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Investigating the Stability of Observed Low Semi-major Axis Exoplanetary Systems With Hypothetical Outer Planets Using The Program Mercury6

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INVESTIGATING THE STABILITY OF OBSERVED LOW SEMI-MAJOR AXIS EXOPLANETARY SYSTEMS WITH HYPOTHETICAL OUTER PLANETS USING THE PROGRAM MERCURY6

by

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ABSTRACT

This project investigates the stability of observed planetary systems, and whether this stability remains in the presence of additional outer planets. This made use of the program Mercury6, an n-body integrator that computes the changes in planetary orbits over time. The Systems HD 136352, GJ 9827, and HD 7924 were studied with initial conditions taken from the available observational data. This information was curated using the online NASA Exoplanet archive of confirmed exoplanets. With these initial conditions, Mercury6 computed the changing planetary orbits of each system for 5 million years. For each of these systems, a single outer planet, which was varied in mass and semi-major axis, was added to test the effect on the resultant data. Three distinct cases emerged. For GJ 9827, a system with very low semi-major axes and initial inclinations, complex oscillatory behavior in the inclination of the planets emerged, with variations based on mass and orbital distance of the additional outer planet. For HD 136352, complex oscillatory behavior was observed in the planet eccentricities. For HD 7924, instability occurred in the first 250 thousand years for the observed system. Tested with perturbing bodies, HD 7924 was shown to have stability in a number of cases, which could be evidence that additional unseen planets are present within the system. A potential conclusion from this data is that long-term repetitive oscillations can be indicative of stability, and that computational methods similar to the use of Mercury6 have significant potential in the field of planetary science.

ACKNOWLEDGEMENTS

I would like to thank my advisor, Dr. Neil Comins, for guiding me through the course of this project and helping come up with the initial idea for the research using Mercury6. I am also very thankful for the help of Kathleen Schultz, a current Graduate student at the University of Maine, who has helped me in countless ways over the course of this past year. I also credit her for allowing me to utilize her python program that converted the Mercury output files into graphs for analysis. I am uncertain whether or not I would have as complete a project without her contributions.

The advanced computing group was also vital to the completion of this project. Without their computer supercluster on campus, it would have been significantly more challenging to complete the necessary computational data on these systems of exoplanets.

Finally, I am grateful for the other members of my Thesis Committee: Dr. Clark, Dr. Batuski, Dr. Farooq, and Dr. Nadeau. I am very appreciative that they would agree to read my thesis/reading list and assess my performance as an Honors Student at the thesis defense.

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INTRODUCTION

The study of exoplanetary systems is a reasonably new field of astronomical research. Most of the stellar systems we study are tens, if not hundreds of light-years away, which creates significant limitations on the data we can gain from such systems. Due to these limitations, it is natural that we would use computer simulation to study what is unknown about the systems we observe. What makes this simpler is the fact that generally speaking, there are two sets of information that dominate the movement of bodies within these systems gravitationally: the mass of the central star and the masses/positions of the planets around said star. Most systems also have fields of asteroids, comets, and moons orbiting the more massive planets. However, these smaller bodies are mostly irrelevant to the study of more significant movements of the planets. This generalization is valid because moons and asteroids have significantly less mass than the larger bodies of a system, and therefore have less of a gravitational effect on the movements of other objects. If a planetary system is stable with its planets alone, it is likely to maintain said stability with the addition of potentially millions of comets and asteroids. Unfortunately, in many systems, we only observe a few exoplanets directly, despite estimating the percentage of sun-like stars with 7 or more exoplanets to be at 42 percent (Mulders et al., 2018).

For this project, we used the program Mercury6 to study three systems of exoplanets. Mercury6 uses numerical integration and Newton's Laws to compute changing planetary orbits over time. Mercury6 and its uses are discussed more thoroughly on page 6. Using this program, we are free to change the parameters of planets and to add additional planets into systems to model how they perturb or change

the orbital parameters of the others. Before we describe this project further however, we need to develop an understanding of the orbital parameters used within Mercury6.

System Parameters and Asteroidal Coordinates

There are several significant parameters that we need to understand in order to study them computationally. These are the mass and radius of the central star; the masses and densities of the large bodies within the system; and, the set of Asteroidal coordinates for each of these planets. The vital Asteroidal coordinates, in this case, are the following:

Semi-Major Axis – Half of the major axis of each planet's elliptical orbit around the star. In laymen's terms, we think of this as the planet's distance from its star.

Eccentricity – The elongation of a planet's orbit. An eccentricity of 0 results in a perfect circle. Note that this value must remain less than 1 and greater than or equal to 0 to remain a valid orbit.

Inclination – Refers to the angular tilt of an orbit relative to a reference plane.

The Longitude of Ascending Node – Angular position at which the planet ascends from below the reference plane to above it.

The Argument of Pericenter – The angle from the ascending node to the pericenter, or tip of the elliptical orbit.

The Mean Anomaly – The angular coordinate related to the planets position. For this, we are using randomized starting positions.

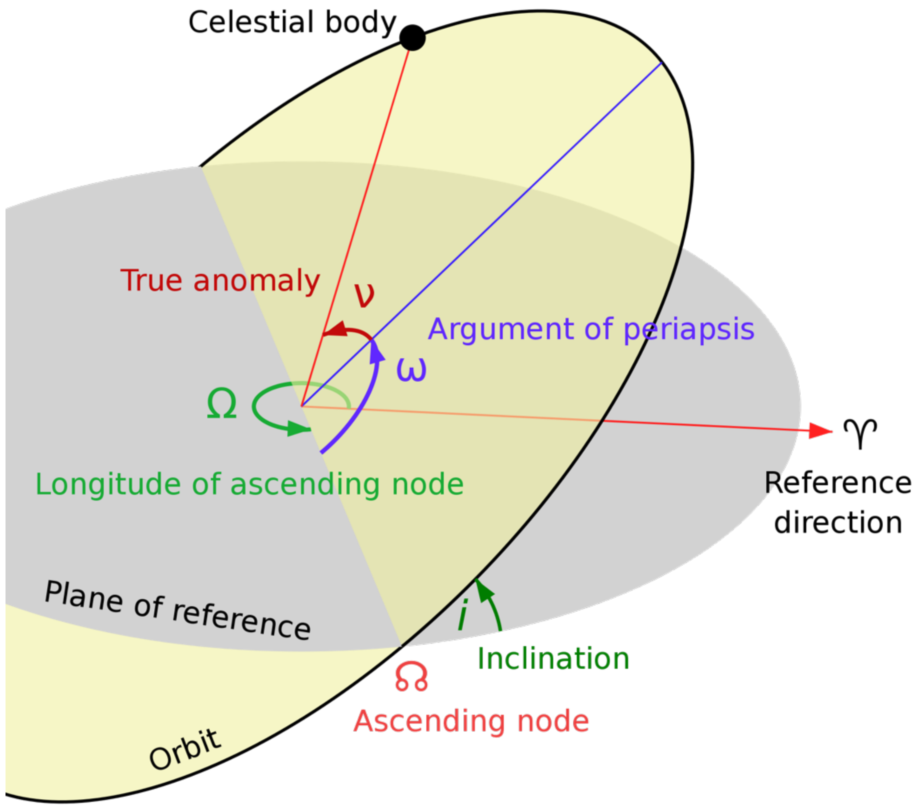


Figure 1: Asteroidal Coordinates

In Figure 1 above, the argument of pericenter is labeled as the argument of periapsis. Eccentricity, inclination, and semi-major axis can be considered the primary Asteroidal Coordinates in understanding system geometry. These relate to the shape of a planets orbit around its central star. The three angular coordinates describe the relative angular positions and orientations of each planetary orbit. For the orbital parameters that are not available in the data for an observed system, we will attempt to make as few assumptions as possible.

The parameters described above will be used by Mercury6 to output information on the changing orbits of planets within each system. As found on the NASA Exoplanetary Archive, we are focusing on systems GJ 9827, HD 7924, and HD 136352. Each has 3 observed planets, all with semi-major axes less than 0.45 AU. GJ 9827 has planets very close to the central star (<0.1 AU) and observed inclinations, but no eccentricity (e=0). Inversely, both of the other systems' planets have observed eccentricities with no observed inclination. In the NASA Exoplanetary archives, some orbital parameters are missing from much of the observational data: the argument of pericenter, the longitude of ascending node, the mean anomaly, the density of some planets, and depending on the system, eccentricity or inclination. Due to this, our initial conditions for each system require a few assumptions within the input files of Mercury6. No assumption should be made that these are perfect representations of the systems in question. Instead, the purpose of this project is to model these systems as well as possible with the available information and to test how they change based on the addition of outer planets.

For each of these systems, we have tested the change in system dynamics given an additional outer planet with simple orbital properties (no Inclination/Eccentricity), varying the outer planet in both mass and semi-major axis. This allows us to explore the plausibility that additional planets could exist in these systems without significantly affecting the orbits of the observed bodies.

Motivation and Problem Statement

Given what we know about exoplanetary systems via direct observation, what is the use in computationally testing the effect that additional perturbing bodies have on the

orbits of planets that we've already observed? One benefit of computation in the study of planetary systems is the ability to answer questions that we would not be able to investigate otherwise. One example of this is that a large number of observed exoplanets are within the orbit of Mercury, which is interesting when compared to the lack of planets in that region for our own solar system. There is also good reason to believe that many planets in the systems we observe go unseen (Winn & Fabrycky, 2015). When determining whether or not additional planets are orbiting the central star of an observed system, researchers often have to look for periodicities in their data that are not explained by the bodies already observed (Beaugé et al., 2012). This, naturally, can be quite difficult for systems with limited data. Due to this, it is valuable to test the effects of additional bodies on observed systems to determine the plausibility that some planets have gone unseen.

In this project, we are attempting to answer two questions: First, what is the plausibility that unseen outer planets exist within these observed exoplanetary systems; and second, how does the addition of perturbing bodies affect their stability? It is a reasonable hypothesis that the computational study of these systems will reveal that perturbing bodies have a variety of effects on the orbits of observed planets and that specific systems are likely to have additional unseen planets.

METHODS

This thesis is based entirely on the use of computational methods to study systems of planets. Three systems of planets, as mentioned earlier, were taken off of the NASA Exoplanetary Archive, and the available information is used within the input files for Mercury6. To maximize efficiency, the University of Maine Computer Supercluster (viz3) was used. The supercluster allows for the remote analysis of data and has capabilities that allowed me to have many sets of integrations on Mercury6 running at the same time. Below, each of these methods are explained in more depth.

A Description of Mercury6

Mercury6 is an n-body integrator, which allows for the input of as many planets and bodies as desired. Put simply, this program takes the input of initial conditions for the bodies within a system and the mass/size of the central star. From there, it uses numerical integration to model changes within the system. Numerical integration can be used to compute how a value of interest (velocity, for example) changes based on differential equations or rates of change (acceleration, for example). Extending this further, we can describe one of the more straightforward methods of numerical integration, the Euler Method.

$$v_{i+1} = v_i + a_i \Delta t \qquad (1)$$

Here v_i is the velocity of the current timestep, a_i is the acceleration at the current timestep, and Δt is the finite time step used to calculate the next value of velocity. This becomes more accurate the smaller Δt is chosen to be. Moving forward, note that

methods of numerical integration tend to be more complex based on the differential equations being used and on the problem that is being solved. Mercury6 uses the Bulirsch-Stoer Algorithm, which is described very briefly on page 8.

The data input into Mercury6 is entirely included within two files: big.in and param.in, which describe the relevant parameters of all bodies within the system and the details of the integration.

```
)O+_06 Big-body initial data  (WARNING: Do not delete this line!!)
) Lines beginning with `)' are ignored.
)---------------------------------------------------------------------
 style (Cartesian, Asteroidal, Cometary) = Asteroidal
 epoch (in days) = 0.0D0
)---------------------------------------------------d or D is not matter--0d0 is possible
too
Planet_1 m=1.0D-07 r=20.D0 d=5.D0
  0.5 0.1D0 12.D0
  0.D0 90.D0 125
  0.D0 0.D0 0.D0
```

Figure 2: Planetary Input File

Figure 2 is an example of a big.in file that would be used in an integration. Note that here "D" multiplies a value by ten to a specified power. The important part of the file are the four lines at the bottom, which contain all of the orbital parameters for a planet. The first of these has the planets title labeled "Planet_1", followed by the mass m in solar masses, the radius r that details a close encounter in Hill radii (distinct from 'mutual Hill radii', which we'll use later), and the density d of the planet in grams per cubic centimeter. The following line contains three values, each separated by a space, that represent the semi-major axis (in AU), the eccentricity, and the inclination. Following that is the line containing the longitude of ascending node, the argument of pericenter, and mean anomaly, all input as angular values in degrees. The final line is not used for Asteroidal coordinates. Initial conditions for each planet are input in this way at the bottom of the big.in file.

There is also a parameter file called param.in that contains the initial timestep for integration (for which I used 1/20th the smallest planetary orbital period), the length of integration, the length of time between data output, and the mass/radius of the central star. With all of this information complete, an integration can be initiated. After the completion of an integration, another program, called element6, is used to convert data into readable files for the user.

The Bulirsch-Stoer Algorithm is our chosen method of numerical integration within Mercury6. As understanding this algorithm is not necessary to analyze the results of our experiments, a prolonged discussion into the process by which the algorithm works is not necessary. Briefly, the Bulirsch-Stoer Method computes its desired values using the “modified midpoint rule” of numerical integration, adjusting the substeps to maximize efficiency and accuracy (Kirpekar, 2003). This method is described thoroughly in the MIT Paper on the Implementation of the Bulirsch-Stoer method as referenced.

The Computer Supercluster

A vital element of this research was gaining the use of the HPC (High-Performance Computing) computer supercluster through the advanced computing group at the University of Maine. In essence, this supercluster contains a large number of computing nodes of varying speeds that can be accessed remotely. When an account is created, files can be transferred back and forth between a virtual desktop. From this remote account, computing jobs can be input into the HPC system to run on their own, independently of the physical computer of the user.

Using the cluster, I have been able to run many Mercury6 integrations simultaneously, which significantly improved the time it took to get data back from each

experiment. Some experiments that would have taken years if I had to use my own computer to run each trial could now be done in weeks. Once the program is finished running, the aei data files can be downloaded for analysis.

These completed data files contain the values for all 6 of the Asteroidal coordinates output at specified time intervals. For analysis of this data, I used a python program that transfers each column of the data into arrays and graphs of each relevant parameter over time.

Mutual Hill Radii

When studying the interactions between planets in orbit, it is natural to consider the strength of gravitational effects between planets. One method of considering this is through the use of 'mutual Hill radii,' which is considered to be the natural unit of gravitational interaction (Weiss et al., 2018). The calculation of mutual Hill radii between two orbiting bodies is as follows:

$$R_H = (\frac{m_j+m_{j+1}}{3M_*})^{\frac{1}{3}} \frac{(a_j+a_{j+1})}{2} \qquad (2)$$

Here, R_H stands for the mutual hill radii between two planets, m_j and m_{j+1} are the masses of the two planets being considered together, a_j and a_{j+1} are the semi-major axes of those planets, and M_* is the mass of the central star of the system.

In our analysis section, we can use the spacing between planets in hill radii as a method of determining how strongly two planets should interact gravitationally. In general, observed exoplanets are about 20 hill radii apart (Weiss et al., 2018). For reference, it is also found that when we control for systems with 4 or more planets, we generally observe planetary spacings to be around 12 hill radii (Bu & Pu, 2018).

