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Jeffrey Todd Morgan

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**INVESTIGATING HOW STUDENTS THINK ABOUT AND LEARN  
QUANTUM PHYSICS: AN EXAMPLE FROM TUNNELING**

By

Jeffrey T. Morgan

B.S. Walla Walla College, 1997

A THESIS

Submitted in Partial Fulfillment of the

Requirements for the Degree of

Doctor of Philosophy

(in Physics)

The Graduate School

The University of Maine

May, 2006

Advisory Committee

Michael C. Wittmann, Assistant Professor of Physics, Advisor

François G. Amar, Associate Professor of Physical Chemistry

James P. McClymer, Associate Professor of Physics

John R. Thompson, Assistant Professor of Physics

William N. Unertl, Professor of Physics

External Reader

James P. Vesenska, Assistant Professor of Physics, University of

New England

# INVESTIGATING HOW STUDENTS THINK ABOUT AND LEARN QUANTUM PHYSICS: AN EXAMPLE FROM TUNNELING

By Jeffrey T. Morgan

Thesis Advisor: Dr. Michael C. Wittmann

An Abstract of the Thesis Presented  
in Partial Fulfillment of the Requirements for the  
Degree of Doctor of Philosophy  
(in Physics)  
May, 2006

Much of physics education research (PER) has focused on introductory courses and topics, with less research done into how students learn physics in advanced courses. Members of The University of Maine Physics Education Research Laboratory (PERL) have begun studying how students in advanced physics courses reason about classical mechanics, thermal physics, and quantum physics. Here, we describe an investigation into how students reason about quantum mechanical tunneling, and detail how those findings informed a portion of a curriculum development project.

Quantum mechanical tunneling is a standard topic discussed in most modern physics and quantum physics courses. Understanding tunneling is crucial to making sense of several topics in physics, including scanning tunneling microscopy and nuclear decay.

To make sense of the standard presentation of tunneling, students must track total, potential, and kinetic energies. Additionally, they must distinguish between the ideas of energy, probability density, and the wave function. They need to understand the complex nature of the wave function, as well as understand what can and cannot be inferred from a solution to the time-independent Schrödinger equation.

Our investigations into student understanding of these ideas consisted of a series of interviews, as well as a survey. Both centered around asking students to reason about energy, probability, and the wave function solutions for the standard square potential energy barrier scenario presented in most textbooks. We describe ideas that students seem to successfully learn following standard instruction, as well as common difficulties that remain. Additionally, we present multiple data points from a small population of physics majors over three years and describe how some of their reasoning about tunneling changed, while other portions seemed to remain unaffected by instruction.

We used the results of these investigations to write tutorials on tunneling and applications of tunneling. The tutorials were part of a course on introductory quantum physics for non-science majors. In this course, most of the ideas were introduced in the small-group, student-centered tutorial-labs. We present evidence that this population can learn some basic ideas of

quantum physics, and on certain tunneling questions perform as well or better than advanced undergraduate students.

## DEDICATION

To Kelly

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## Chapter 1

### INTRODUCTION AND OVERVIEW

#### 1.1. Introduction

The field of Physics Education Research (PER) has grown tremendously over the past two decades as researchers study student understanding of various physical phenomena, create models to describe student thought processes, develop curriculum to improve student understanding, and evaluate the affect of modified forms of instruction. Much of the work in the field, however, has been done on student understanding of topics in introductory physics courses,<sup>1</sup> while comparatively little work has focused on ideas covered in advanced courses for undergraduate and graduate physics students. That trend is beginning to reverse, however, with expanding numbers of research groups,<sup>2</sup> and an increased focus on student ideas in classical mechanics,<sup>3</sup> thermal physics,<sup>4</sup> advanced electricity and magnetism,<sup>5</sup> and quantum mechanics,<sup>6</sup> among others.

This dissertation describes investigations into student understanding of quantum mechanical tunneling undertaken by members of the Physics Education Research Laboratory (PERL) at The University of Maine (UMaine) since 2002. Through interviews, a survey, and student responses to examination questions, we have analyzed student reasoning about the phenomena and identified common difficulties that students encounter.

Using these results, we have developed a set of tutorials introducing students to one-dimensional tunneling models. These tutorials are part of a course on introductory quantum physics for non-science majors that utilizes a primarily conceptual, tutorial-based method of instruction and emphasizes qualitative reasoning.

Tunneling provides a unique opportunity to study student adoption of the ideas of quantum mechanics. Quantum mechanics allows for the possibility of tunneling through potential energy barriers, something forbidden by the laws of classical physics. Therefore, the expected behavior of particles differs dramatically depending on whether one is reasoning with classical or quantum ideas. Tunneling and other quantum mechanical phenomena require probabilistic interpretations of systems, something not commonly encountered during the study of classical physics. Vocabulary including “barrier” and “decay,” with their associated common usage, provide difficulties for many students as we discuss in Chapter 4. Wave function representations, which appear at first glance to closely resemble representations of mechanical waves, provide further challenge as we show in Chapters 4 and 6.

We note that we have assumed ideas consistent with the Copenhagen interpretation of quantum physics in our work.<sup>7</sup> This is for two reasons – first, it is the interpretation that we are most familiar with, and second, it is the interpretation presented to the students we have studied in their textbooks and

courses. Other interpretations of quantum mechanics, such as the Bohmian interpretation,<sup>8</sup> do not present some of the same difficulties in interpreting the standard tunneling scenario we describe. However, we have not explored these alternate interpretations, nor what challenges they present to students studying them.

## **1.2. Overview of Dissertation**

In Chapter 2, we present a standard solution to the one-dimensional tunneling problem, similar to that presented in many introductory quantum physics textbooks. Additionally, we present an overview of the discussion of tunneling present in the two textbooks used by the majority of students in our interview and survey populations and a brief history of tunneling theory and applications.

In Chapter 3, we review previous work in the field that informs this project. In addition to presenting work on student ideas about modern physics topics, we review relevant researches on student understanding of waves. The chapter concludes by presenting other work on student understanding of quantum mechanical tunneling.

In Chapter 4, we describe the development of our interview protocol on quantum mechanical tunneling. Because the initial set of questions revealed student difficulties with electric fields and potentials, but did not provide sufficient nor significant insight into student reasoning about tunneling, the protocol was revised several times in an iterative process. We discuss the

various forms of the interview tasks, as well as the rationale behind the revisions that took place.

In addition, we discuss the interview findings from populations of students who completed the introductory quantum physics course, and those who completed both the introductory course as well as the senior-level quantum physics course. We also present contrasting findings from interviews with a small population of graduate students.

The initial rounds of interviews suggested several areas of student difficulty. To better understand the prevalence of these ideas, we developed a survey, and administered it to several classes of students at UMaine. In Chapter 5, we discuss the development of the initial survey, and how the survey was revised based on the first year's results. We present the results from both surveys, discuss conclusions we have made from the data, and contrast the findings from UMaine students to those from students from other institutions.

While a small advanced undergraduate population of physics majors presented some challenges to the research, it also provided a unique opportunity. Over the three years of data collection, we had a small number of students who were interviewed twice, took the survey at least once, and answered questions about quantum mechanical tunneling on exams in their senior quantum physics course. In Chapter 6, we present case studies of three

students, and examine which ideas changed over time, and which remained fixed.

Physics education research has long advocated the use of non-traditional forms of instruction, such as tutorials. Because the opportunity to revise curriculum and instructional methods in the advanced undergraduate quantum physics courses offered by the department did not exist, the curriculum development phase of this project took place in an introductory quantum physics course for non-science majors, developed by members of the Physics Education Research Laboratory. In Chapter 7, we discuss the structure of the course and populations of student enrolled. We present brief synopses of each tutorial-lab's activities, as well as an in-depth discussion of the development of two weeks of activities on quantum mechanical tunneling. We present post-test results from these populations, and compare and contrast the findings with the results gathered from physics majors.

In the final chapter, we summarize our findings on student understanding of quantum mechanical tunneling, and discuss implications for future work.

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<sup>1</sup> Much of the work done prior to 1998 is documented in L. C. McDermott and E.F. Redish, "Resource Letter: PER-1: Physics Education Research," *Am. J. Phys.* **67**, 755-767 (1999).

<sup>2</sup> A list of physics education research groups around the world can be found at the University of Maryland Physics Education Research Group's website, available at <<http://www.physics.umd.edu/perg/homepages.htm>>.

<sup>3</sup> See, for example B. S. Ambrose, "Investigating student understanding in intermediate mechanics: Identifying the need for a tutorial approach to instruction," *Am. J. Phys.* **72**, 453-459 (2004). Additional work on student use of mathematics in an intermediate mechanics course is underway at UMaine.

