Accessible Ratio		
				Early Bird Path	Procrastin- ator Path	Random Path
<i>Remain at Center</i>	$C_0 \rightarrow C_5$	63	47	1.00	1.00	0.91
	$C_0 \rightarrow C_{15}$	986	726	1.00	1.00	0.96
	$C_0 \rightarrow C_{30}$	4,496	3,786	1.00	1.00	0.89
<i>Remain at Boundary</i>	$B_0 \rightarrow B_5$	42	36	0.94	0.94	0.86
	$B_0 \rightarrow B_{15}$	612	512	0.87	0.87	0.93
	$B_0 \rightarrow B_{30}$	3,696	3,176	0.87	0.87	0.93
<i>Boundary to Center</i>	$B_0 \rightarrow C_5$	21	21	1.00	1.00	1.00
	$B_0 \rightarrow C_{10}$	267	219	1.00	0.70	0.91
	$B_0 \rightarrow C_{15}$	836	671	1.00	0.75	0.95
	$B_0 \rightarrow C_{20}$	1,731	1,416	1.00	0.80	0.96
	$B_0 \rightarrow C_{25}$	2,901	2,436	1.00	0.83	0.89
	$B_0 \rightarrow C_{30}$	4,346	3,731	1.00	0.86	0.96
<i>Center to Boundary</i>	$C_0 \rightarrow B_5$	21	21	1.00	1.00	1.00
	$C_0 \rightarrow B_{10}$	237	189	0.97	0.81	0.97
	$C_0 \rightarrow B_{15}$	711	546	0.93	0.93	0.97
	$C_0 \rightarrow B_{20}$	1,481	1,166	0.89	0.97	0.93
	$C_0 \rightarrow B_{25}$	2,526	2,061	0.88	0.98	0.98
	$C_0 \rightarrow B_{30}$	3,846	3,231	0.88	0.99	0.92
<i>Boundary to Boundary</i>	$B_{10} \rightarrow B_{2_{10}}$	66	66	1.00	1.00	0.17
	$B_{10} \rightarrow B_{2_{15}}$	561	481	0.99	0.59	0.93
	$B_{10} \rightarrow B_{2_{20}}$	1,331	1,111	0.96	0.70	0.95
	$B_{10} \rightarrow B_{2_{25}}$	2,376	2,006	0.94	0.77	0.94
	$B_{10} \rightarrow B_{2_{30}}$	3,696	3,176	0.92	0.82	0.97

Table 8-1: Accessibility metrics for linear scenarios without barriers.

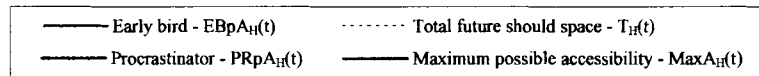
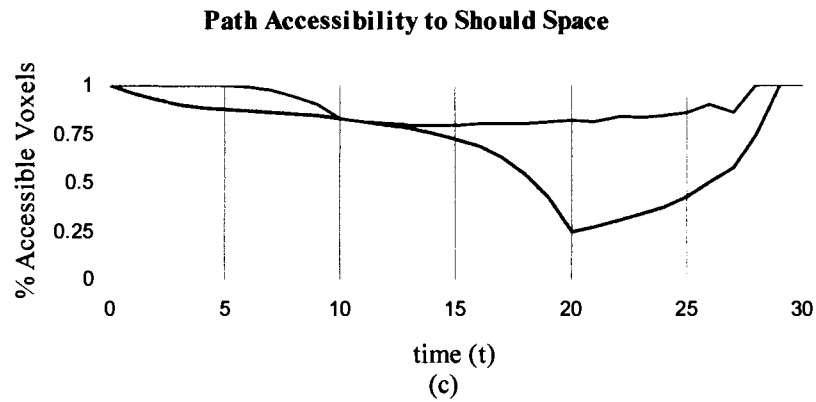
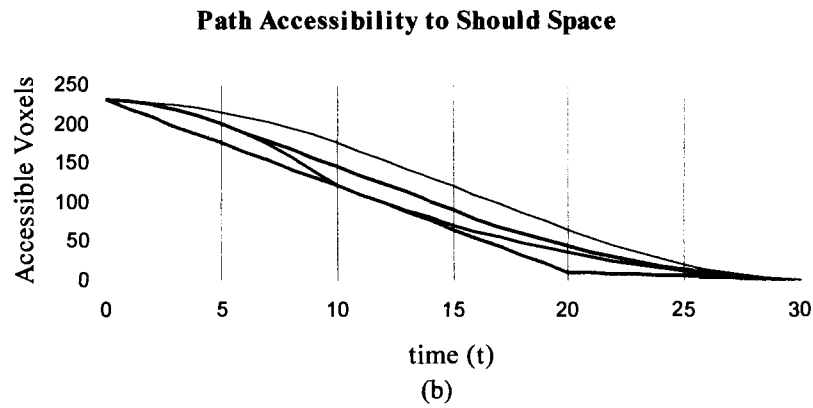
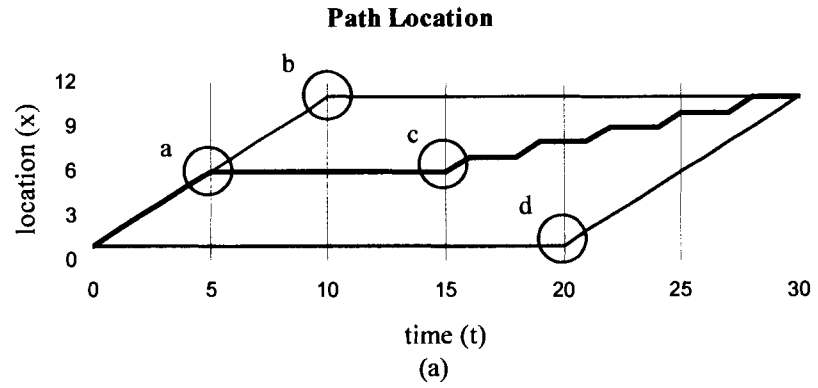


Figure 8-14: Accessibility plots over time for a linear scenario moving from a boundary to the opposite boundary ($B1_0 \rightarrow B2_{30}$): (a) path location through time, (b) path accessibility to *should-space* in voxels, and (c) path accessibility to *should-space* as percentage of maximum possible.