EXPERIMENTAL DESIGN

As already mentioned, three systems of planets were chosen off of the NASA Exoplanet archive. If no eccentricity was listed, an eccentricity of zero was input into Mercury6. Likewise, if no inclination was listed on the database, an inclination of zero was input into Mercury6 as well. In the selection of potential systems to be used in this project, two requirements were satisfied: first, that the system contains three observed planets; and, for inclination and eccentricity, that each system had null values for only one of the two. If both of these parameters were blank on the NASA database, the system was ignored. Also, for GJ 9827, the density of each planet was specified, but for the other two systems, there were no observed values for planetary densities. Due to this, a general density of 6.0 grams per cubic centimeter was used, which can be considered a reasonable density for a terrestrial planet. This is a potential source of error however, as it is unknown whether these observed planets are terrestrial (rocky/solid surface, similar to Mercury, Venus and Earth) or jovian (gaseous exterior, similar to Jupiter and Saturn). If issues arise in the analysis of each system, this assumption could be revisited and rethought. For the angular parameters, it is assumed that at the beginning of the integration, each planet's orbit is similarly oriented relative to a reference direction. This is also an assumption that could be revisited if issues arise.

GJ 9827 is a system of three planets orbiting in very close proximity to their central star, as seen in Table 1 below. The central star in this system has a mass of 0.606 solar masses and a radius of 0.002801 AU. Uniquely, the density of each planet here has a recorded value on the database: GJ 9827 b has a density of 6.93 grams per cubic

centimeter; GJ 9827 c has a density of 2.42 grams per cubic centimeter; and, GJ 9827 d has a density of 2.69 grams per cubic centimeter.

System	Planet	Mass (Solar Masses)	Semi-Major Axis (AU)	Eccentricity	Inclination (degrees)
GJ 9827	b	1.4695×10^{-5}	0.01880	0	1.965
	c	2.48105×10^{-6}	0.03925	0	0.155
	d	1.2119×10^{-5}	0.05591	0	0.455

Table 1 – GJ 9827

As seen in Table 1, each of these planets has comparatively small orbital distances, with none of the planets' semi-major axes exceeding 0.06 AU. For reference, remember that the semi-major axis of earth and mercury are 1.0 AU and 0.3871 AU, respectively. As a final note on these orbital parameters, note that the inclinations were input to portray the listed data on the planets as relative to each other, varying around a central average. For example, planet c had an inclination under the central average and therefore was rotated 180 degrees in the input files to represent this accurately.

For HD 7924, each planet again has a semi-major axis lower than any in our solar system. These planets, however, have significantly larger orbits than that of our first system, with semi-major axes ranging from 0.06 AU to 0.1551 AU. The central star in HD 7924 has a mass of 0.65 solar masses and a radius of 0.003629 AU. For these planets, there was not listed density or planetary radii, so the density used for each planet was 6 grams per cubic centimeter, as discussed above. In Table 2 below, all available data for the planets in the system is detailed.

System	Planet	Mass (Solar Masses)	Semi-Major Axis (AU)	Eccentricity	Inclination (degrees)
HD 7924	b	1.9086×10^{-5}	0.06	0.06	No Inclination Information Available
	c	2.3571×10^{-5}	0.1134	0.098	
	d	1.9372×10^{-5}	0.1551	0.21	

Table 2 – HD 7924

Due to a lack of inclination in this system, its primary characteristic is the eccentricity of each planet. As was done for all three systems, the angular parameters for the system were chosen such that each planet's orbit is initially similarly oriented.

For HD 136352, the mass of the central star is 0.81 solar masses, with no recorded value for its radius. Based on this mass in comparison our other systems, a radius of 0.78 solar radii was used, which came out to 0.00363 AU. This value is simply to maintain a reasonable value for stellar density as compared to our other systems.

System	Planet	Mass (Solar Masses)	Semi-Major Axis (AU)	Eccentricity	Inclination (degrees)
HD 136352	b	1.44099×10^{-5}	0.0934	0.14	No Inclination Information Available
	c	3.24462×10^{-5}	0.1666	0.04	
	d	2.57661×10^{-5}	0.4128	0.09	

Table 3 – HD 136352

This system, as seen in Table 3, is similar to HD 7924 in that our observational data gives us eccentricity values for each planet in the system, but no values for inclination. Due to this, we again assume that these planets have no inclination (are coplanar), and can use this system to observe how the eccentricities of each planet vary relative to each other.

For all of these systems, it is essential to recognize that the masses of these planets are uniquely large for planets with low semi-major axes. For example, all of the terrestrial planets in our own solar system are fairly low-mass compared to these; Mercury's mass is about 1.652 x 10^{-7} solar masses. As compared to the masses of the planets in Tables 1, 2, and 3, Mercury is orders of magnitude lower in mass. Due to this, Mercury's mass is our reference point for the masses of our outer planets that we add to each system. As seen in Table 4 below, the mass of the outer planet is doubled in each step, ranging from 2 to 64 Mercury Masses. This way, we can observe the effects of both small exterior planets on the orbits of the interior planets, as well as the effects of planets more similar to those already in the system. For semi-major axes, a range was chosen from just above the orbit of Mercury (0.387 AU) to just under the orbit of Mars (1.524 AU). This ended up as a range from 0.4 AU to 1.4 AU in steps of 0.2 AU.

<table>
<tr><th></th><th>2 Mercury Masses</th><th>4 Mercury Masses</th><th>8 Mercury Masses</th><th>16 Mercury Masses</th><th>32 Mercury Masses</th><th>64 Mercury Masses</th></tr>
<tr><td>0.4 AU</td><td colspan="6" rowspan="6">These parameters combine to describe each perturbing planet added to the systems described above</td></tr>
<tr><td>0.6 AU</td></tr>
<tr><td>0.8 AU</td></tr>
<tr><td>1.0 AU</td></tr>
<tr><td>1.2 AU</td></tr>
<tr><td>1.4 AU</td></tr>
</table>

Table 4 – Set of Outer Planets

Parameters in Table 4 were used to add outer planets to each system. Note that 0.4 AU was not used for system HD 136352, because its outermost planet has a semi-major axis of 0.4128 AU. To have 5 semi-major axis steps for each system, 1.4 AU was not used for the outer planets in systems GJ 9827 or HD 7924. These small differences in

experimental design are clarified in Table 5 below. For these outer planets, their orbits were chosen to be as simple as possible, with no initial eccentricity or inclination. These outer planets were also given a density of 5 grams per cubic centimeter, a reasonable density for a terrestrial planet.

<table>
<tr><th></th><th>GJ 9827</th><th>HD 7924</th><th>HD 136352</th></tr>
<tr><td rowspan="5">Semi-major Axis of Each Perturbing Planet</td><td colspan="2">0.4 AU</td><td>0.6 AU</td></tr>
<tr><td colspan="2">0.6 AU</td><td>0.8 AU</td></tr>
<tr><td colspan="2">0.8 AU</td><td>1.0 AU</td></tr>
<tr><td colspan="2">1.0 AU</td><td>1.2 AU</td></tr>
<tr><td colspan="2">1.2 AU</td><td>1.4 AU</td></tr>
</table>

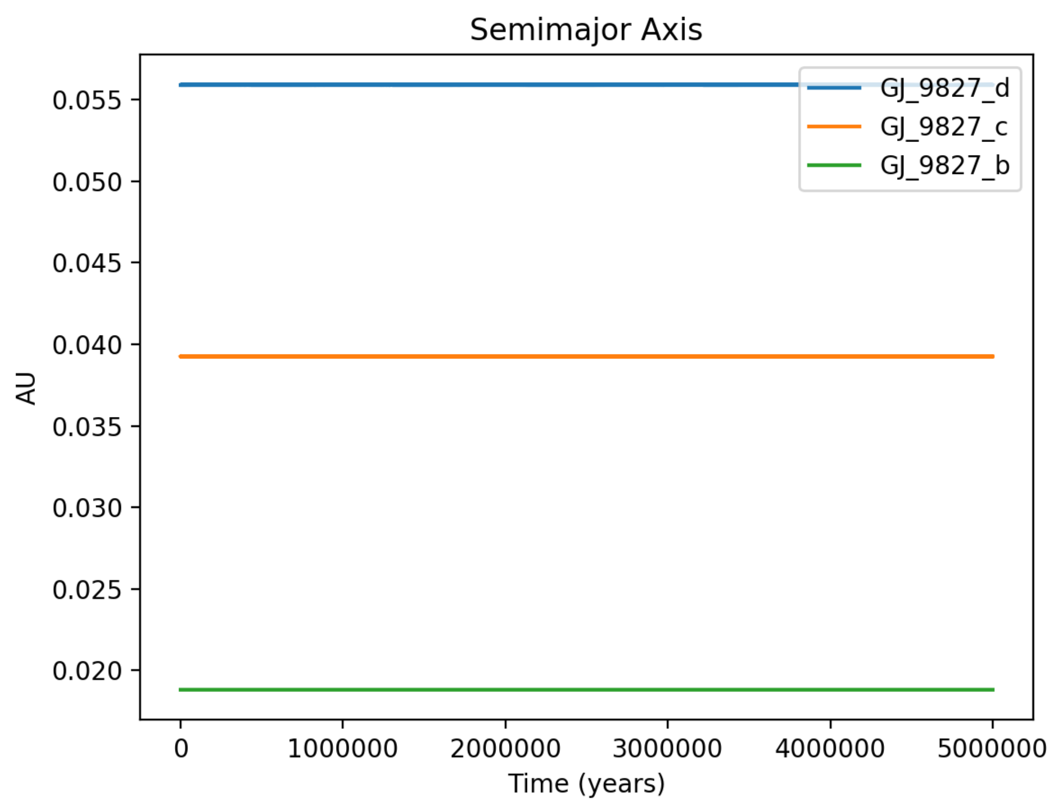
Table 5 – Differences in Added Perturbing Bodies

The program was set to simulate each system over 5 million years, outputting data at 10-year intervals, yielding 500 thousand data points for the orbital parameters in each system. The initial timestep was chosen to be one-twentieth the orbital period of the innermost planet. This was done to maintain numerical accuracy for each integration. These initial timestep values came out to be 0.06 days for GJ 9827, 0.269 days for HD 7924, and 0.579 days for HD 136352. With each set of runs set up, the resulting data is useful for understanding how the addition of outer planets affects the orbits of the observed inner-planets.

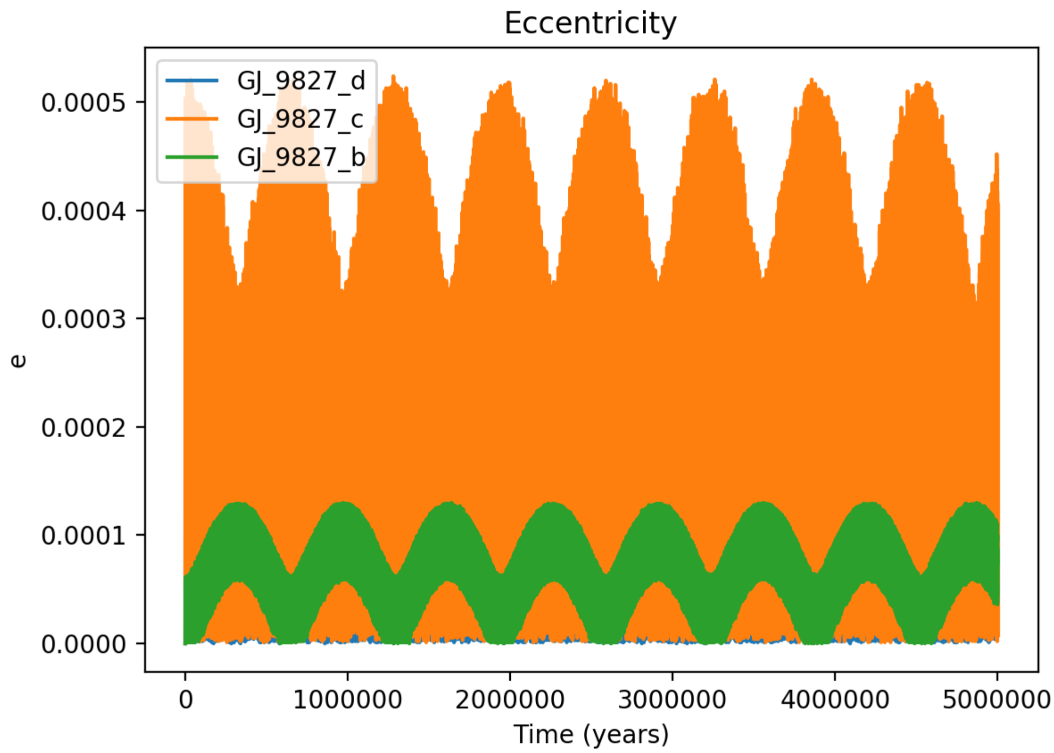
RESULTS

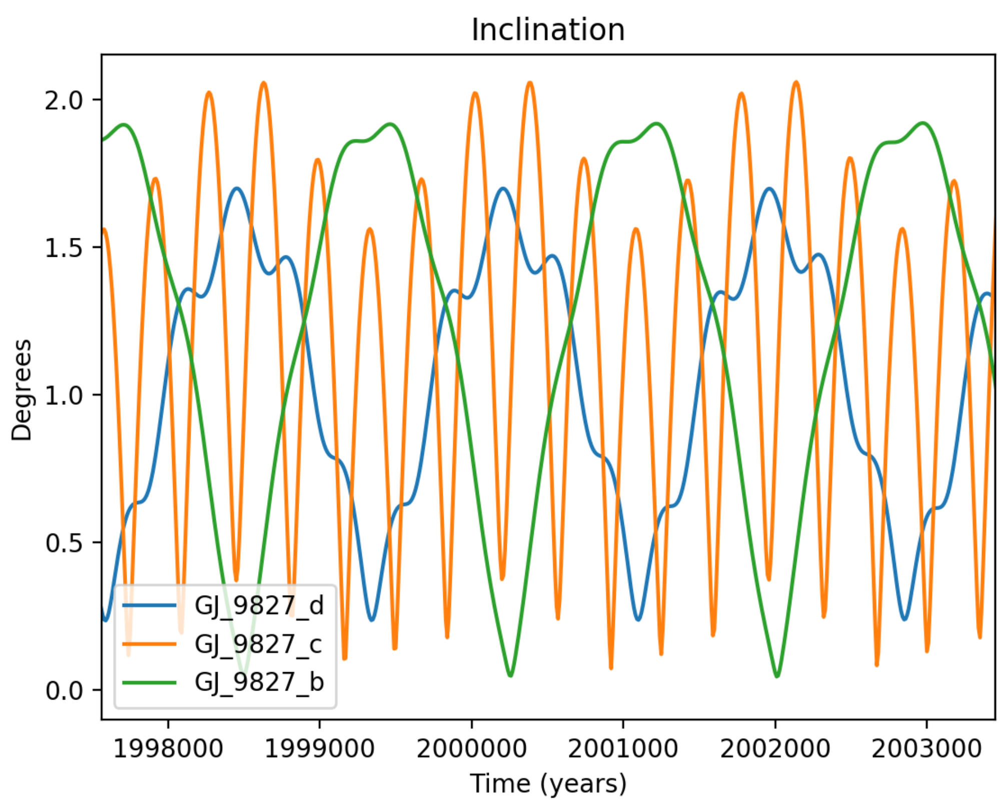
For each system, Mercury6 was run first with the observed planets on their own as a test to be compared with each set of results with a perturbing planet. Due to the large number of tests with each of the three systems, this experiment resulted in the output of hundreds of data sets and nearly a thousand graphical images for semi-major axes, eccentricities, and inclinations. Due to the sheer size of results, graphs, and data points were chosen from this experiment to represent its results most effectively. For each system, the results for the observed planets alone are shown first, with selected results from the rest displayed after.