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<sup>4</sup> See, for example B. R. Bucy, J. R. Thompson, and D. B. Mountcastle, "What is Entropy: Advanced Undergraduate Performance Comparing Ideal Gas Processes," AIP Conf. Proc. **818**, 81-84 (2006); J. R. Thompson, B. R. Bucy, and D. B. Mountcastle, "Assessing Student Understanding of Partial Derivatives in Thermodynamics," AIP Conf. Proc. **818**, 77-80 (2006); D. E. Meltzer, "Student Learning in Upper-Level Thermal Physics: Comparisons And Contrasts With Students In Introductory Courses," AIP Conf. Proc. **790**, 31-34 (2005).

<sup>5</sup> See, for example B. R. Patton, "Jackson by Inquiry," APS Forum on Education Newsletter, Summer 1996. Research on student understanding of Gauss' law is currently underway at UMaine.

<sup>6</sup> References for other work on student understanding of quantum mechanics are discussed in Chapter 3 of this dissertation.

<sup>7</sup> The Copenhagen interpretation is discussed in many quantum physics textbooks. See, for example H. C. Ohanian, Principles of Quantum Mechanics, (Prentice-Hall, Englewood Cliffs, NJ, 1990).

<sup>8</sup> A discussion of Bohm's interpretation of quantum mechanics can be found in D. Bohm and B. Hilley, The Undivided Universe, (Routledge, London, 1993).

## Chapter 2

### THE PHYSICS OF TUNNELING

Before we can discuss student reasoning about the phenomena of quantum mechanical tunneling, it is important to describe what we want students to learn in courses where tunneling is discussed. Most introductory text presentations of tunneling consider only one-dimensional scenarios. Though potential energies for real-world phenomena are often complicated, a majority of introductory texts discuss tunneling through a square potential energy barrier, and follow up with brief discussions on various approximation techniques. Many textbooks present wave function solutions to the time-independent Schrödinger equation using position space representation.<sup>1</sup>

#### 2.1. Mathematical Solution to the Schrödinger Equation

Most quantum mechanics textbooks introduce the reader to the Schrödinger equation:

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) \quad (2-1)$$

If the potential,  $V$ , is independent of time, the Schrödinger equation can be solved using separation of variables, assuming a solution of the form

$$\Psi(x,t) = \psi(x)\tau(t). \quad (2-2)$$

Inserting this solution into the Schrödinger equation and dividing through by  $\psi(x)T(t)$  yields

$$i\hbar \frac{1}{\tau(t)} \frac{d\tau(t)}{dt} = -\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{d^2\psi(x)}{dx^2} + V(x). \quad (2-3)$$

Because the left side of the equation is a function of time alone and the right side of the equation is a function of position alone, both sides must equal a constant. It can be shown that this constant is the total energy  $E$ . Solutions of the left hand side of the equation have form

$$\tau(t) = Ke^{\frac{-iE}{\hbar}t} \quad (2-4)$$

where  $K$  is some constant. The right hand side of the equation is commonly called the time-independent Schrödinger equation:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x). \quad (2-5)$$

Much of the work in introductory quantum mechanics courses involves finding solutions to the time-independent Schrödinger equation.

To introduce tunneling, many texts discuss a rectangular potential “barrier” of the form:

$$V(x) = \begin{cases} 0 & x < 0 & \text{Region I} \\ V_0 & 0 < x < L & \text{Region II} \\ 0 & x > L & \text{Region III} \end{cases} \quad (2-6)$$

If one assumes the beam of identical particles with kinetic energy  $E < V_0$  incident from the left on the potential barrier, the solutions to the Schrödinger equation in Regions I and III are

$$\psi_I(x) = Ae^{ikx} + Be^{-ikx} \quad (2-7)$$

$$\psi_{III}(x) = Fe^{ikx} + Ge^{-ikx} \quad (2-8)$$

where

$$k = \frac{\sqrt{2mE}}{\hbar}. \quad (2-9)$$

The first term in the solution in Region I represents the incident beam of particles,

$$\psi_{I+}(x) = Ae^{ikx}, \quad (2-10)$$

while the second term represents the particles reflected by the barrier,

$$\psi_{I-}(x) = Be^{-ikx}. \quad (2-11)$$

Because the incident beam is specified as traveling from the left, there can be no incident beam in Region III, and  $G = 0$ . The transmitted wave function is thus

$$\psi_{III+}(x) = Fe^{ikx}. \quad (2-12)$$

The transmission probability  $T$ , given by

$$T = \frac{|\psi_{III+}(x)|^2}{|\psi_{I+}(x)|^2} = \frac{F^* F}{A^* A} \quad (2-13)$$

describes the fraction of incident particles that succeed in tunneling through the barrier.

The wave function inside the barrier can also be found. Here, the solution is of the form

$$\psi_{II}(x) = Ce^{-\kappa x} + De^{\kappa x} \quad (2-14)$$

where

$$\kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}. \quad (2-15)$$

To find the transmission probability  $T$  for a given potential energy barrier, we must apply appropriate boundary conditions at the edges of the regions.

Both the wave function and its first derivative must be continuous everywhere, so at  $x = 0$  we require

$$\psi_I(x)\Big|_{x=0} = \psi_{II}(x)\Big|_{x=0} \quad (2-16)$$

$$\frac{d\psi_I(x)}{dx}\Big|_{x=0} = \frac{d\psi_{II}(x)}{dx}\Big|_{x=0} \quad (2-17)$$

Similarly, at  $x = L$  we require

$$\psi_{II}(x)\Big|_{x=L} = \psi_{III}(x)\Big|_{x=L} \quad (2-18)$$

$$\frac{d\psi_{II}(x)}{dx}\Big|_{x=L} = \frac{d\psi_{III}(x)}{dx}\Big|_{x=L} \quad (2-19)$$

Substituting in the previously stated solutions (2-7), (2-8), and (2-14) yields four equations:

$$A + B = C + D \quad (2-20)$$

$$ikA - ikB = -\kappa C + \kappa D \quad (2-21)$$

$$Ce^{-\kappa L} + De^{\kappa L} = Fe^{ikL} \quad (2-22)$$

$$-\kappa Ce^{-\kappa L} + \kappa De^{\kappa L} = ikFe^{ikL} \quad (2-23)$$

which may be solved to find

$$\left(\frac{A}{F}\right) = \left[\frac{1}{2} + \frac{i}{4}\left(\frac{\kappa}{k} - \frac{k}{\kappa}\right)\right] e^{(ik+\kappa)L} + \left[\frac{1}{2} - \frac{i}{4}\left(\frac{\kappa}{k} - \frac{k}{\kappa}\right)\right] e^{(ik-\kappa)L} \quad (2-24)$$

If we assume that the potential energy of the barrier is high compared to the energy of the incident particles, then

$$\frac{\kappa}{k} - \frac{k}{\kappa} \approx \frac{\kappa}{k} \quad (2-25)$$

Also, if the barrier is sufficiently wide so that  $\psi_{II}(x)$  drops off a great deal, then  $\kappa L \gg 1$ , and  $e^{\kappa L} \gg e^{-\kappa L}$ . This reduces equation (2-24) to

$$\left(\frac{A}{F}\right) = \left[\frac{1}{2} + \frac{i}{4}\frac{\kappa}{k}\right] e^{(ik+\kappa)L} \quad (2-26)$$

We can now approximate the transmission probability  $T$ , since

$$T = \frac{|\psi_{III+}(x)|^2}{|\psi_{I+}(x)|^2} = \frac{F^* F}{A^* A} \approx \left[\left(\frac{4k}{2k - i\kappa}\right) e^{(ik-\kappa)L}\right] \cdot \left[\left(\frac{4k}{2k + i\kappa}\right) e^{-(ik+\kappa)L}\right] \quad (2-27)$$

$$T \approx \left[\frac{16}{4 + (\kappa/k)^2}\right] e^{-2\kappa L}. \quad (2-28)$$

Because

$$\left(\frac{\kappa}{k}\right)^2 = \frac{2m(V_0 - E)/\hbar^2}{2mE/\hbar^2} = \frac{V_0}{E} - 1 \sim 1 \quad (2-29)$$

the exponential term will dominate the behavior. Therefore, we can approximate the transmission probability as

$$T \approx e^{-2\kappa L} = e^{-2\frac{\sqrt{2m(V_0-E)}}{\hbar}L} \quad (2-30)$$

Thus, the tunneling probability drops off with an increase in (i) the potential energy of the barrier, relative to the particle energy, or (ii) an increase in barrier width.<sup>2</sup>

## 2.2. Graphical Representations of Tunneling Solutions

Often, textbooks include sketches of the potential energy barrier and the wave function for the square-barrier tunneling scenario. We assume the potential energy is identical to that given in the previous section, as shown in Figure 2-1.