8.3.2 Linear Scenarios with Barriers

Adding temporary barriers to the basic linear scenario of moving from one boundary to another ($B1_0 \rightarrow \neg M_{(x,m,n)} \rightarrow B2_{30}$), tests the impact of intermittently restricting travel. The *early bird path* failed to maximize accessibility in every barrier configuration (Table 8-2). A number of observations can be made from the data.

Observation 5: The early bird path's percentage of maximum accessibility in scenarios with barriers is better than in scenarios without barriers.

In every scenario with barriers, the *early bird path* performed better than the same scenario without barriers (Table 8-2). The increase in the percentage of maximum accessibility occurs because barriers decrease the amount of *should-space*. Less *should-space* indicated lower travel possibilities and, as a result, the difference between the maximum accessibility and the *early bird path* decreases. This observation suggests that in scenarios with numerous temporary restrictions, the *early bird path* will perform adequately.

Observation 6: Early bird paths demonstrate a dip in performance prior to the appearance of intermittent barriers.

This observation becomes apparent by considering accessibility over time of three different barrier existence configurations: (1) a barrier lasting until $t = 10$, (2) a barrier beginning at $t = 20$, and (3) a barrier beginning at $t = 10$ and ending at $t = 20$ (Figure 8-15). In all three scenarios, the *early bird path* overall performs nearly the same, approximately 95%. When viewed over time, though, the *early bird path's* instantaneous performance dips briefly to 70%, prior to the appearance of the intermittent barrier (Figure 8-15i). This sharp drop does not occur when barriers remain active until the end of the scenario. This observation suggests that a slight modification in the *early bird path* to account for this drop would greatly improve path performance.

Hypothesis Test Summary						
Linear Scenarios without Barriers		Should-Space Accessibility				
		Total Voxels Summed over Time	Maximum Accessible Voxels Summed over Time	Max Voxels Accessible Ratio		
				Early Bird Path	Procrastin- ator Path	Random Path
<i>Barrier until sometime</i> $B1_0 \rightarrow \neg M_{(x,0,n)}$ $\rightarrow B2_{30}$	$\neg M_{(4,0,10)}$	2,992	2,696	0.97	0.86	0.98
	$\neg M_{(4,0,20)}$	1,392	1,336	0.99	0.92	0.96
	$\neg M_{(6,0,10)}$	3,300	2,924	0.95	0.84	0.97
	$\neg M_{(6,0,20)}$	3,700	3,359	0.97	0.95	0.98
	$\neg M_{(8,0,10)}$	3,520	3,072	0.94	0.83	0.98
	$\neg M_{(8,0,20)}$	2,800	2,528	0.95	0.86	0.98
<i>Barrier after sometime</i> $B1_0 \rightarrow \neg M_{(x,m,30)}$ $\rightarrow B2_{30}$	$\neg M_{(4,10,30)}$	2,736	2,488	0.95	0.88	0.95
	$\neg M_{(4,20,30)}$	3,216	2,804	0.94	0.83	0.98
	$\neg M_{(6,10,30)}$	2,076	1,948	0.97	0.92	0.96
	$\neg M_{(6,20,30)}$	2,736	2,408	0.96	0.83	0.98
	$\neg M_{(8,10,30)}$	1,296	1,256	0.99	0.98	0.57
	$\neg M_{(8,20,30)}$	2,096	1,856	0.97	0.82	0.98
<i>Barrier Variable start and end</i> $B1_0 \rightarrow \neg M_{(6,m,n)}$ $\rightarrow B2_{30}$	$\neg M_{(6,5,10)}$	3,300	2,924	0.95	0.84	0.98
	$\neg M_{(6,5,15)}$	2,805	2,549	0.97	0.86	0.98
	$\neg M_{(6,5,20)}$	2,160	2,008	0.97	0.89	0.97
	$\neg M_{(6,5,24)}$	1,536	1,420	0.95	0.94	0.95
	$\neg M_{(6,10,15)}$	3,596	2,994	0.94	0.81	0.97
	$\neg M_{(6,10,20)}$	3,501	2,785	0.95	0.77	0.98
	$\neg M_{(6,10,25)}$	2,076	1,948	0.97	0.92	0.96
	$\neg M_{(6,15,20)}$	3,556	3,000	0.92	0.80	0.97
	$\neg M_{(6,15,25)}$	2,331	2,121	0.97	0.86	0.97
	$\neg M_{(6,20,25)}$	2,736	2,408	0.96	0.83	0.99
$B1_0 \rightarrow B2_{30}$	\emptyset	3,696	3,176	0.92	0.82	0.97

Table 8-2: Accessibility metrics for linear scenarios with barriers.