GJ 9827



GJ 9827 Test Data

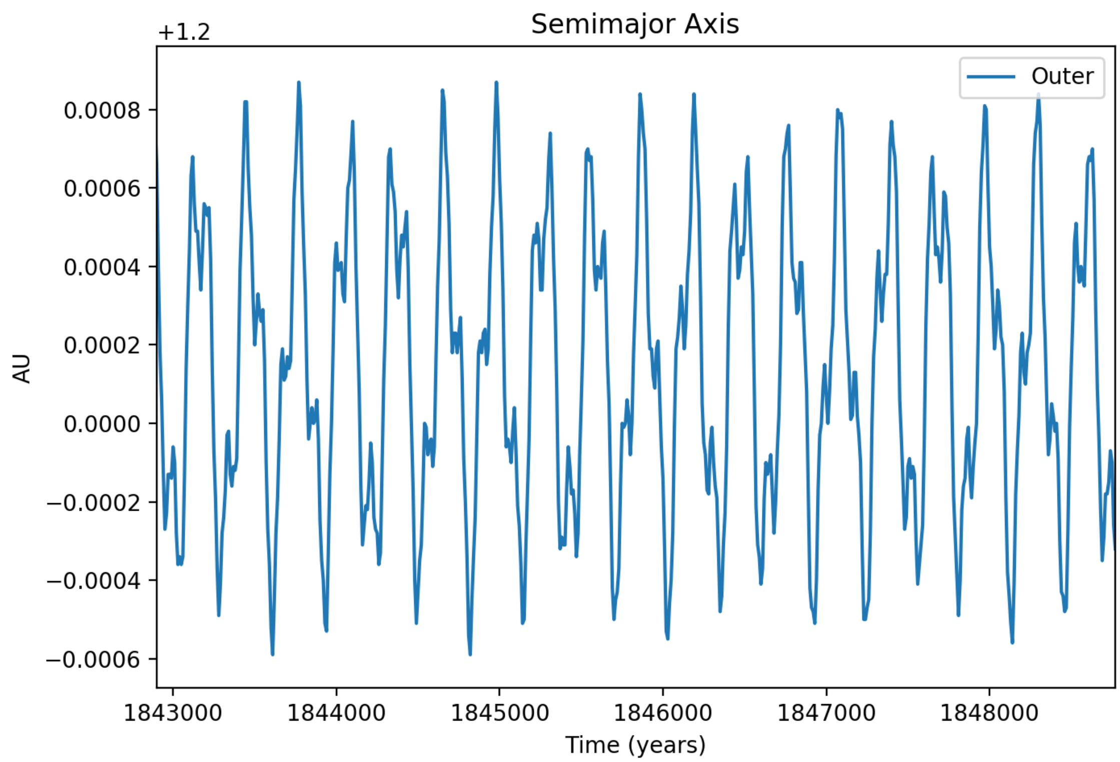




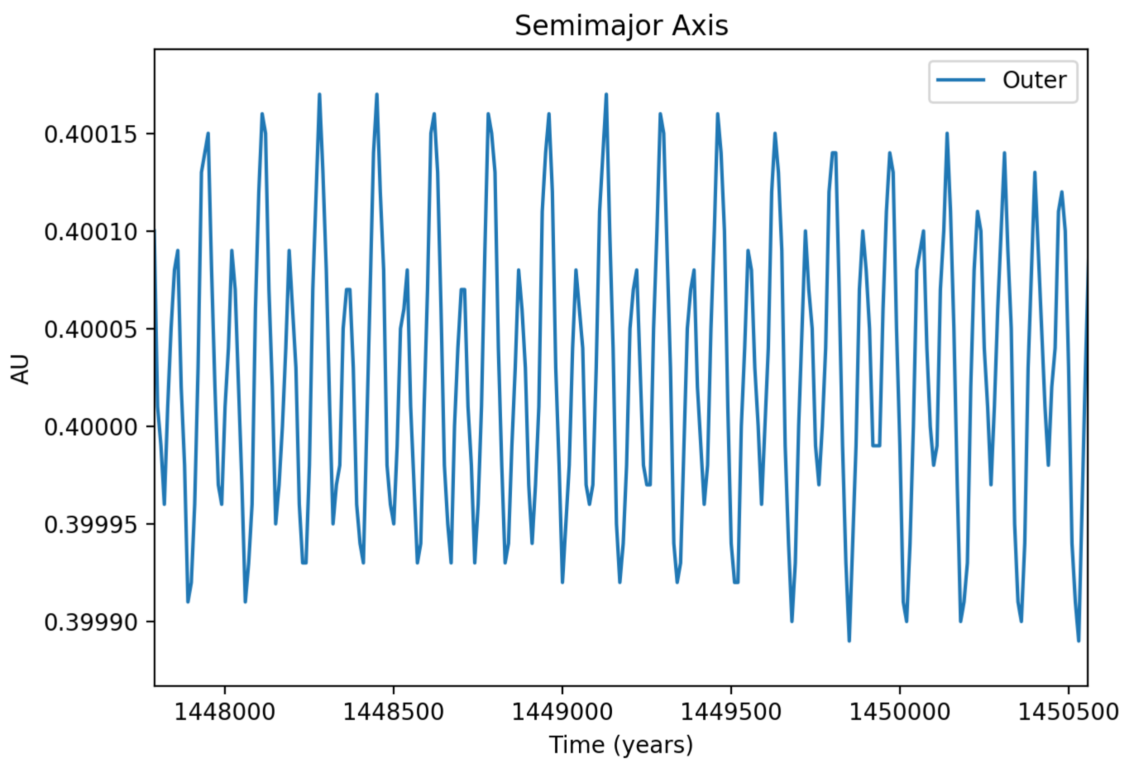
GJ 9827 Test Data Continued

The semi-major axis for system GJ 9827 is extremely stable, with no noticeable variation over time. Eccentricity is less important for this system, as none of the planets have any initially. However, as is observed, the planets develop some eccentricity due to their interaction over time, exhibiting oscillatory behavior. In the third graph above, we have a zoom into the inclinations of the three planets of GJ 9827, which all develop complex repetitive oscillations as they interact with one another.

First let's look at the changing semi-major axis of our perturbing planet as we adjust its distance from the central star. Both graphs are with an outer planet 4 times the mass of mercury.



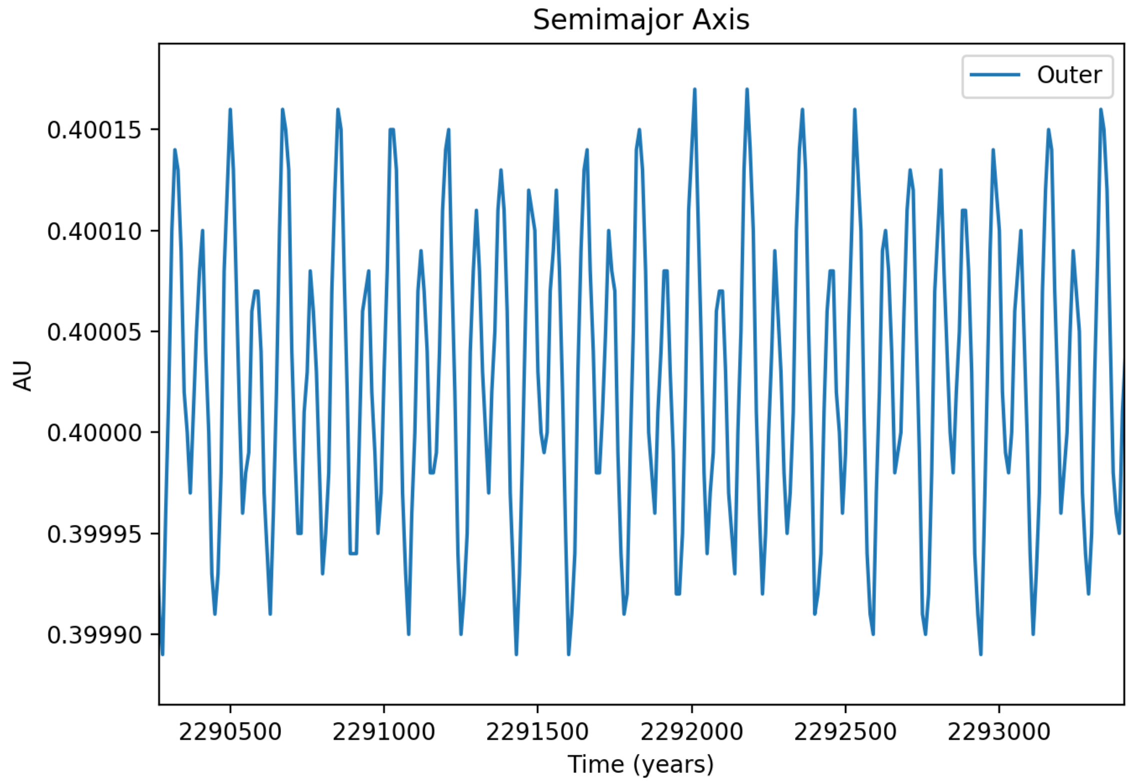
Outer Planet (4 mercury masses) Semi-Major Oscillation (1.2 AU)



Outer Planet (4 mercury masses) Semi-Major Oscillation (0.4 AU)

This demonstrates that the semi-major axis of the outer planet, in this case, oscillates with a greater range when the planet is at a greater distance from the inner planets. This could be because the gravitational effect of the star dominates the motion of the planet more when it is closer to the star.

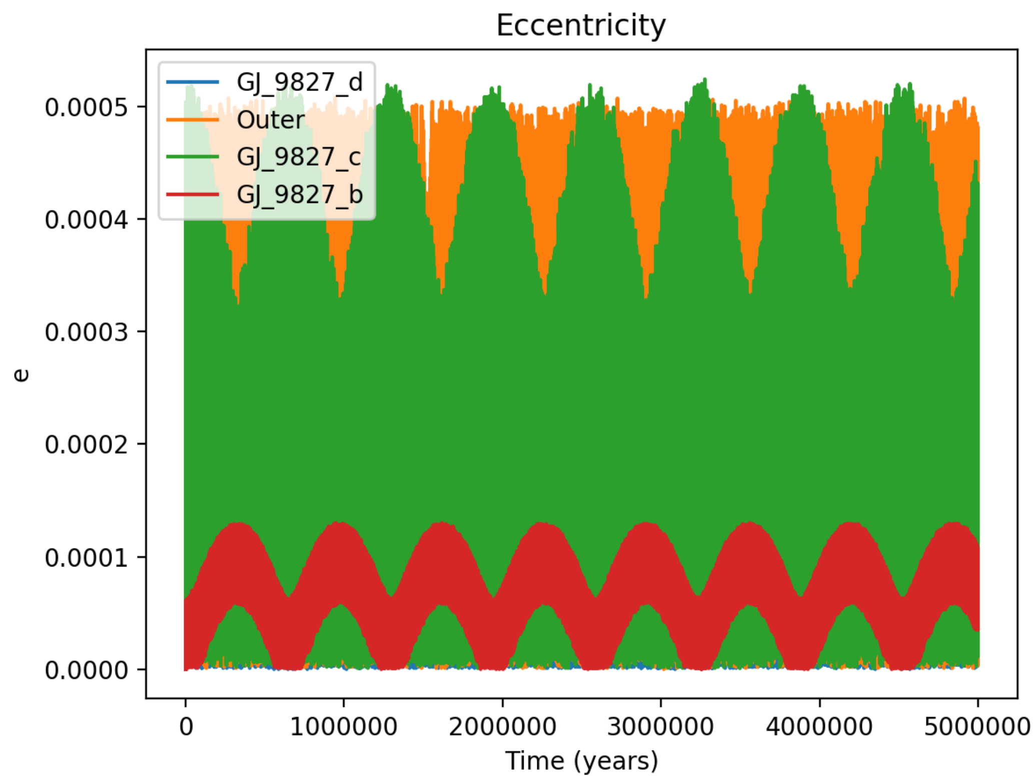
Next, we will look at the differences in semi-major axis oscillation as we increase the mass of our outer planet. Compare the following graph to that above at 0.4 AU semi-major axis and 4 mercury masses.



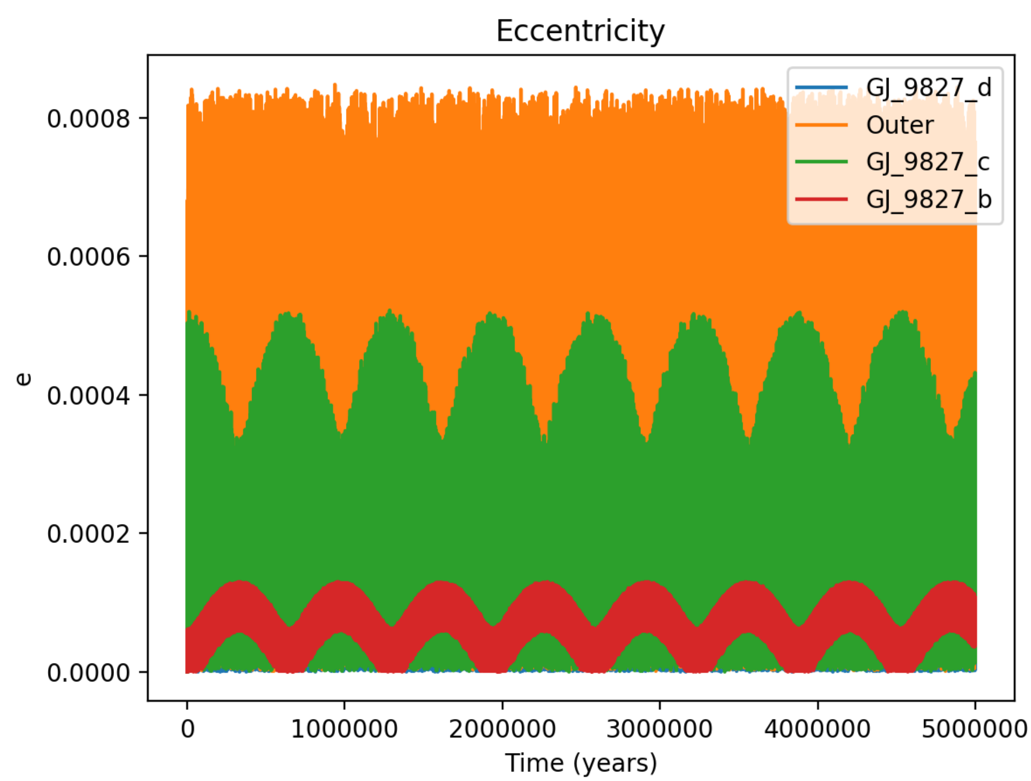
Outer Planet (64 mercury masses) Semi-Major Oscillation (0.4 AU)

This figure illustrates that with greater mass, the semi-major axis oscillations of the perturbing planet remain about the same. Apart from these small oscillations, the semi-major axes of these outer planets are incredibly stable in all combinations of mass and orbital distance.