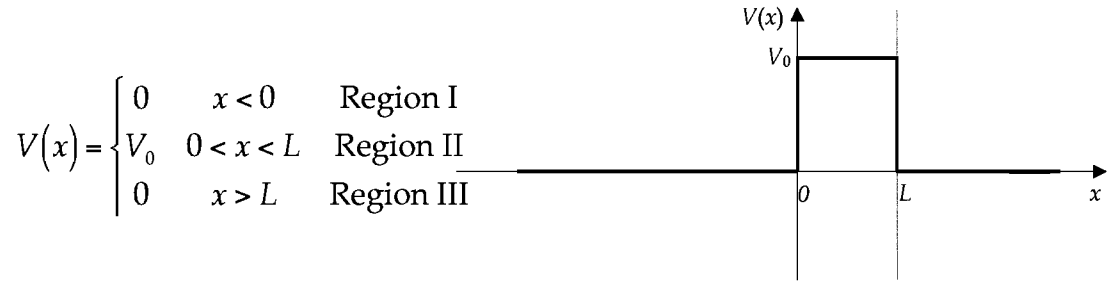


Figure 2-1: Square potential energy barrier.

As was shown previously, the wave function solutions in Regions I and III are sinusoidal, and the solution in Region II is a decaying exponential. Thus, a graph of the real portion of the wave function as a function of position could look like that shown in Figure 2-2.

There are several things to note about this graphical solution. First, the amplitude of the wave function in Region III is less than the amplitude of the wave function in Region I,

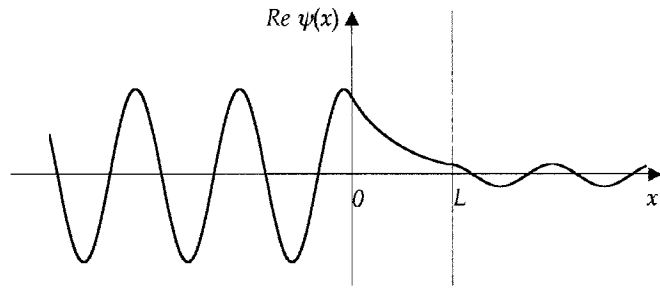


Figure 2-2: Real portion of the solution to Schrödinger's equation for the square potential energy barrier.

corresponding to a lower probability of detecting particles in Region III. (Alternately, one may picture many of the incident particles reflected by the potential barrier, while only a fraction of the incident particles successfully tunnel to Region III.) The wavelengths of the wave functions in Regions I and III are identical, since no energy is lost in tunneling. (This fact is assumed when writing down the solutions to the Schrödinger equation in each region.)

The exponential solution pictured in Region II is only the decaying exponential portion of the solution stated in the previous section. While an increasing exponential solution is mathematically allowed, it cannot be such that the amplitude of the wave function increases, as this solution doesn't match experiment. Therefore, the decaying exponential must be the dominant term.

Finally, the pictured solution is one (i) for many particles, and (ii) to the time-independent Schrödinger equation. In addition, the graph shows only

the real portion of the solution. If one calculates the probability density for this system using

$$P(x)dx = |\psi(x)|^2 dx \quad (2-31)$$

one must multiply the wave function by its complex conjugate. For example:

$$\psi_{I+}^*(x)\psi_{I+}(x) = (A^* e^{-ikx})(A e^{ikx}) = A^* A \quad (2-32)$$

Thus, the probability of detecting particles anywhere in Regions I or III is generally non-zero and constant.

Depending on the phase shift of the reflected portion of the wave function, the probability

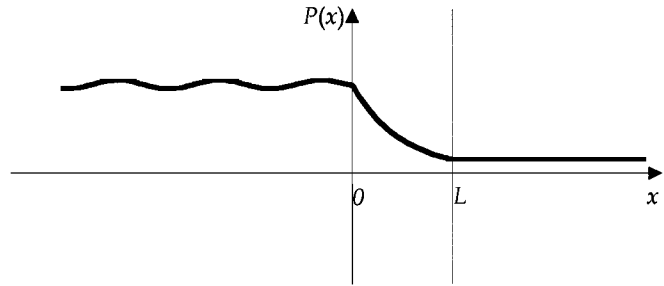


Figure 2-3: Probability density in the square potential energy barrier region.

density in Region I may vary sinusoidally, but still be non-zero everywhere, as shown in Figure 2-3. One cannot use graphical representations such as that given in Figure 2-2 to reason about what happens to the stream of particles over time; for this, solutions to the time-dependent Schrödinger equation are required.

## 2.3. History and Applications of Tunneling

### 2.3.1. Alpha decay

Although radioactivity had been discovered at the end of the 19<sup>th</sup> century, and Rutherford introduced the idea of half-life in 1900, no theoretical model

satisfactorily matched the experimental data.<sup>3</sup> Although alpha particles escaping the nucleus had been experimentally observed, theory could not explain the characteristic energies of emitted alpha particles, nor the wide variation in half-lives for various nuclei. In the 1920s Gamow combined the attractive nuclear forces with the Coulomb repulsion, forming an effective barrier for the alpha particles, and solved the Schrödinger equation for this barrier.<sup>4</sup> Independently, Condon and Gurney published the same result utilizing the Wentzel, Kramers, Brillouin (WKB) approximation.<sup>5</sup>

A model for the potential barrier for alpha decay is shown in Figure 2-4.

Outside the nuclear radius  $R$  the potential energy falls off as  $1/r$ . A particle of energy  $E$  sees a potential barrier of approximate width  $a - R$ . A

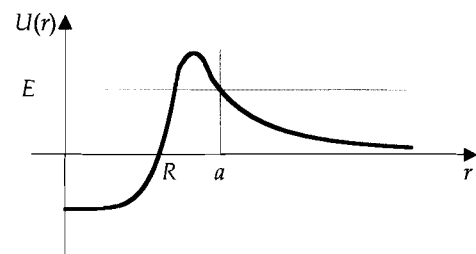


Figure 2-4: One-dimensional model of the potential energy barrier of an atomic nucleus.

lower energy particle must tunnel through a much wider barrier, therefore the probability of tunneling for lower energy particles is much less than for higher energy particles. This corresponds to experimentally observed greater half-lives for low-energy alpha particles.

### 2.3.2. Tunnel diodes

Tunnel diodes, formed by joining highly doped p- and n-type materials, were first discovered by Leo Esaki in 1957,<sup>6</sup> earning him the Nobel Prize in 1973. They originally generated considerable interest in the scientific

community as they had picosecond switching times, whereas transistors of the time were occasionally achieving millisecond switch times. However, the tunnel diode's two-terminal nature (making it difficult to use as an amplifier) and low impedance characteristics caused it to fade quickly from the electronics forefront. What few tunnel diodes exist today are used for microwave detectors and picosecond pulse generators.<sup>7</sup>

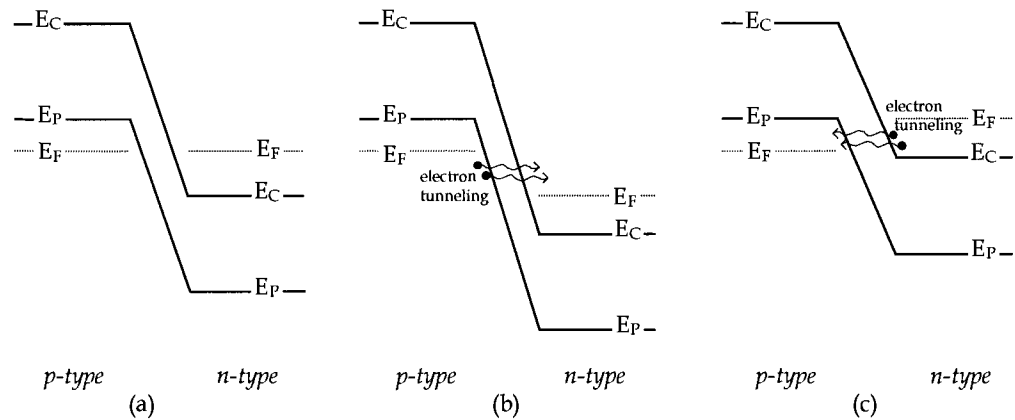


Figure 2-5: Energy level model of a tunnel diode (a) at equilibrium, (b) reverse biased, and (c) forward biased.

In heavily doped p- and n- type semiconductors, the Fermi level (the highest energy level occupied by electrons) lies within the valence and conduction bands, respectively.<sup>8</sup> If a p-n junction is fabricated from heavily doped p- and n-type materials, the valence band of the p-type material lies energetically higher than the conduction band of the n-type material, and the Fermi levels coincide. Since filled states are opposite filled states, equilibrium is achieved, and no tunneling occurs. At equilibrium, there is no net current, as shown in Figure 2-5(a).