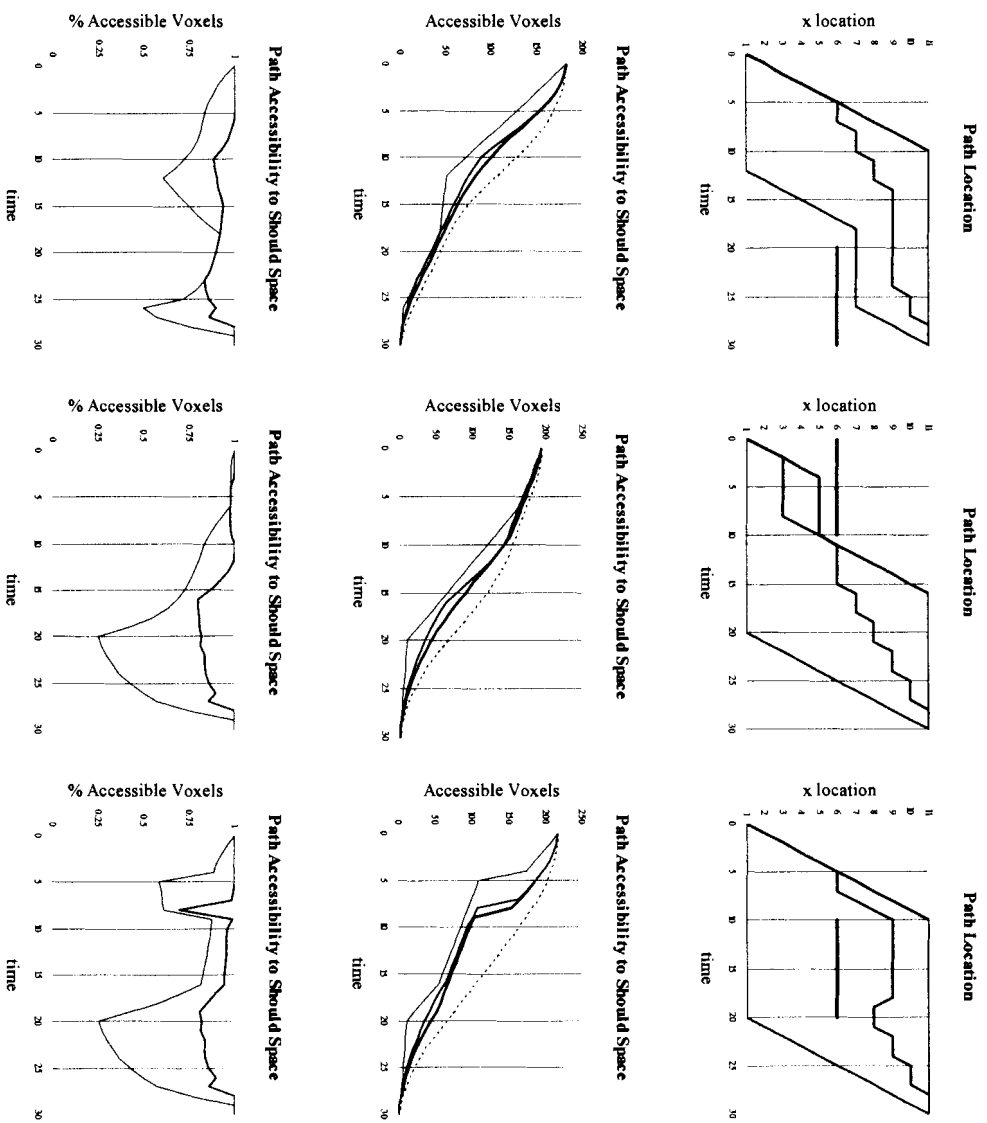


Figure 8-15: Linear scenario with a barrier in the center lasting until $t = 20$ when the requirement is at $t = 30$.

Observation 7: When barriers exist and then disappear, early bird paths perform best when the barrier is near the start. Conversely, when barriers initially do not exist and then appear, early bird paths perform best when the barriers is near the requirement. In both cases, the early bird path performs better when the barrier lasts longer.

Graphing the percentage of maximum accessibility for various barrier configurations demonstrates this observation (Figure 8-16). Barriers existing until some time and located near the start point, create very restrictive conditions. The extreme case is a barrier blocking movement from the start point until the last possible moment, causing the wayfinder to travel at maximum speed directly to the requirement.

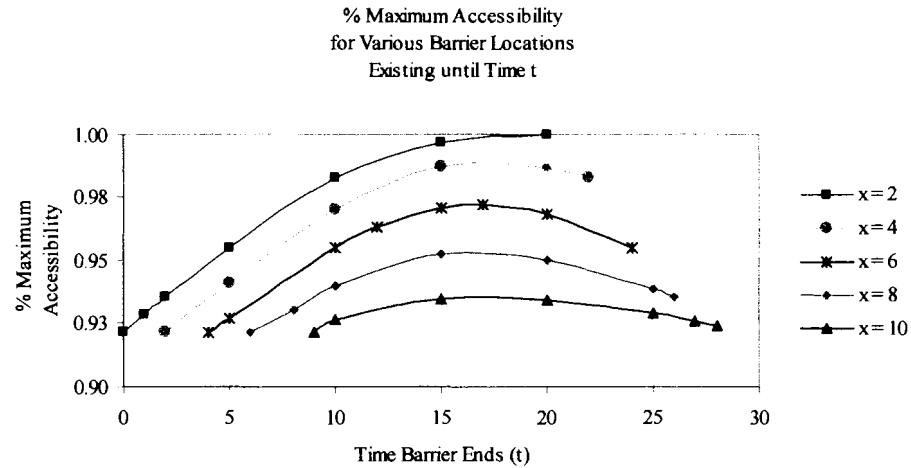


Figure 8-16: Percentage of overall maximum accessibility for *early bird paths* traveling in linear scenarios with barriers located at x and existing until t.

8.3.3 Planar Scenario Validation of Linear Findings

To validate that linear results generalize to higher dimensions, a subset of the linear scenario configurations are re-evaluated in a planar environment. In each test, the requirement time is set to 15. The results are all within 1 percentage point of the same scenario in the linear environment (Table 8-3). This suggests that the basic findings—the *early bird path* performs best when traveling away from boundaries—do generalize to higher dimensions.

Hypothesis Test Summary						
Planar Scenarios without Barriers		Should-Space Accessibility				
		Total Voxels Summed over Time	Maximum Accessible Voxels Summed over Time	Max Voxels Accessible Ratio		
				Early Bird Path	Procrastin- ator Path	Random Path
<i>Remain at Center</i>	$C_0 \rightarrow C_{15}$	8,296	5,032	1.00	1.00	0.86
<i>Remain at Boundary</i>	$B_0 \rightarrow B_{15}$	4,964	3,390	0.87	0.87	0.86
<i>Boundary to Center</i>	$B_0 \rightarrow C_{15}$	6,660	4,509	1.00	0.75	0.87
<i>Center to Boundary</i>	$C_0 \rightarrow B_{15}$	5,801	3,646	0.94	0.92	0.91
<i>Boundary to Boundary</i>	$B_{10} \rightarrow B_{215}$	3,995	2,883	0.99	0.54	0.87

Table 8-3: Accessibility metrics for basic scenarios without barriers in 2D.