Moving on from the semi-major axis, oscillations in small levels of eccentricity also occurred in each system, nearly identical to that of the GJ 9827 test data, as shown on page 16. It appeared that the oscillations of eccentricity in the inner planets remained the same regardless of the mass or orbital distance of the outer planet. This phenomenon is demonstrated with the eccentricity data for 2 systems on the next page: one with a 16-mercury mass outer planet at 0.4 AU and one with a 4-mercury mass outer planet at 1.2 AU.

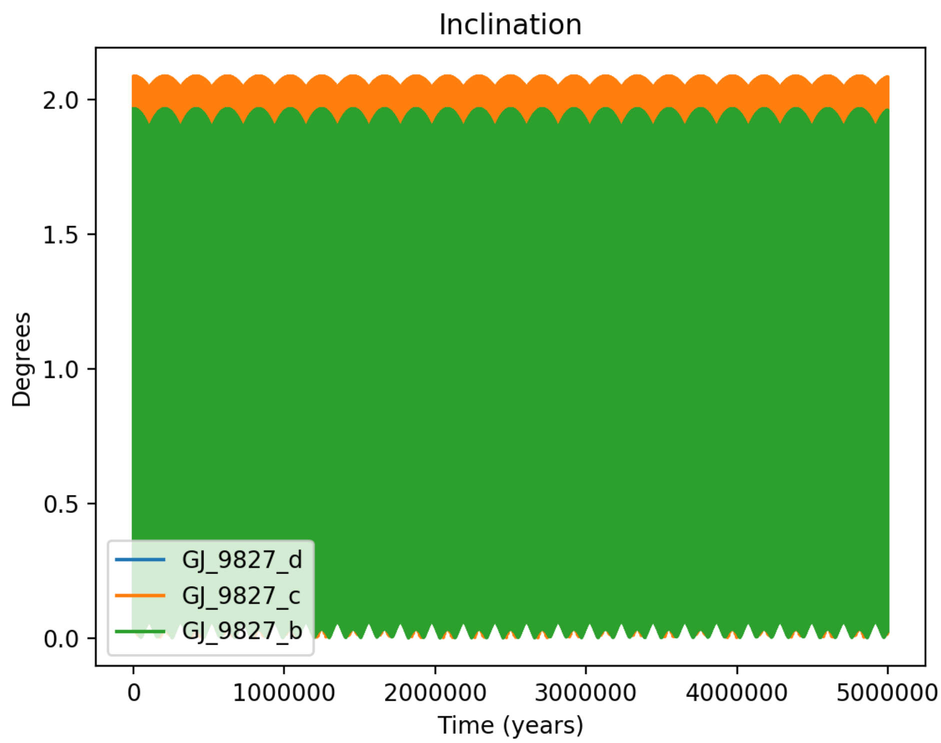


Eccentricity (16 Mercury Masses at 0.4 AU)



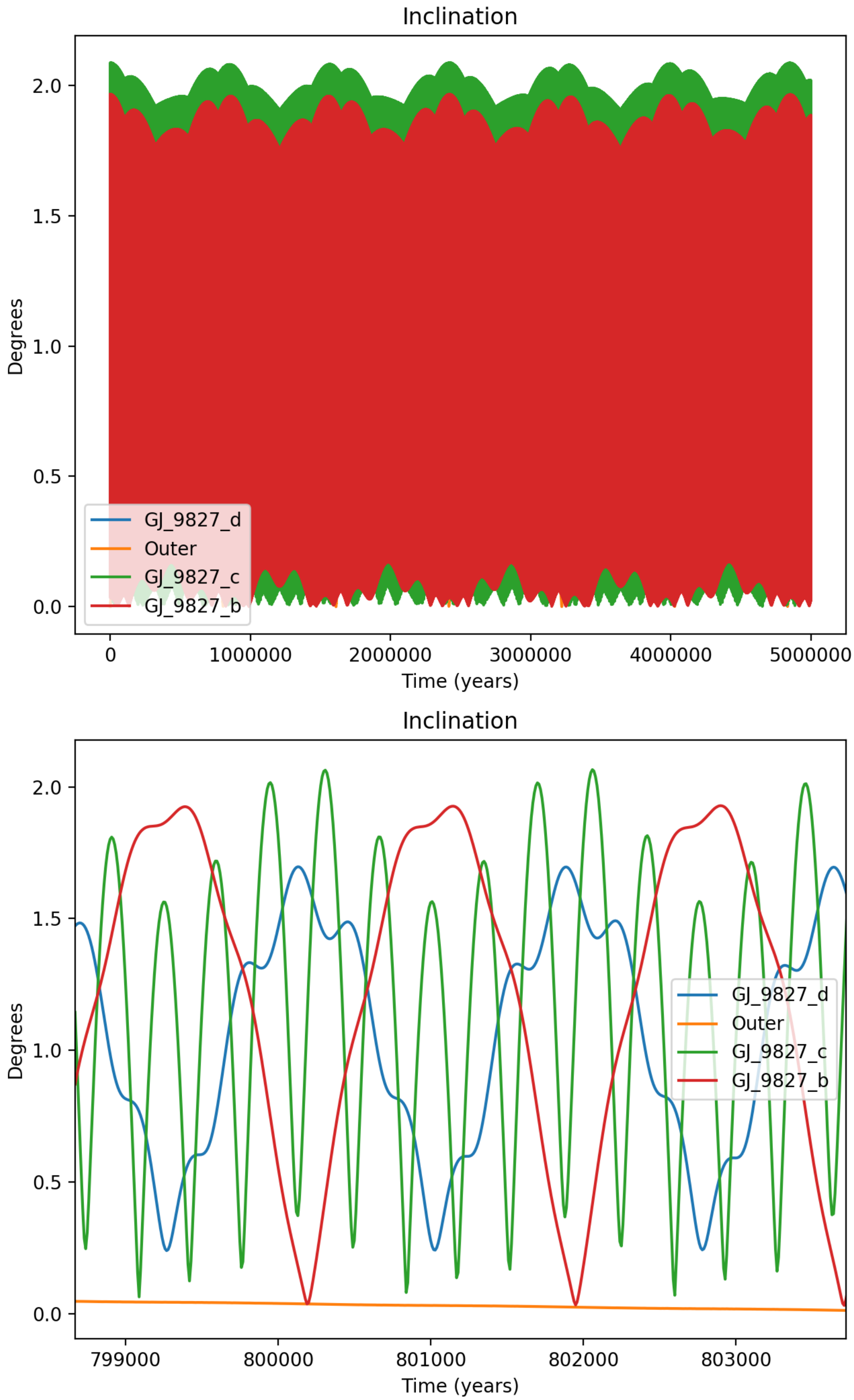
Eccentricity (4 Mercury Masses at 1.2 AU)

Due to the density of data, the eccentricity curves for each planet covers some of the data for the others. Despite this, it is clear that the eccentricity oscillations of the inner planets are barely distinguishable between the two figures on the previous page and the Eccentricity of the planets in our test system. More significant for the case of this system, however, is the oscillation of inclination. As seen in the test system graphs for GJ 9827, each planet exhibited complex oscillations every few thousand years. On top of this, larger long-term oscillations can be seen over the entire range of 5 million years.

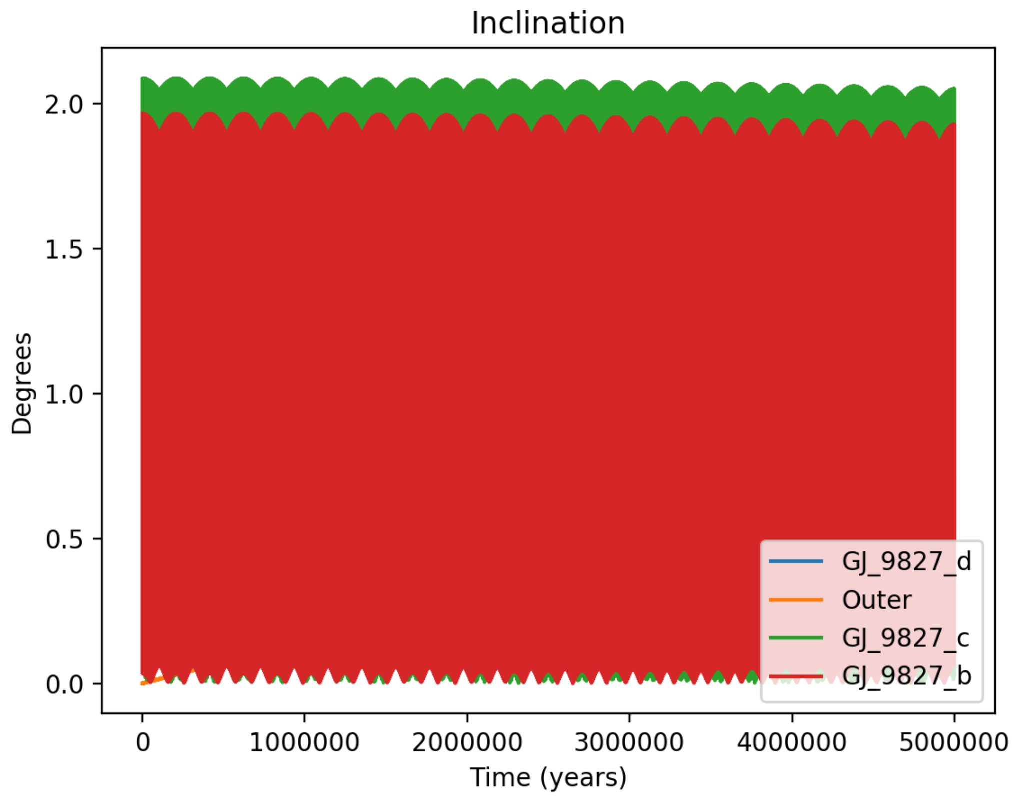


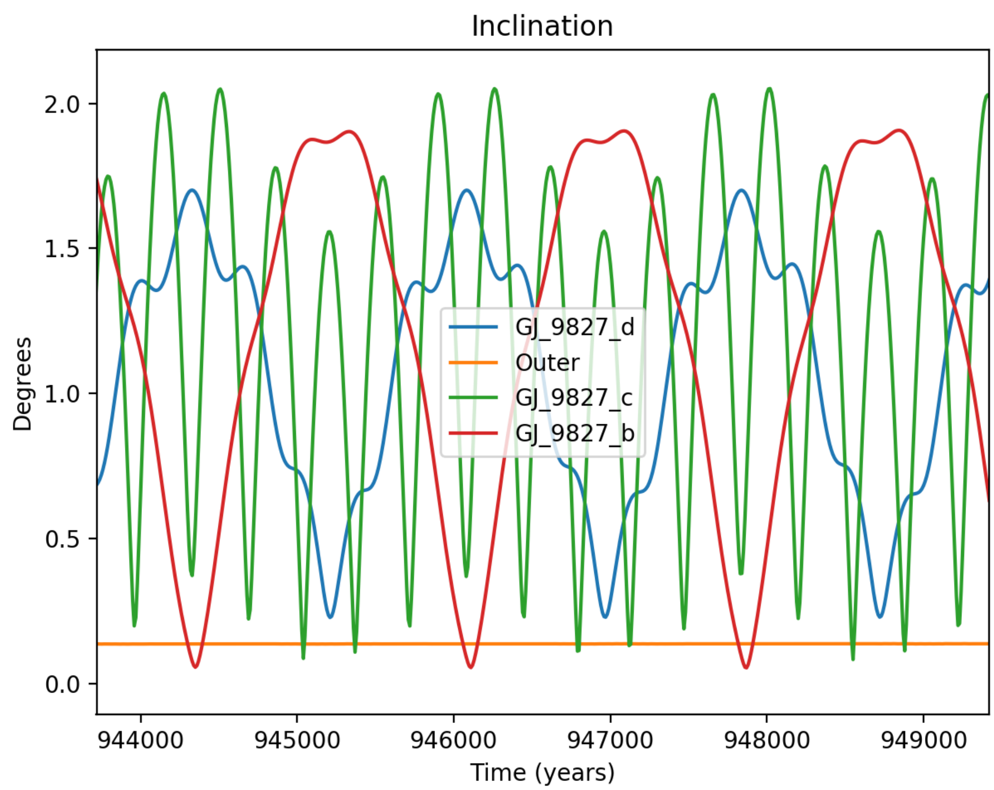
GJ 9827 Full Inclination

Here, in addition to the oscillatory behavior demonstrated in the zoomed inclination graph, the system also sees repetitive oscillations for Inclination over larger time scales. In order to compare to our many experimental cases, the next few pages will contain comparative examples for these graphs with various perturbing planets.

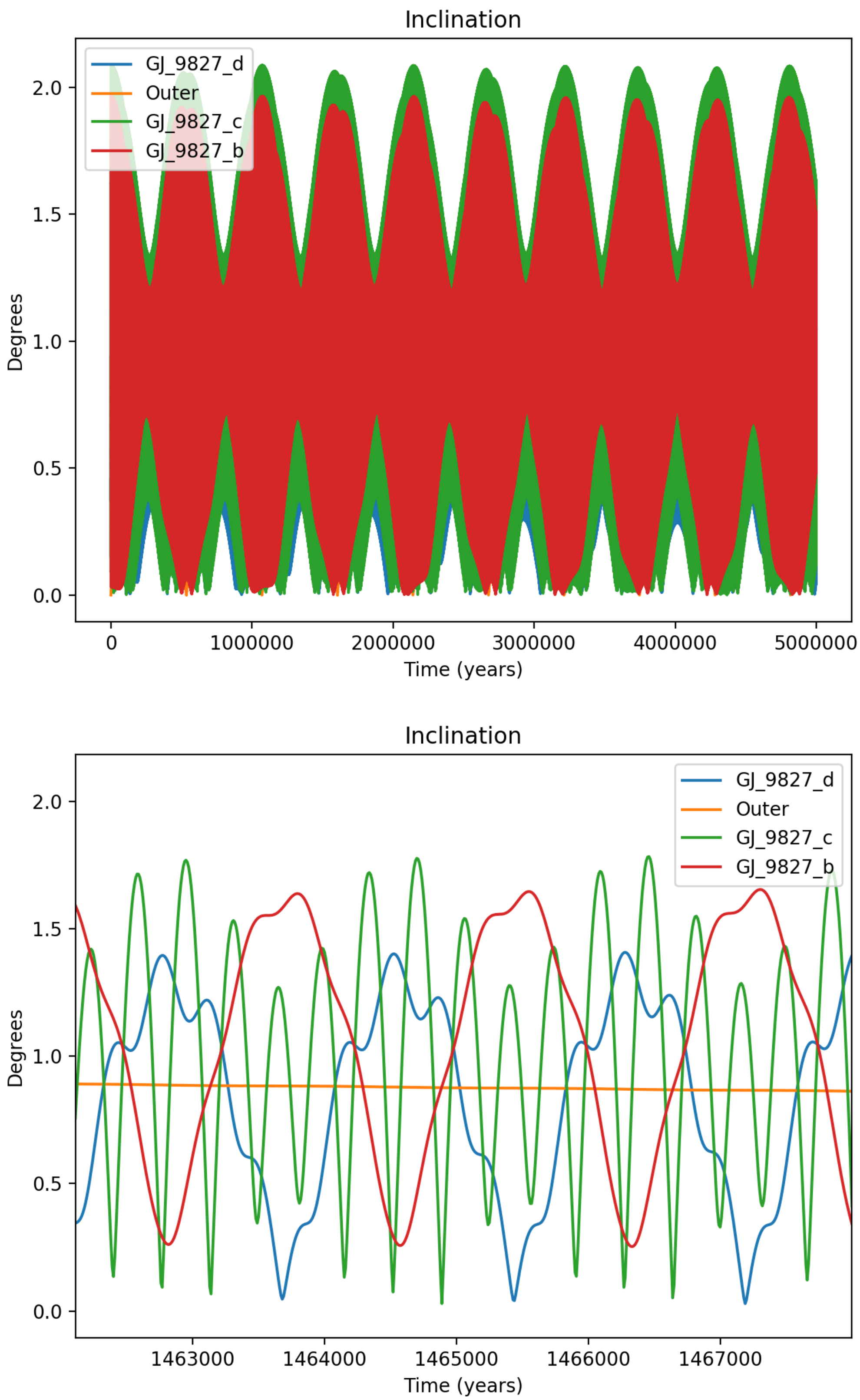


GJ 9827 Inclinations (Outer Planet at 4 mercury masses, 0.4 AU)