When the p-n junction is reverse biased, the energy levels of the n-type material are shifted lower relative to the p-type materials, as shown in Figure 2-5(b). Thus, there are empty states in the conduction band of the n-type material opposite to filled states in the p-type material, and electrons can tunnel from p to n (or holes from n to p) if the transition region is sufficiently narrow. Thus, there is a net current from n to p. As the reverse bias increases, it exposes a greater number of empty states to filled states, and tunneling increases, increasing the reverse current.

If the diode is forward biased, the energy bands of the n-type material shift upwards relative to their counterparts in the p-type material, modeled in Figure 2-5(c). Now, filled states below the Fermi level in the conduction band of the n-type material are opposite empty states above the Fermi level in the valence band of the p-type material, and tunneling of electrons can occur from n to p (or holes from p to n), resulting in a forward tunneling current.

### 2.3.3. Cold emission and scanning-tunneling microscopes

The tunneling phenomena can also be used to describe the behavior of electrons in a metal. Electrons are held in a metal by an attractive potential, which can be modeled as a finite potential well, one portion of which is shown in Figure 2-6. At absolute zero, electrons fill up all available energy

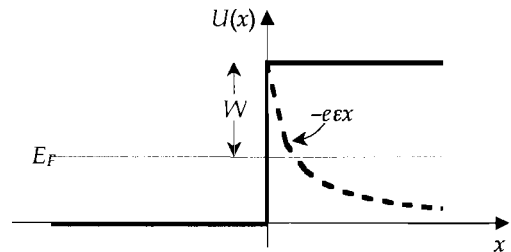


Figure 2-6: One side of a finite potential energy well.

levels in pairs, as described by the Pauli exclusion principle. As the temperature increases, some electrons are excited into higher energy states, but even at room temperature the number of excited electrons is rather small. Thus, there exists some work function  $W$  that separates the Fermi level from the top of the well; electrons must receive that much energy to be removed from the metal, either through photon absorption or heating.

If the metal is placed in an external electric field  $\epsilon$ , the potential energy outside the well seen by the electrons,  $W$ , is reduced by an amount  $-e\epsilon x$ . This creates a finite-width potential energy barrier that electrons have a non-zero probability of tunneling through.<sup>9</sup>

In 1981, Gert Binnig and Heinrich Rohrer invented the scanning-tunneling microscope, for which they shared the 1986 Nobel Prize with Ernst Ruska, who invented the electron microscope.

The scanning tunneling microscope uses the idea of cold emission to map surfaces, something not possible with traditional electron microscopes since the electron energy is too high.

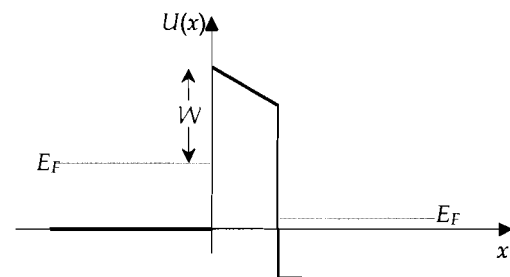


Figure 2-7: Energy model of a scanning-tunneling microscope.

In a scanning-tunneling microscope, a small tip, sharpened further by atoms “pulled out” by the large electric fields involved, passes over the surface being studied. The separation of the tip and surface is a few angstroms. If no potential difference exists between the tip and the surface,

the Fermi levels in both materials coincide, and little tunneling takes place. However, with an applied potential difference, the Fermi level on one side is shifted lower (as shown in Figure 2-7), leaving available energy states coincident with filled energy levels in the other material, and a tunneling current exists.

As the tip is passed over the surface, variations in the tip-to-surface separation will change the potential energy barrier, and as a result reduce or increase the tunneling current. By keeping either the current or the tip height constant, one can experimentally map out the surface of the material being studied.

#### **2.4. Surveying the Presentation of Tunneling in Texts**

The applications of alpha decay and scanning-tunneling microscopy were discussed during the interview sessions, as will be described in Chapters 4 and 6. To ensure that our questions were appropriate to what students were expected to learn in their courses, we surveyed the texts used in their courses.

The students we interviewed and surveyed came primarily from two populations – students who completed the sophomore-level introductory quantum physics course, and students who completed the senior-level quantum physics course. In both courses, reading of the text was required, and homework from selected chapters was assigned. So that the reader is familiar with the presentation of quantum tunneling in the texts utilized in these courses, we briefly outline the discussion in each.

#### **2.4.1. Modern physics (sophomore and junior populations) text**

The sophomore and junior students interviewed as part of this project had all completed the introductory quantum mechanics course at the University of Maine using Beiser's *Concepts of Modern Physics*.<sup>10</sup> In the first chapter on quantum mechanics, the author introduces the wave function, general wave equations, and the time-dependent Schrödinger equation. Within this chapter, he uses separation of variables to find what he terms the "steady-state form" of the Schrödinger equation. Subsequent sections introduce the one-dimensional particle in a box and the finite potential well.

Beiser subsequently introduces tunneling in a rather qualitative fashion. There is a brief discussion of the phenomena of alpha decay. The approximation for the transmission coefficient is shown, and used for a calculated example of 1.0 eV and 2.0 eV electrons tunneling through a 10.0 eV, 50-nm barrier. An applications section describes the scanning-tunneling microscope (STM) and the atomic force microscope (AFM). The author does demonstrate how to find the transmission coefficient in an appendix to the chapter, in a fashion similar to that shown in the first section of this chapter.

#### **2.4.2. Senior-level quantum physics text**

The senior quantum mechanics course at the University of Maine is taught using Griffiths' popular text.<sup>11</sup> The book begins with an introduction of the Schrödinger equation. In chapter 2, Griffiths uses separation of variables to introduce the time-independent Schrödinger equation. That equation is used

to analyze a number of scenarios, including the infinite square well (particle in a box), the harmonic oscillator, and the free particle. He introduces tunneling in a section discussing delta-function potential barriers and wells, including the possibilities of transmission and reflection in each of those cases.

Griffiths returns to tunneling in the eighth chapter, discussing use of the WKB method for obtaining approximate solutions. Tunneling is discussed in the situation of a rectangular barrier with an uneven top. Later, Griffiths introduces Gamow's theory of alpha decay, and uses it to deduce an approximate lifetime of the parent nucleus.

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<sup>1</sup> A survey of over twenty quantum physics textbooks revealed a similar discussion involving wave functions in all of the texts.

<sup>2</sup> The characteristics of the wave function are easier to see if one makes the described approximations. If one does not approximate, one finds the

transmission coefficient to be  $T = \frac{16k^2\kappa^2 e^{2\kappa L}}{(e^{2\kappa L} - 1)^2 (k^2 + \kappa^2)^2}$ , which also drops off with

increases in  $\kappa$  or  $L$ .

<sup>3</sup> M. Razavy, Quantum Theory of Tunneling, (World Scientific, Singapore, 2003), pp. 1-4.

<sup>4</sup> G. Gamow, "Quantum theory of nuclear disintegration," *Nature* **122**, 805 (1928).

<sup>5</sup> R.W. Gurney and E.U. Condon, "Wave mechanics and radioactive disintegration," *Nature* **122**, 439 (1928).

<sup>6</sup> L. Esaki, "New phenomenon in narrow germanium *p-n* junctions," *Phys. Rev.* **109**, 603-604 (1958).

<sup>7</sup> G. Rostky, "Tunnel Diodes: The Transistor Killer", *EETimes*, online at <[http://www.eetimes.com/special/special\\_issues/millennium/milestones/holonyak.html](http://www.eetimes.com/special/special_issues/millennium/milestones/holonyak.html)>.

<sup>8</sup> B. G. Streetman, Solid State Electronic Devices, 4th ed., (Prentice Hall, Upper Saddle River, NJ, 1995), pp. 208-212.

<sup>9</sup> S. Gasiorowicz, Quantum Physics, 2nd ed., (Wiley, New York, 1996), pp. 82-85.

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<sup>10</sup> A. Beiser, Concepts of Modern Physics, 6th ed., (McGraw-Hill, New York, 2003). Chapter 5 in the text discusses quantum mechanics, including tunneling.

<sup>11</sup> D. Griffiths, Introduction to Quantum Mechanics, 1st ed., (Prentice Hall, Upper Saddle River, NJ, 1995).