A subset of the barrier configuration tests in the 1-dimensional environment are also re-evaluated in two dimensions. Four scenarios are tested, each with a temporary linear barrier located half way between the start and the requirement, which is set at $t = 30$: (1) a barrier existing until $t = 10$, (2) a barrier existing until $t = 20$, (3) a barrier existing after $t = 10$, and (4) a barrier existing after $t = 20$ (Table 8-4). In these scenarios, the early bird path performed nearly the same in the 2-dimensional environment as in the 1-dimensional environment, indicating that linear results can generalize to higher dimensions.

Hypothesis Test Summary						
Planar Scenarios with Barriers		Should-Space Accessibility				
		Total Voxels Summed over Time	Maximum Accessible Voxels Summed over Time	Max Voxels Accessible Ratio		
				Early Bird Path	Procrastin- ator Path	Random Path
<i>Barrier until sometime</i>	$\neg M_{(6,0,10)}$	32,560	26,069	0.97	0.84	0.92
$B_{10} \rightarrow \neg M_{(x,0,n)} \rightarrow B_{230}$	$\neg M_{(6,0,20)}$	20,020	16,481	0.97	0.90	0.96
<i>Barrier after sometime</i>	$\neg M_{(6,10,30)}$	18,423	16,062	0.98	0.93	0.96
$B_{10} \rightarrow \neg M_{(x,m,30)} \rightarrow B_{230}$	$\neg M_{(6,20,30)}$	25,647	20,594	0.97	0.81	0.94

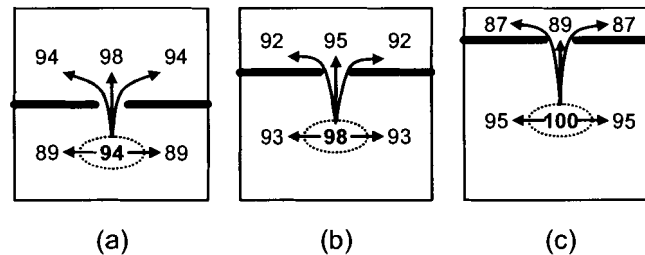
Table 8-4: Accessibility metrics for basic scenarios with barriers in 2D.

8.3.4 Gaps in Linear Barriers

To evaluate the performance of the *early bird path* in scenarios with linear barriers that include gaps, 216 different configurations are tested (Figure 8-18). For comparison, 36 barrier-free configurations are also tested (row \emptyset , Figure 8-18). In most cases, the *early bird path* did not maximize accessibility to possible additional requirements and performed as low as 68%.

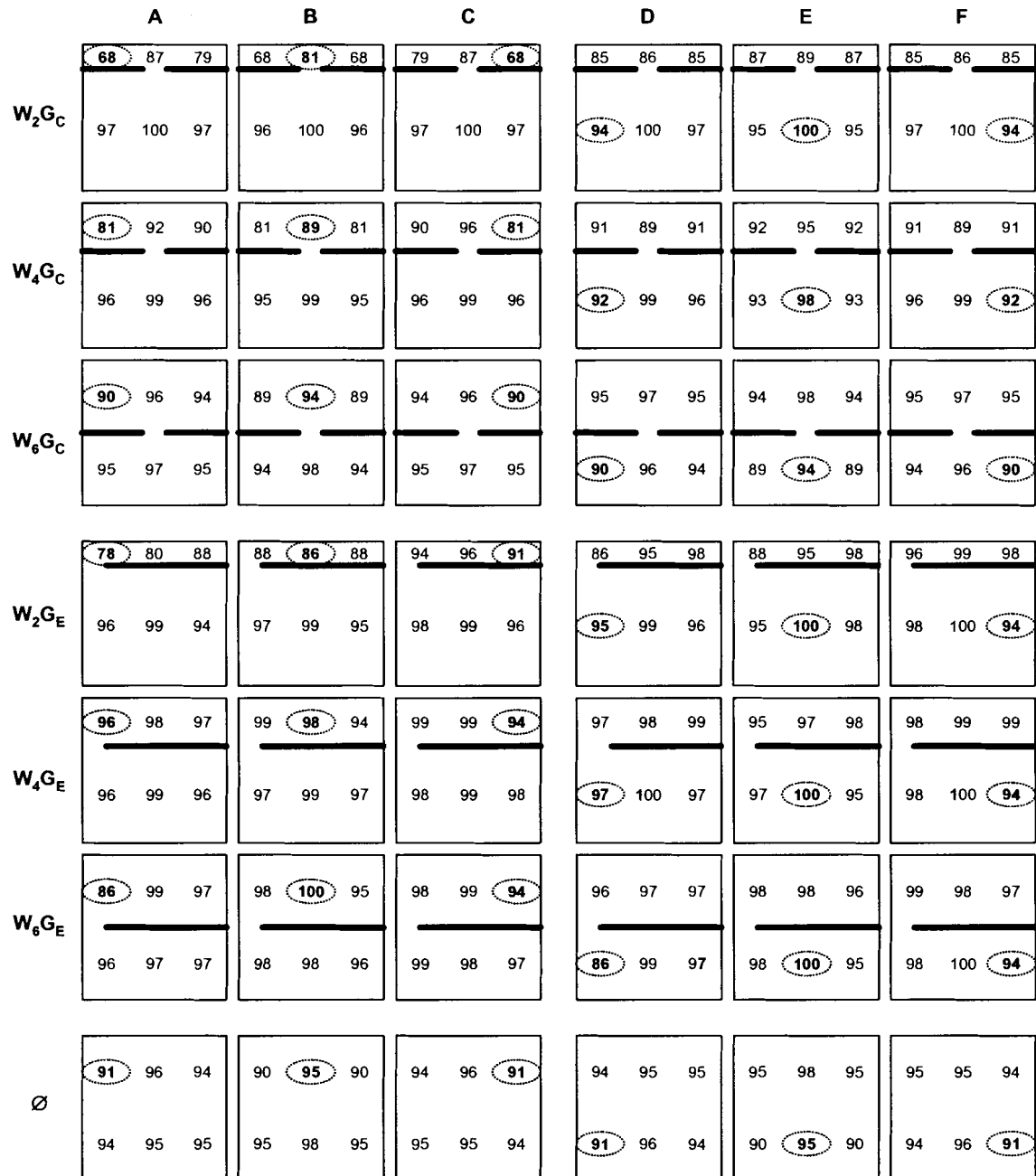
Observation 8: When moving from large to small regions separated by a linear barrier with a gap, early bird path performance decreases as the small region's size decreases.