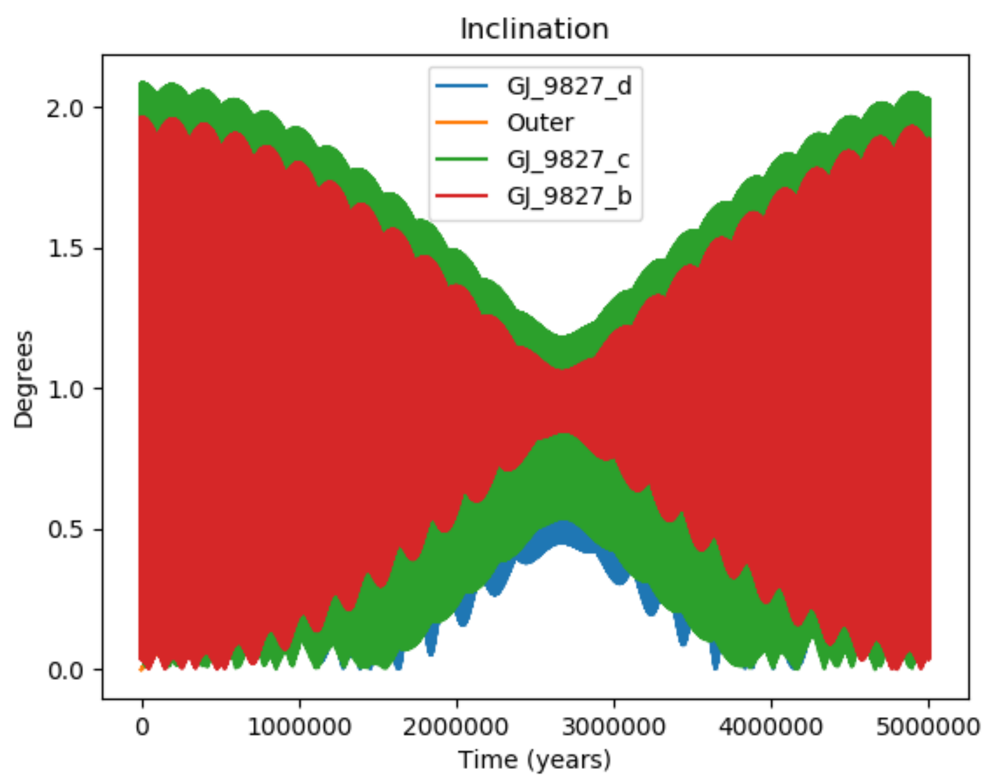
GJ 9827 Inclinations (Outer Planet at 4 Mercury Masses, 1.2 AU)



GJ 9827 Inclinations (Outer Planet at 32 mercury masses, 0.4 AU)

These images reveal that for each added planet chosen here, the three observed inner planets of GJ 9827 continue to have oscillating inclinations similar to that of the system on its own. These are complex, repetitive oscillations that are affected very little by the changing orbital parameters of the added perturbing planet. It also appears that the inclination of the perturbing planet is changing very slowly.

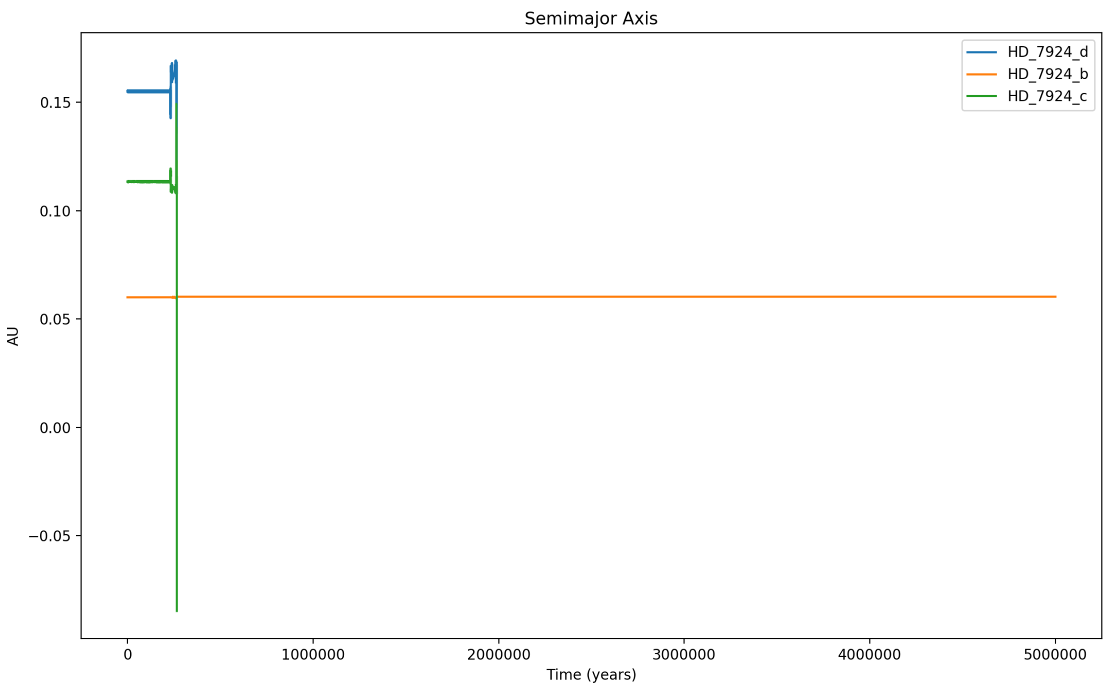
Moving on to the full inclination graphs, it appears that greater long-term oscillatory properties emerge both when the perturbing planet has a lower semi-major axis, as well as when it has a greater mass. This is demonstrated by the graphs for the outer planet at 4 mercury masses and 0.4 AU semi-major axis as well as that for the outer planet at 32 mercury masses at 0.4 AU semi-major axis. These larger oscillations in inclination also decrease in frequency with increasing distance of the perturbing planet from the observed bodies, as demonstrated in the figure below.



GJ 9827 Inclinations (Outer Planet at 32 Mercury Masses, 0.8 AU)

HD 7924

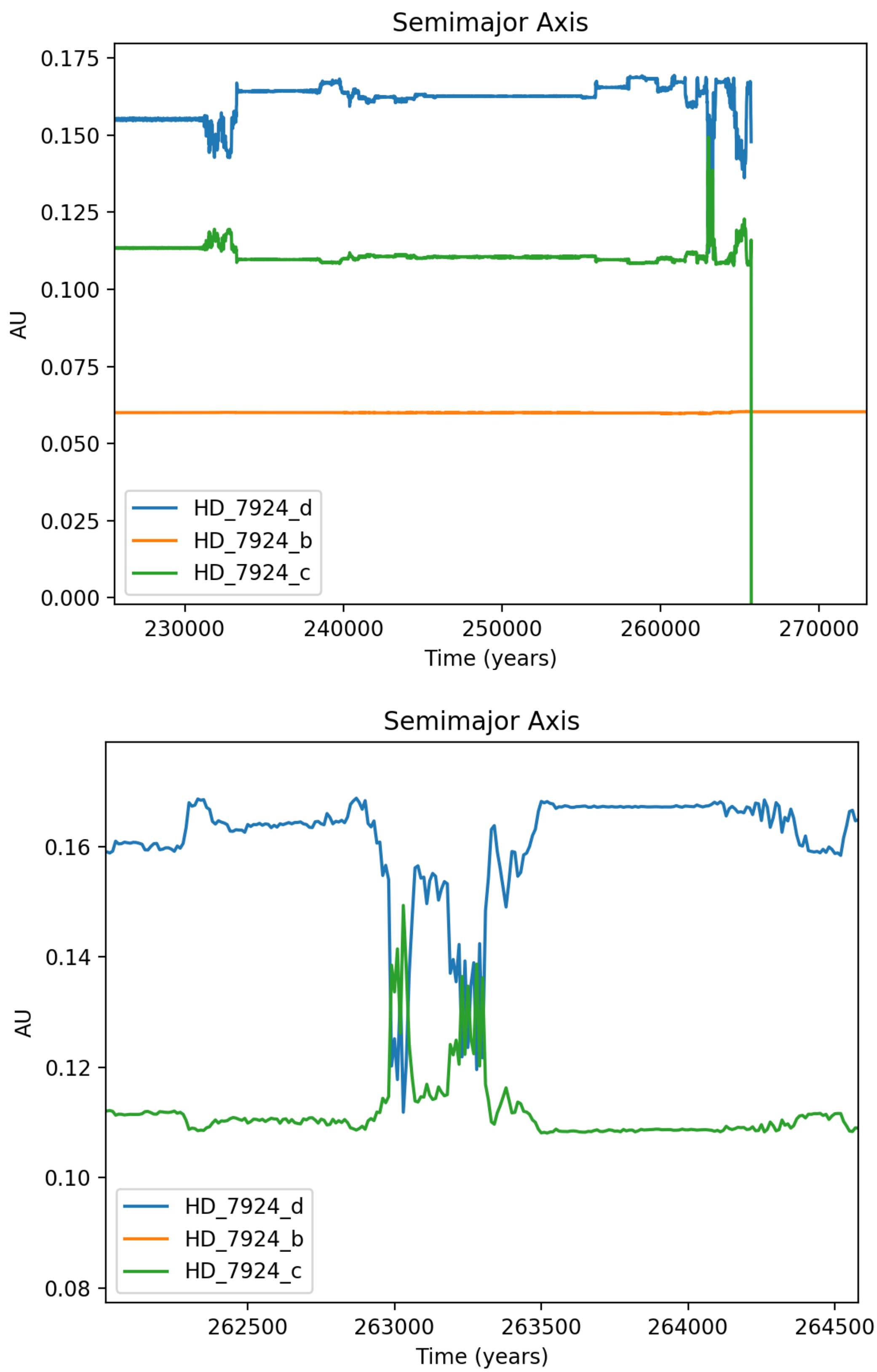
In the case of HD 7924, the system faced instability within the first 200 to 250 thousand years of the integration. This may be due to a number of reasons that will be discussed in our analysis section, but the graphical data demonstrating this instability is shown below.



HD 7924 Semi-major Axes

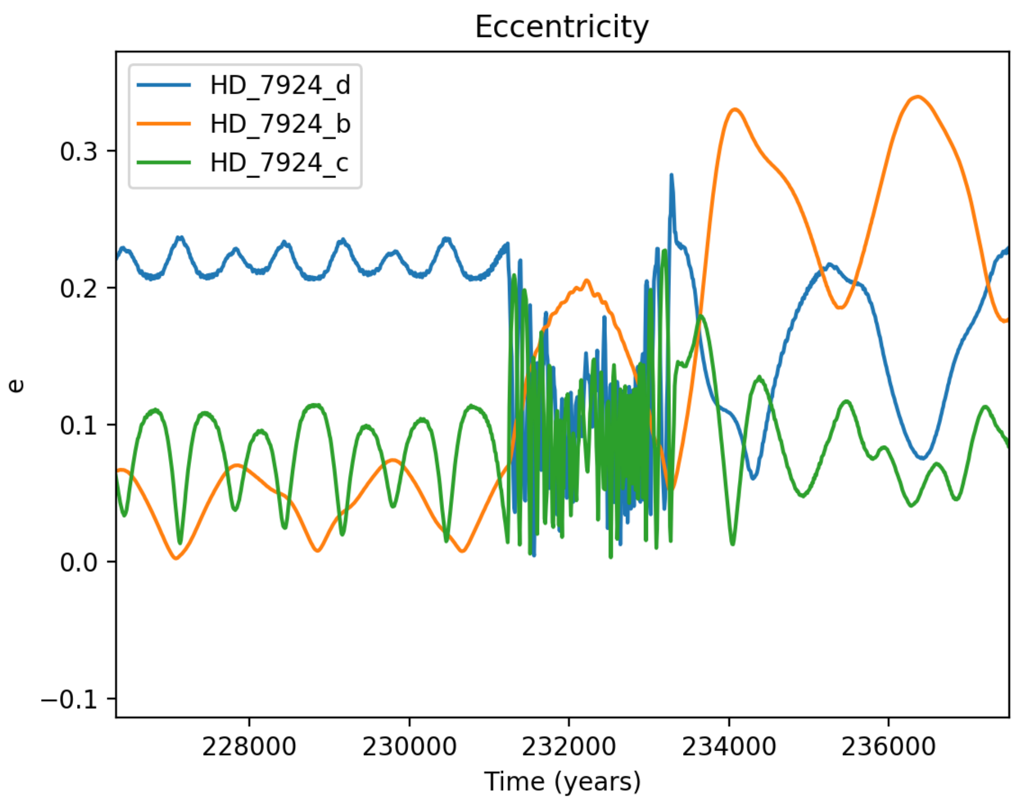
It is clear here that for the test system on its own, HD 7924 does not appear to demonstrate long-term stability as it was input into mercury6. This Instability begins at about 230 thousand years and continues, with a number of close encounters between planets HD 7924 d and HD 7924 c before their eventual destruction/ejection. In the info file that is output by Mercury6 after an integration, it was noted that a collision occurred between these two planets, leaving only HD 7924 in orbit.

Finer graphs of the semi-major axes of these planets during this period of instability are on the following page.



HD 7924 Semi-major axes during period of Instability

In these figures we observe erratic changes in the semi-major axes of both planets in question, with a short period of overlapping orbits before returning to their positions and eventual collision. Looking at the eccentricity graph in this system pre-instability is also valuable here, as there are simple oscillatory changes before instability, followed by erratic changes in eccentricity at the point where the system goes unstable.



HD 7924 Eccentricity at the Boundary of Instability

As seen above, all three planets exhibit fairly simple oscillatory behavior up until the point of instability where the eccentricity of the planets changes rapidly and erratically. This lack of predictability that is paired with instability will be discussed more thoroughly in the analysis section.