## Chapter 3

### A REVIEW OF THE LITERATURE ON STUDENT UNDERSTANDING OF QUANTUM PHYSICS

A large portion of the work done in Physics Education Research has focused on student understanding of topics covered in a typical introductory physics course.<sup>1</sup> Though less attention has been given to topics in advanced physics courses, there are a growing number of researchers beginning to study these populations.<sup>2</sup> Modern physics and quantum mechanics is no exception, with a growing body of investigations into student thinking about these subjects.<sup>3</sup>

In the first section of this chapter, we provide an overview of surveys, interviews, and other investigations into student understanding of topics in quantum mechanics. In the second section, we review published descriptions of new curriculum that has been developed for teaching modern physics topics, as well as innovative efforts in teaching quantum courses. In the third section, we discuss previous work done on student understanding of waves and wave functions, as our work on tunneling revealed that many students use classical wave ideas when discussing wave functions. Finally, in the fourth section, we examine previous research on student understanding of quantum tunneling, as this provides a context for the comparison of our work to that of others.

### 3.1. Research into Student Ideas about Quantum Mechanics

In this section, we describe work on student understanding of various topics in quantum mechanics, but exclude investigations specifically focused on tunneling. These are discussed in the last section of the chapter.

#### 3.1.1. Common misconceptions in quantum mechanics

Styer reported on fifteen common misconceptions in quantum mechanics.<sup>4</sup> His catalog is based on observations of his students, other instructors, various writings, and a self-assessment. Though his list is not supported by interviews, surveys, or any of the other traditional forms of PER, several of the catalogued items show up in the interview data we have gathered, and are discussed here.

He points out that many students believe that energy eigenstates are the only allowed states, which is similar to the correct idea that energy eigenvalues are the only allowed energies, but if true would leave quantum mechanics with no classical limit. He observes that this misconception is bolstered by overemphasis on the analysis of problems using the time-independent Schrödinger equation, which he renames the “energy eigenproblem” to suggest a lesser role to the time-dependent Schrödinger equation, which he simply calls the “Schrödinger equation.”

A second misconception is that some quantum state  $\psi(\mathbf{x})$  is completely specified by  $|\psi(\mathbf{x})|^2$ , its associated probability density. Though understanding the probability density can be useful for some descriptions of systems, it says

nothing about the expected momentum of a particle, and therefore nothing about what the probability density will be at a later time. As will be shown in more detail in our description of interview results in Chapters 4 and 6, we observed students who claimed that they could only describe the characteristics of a system via its probability density.

Styer also lists the tendency of students to think of the wave function  $\psi(\mathbf{x})$  as a function of regular three-dimensional space, when it is in fact a function of configuration space. In our interviews, we repeatedly observed students thinking of both the wave function and potential energy barriers as representations in space.

A final misconception relevant to our work that Styer catalogues is the tendency for students to use a wave function or state vector to describe a single system averaged over some amount of time. Furthermore, he states that anyone asking, “how does a particle get through a node in its wave function?” and saying, “when a particle tunnels through a potential barrier, it never appears under the barrier...it just disappears from one side and reappears on the other” is likely using this idea. We repeatedly saw evidence of this thinking during our interviews, as we describe in Chapters 4 and 6.

### **3.1.2. Resources in quantum mechanics**

Oliver and Bao have done some preliminary work on student meta-resources in quantum mechanics.<sup>5</sup> They briefly describe Hammer’s work on resources,<sup>6</sup> where he describes the many resources available to a computer

programmer, and how the programmer assembles various procedures, functions, and subroutines as needed to create a computer program. By analogy, students bring a library of productive resources with them to the physics classroom.

Oliver and Bao describe how students bring resources about both waves and particles to the quantum physics class, though the students have not previously been asked to merge the two. They also describe how the “deciding” resource, as well as analogies to social situations, can be both useful and dangerous when reasoning about quantum mechanics. They define “meta-resources” as “the resources students can use to evaluate and control their own thought processes.” They then argue that instead of asking students to develop a new set of resources to learn quantum mechanics, the instructor’s focus should be on identifying the resources students enter class with, and figuring out how to use them productively.

In our interviews, we occasionally observed students reasoning with analogies, as we describe in Chapters 4 and 6. However, they usually lacked the physics knowledge to realize a conflict and reason about it.

### **3.1.3. Properties of quantum objects**

It is often assumed by instructors that, following instruction, students have a grasp of the properties of fundamental objects, such as electrons and photons. R. Müller and Wiesner asked hundreds of German gymnasium (roughly equivalent to American high school) students to share their ideas on

the contrast between classical and quantum physics.<sup>7</sup> Students overwhelmingly listed mass and/or weight as the most essential property of a classical object; velocity and momentum also ranked quite high. Position and energy were lowest on the list. When asked about the essential properties of quantum objects, charge was frequently mentioned, whereas it was rarely listed in the classical regime. Mass was not given nearly as often as it had been for classical objects. Energy was more than twice as common a response to the quantum question compared to the classical question.

According to the researchers, students performed quite well on citing differences between classical and quantum objects. The most frequently seen explanation was that quantum effects show up in very small objects, whereas classical mechanics works to describe larger objects. The second highest percentage of students gave some explanation of duality.

Students were also asked to describe a photon. While one-third described it as a particle of light with wavelike properties, many incorrectly described the motion of a photon in terms of a wave. Interviews accompanying the surveys revealed that several students often mistake the symbol listed on Compton effect diagrams as representative of the actual motion of the photon.

Researchers at the University of Washington and the University of Maryland found similar results when questioning students about photons.<sup>8</sup> Some students sketched photons traveling up and down sinusoidal paths. Others describe part of the amplitude of light as being “cut off” when passed

through a narrow slit. Other descriptions of the phenomena include the ability of a polarizing lens to affect the magnetic part of the wave and not the electric, and vice-versa.

#### **3.1.4. Structure of atoms**

Müller and Wiesner found that student models of atomic structure are often incomplete. Many students keep an incorrect, classical model even after receiving instruction to the contrary. When they asked students about the structure of atoms, a large percentage of the students studied suggested either a Bohr model or a picture of orbitals.<sup>9</sup> More than half claimed that electrons had either definite positions or were localized in certain regions. Students admitted in interview sessions that even though they had been told the Bohr model was incorrect, it became the dominant mental picture they had of an atom since their instructors frequently used it in explanations.

A second German group found similar results.<sup>10</sup> In a survey of 236 high school students, researchers found that prior to instruction, nearly 70% described the atom with a planetary-orbit model. After traditional instruction in introductory quantum mechanics, the percentage that held a planetary model dropped to just under 60%, but the Bohr model remained as the dominant atomic model.

#### **3.1.5. Models of conduction**

Students also express incomplete or incorrect models when asked to describe the behavior of various electrical devices, such as resistors, insulators,

semiconductors, and diodes. For example, to correctly reason about conductivity, students need to be able to combine a semi-classical and wave physics approach. Researchers at the University of Maryland found that the conductivity models of the students that they studied were insufficient for describing the systems students were asked to describe. Half of the students interviewed held a model of conductivity requiring a “minimum voltage” to create a current.<sup>11</sup> Others use the size of physical constrictions electrons move through to explain differences in conductivity of various materials. Neither were sufficient for explaining the conductivity characteristics of semiconductors.

Although we did not study student reasoning about conductivity, the physical systems presented in the first two versions of the interviews required some knowledge of electric potentials and electric fields. As we describe in Chapter 4, we observed students struggle to correctly describe the energy nature of these systems, which created difficulties in asking them about tunneling problems.

### **3.1.6. Location of quantum objects**

Müller and Wiesner also found that students have trouble reasoning about localization.<sup>12</sup> When asked about whether or not electrons hold definite positions in atoms, many students replied that the electrons possessed definite positions; the positions were merely unknown. This idea was closely related to student ideas about the uncertainty principle. Many felt that the

uncertainty principle defined regions of localization. Others used the uncertainty principle to establish causality, and reasoned that a precise measurement of the position of an electron somehow changed the electron's momentum.

### **3.1.7. Probability**

Probability is used frequently in statistical mechanics and quantum physics, but research suggests that large numbers of students have difficulty reasoning about probability. In administering a probability pretest to two upper-division quantum physics classes, Bao and Redish found that the majority of students believed the gambler's fallacy – that the outcome of previous coin flips affects the probability of the outcome of future flips.<sup>13</sup> A significant percentage believed that tossing a coin 100 times would yield an exact 50-50 split of heads and tails. Two-thirds of surveyed students thought that knowing one person's exam score affected the probable average of other students' exam scores. In Chapter 7 we describe materials we have developed for teaching probability ideas to introductory students that specifically address these misconceptions.

### **3.1.8. Quantum measurement**

Much of the initial work described in the previous sections involves students at the introductory quantum physics level. Some researchers are looking at student understanding at more advanced levels, including a group at the University of Pittsburgh. Surveying advanced undergraduate physics

majors, Singh found that the majority of students were able to correctly answer two questions on measurements of a system.<sup>14</sup> 76% correctly stated that a measurement of some physical observable immediately following another measurement would yield the same result, and 83% were also able to correctly respond that a measurement on 100 identically prepared systems would likely not yield the same result.