When barriers bisect space, the *early bird path* performs better when moving through the gap than not crossing the gap (Figure 8-17a). When the barrier divides space unequally, the *early bird path's* performance is worse when traveling through the gap than staying at the start point (Figure 8-17b). As the barrier partitions space more unequally, the *early bird* performance is even worse (Figure 8-17c).



Start point is indicated with a dashed oval and values indicate percentage of the early bird's path to maximum accessibility.

Figure 8-17: Effects of barrier position on *early bird's* percentage of maximum accessibility: (a) bisected space, (b) barrier dividing space in a 3 to 7 ratio, and (c) barrier dividing space in a 1 to 10 ratio.



The time of the requirement is 20. A dashed oval indicates the start point and percentage values are listed for each requirement location.

Figure 8-18: Early bird path percentage of maximum possible accessibility to should-space for various configurations.

Observation 9: Early bird path performance is not necessarily symmetric in respect to the overall percentage of maximum accessibility.

Every space has a point that minimizes the sum of the distances from that point to each of the others. Solving for this point is often referred to as the Post Office Problem. Barriers influence the location of this point and as a result affect the success of *early bird paths*. In a barrier free environment, two points, equidistance from this point, are symmetric in respect to the overall percentage of maximum accessibility. A linear barrier with a gap changes the location of this point. As a result, paths moving towards this point will have a higher success rate than those going in the opposite direction.

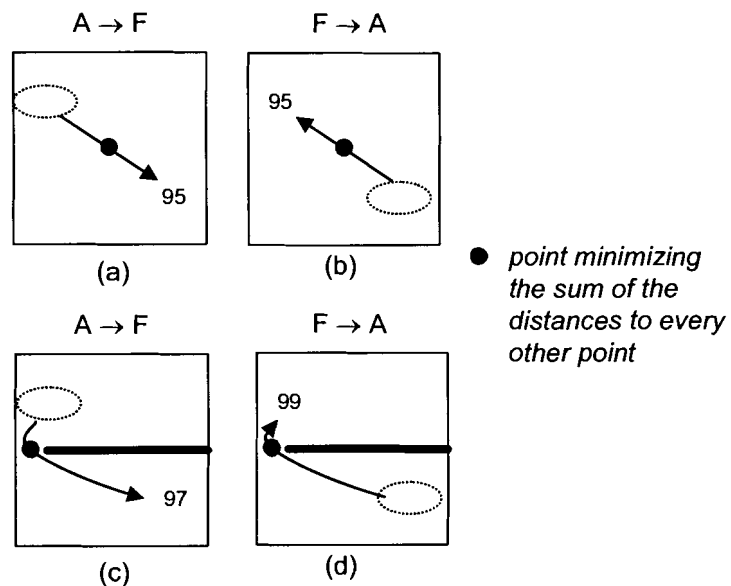


Figure 8-19: A barrier's impact on the *early bird path's* percentage of maximum accessibility.

8.4 Discussion

As can be observed from the evaluation the *early bird path* does not maximize accessibility to a possible additional requirement; therefore the hypothesis must be rejected. It was found instead that the *early bird path's* ability to maximize success is dependent on the path's relationship to changing spatio-temporal characteristics of barriers. Paths that maximize accessibility spend more

time away from barriers. The complex nature of changing conditions through time and the particular scenario considered make the calculation of a maximum accessibility path difficult.

8.5 Summary

This chapter described the results of the hypothesis and the travel possibility calculator prototype, which served as the testing environment. The hypothesis was not supported, but instead it was found that an arrival minimization path's ability to react to possible additional requirements is heavily dependent on its relation to barriers. The chapter also highlighted the required adjustments to a standard 3-dimensional voxel data structure to account for movement in space and time, along with algorithms to calculate accessibility, partition space-time, and generate *early bird* and *procrastinator paths*.

CHAPTER 9

CONCLUSIONS AND FUTURE WORK

This thesis focused on providing support to wayfinders selecting paths in dynamic and uncertain environments. A conceptual model was presented for calculating travel possibilities through space and time analogous to categorizing terrain features according to their effect on movement. Arrival time minimization paths were evaluated in their ability to react to uncertainties in the form of possible additional requirements. The results of this evaluation highlight the complexities of path selection in dynamic and uncertain environments. As a mechanism for hypothesis evaluation, a prototype application was developed that also served to demonstrate an implementation of the presented concepts. This chapter summarizes the thesis (Section 9.1) and highlights the major findings (Section 9.2). The thesis ends by describing possible future research efforts related to this work (Section 9.3).

9.1 Summary of Thesis

Path selection in dynamic environments is a complex task. This complexity increases when a wayfinder's understanding of future characteristics is ill defined or incomplete. To address this complexity, the goal of this research is a better understanding of the impact of dynamic and uncertain environments on wayfinding travel possibilities. The fundamental concept employed to accomplish this goal is the development of a wayfinding model that integrates space and time in a manner similar to that used in time geography. The wayfinder is represented in this space-time framework as a point object, which over time traces a path. Additional objects exist as primitives within the framework and define a wayfinding scenario: (1) maximum travel speed, (2) start point, (3) temporary barriers, and (4) requirements. Successful wayfinding is modeled by considering the intersection of the wayfinder's space-time path with the objects representing the

wayfinding scenario's requirements. Fourteen wayfinder-requirement intersection realizations are organized into a lattice according to travel flexibility.