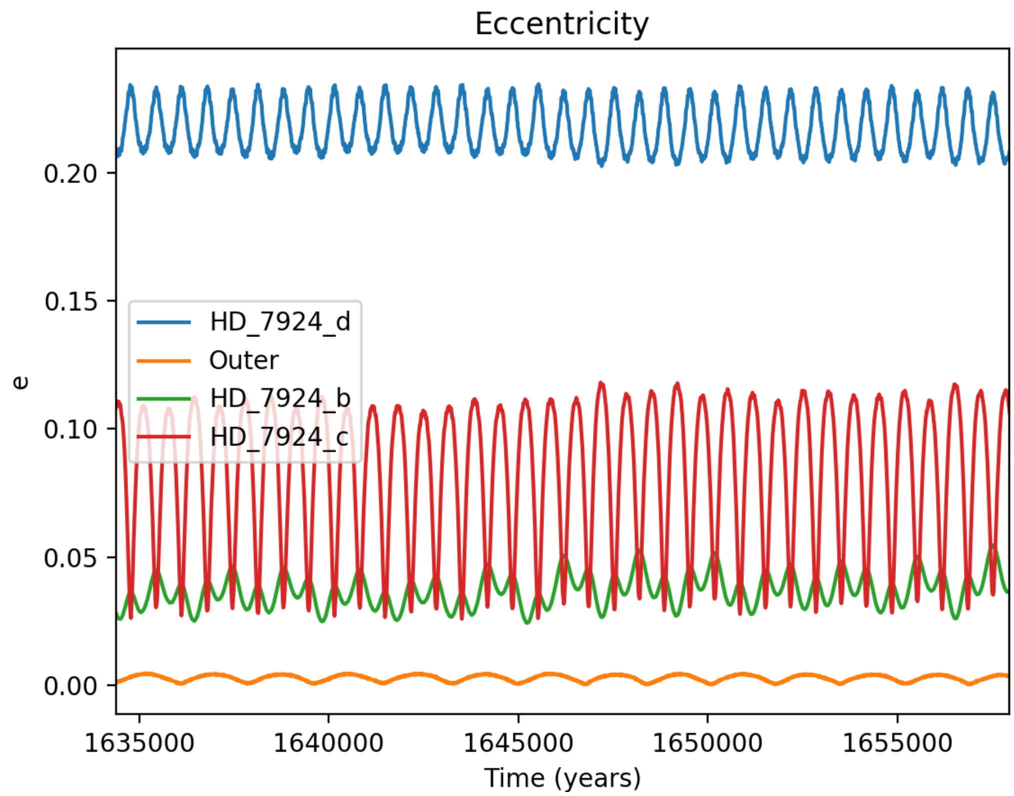
Testing this system with additional outer planets, as was done with both of the other two systems, yielded varying results. The table below outlines which of the cases with a perturber resulted in a stable system, with a U representing an unstable system and an S representing a system that maintained its stability over the 5-million-year integration.

	0.4 AU	0.6 AU	0.8 AU	1.0 AU	1.2 AU
2 Mercury Masses	S	U	U	U	U
4 Mercury Masses	S	U	U	U	U
8 Mercury Masses	U	S	U	S	U
16 Mercury Masses	S	U	U	U	S
32 Mercury Masses	S	U	U	S	U
64 Mercury Masses	U	U	S	U	U

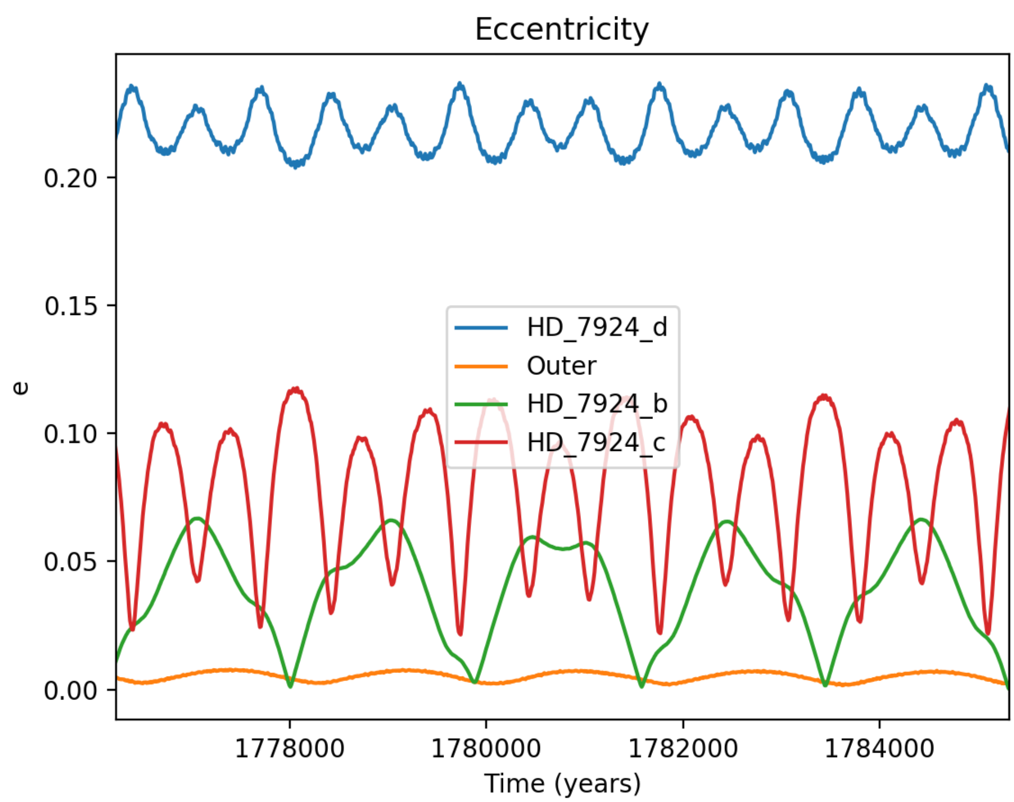
Table 6 – Perturber Properties and resultant stability for HD 7924

Here we see that, in many cases, the addition of a perturbing planet into HD 7924 results in a system that is more stable than the set of observed planets in the system alone. The systems that maintained stability throughout the course of these tests exhibited varying oscillatory properties in eccentricity and very little perturbation in the semi-major axes of each body, as seen in the results from other stable systems.

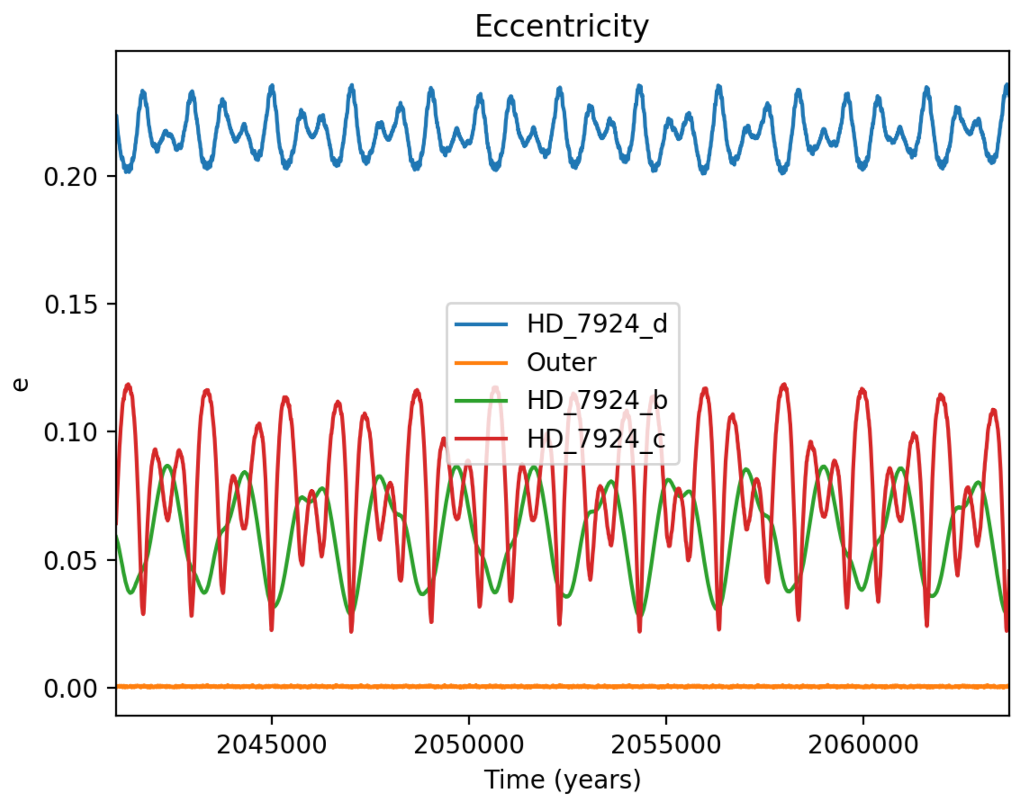
For the rest of the planets that resulted in instability, similar results to that of HD 7924 on its own were observed. Eccentricity graphs from a few of these observed stable systems are on the next few pages.



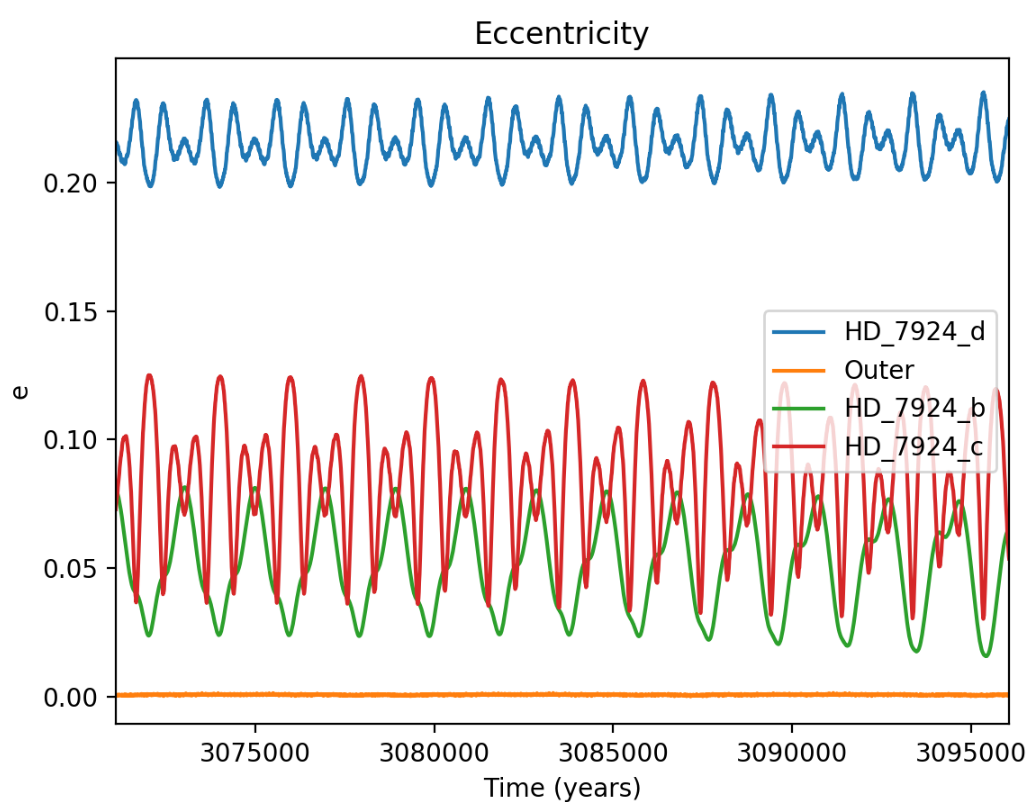
HD 7924 Eccentricity (2 Mercury Masses, 0.4 AU)



HD 7924 Eccentricity (4 Mercury Masses, 0.4 AU)



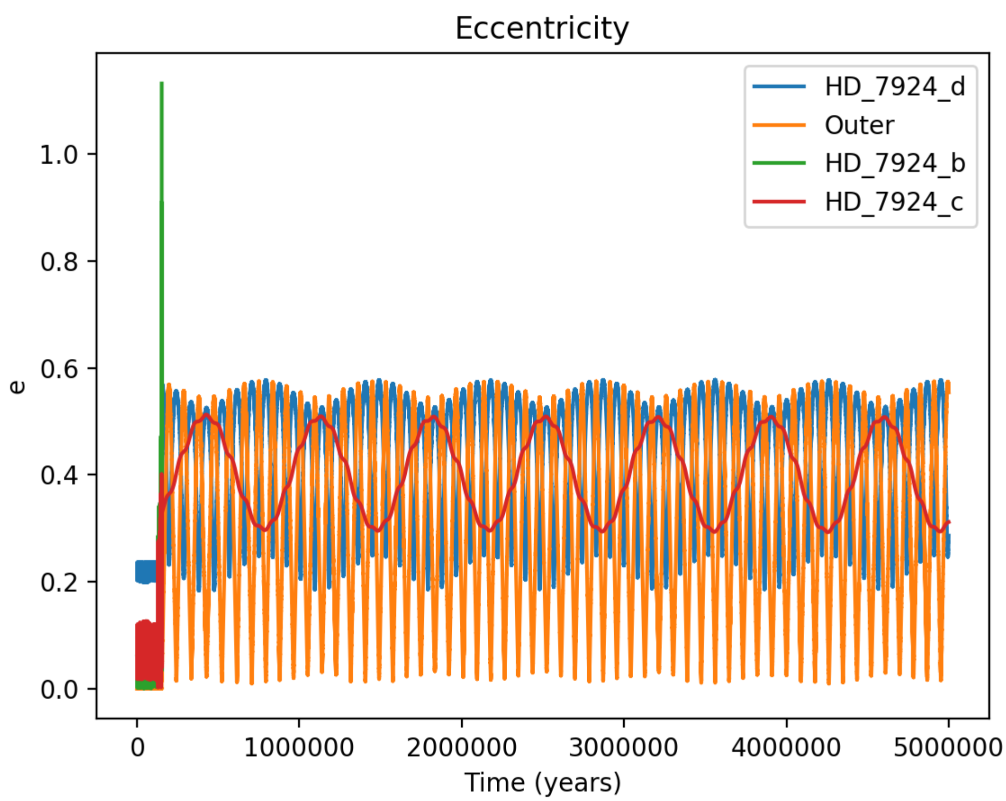
HD 7924 Eccentricity (32 Mercury Masses, 1.0 AU)



HD 7924 Eccentricity (64 Mercury Masses, 0.8 AU)

The figures on the previous two pages all represent systems that remain intact throughout their 5 million-year integrations. In general, the systems that remained intact were the ones that eventually reached oscillatory behavior like that of the systems in these graphs. The unstable systems exhibited erratic changes in eccentricity and semi-major axis before eventual ejection of a planet from the system.