The same students did significantly worse in analyzing the time-dependence of operator expectation values. Asked to analyze a particle in a one-dimensional oscillator, only 11% correctly identified that the expectation value of an operator would depend on time if the particle were initially in a momentum eigenstate, and just 17% correctly identified that the expectation value would not depend on time were the particle initially in an energy eigenstate.

Asked to analyze an electron at rest in a magnetic field, most students did well on initial questions regarding possible results and probabilities for spin measurements in the  $x$ -,  $y$ -, and  $z$ -directions. However, when asked about the time dependence of the expectation values of spin operators given the initial spin eigenstates, performance was much worse, with only 7% of students surveyed answering all four questions on time dependence correctly.

### **3.1.9. Student conceptual and visual understanding of quantum mechanics**

In an attempt to begin describing the changes in conceptual understanding students go through as physics undergraduates and graduates, Cataloglu and

Robinett developed a survey to probe student understanding of basic concepts and visual representations in quantum mechanics.<sup>15</sup> Titled the *Quantum Mechanics Visualization Instrument* (QMVI), the 25-question survey was administered to three populations of students – undergraduates in the sophomore-level modern physics course, undergraduates in the junior-senior level quantum mechanics course, and graduate students in graduate quantum theory courses.

Each multiple-choice question was graded for both the answer and the explanation given; the maximum possible score was 100. Over the three semesters reported, sophomore-level students averaged in the mid to high twenties, juniors and seniors in the advanced quantum mechanics course averaged in the mid to high forties, and the graduate populations had an average score of 55. They report one aberration in the data, when an undergraduate quantum class taught using a text that emphasizes conceptual understanding and visualization of phenomena scored a 58. Thus, the authors conclude in general that conceptual understanding of quantum mechanics increases throughout an undergraduate and early graduate career. Although we did not study a wide spectrum of quantum understanding, we also observed an increased understanding in a small sample of students, as we describe in Chapter 6.

The authors also analyze clusters of questions and speculate on reasons for the observed performance. Five of the questions were designed to probe

student understanding of wave functions found in different potential energy situations. The three questions that dealt with bound states in position space in general followed the overall scoring trends of the population, but the question on one-dimensional scattering (tunneling) showed significantly lower scores. (The fifth question, involving momentum-space representations, was even worse, scoring lower than random guessing would predict.) An analysis of the reasoning shows that students most frequently tried to use reasoning about transmitted and reflected fluxes, rather than focusing on the wave function itself.

#### **3.1.10. Conceptual change in quantum mechanics**

Fletcher and Johnson developed a four-question survey covering fundamental concepts in quantum mechanics: the photoelectric effect, the meaning of uncertainty, the nature of waves, and the nature of energy levels.<sup>16</sup> All questions contained multiple-choice responses in addition to asking students to explain their reasoning. When examining and coding student responses, rather than assuming categories based on an expert model of the physics, they created categories of student responses based on the set of responses themselves. Additionally, they studied the context of the student responses, the content presented, and the level of correctness of their answers.

The first question asked students to describe the arrival time of a bus having an associated Heisenbergian uncertainty. The analysis revealed that a third of the students picked the correct response, but only 9% provided correct

reasoning. Studying the student explanations, the researchers concluded that students' did not seem to view uncertainty as a new quantum concept, but rather used their understanding of uncertainty in other contexts.

The second question posed an imaginary conversation among classmates regarding energy levels, then asked students to describe energy levels and what is meant by "wavelengths fitting into an atom." Analysis revealed that nearly half of the students possessed a concrete, orbit/shell model of an atom, while the second largest subset possessed a more abstract, discrete energy model. In describing the "fitting in" phenomenon, students talked about waves fitting in a certain space, integer numbers of wavelengths, and certain energies. A small subset talked about electrons moving about the nucleus in wave-like paths.

The third question gave students information about what constitutes a particle, and then asked them to check the box that most clearly described what is meant by "something is a wave." Previously, 70% of first-year students correctly chose interference/diffraction. However, in the survey presented, this option was not given, and instead required students to check "none of the above" and describe their reasoning. Only 30% of the students checked the appropriate box, and only 12% of that subset provided correct reasoning. The largest distracter was the option that "everything is a wave," which the authors point out may be influenced by the wave nature of matter stressed in their course.

The final question focused on experimental observations of the photoelectric effect and their relation to the wave/particle nature of light. Students were given a standard textbook description of experimental observations, including (i) the idea that the number of electrons ejected from a surface was proportional to the intensity of the light on the surface, and (ii) that no electrons were ejected when the light's frequency fell below some threshold, regardless of the intensity. They were asked to think about a "bird on a wire" analogy for this observation, where the wave model suggests removing the bird by shaking the wire, and the particle model suggests removing the bird by firing particles at it. Students were asked which analogy was more appropriate for each portion of the observation.

While roughly 45% of the students chose the correct option for each observation, only 18% chose the correct option for both observations. When examining written explanations, the researchers found only one student in 205 sampled who correctly explained both parts of the question. From this data, the researchers suggest students have difficulty applying their mental models in new situations.

### **3.1.11. Summary of research results**

A limited amount of research has been performed on student conceptions of major ideas in quantum mechanics, but several themes are developing, including:

- Students seem reluctant to abandon familiar ideas from classical mechanics in favor of a quantum model.
- When thinking about the atom, a majority of students hold a Bohr model of atomic structure and behavior, even after explicit instruction in quantum mechanics.
- Students have difficulty when describing systems probabilistically.
- Although students are aware of the uncertainty principle, they have trouble describing the meaning and applying it to given situations.
- Analyzing the behavior of quantum systems over time is difficult for most students.

### **3.2. Curriculum Innovations in Quantum Mechanics**

In this section, we describe three types of innovations in the teaching of quantum physics; using computer simulations to help students visualize behavior, introducing tutorials, and revising entire courses.

#### **3.2.1. Computer simulations**

##### **3.2.1.1. Visual Quantum Mechanics**

Researchers at Kansas State University have developed a series of computer-based instructional materials called *Visual Quantum Mechanics* (VQM).<sup>17</sup> The six instructional units, titled (i) Solids & Light, (ii) Luminescence, (iii) Waves of Matter, (iv) Seeing the Very Small: Quantum Tunneling, (v) Potential Energy Diagrams, and (vi) Making Waves, were originally designed for use with high school students and non-science

students at the college level. Since then, the target group has been expanded to include advanced undergraduate majors, science and engineering students, medical students, and in-service teachers.

Analysis of student learning has yielded mixed results. Following a semester course utilizing the VQM units, Rebello and Zollman found that most students did not relinquish a planetary model of an atom.<sup>18</sup> Students were able to sketch probability densities given a wave function (and the reverse), but don't have a complete understanding of the relationship between probability and probability density. Students were also able to sketch correct wave functions for tunneling phenomena given a tunneling probability. However, when asked to extend this knowledge to an explanation of why a cart cannot tunnel through a barrier, most used the reasoning that the total energy of the cart is less than the energy of the barrier. Only a few correctly reasoned that the de Broglie wavelength of the cart is much smaller than the barrier width.

The researchers also asked students to sketch a concept map of the central ideas of quantum mechanics at the completion of the course. They found that the student's maps were rather fragmented. Often, the only ideas that were connected were ideas common to an instructional unit.

The researchers conclude that comparing the effectiveness of VQM with traditional courses is difficult, since (i) no established instrument for measuring students' conceptual understanding of modern or quantum

physics exists, and (ii) the material is taught in a manner different from the traditional format with different emphases.

### **3.2.1.2. Computer-based laboratory experiments**

Other researchers have written computer-based laboratory experiments for use in redesigned courses. R. Müller and Wiesner discuss two simulations used in their course designed for gymnasium students.<sup>19</sup> The first simulates a Mach-Zehnder interferometer. The two arms of the interferometer are of different lengths, and the simulation allows single photons to pass through. Filters are then placed in each arm. With the filters parallel, students discover that the interference pattern on the screen builds up in the same way as it does when the beams are unfiltered. However, when the filters are orthogonal, no interference pattern is observed. Students are led to conclude that photons within the interferometer are not localized.

A second simulation uses a double-slit apparatus that electrons pass through. Students observe an interference pattern gradually developing as single electrons are passed through the apparatus. Following an introduction of the wave function to describe electrons, students then experiment with closing one slit at a time, and observe that no interference pattern emerges. Thus, they are led to again conclude the absence of locality, this time for electrons.