To create categories of spatio-temporal travel possibilities, wayfinding primitives combine to partition space-time according to accessibility. When selecting paths, wayfinders view and query these partitions to increase their situational awareness of future travel restrictions. The maximum travel speed defines accessibility through time and combines with each of the remaining three primitives to create partitions of space-time. Sequentially partitioning these primitives results in four basic travel possibility categories, each described with a modal verb: *cannot*, *may not*, *should not*, and *should*.

To address more realistic wayfinding scenarios, concepts of travel possibility partitioning extend to include considerations of uncertainty. Uncertainty results when the wayfinder cannot determine the spatio-temporal characteristics of wayfinding primitives. A wayfinder, for instance, may be unsure about the existence of barriers and requirements. Uncertainty may also result when the actual location and time of primitives cannot be determined from some set of possibilities. Uncertainty is modeled with a three-valued logic where objects or effects: exist, do not exist, or possibly do not exist. Sequentially partitioning uncertain primitives expands the four basic travel possibility categories to fifteen. The fifteen categories generalize into two separate three-valued partition spaces: (1) where paths are possible (*valid-space*), and (2) where paths can meet requirements, (*successful-space*).

When wayfinding scenarios include requirement combinations, additional complexities arise. Individual requirement uncertainty can spread to other requirements in the combination. Even when individual requirements are well defined, uncertainty can exist when the wayfinder is unsure about the combination operation itself. Three categories of uncertainty in the requirement operation were demonstrated: (1) uncertainty in the combined requirement's existence, (2) uncertainty in whether a subsequent requirement will be added, and (3) uncertainty in whether a subsequent requirement will replace an initial requirement. Two special uncertainty scenarios are

described: the *police officer scenario* and the *repairmen scenario*. The *repairmen scenario* considers a well defined requirement with the possibility that a subsequent one will be added somewhere beforehand. The *police officer scenario* also includes a well-defined requirement, but in this case the possibility exists where a subsequent requirement replaces the initial requirement.

The purpose behind developing travel possibility partitions is to provide information to wayfinders when selecting paths. Two particular paths often returned from path selection algorithms were described: (1) the *early bird path* minimizes arrival time to requirements, and (2) the *procrastinator path* maximizes the departure time from the start point. The effects of uncertainty on these and other paths introduces the concept of *valid paths* and *successful paths*. *Valid paths* were defined as paths that wayfinders have the capability to travel along. *Successful paths* were more stringently defined as those paths that are *valid* and also meet the wayfinding scenario's requirements. Uncertain scenarios, however, often include conditions where *successful paths* are not available, and wayfinders must select *possibly unsuccessful paths*. To maximize the probability of success in these settings, a set of metrics were presented to measure a path's accessibility to possible requirements. As a demonstration, the metrics measured the accessibility of the *early bird* and *procrastinator paths* in a basic *repairmen scenario*. The data from this example appeared to indicate that the *early bird path* maximizes accessibility to possible additional requirements and provided motivation for the hypothesis that arrival minimization paths also maximize accessibility to the possible additional requirements.

To test this hypothesis, *early bird* and *procrastinator paths*, along with a random path, were evaluated in various linear and planar wayfinding scenarios. The testing occurred within the prototype Travel Possibility Calculator (TPC) application, developed as part of this research. The data from these tests rejected the hypothesis that the *early bird path* also maximizes accessibility to possible additional requirements. It was found, instead, that arrival minimization paths for the *repairmen scenario* only maximize accessibility to possible additional requirements when traveling to or remaining at the center of some space. The *early bird path* performed well below

maximum, when traveling towards a boundary. It was found that paths maximizing accessibility to possible additional requirements are heavily dependent on the spatio-temporal configuration of barriers and requirements in the wayfinding scenario.

9.2 Results and Major Findings

To understand better the impacts of change and uncertainty on wayfinding travel possibilities, this thesis develops an integrated model of wayfinding and the tests time minimization paths within a prototype application. It was demonstrated that an integrated space-time approach to model dynamic wayfinding scenarios is a plausible approach in representing the impact of changing conditions on travel possibilities. The novel technique of sequentially partitioning space-time into four travel possibility categories, and the use of modal verbs to describe these categories, provides a cognitively straightforward method to represent future travel possibilities to wayfinders selecting paths.

An extension with uncertainty considerations leads to a three-valued broad-boundary technique that represents a wayfinder's indiscernibility of future possibilities. It was shown that sequentially partitioning with uncertainty could result in up to fifteen travel possibility categories. As a result, two generalization schemes were introduced to provide information to the wayfinder about where travel is possible (*valid-space*) and where travel can be successful (*successful-space*).

Uncertainty in the future states of barriers and requirements can create scenarios where the only valid paths are those that may not succeed in meeting all requirements. As a result, this thesis developed a set of metrics to measure a path's accessibility to requirement possibilities over time and a metric of overall path accessibility. With these metrics, wayfinders can compare various paths to select the one with the greatest probability of being successful in meeting requirements. Testing of paths in various realizations of the *repairmen scenario* showed that

arrival minimization paths fail to maximize accessibility to possible requirements, but instead are impacted greatly by the changing spatio-temporal characteristics of barriers.

The development of a Travel Possibility Calculator (TPC) prototype application demonstrated the feasibility of the presented concepts. By integrating space and time in a discrete voxel-based spatio-temporal data structure, wayfinding speed options were shown to be limited when using traditional raster- and voxel-based path algorithms. As a result, a method of accounting for various travel speeds when creating paths in voxel-based spatio-temporal data structures is presented.

9.3 Future Work

This research exposed a number of interesting research questions and highlighted fruitful areas of further research related to this topic. Practical questions relate to the implementation of these concepts and include: transferring these concepts into a linear network and the development of more efficient accessibility algorithms and approximation methods. In addition, two specific examples are explored where initial simplifications can be expanded into more sophisticated models of wayfinding allows requirements to occupy a spatial extent, and introducing variable probability to possible requirements. Additional research can extend the basic concepts introduced in this thesis to include the consideration of multiple wayfinders and danger areas.

9.3.1 Movement through Networks

The framework used to build a conceptual model of travel possibilities assumes a continuous volume of space-time. The prototype implemented these concepts with a 3-D voxel-based spatio-temporal data structure. Many applications rely, instead, on a network representation of space (Miller and Shaw 2001).