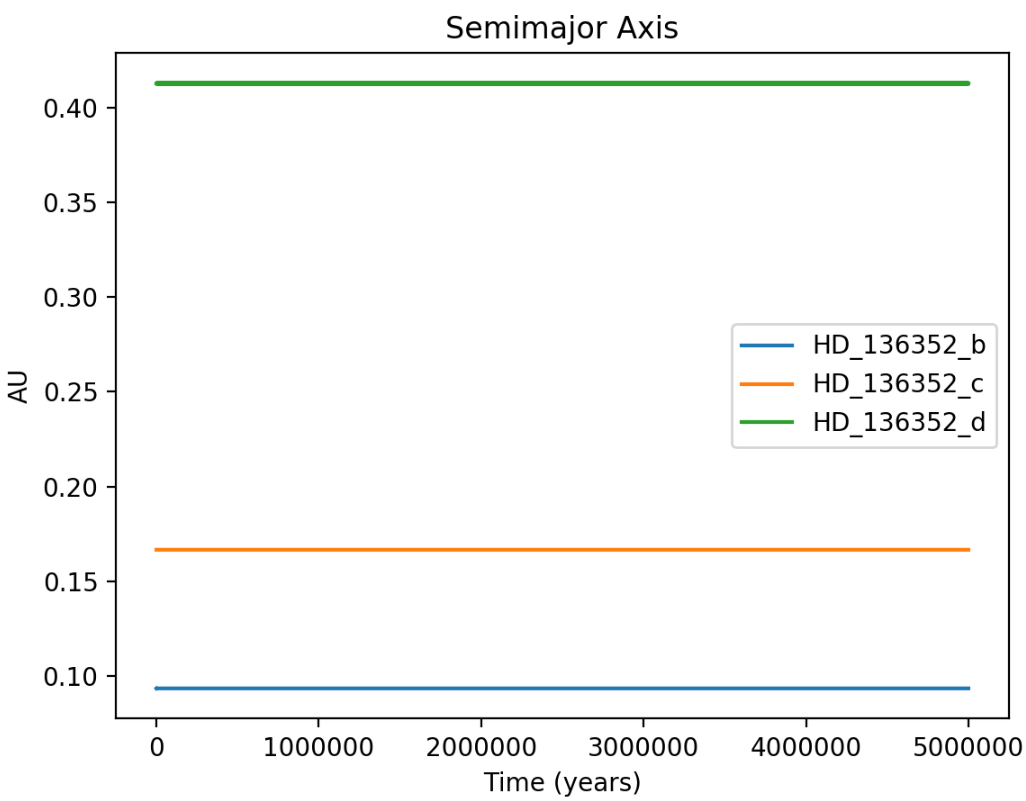
A curious case (albeit an unstable case for HD 7924), is demonstrated in the following figure, in which HD 7924 b was ejected from the system. After this ejection the system fell into a very stable oscillation of eccentricities.



HD 7924 Eccentricity (64 Mercury Masses, 1.0 AU)

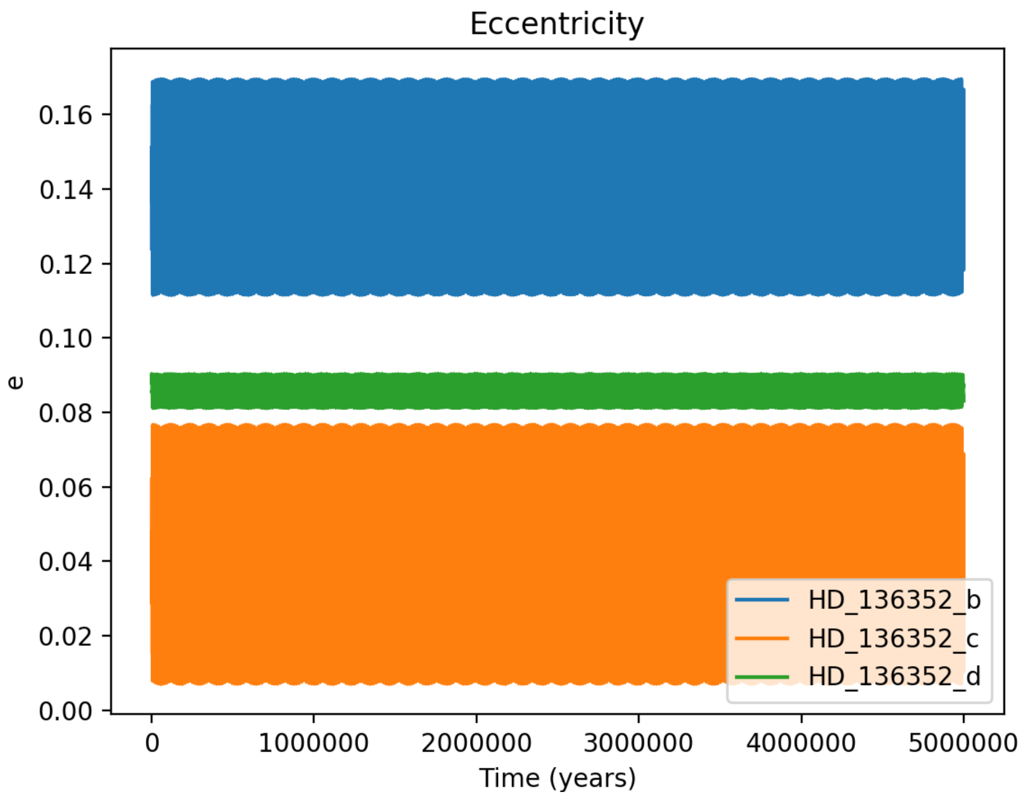
HD 136352

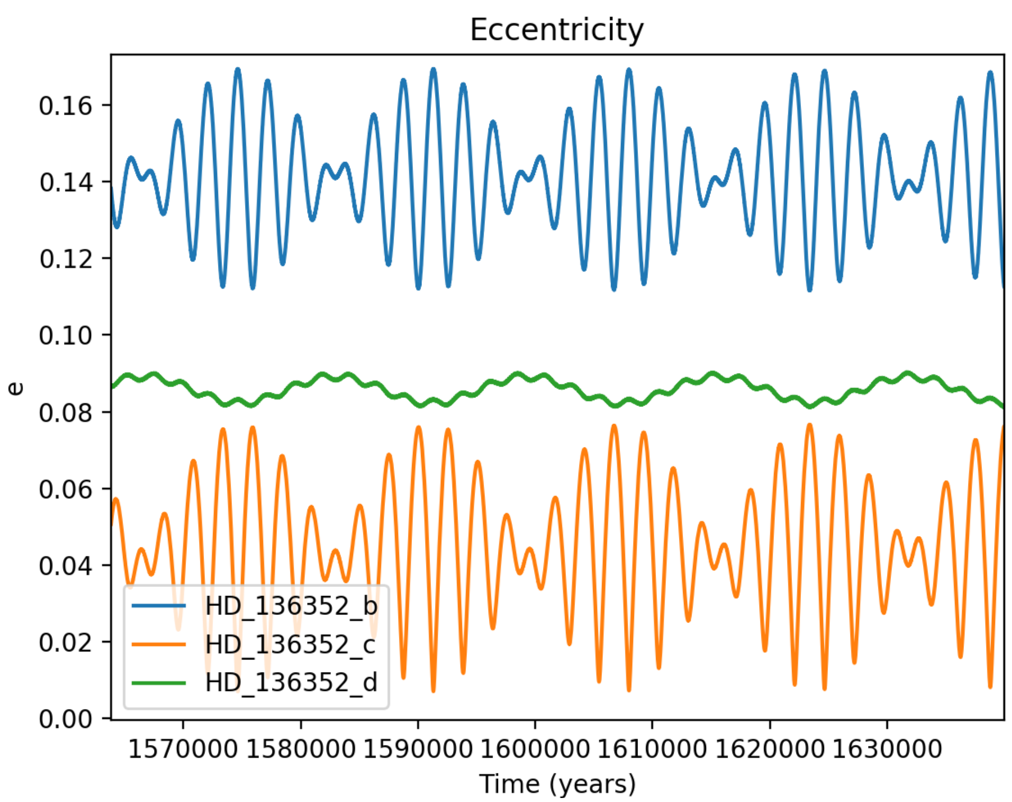
Like HD 7924, the system HD 136352 contains 3 observed planets, all with observed eccentricity and no inclination information available. Distinct from HD 7924 however, we see no instability develop over the 5-million-year period over which our Mercury6 simulation is run.



HD 136352 Test Data

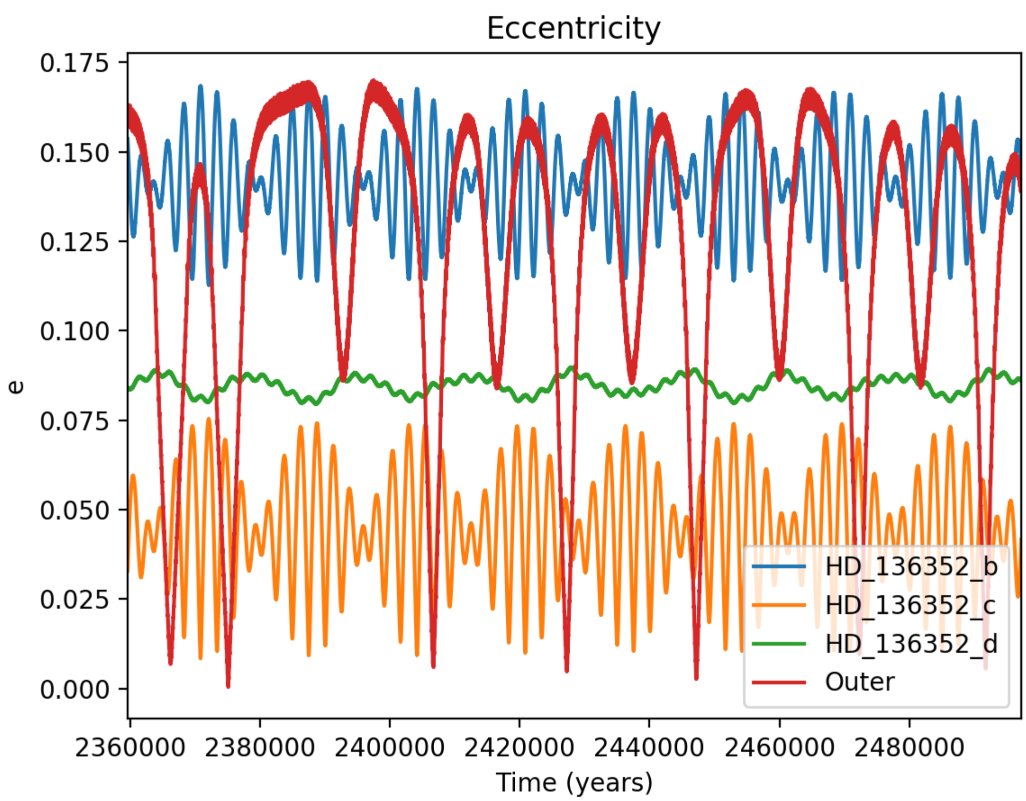
This, as with GJ 9827, shows stability and constancy in the semi-major axes of each of the planets within HD 136352. The test data for eccentricity is shown on the next page.





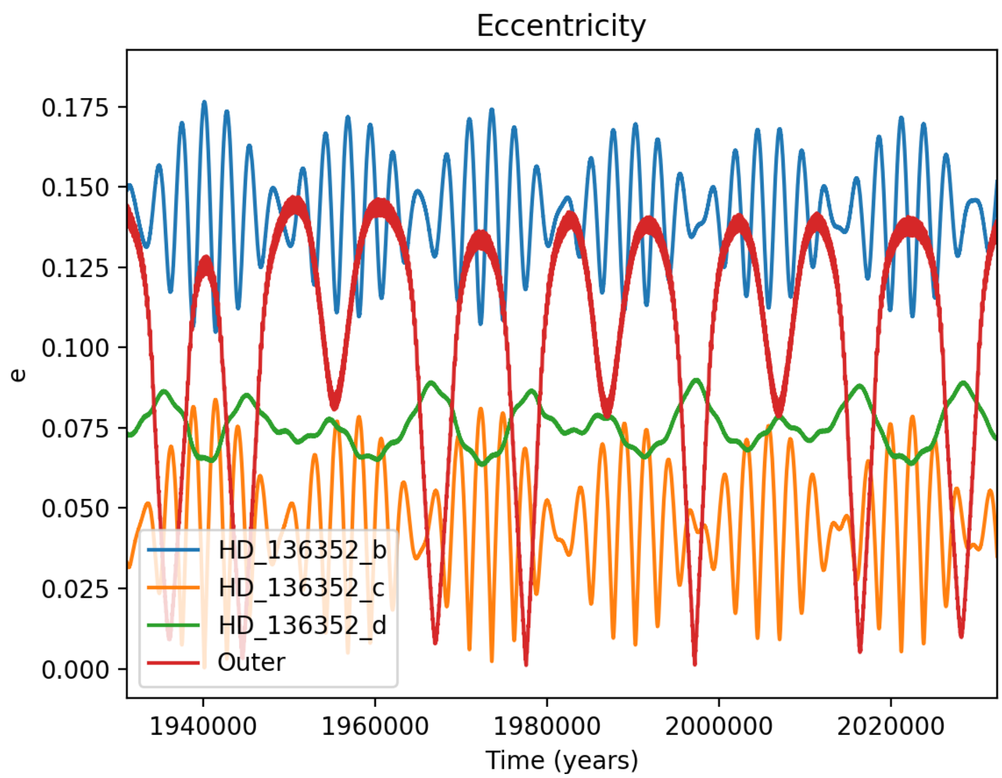
HD 136352 Test Eccentricity

These oscillations in eccentricity are repetitive and predictable throughout the 5 million-year integration. For the presentation of the data due to the addition of perturbing planet, we will focus on how this oscillating eccentricity is affected. The next few graphs are similar to the 2nd Eccentricity graph on the previous page, zoomed into a time scale on the order of tens of thousands of years.