In contrast to the Kansas State researchers, the University of Munich group did attempt to analyze their students' conceptual understanding in

comparison to a control group. Researchers created a questionnaire on students' conceptions that included statements like "An atom has a similar structure as the solar system (planets that orbit the sun)" and "In principle, quantum objects can simultaneously possess position and momentum." Students were asked to indicate how much they agreed with each statement on a scale of 1-5. A statistical index  $C$  was calculated from 29 items on the questionnaire, where a +100 would indicate full quantum mechanical conceptions, and a -100 indicated conceptions that contradicted strongly with quantum mechanics. Students in the new course had an average index value  $C$  of +55.8. By contrast the control group, 35 first-year university students who had taken quantum physics courses in gymnasium had an average index value  $C$  of +35.2.

### **3.2.1.3. Wave packet simulations**

Styer has also developed tools for use in visualizing quantum physics. He presented a talk on visualization at the 2000 winter meeting of the American Association of Physics Teachers.<sup>20</sup> Though he acknowledges that some pioneers of quantum mechanics, notably Heisenberg, argued against visualization, Styer argues that it can be a useful tool if its limitations are recognized and discussed.

To that end, Styer developed different forms of computer simulations to address the scenario of a wave packet encountering a potential barrier. In the first, the probability density of the wave packet is displayed, which does show

the interference between the incident and reflected portions of the wave packet near the barrier. However, this representation lacks information about the phase. In the second simulation, phasors are plotted, based on techniques developed by Feynman<sup>21</sup> and Taylor.<sup>22</sup> Though phasors in the incoming and outgoing wave packets are oriented in various directions, all phasors are nearly parallel when the wave packet is near the barrier. In a third simulation, color is used to represent positive and negative parts of the real and imaginary portions of the wave function. Again, when the wave packet encounters the barrier, all portions of are nearly in phase.

Styer concludes by arguing that visualization can prove a useful tool for beginning to develop intuition. He cautions, however, against using it as an end, as this would encourage students to not investigate or analyze the simulation and explore its limitations. He argues that game-playing using visual representations might provide a useful vehicle for developing quantum intuition.

#### **3.2.1.4. Quantum Science Across Disciplines software**

Robblee, Garik, and Abegg studied a group of teachers that underwent a summer workshop utilizing Quantum Science Across Disciplines (QSAD) software and instructional materials.<sup>23</sup> The teachers were interviewed to discover both the extent of their knowledge about quantum science and their knowledge about how to teach quantum science effectively. In the workshop the teachers learned to use the software and interacted with and questioned

the scientists and programmer who developed the materials. During the following school year, the teachers implemented portions of the curriculum.

The authors present a case study of one veteran high school chemistry teacher. The teacher implemented portions of the software not emphasized during the workshops, and wanted students to explore the properties of electron densities. In follow-up interviews, the teacher discussed how the software had been useful in altering his own perceptions of atomic nature, and that it provided a useful venue for students to explore the fundamental nature of matter.

### **3.2.2. Tutorials**

In contrast to the computer simulation environment, where students are free to explore topics in a laboratory-style setting, other groups have worked on the development of University-of-Washington-style tutorials.<sup>24</sup> Tutorials are a series of carefully crafted questions, sequenced with the intention of showing students where their current models fall short of explanation, and helping them to develop more robust models. To help students build ideas about the probability density of systems, University of Maryland researchers developed a series of tutorial activities that deal with one-dimensional systems.<sup>25</sup>

In the sophomore-level tutorial on probability, balls roll down a stepped track with two levels, where a single ball is in the system at any given time. Students use the activity to think about the relation between probability and

different (but constant) velocities. A second activity involves using a glider on an air track with spring bumpers on the end to simulate a potential well.

At the more advanced level, the tutorial involves examining time exposure photographs of a pendulum bob, which reflects the probability density for the bob to be found in various locations. Students then use digital video of a glider in harmonic oscillation, and examine a pseudorandom selection of frames, using the data to create plots of the probability of finding the glider in any given location.

Researchers administered a conceptual quiz involving wave functions associated with various energy levels to both an experimental and control group of the sophomore-level modern physics course. Researchers found that 30% of students in the experimental group used the correct spatial dimension to represent position, as opposed to 9% in the control group. Also, 27% of the experimental group could use velocity to describe the probability of finding an electron in a given region, while none of the students in the control group could. Also, three times as many students in the experimental group were able to explain their reasoning regarding the probability of finding a particle in a given state, when compared to responses from the control group.

Results from a second tutorial, dealing with tunneling, are presented in Section 3.4 of this chapter. Additionally, we used this tutorial to develop the second version of our tunneling interview protocol, as is described in Chapter 4.

### 3.2.3. Revised courses

Some attempts have been made to revise the order in which topics are presented to students. Faced with evidence that suggests how strongly students hold to a Bohr model of the atom, think that photons have particle characteristics, and have problems with duality, a research group in Berlin decided to create an introductory quantum mechanics unit that made no mention of classical phenomena, and started the course instead with discussion of the electron.<sup>26</sup> The teaching unit was tested in 11 courses. The researchers then compare answers to a conceptual questionnaire administered to the experimental group, as well as students in 14 traditionally taught courses that served as the control group.

The researchers then analyzed the responses, and compared them to students' pretest responses. In the control group, 71% of students exhibited no conceptual change on the behavior of electrons and the structure of the atom, compared to 6% in the experimental group. 67% of students in the experimental group were categorized as exhibiting "satisfactory" or "complete" change, while only 2% of the control group was placed in either of these categories.

Another German group examined future physics teachers' concepts regarding modern physics.<sup>27</sup> On a pretest, students in a quantum physics seminar course were asked to describe knowledge of quantum physics, such as their model of a hydrogen atom, the locality of electrons, the uncertainty

principle, and their exposure to terms in quantum mechanics such as “Schrödinger’s cat” and the “Einstein-Podolsky-Rosen Paradox.” Based on their answers, students were classified as using classical models, quantum models, or a mixture. They found the majority of students held a classical view.

The researchers then split the students into two groups, and the experimental group took part in three special sessions on concepts and models in modern physics. Both groups went through the normal seminar course. They then administered the same posttest to both populations. On the posttest, students were asked to judge the correctness of 25 questions on a five-point scale, from “correct” to “incorrect.” Students in the experimental group averaged 14.9 correct answers, while students in the comparison group averaged 10.5 correct answers. Additionally, the researchers examined the deviation of the incorrect answers. For example, if the correct answer is “5-not correct” and a student circles “1-correct,” the deviation is 4. The experimental group had an average summed deviation of 18.4, while the control group had an average summed deviation of 28.5.

Niedderer and Deylitz authored a manuscript aimed at introducing high school students to ideas in modern physics.<sup>28</sup> Topics included light and electron as quanta, classical standing waves, the hydrogen atom, and higher order atoms. The modified instruction took place in three high schools in Bremen, Germany, where each of the teachers received special training.

Following the modified instruction, the 27 students were given a questionnaire and interviewed. The researchers evaluated their performance in six domains: the atom, the  $\psi$ -function, their notion of state, use and understanding of the Schrödinger equation, relating measurement to theory, and higher order atoms. Responses were coded on a three-point scale.

The best results were found in the atom domain, which involved using an orbital model to describe physical phenomena. Students also performed reasonably well on the notion of state domain, where they used the concept of state to describe the model of an atom, and explain processes like emission and absorption. The poorest performance came in the Schrödinger equation domain, despite concerted efforts to foster a qualitative understanding of the equation and using graphical computer models.

The researchers also compared the results by institution, and found that students in one school outperformed their peers at the other institutions in every domain. The authors note that the teachers differed in their acceptance of the new approach and their physics background, and that possible language difficulties existed at one of the other institutions.

#### **3.2.4. Summary of curriculum innovations**

Paralleling research into student understanding of quantum mechanics has been an effort to create new curriculum for use in the teaching of quantum mechanics.

Several individuals or research groups have reported on computer-based simulations they have developed to aid students' visualization of quantum phenomena, including tunneling. While there is limited evidence that students taught using these simulations perform well on some tasks, there is no data comparing learning in a course that utilizes simulations to learning in a traditional course.

Others have developed tutorials for use in courses with modified instruction. There is some evidence that students taught with this style of instruction have improved models and are better able to explain physical phenomena.<sup>29</sup>

Still other researchers have analyzed the content and order of presentation of traditional quantum mechanics courses and worked on revising the order and topics of presentation while keeping a traditional format. Improved conceptual understanding is observed in reform-based classes, but difficulties with topics such as the Schrödinger equation and atomic models remain.

### **3.3. Student Ideas about Waves**

In our interviews with students described in Chapters 4 and 6, we often saw them discussing classical wave attributes when asked to reason about wave functions. To help us understand their reasoning, we investigated what the research has revealed about student understanding of waves. In this section, we summarize several important findings.