One potential method to create a space-time volume from a network data model is to recreate an instance of the network for every time increment. This approach, however, is contrary to the

advantage of a network data structure's efficient coding of important spatial information relative to raster structures. A more desirable approach would be to model change through time in uneven steps and only create structure as needed. In some regards this would be analogous to the quadtree coding of uneven spatial distributions (Samet 1990).

9.3.2 Efficient Accessibility Algorithms and Approximation Methods

Regardless of whether space-time is model discretely with voxel space or with some form of a network graph structure, there is a heavy reliance on accessibility calculations. To realistically model dynamic and uncertain scenarios requires fast and efficient algorithms. Though considerable research is underway in computation geometry (Lee and Preparata 1984; Hershergery and Suri 1999; Mitchell 2000; Sellen *et al.* 2000) and other fields (Douglas 1994; Stefanakis and Kavouras 1995; Zhan and Noon 1998; Duckham and Kulik 2003), path generation in space-time volumes is lacking and research in this area is needed.

As opposed to exclusively focusing on faster and more efficient algorithms, the potential exists for good enough approximations. For example, in the evaluation of the hypothesis, the maximum accessibility path relied on accessibility volume exhaustively calculated for every voxel. This results in hundreds of accessibility calculations and is not efficient and an approximation algorithm is desirable.

An approximation of accessibility to future possibilities that holds promise is similar to the accumulation of stream flow over digital elevation models (Tomlin 1990; Jones *et al.* 2002) or methods to maximize the line of sight over a path (de Floriani and Magillo 1999; O'Sullivan and Turner 2001). The procedure begins by assigning a value of 1 to all possible locations of an uncertain requirement (Figure 9-1a). A sweep algorithm runs backward through time accumulating values (Figure 9-1b). Values do not directly indicate the number of accessible possibilities, but are a relative measure, from which maximum accessibility paths can be generated. Preliminary tests suggest that this approach holds promise, but currently generates

localized maximum values when encountering narrow gaps. Additional testing is warranted to formalize this algorithm and tests its effectiveness.

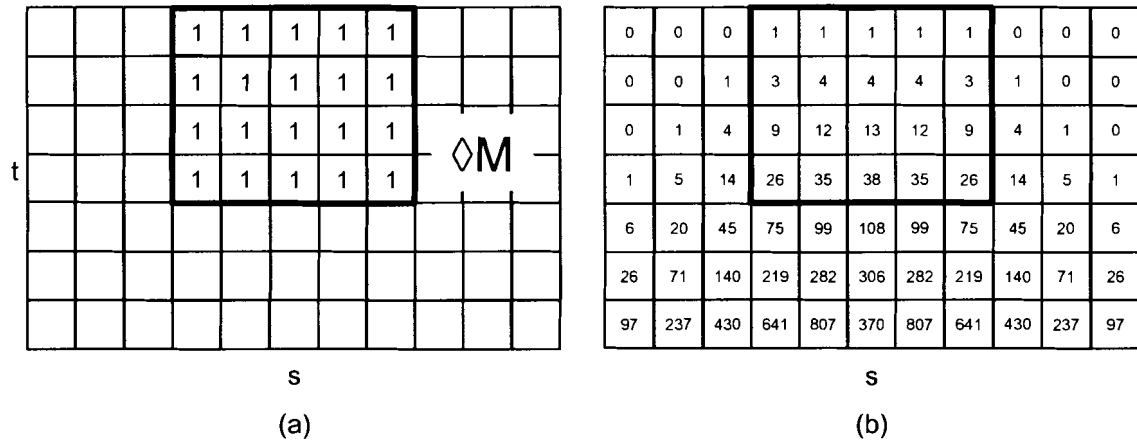


Figure 9-1: Accessibility approximation method: (a) assignment of values to possibility spaces, and (b) accumulation of values backwards in time.

9.3.3 Modeling Requirements with a Spatial Extent

This thesis only considered point requirements. Many requirements, however, occupy an extended spatial region and are not modeled well as points. For example, the requirement to be within the boundaries of a town is best modeled as a spatial region. The spatial components of requirements with a spatial extent consist of a spatial interior, M° , and a spatial boundary, ∂M (Egenhofer 1993a). The directed graph indicating possible wayfinder path interactions with a point requirement (Figure 3-5c) can be extended to include an additional node representing the requirement's spatial boundary (∂M) (Figure 9-2).

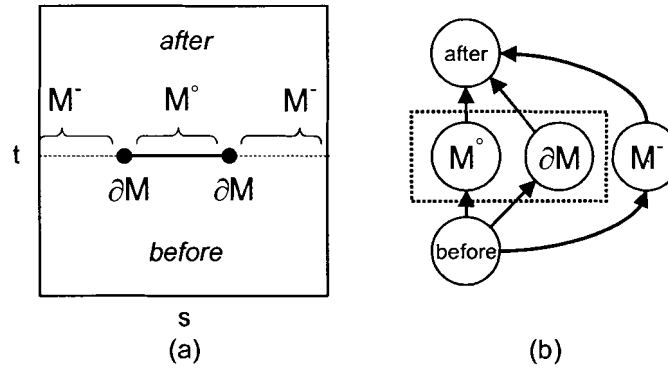


Figure 9-2: A requirement with a spatial extent: (a) the three components of the requirement, (b) the wayfinder-requirement interaction graph (G) of possible wayfinder space-time path intersections.

Requirements with both a temporal and spatial extent are modeled as polygons extruded through time. Combining the temporal and spatial elements results in a general space-time requirement composed of six components (Figure 9-3a). The first is an *interior* (M°) representing the requirement's spatial interior during the course of the requirement, and a *spatial boundary* (∂M). At the start of the requirement, the interior ($\partial_s M$) and boundary ($\partial_s \partial M$) exist. In a similar manner at the end of the requirement and interior ($\partial_e M$) and boundary ($\partial_e \partial M$) exist. The point object with a temporal extent's directed graph (Figure 3-6c) expands with these additional spatial components to nine nodes with eighteen directed edges (Figure 9-3b). There are many more traversals of the wayfinder's path with this more graph than the fourteen traversals of a point requirement with a temporal extent. Additional research can identify the importance of these traversals and provide some form of organization.