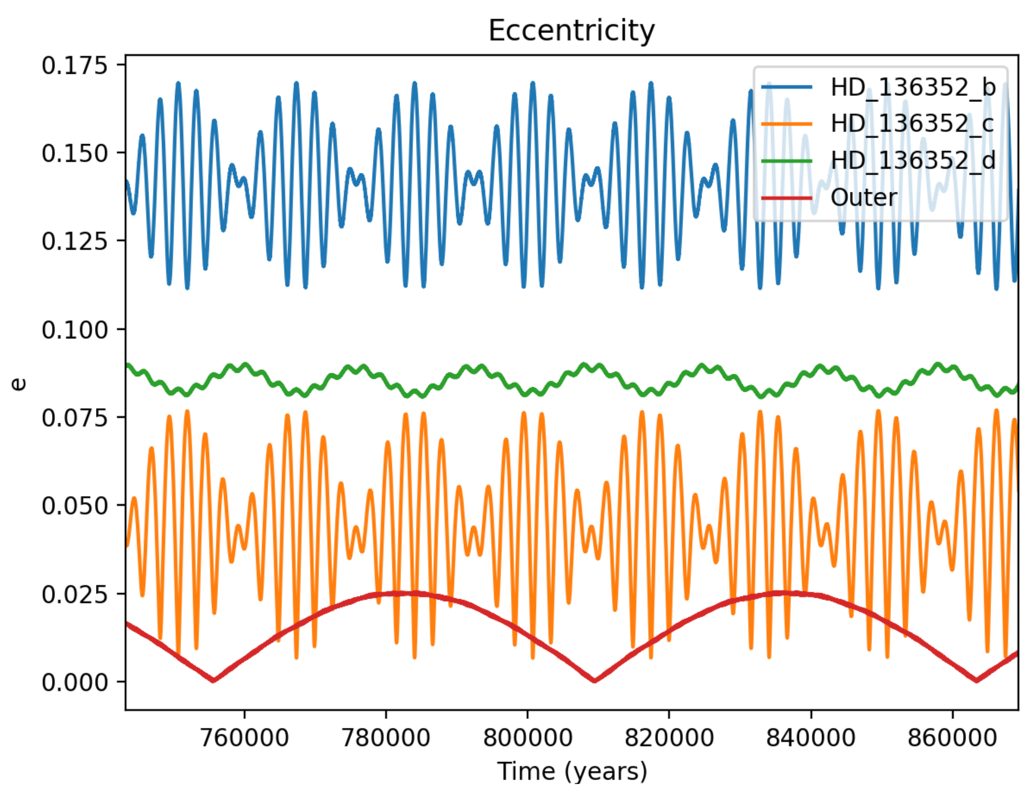


HD 136352 Eccentricity (2 Mercury Masses, 0.6 AU)

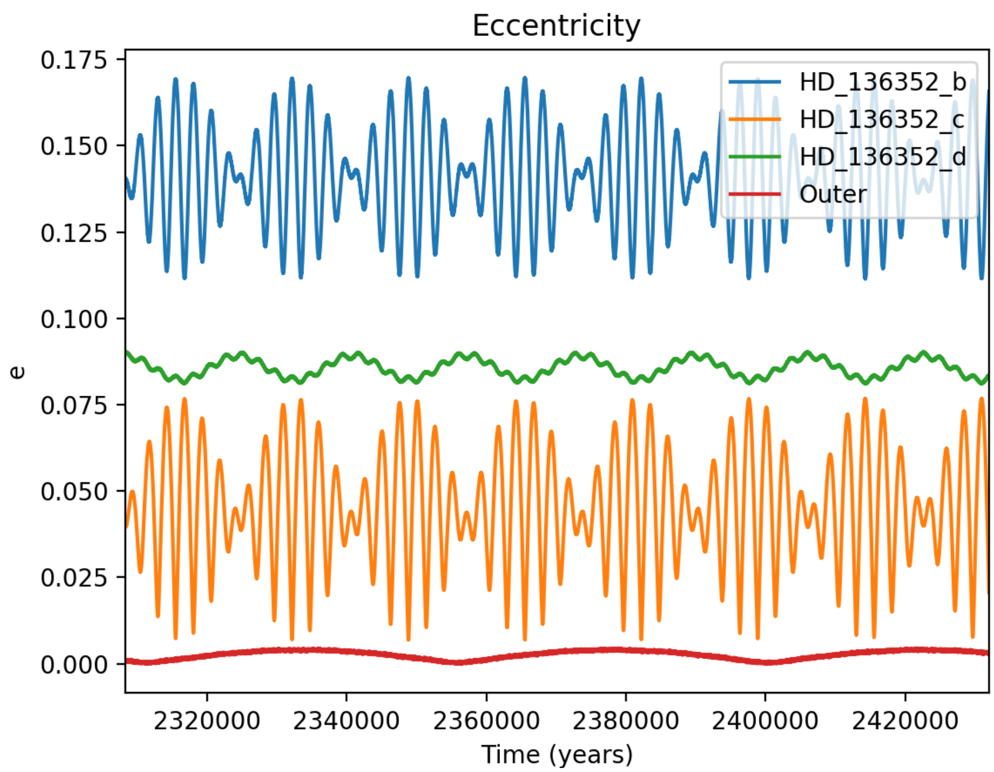
At 2 mercury masses and an initial semi-major axis of 0.6 AU, we observe similar periodic oscillations for each of the three inner planets. For our outer planet, we observe a more complex series of oscillations. On the following two pages, 4 more cases will be displayed.



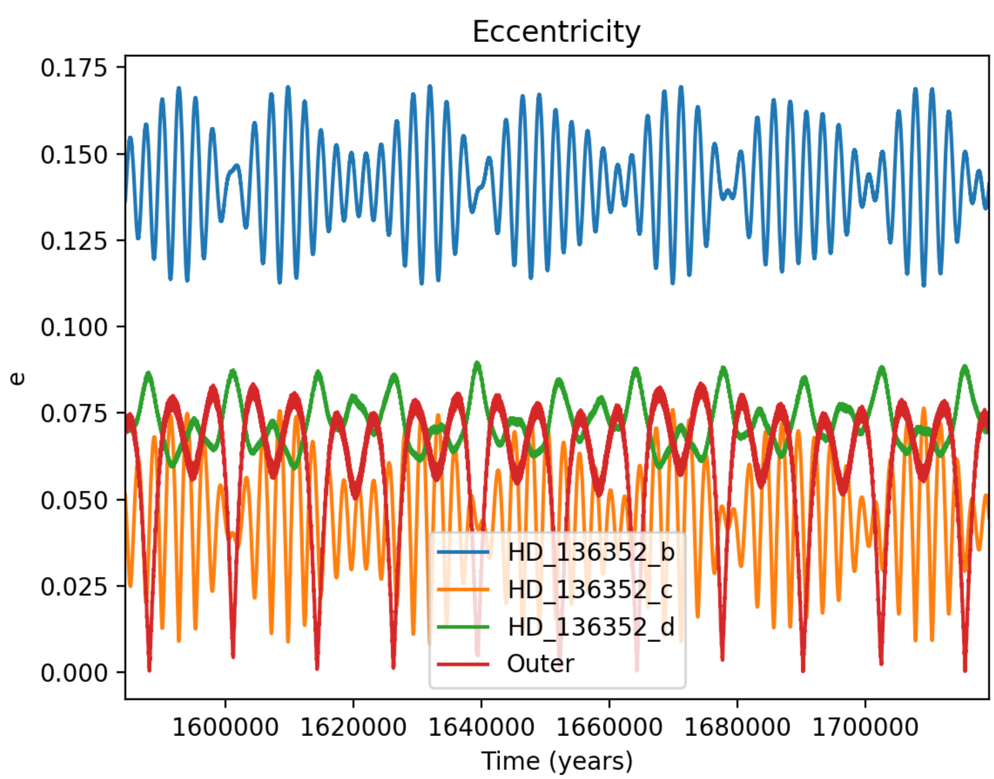
HD 136352 Eccentricity (16 Mercury Masses, 0.6 AU)



HD 136352 Eccentricity (16 Mercury Masses, 1.0 AU)



HD 136352 Eccentricity (16 Mercury Masses, 1.4 AU)



HD 136352 Eccentricity (64 Mercury Masses, 0.6 AU)

As is can be seen in the previous 5 eccentricity graphs, the oscillations of eccentricity for the four planets in the system and their interactions change depending on the mass and initial semi-major axis of the outer planet. These changes will be discussed more thoroughly in the Analysis section.

ANALYSIS

There are many variables and parameters that can affect the movement of planetary systems: masses, distances, eccentricities, inclinations, and the orientation of each planet within a system. There are pros and cons in working with simplified versions of systems. On one hand, when a system is simple, it can be much easier to ascertain the variables that are actually causing differences in the motions of planets. On the other hand, one can recognize that eliminating the complex elements of a system can also make a system less realistic, and therefore less valuable scientifically.

As mentioned already, we must recognize that our models input into Mercury6 cannot perfectly represent the actual nature of these systems. Instead, the goal is to model the inner planets as realistically as possible. Many choices were made about the parameters that were not described within the NASA Exoplanet Archive database. For example, when inclination was not given for a system, it was assumed in the Mercury inputs that inclination was equal to zero. This choice was made to simplify the study of these systems and to avoid making unnecessary assumptions about the relative inclinations of each planet in these systems. Unfortunately, though, this may not be fully realistic or accurate.

Another assumption made in the input files of these systems was the density of the planets in HD 136352 and HD 7924. This may not have an extraordinary effect on the movements of the planets (barring close encounters between the planets), but is still valuable to recognize as an assumption that may contribute to the inaccuracies in the representation of these planets.

In general, a goal of this project was not to perfectly represent these systems, but instead to create working models for each system that can be used to understand them with more complexity. Many questions can be asked and answered once we have a working model for a system. For example, we can test what sizes of perturbing planets would significantly alter the orbits of the observed planets in each system, a method we have attempted in this project.

Hill Radii Analysis

Using equation 2 from our methods section, we can determine the Hill radii spacing between the planets in each of these systems, which may allow us to understand why, gravitationally, some systems were significantly more stable than others. Working with this equation, and the spacing between planets, the following allows us to determine the number of Hill radii between two planets.

$$N_H = \frac{a_{i+1} - a_i}{R_H} \qquad (3)$$

Here N_H is the number of Hill radii between two planets, and the remaining variables are as they were described for equation 2 in the methods section.

Now, combining equations 2 and 3, we can compute the Hill radii spacings for the planets in each system.

GJ 9827 b & c	33.328 Mutual Hill Radii
GJ 9827 c & d	17.485 Mutual Hill Radii
HD 7924 b & c	22.023 Mutual Hill Radii
HD 7924 c & d	11.082 Mutual Hill Radii
HD 136352 b & c	20.998 Mutual Hill Radii
HD 136352 c & d	29.481 Mutual Hill Radii

Table 7 – Mutual Hill Radii Separation Between Planets

Given that the two planets that interacted strongly enough to set HD 7924 into instability were HD 7924 c & d, it makes sense that they would have the smallest value of mutual Hill radii separation. This spacing gives these planets the greatest strength of gravitational interaction and therefore the greatest likelihood of instability.

Oscillations and Stability

In the case of both GJ 9827 and HD 136352, orbital parameters oscillated in a predictable, periodic fashion. This was the case for the entire range of perturbing planets that we added to both of these systems. For systems that are only initially stable, HD 7924 for example, you would expect that unpredictable, erratic changes in semi-major axis, eccentricity, and inclination should lead to eventual instability, which is what we have observed. This could potentially mean that uniform predictable oscillations are an indicator of stability.

For the case of system GJ 9827, the figures on pages 17 through 19 demonstrate that the inner planets in this system have a gravitational tug on our chosen perturbing planet, giving it small oscillations in its semi-major axes without compromising stability. On page 20, our graphs for eccentricity show that in each case small amounts of eccentricity (nearly negligible) is developed in each planet's orbit. Furthermore, the addition of an outer planet does not have a noticeable effect on this eccentricity oscillation of the inner planets in this case.

The primary orbital parameter in GJ 9826 is inclination, which was shown to have very complex oscillations at small time scales (on the order of thousands of years), demonstrated in the second graphs on pages 22, 23, and 24. Here the inclinations of the three inner planets oscillate similarly regardless of the properties of the outer planet. The

full-length inclination graphs on these pages however illuminate that the mass and orbital distance of this additional outer planet do have a long-term effect on the changing inclination of the planets within each system. This appears to be an oscillation in the maximum and minimum inclination for each planet. If the outer planet is larger in mass, or closer to the inner planets, this oscillation becomes higher in frequency.

For HD 7924, our test system on its own did not end up maintaining its own stability. As observed from the graphs on pages 26 through 28, both the semi-major axes and eccentricities of HD 7924 c and d became erratic and unpredictable at around 230 thousand years after the beginning of our integration on Mercury6. The data here again supports the statement made at the beginning of this subsection: that predictable oscillatory behavior is indicative of stability. It can also be explained given the relatively small mutual hill radii spacing between these planets as shown in Table 7.

There are a number of reasons why HD 7924 may be unstable in our computations. A simpler explanation in this case is that the parameters for the planetary orbits of the system were simply not accurate, and there is some way to describe these three planets that would not result in this observed instability. Another possibility, and one that we have explored in this experiment, is that there are unseen planets within the system that allow for the stable orbits of these observed inner planets, and therefore our information on the planets within the system is incomplete.

We tested the case of an added outer planet orbiting HD 7924 with 30 combinations of semi-major axis and mass. Table 6 has 8 cases in this set of planets that results in a stable HD 7924. We can interpret this as evidence that there may be an unseen

perturbing planet in this system keeping it stable. Given the results of our experiment, this is an intriguing possibility.

For our final system, HD 136352, we observe simple periodic oscillations of eccentricity for the three inner planets. From each case displayed on pages 35 to 37, we observe that these oscillations are affected by the outer planet to varying extents based on the mass and orbital distance of the outer planet that we had added to the system.

The planet whose orbit is most affected by the outer planet, naturally, is the outermost of the three observed planets, HD 136352 d. When our perturbing body is 16 mercury masses and orbiting at a semi-major axis of 1.4 AU, as seen on page 37, there is very little change in the eccentricity oscillation of these inner planets. When the gravitational effect of the outer planet is greater however, either by increasing its mass or decreasing its semi-major axis, the eccentricity oscillations of HD 136352 d become less predictable. This is due to increased gravitational interaction with our outer planet.

CONCLUSION

The experimental results of this project primarily illuminate that repetitive, predictable oscillations are generally demonstrative of stability. It is logically consistent that orbital parameters in complex systems would change with time, as the gravitational effects of one planet causes changes in the orbits of all other planets within a system. If these gravitational interactions between planets result in oscillation however, we can predict that planets will periodically return to the orbits they had initially been in.

In our study of the systems GJ 9827, HD 7924, and HD 136352, these repetitive oscillations were observed in both eccentricity and inclination, with noteworthy exception in the case of HD 7924, which went unstable a short time after the beginning of the integration in Mercury6. This, as explained in the analysis section, may either be indicative of incorrect/incomplete knowledge of the orbits of these observed planets, or indicative of the existence of additional outer planets that allow for the stability of these inner planets. After testing this system with a range of 30 different perturbing planets, it was discovered that in a number of cases this outer planet allowed for the maintained stability of the observed inner planets. This is evidence for the idea that an outer planet exists within this system.

The goal of this project was to test whether or not observed systems of exoplanets remained stable with the addition of outer planets and to determine the plausibility that additional planets could be in these systems without significantly changing the orbits of these observed interior planets. For systems GJ 9827 and HD 136352, outer planets affected the oscillations of orbital parameters over time but did not cause instability to occur. A safe conclusion from this data would be to say that the less an outer planet

changes the behavior of the other planets in the system, the more plausibly it could exist within said system.

Computational methods like the use of Mercury6 can be extraordinarily valuable in the study of exoplanetary systems. With advanced knowledge of astronomy and data on systems we have already observed, programs similar to this one can be used to ascertain information that would be unavailable if researchers relied on observation alone. These systems, and other similar systems could potentially be studied further by making fewer assumptions about the orbital parameters. We could make fewer assumptions by testing different initial values for mean anomaly, longitude of ascending node, and for the assumed coplanar systems HD 7924 and HD 136352, testing the effects of adding a range of initial mutual inclinations. Due to our interesting results in system HD 7924, work expanding the integration parameters for this system is already underway. As is the nature of Mercury6, adjustments can be made to any orbital parameter that could cause errors within the model of a system. With improved knowledge of observational data and our understanding of planetary science, these experiments could certainly be improved on and repeated.

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Figure 1 Image:
https://en.wikipedia.org/wiki/Longitude_of_the_ascending_node#/media/File:Orbit1.svg

AUTHOR'S BIOGRAPHY

Kendall Butler was born on August 2nd, 1998 in a small town in northeast Connecticut. Early on in his childhood Kendall's family moved to a town called Harwinton, which is where he spent most of my childhood. He had a close relationship with his family and a tightly knit group of friends from the early days of elementary school through high school, and has been able to maintain those friendships even today. Later on in high school Kendall became fascinated with the Physical sciences and ended up deciding to study Physics at the University of Maine. The study of Physics has been of great value to him and his intellectual life, and has culminated in the completion of this Honors Thesis on the stability of exoplanetary systems.

Next fall, Kendall Butler will be moving on to the next stage of his life as a Physics PhD student at Drexel University in Philadelphia, continuing to pursue his interest in the Physical Sciences.