Accessible Space (A-Space)

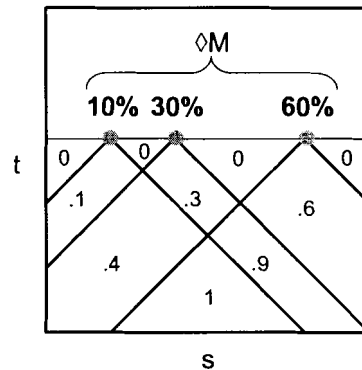


Figure 9-4: A requirement (ϕM) with three possible locations, each with a different probability of occurring, creates an accessibility space (*A-space*) with values representing the probability of meeting a requirement.

9.3.5 Multiple Wayfinders

The focus of this thesis is on travel possibilities of individual wayfinders. There are, however, numerous applications involving multiple wayfinders. This may consist of a group of individuals working together to navigate a ship (Hutchins 1995), or individual wayfinders traveling with a common goal. In some cases, individuals operate with different sets of goals, for instance a search and rescue operation (Heth and Cornell 1998), or a convict apprehension case. In both these instances a rendezvous is the goal of at least one of the individuals (Alpern and Gal 2003).

Rendezvous scenarios can be modeled with this approach, where each wayfinder creates a travel possibility space, and these spaces are combined to indicate possible interaction partitions. Consider for instance two wayfinders with different start points and requirements. Combining the two travel possibility spaces (Figure 9-5a) creates an interaction space, that indicates different interaction possibility categories (Figure 9-5b). Additional research in this area could develop tools to assist the search and rescue community and law enforcement organizations conducting cordon and search operations.

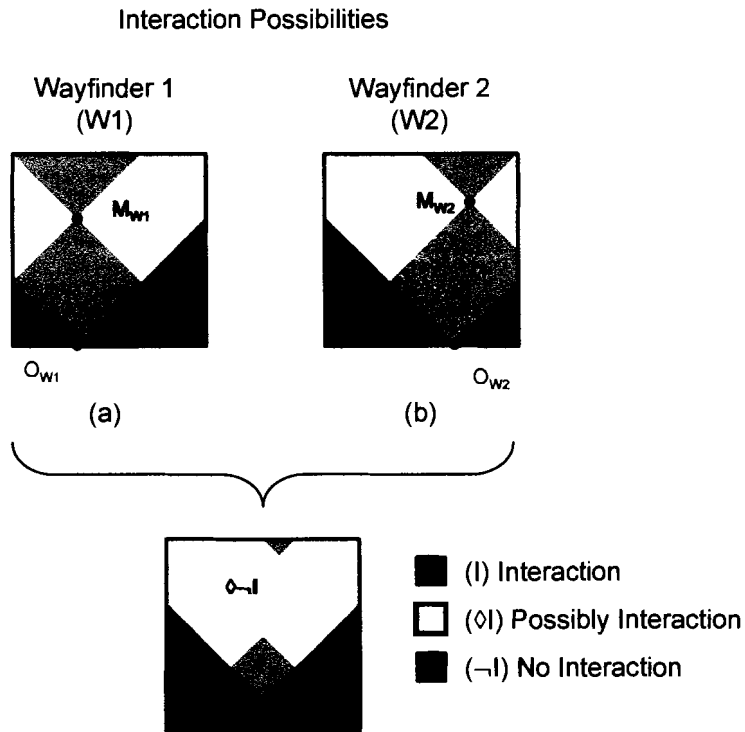


Figure 9-5: Interaction possibility spaces between two wayfinders.

9.3.6 Danger Areas

Wayfinding can occur in dangerous settings. This may be direct physical danger, as the case of traveling in a battle zone or indirect danger, for example the danger of environmental harm by traveling through an area. Danger regions can be identified and avoided by wayfinders. These regions differ from barriers in that barriers do not allow travel, while wayfinder can travel through danger areas if they choose, but may be or will be negatively affected.

As an example of how the partitioning concepts in this thesis can be extended to include danger areas, consider a scenario where a wayfinder is told that until a certain time it would be best if travel past a certain point was avoided. Regions are often classified in this manner to protect bird nesting sites or other environmentally sensitive events. The danger object (D) partitions space-time, in a manner similar to the other wayfinding primitives, into *bad-space* (B) and *not bad-space* ($\neg B$) (Figure 9-6a). This partition of space-time can be combined with the

standard four travel possibility partition space (Figure 9-6b). Using these partitions a wayfinder can determine that to meet the possible requirement ($\Diamond M_2$) a risk will have to be taken.

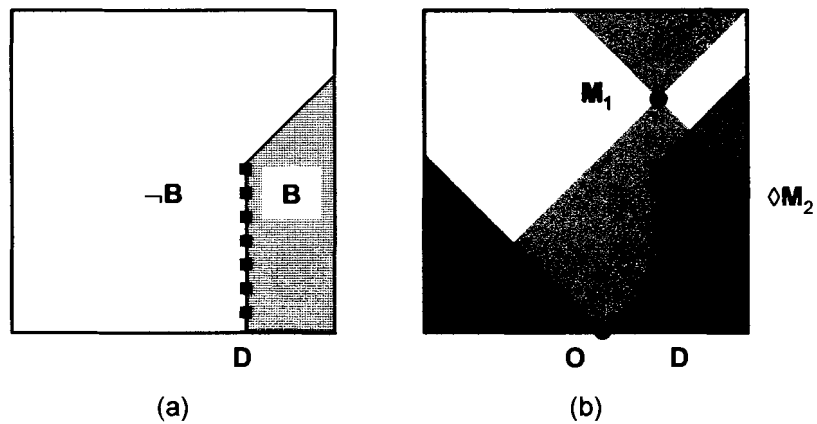


Figure 9-6: Danger areas: (a) a point danger area for a portion of time creates bad space (B) and not bad space ($\neg B$), (b) combining the danger partition with a standard travel possibility partition space.

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