### **3.3.1. Student ideas about mechanical waves**

For his doctoral research at the University of Maryland, Wittmann studied how students learn about mechanical waves.<sup>30</sup> While a significant portion of his work was theoretical and dealt with the mental models students use and develop to reason about mechanical waves, several of the responses given by students illustrate common difficulties students have with using waves to describe physical phenomena.

When asked how to create a faster or slower-moving pulse on a taut string, many students responded that the motion of the hand affects the speed of the wave pulse, and that by flicking the hand more quickly one could create a faster moving pulse. An interviewed student described how putting “a greater force in your hand” will cause the pulse to move faster. These explanations are consistent with what might be given to describe how to throw an object to make it go faster, and Wittmann argues they indicate that students are seeing waves as objects.<sup>31</sup>

In another example, students are asked to sketch the shape of two unequal wave pulses that are pictured traveling toward each other at a time after the waves have overlapped. Students commonly sketch a single pulse of decreased amplitude. If students are using ideas about colliding particles (for example, a perfectly inelastic collision between carts), both move in the same direction following collision at a reduced speed.

Wittmann's work also uncovered a misunderstanding of the mathematical description of a pulse that may shed light on student difficulties reasoning about the exponential portion of the wave function in the potential barrier region. Students were given a Gaussian function that described a single pulse centered at  $x = 0$  and asked to sketch the shape of the string after it had traveled a distance  $x_0$  and find the displacement of the string as a function of  $x$ .<sup>32</sup> Mathematically, this should be an identical pulse centered at  $x_0$ , but many students incorrectly inserted  $x_0$  into the given equation and argued that this reduced the amplitude of the wave pulse.

Wittmann then cites Sherin's "symbolic forms"<sup>33</sup> to explain what students may be doing. Rather than applying the exponential function to the shape of the entire pulse at a single instant of time, they tend to apply the form of the function to the amplitude of the wave function – focusing on a single point – and reason that the peak value must decay over time.

### **3.3.2. Student understanding of electromagnetic wave representations**

In the course of exploring student understanding of physical optics, Ambrose, Heron, Vokos, and McDermott discovered a lack of understanding of light as an electromagnetic wave.<sup>34</sup> This led to an exploration of student understanding of the diagrammatic and mathematical representations of waves.

While our project did not involve investigating student understanding of electromagnetic waves, the reasoning about wave function sketches we observed have many parallels to the Washington findings.

Pretest questions on representations of electromagnetic waves at the University of Washington (specifically traditional text diagrams of perpendicular  $\vec{E}$  and  $\vec{B}$  fields) revealed many student difficulties. Many students thought different points in planes perpendicular to the direction of propagation had different magnitude electric and/or magnetic fields. Alternately, points outside the diagram's sine curve were often identified as points of zero field. The researchers also found that many students thought the fields either increased or decreased as one moved out from the axis of propagation. The literal interpretations of wave diagrams observed in these student populations is similar to some of the explanations given by students during our interviews and on our surveys, as will be discussed later in this dissertation.

The researchers developed a tutorial on electromagnetic waves, modifying it prior to the second use due to subtleties in student understanding revealed on the posttests. The tutorial requires students to rank magnitudes of electric and magnetic fields, and then connects the formalism of the representation to the real world. To accomplish this, questions about an antenna and its orientation are used. Later, a bulb was connected to the wire to elicit specific thinking about the electric currents in the wire.

Posttest results met or exceeded the success levels of teaching assistants on pretests and pretests of physics faculty at national meetings. Additional modifications were made to the tutorial to address misconceptions about polarization, and portions of the wave being “cut off.” Specifically, many students think that certain orientations of polarizing filters will chop off the magnetic portion of the EM wave, leaving the electric field portion intact. Students are led to calculate magnitudes of transmitted  $\vec{E}$  and  $\vec{B}$  fields, and realize the interdependence of these ideas.

### **3.3.3. Student understanding of the wave nature of matter**

Vokos, Shaffer, Ambrose, and McDermott studied student understanding of the wave nature of matter in the context of diffraction and interference patterns, particularly as the subtleties relate to the deBroglie wavelength of various particles.<sup>35</sup> Four populations were studied: algebra-based general physics, calculus-based general physics, second-year modern physics, and third-year quantum mechanics. Pretest and posttest questions that were asked were lumped into two larger categories: type S, and type P. Type S questions involved slit width, separation, or lattice spacing. Type P questions dealt with changes to the momentum of the particle (which was at times expressed by changes in the accelerating potential, changes in the energy, or changes in velocity). At no time was the deBroglie wavelength explicitly mentioned.

On type S questions, calculus-based students with a history of tutorials fared decently on post-lecture, pre-tutorial tests. All populations performed rather poorly on type P questions (pretest). Identified student difficulties were (i) failure to recognize the relevance of the deBroglie wavelength to this scenario, (ii) failure to relate the deBroglie wavelength to the momentum of the particle, and (iii) failure to treat particles with and without mass differently.

The researchers developed a tutorial that was administered in different settings – some during traditional tutorials, some during modified lectures. All populations showed improvement post-tutorial. The researchers contend that this tutorial and others designed show the utility of having students go through the chain of reasoning necessary to develop conceptual understanding, and this approach is more effective toward that goal than lecture or solving traditional problems.

### **3.3.4. Summary of relevant student ideas about waves**

Although the research described in this section investigated student understanding of classical waves, several ideas emerged that parallel observations we have made in student descriptions of the wave function, including:

- Students may describe objects as moving along wave-like trajectories.
- Students often misinterpret wave diagrams.

- At times, students focus on one attribute of a wave diagram, rather than addressing the diagram holistically.
- Students have difficulty understanding and interpreting mathematical representations of waves.

### 3.4. Previous Research on Student Understanding of Tunneling

As we describe in the following sections, previous research on tunneling involved development of a tutorial on tunneling for advanced undergraduate students, investigations into student ideas about energies involved with tunneling through an asymmetric barrier, and investigating student descriptions of probability related to the tunneling problem.

#### 3.4.1. Tutorial instruction on tunneling

Redish, Wittmann and Steinberg studied student ideas about tunneling in a quantum physics course for engineering majors at the University of Maryland.<sup>36</sup> A pretest, administered to 11 students, showed a potential energy diagram, as

illustrated in Figure 3-1, with increased energy in the region  $L < x < 2L$ , and stated that a quantum mechanical particle with energy  $E$  (less than the barrier energy) is

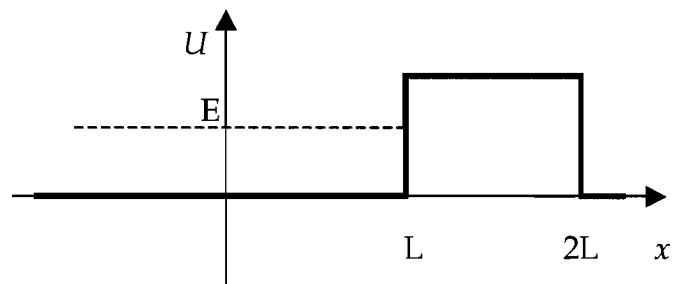


Figure 3-1: Potential energy barrier from University of Maryland pretest.

incident from the left. Students were asked whether the particle energy

increased, decreased, or remained the same, and to explain their reasoning. A second question asked students to compare the energy of the particle in region I ( $x < L$ ), and region III ( $x > 2L$ ), and to again explain their reasoning.

Four of the eleven students answered both questions correctly – the energy of the particle stays the same, and the particle's energy in region III is the same as the energy in region I. Four of the students chose energy loss, and three were inconsistent, meaning they stated the particle loses energy in the barrier, but has the same energy once it passes through.

They then administered the same exam questions in two classes, one traditional lecture-based section with eleven students, and the other a modified-instruction section with thirteen students, where one of the three hours of lecture was replaced with tutorials developed by the Maryland Physics Education Research Group.<sup>37</sup> The students were given a potential energy diagram with energy 0 in region I ( $x < 0$ ), and region III ( $x > a$ ), and potential energy  $U$  in region II ( $0 < x < a$ ). The students were asked to consider a beam of electrons with energy  $E_0$  incident from the left on this barrier, to sketch the shape of the total wave function, to write equations for the wave function in each region (but to leave the normalization constants unspecified), and to compare the energy of electrons in regions I and III.

Eight of the eleven students in the traditional course gave a correct mathematical response, with two making errors and one leaving the question blank. All thirteen students in the modified course gave correct mathematical

responses. On the energy comparison task, however, the differences were striking. Twelve of the thirteen students in the modified course gave the correct response, while only two of the students in the traditional course correctly identified the energy as being the same. Seven of the students in the traditional course stated that energy was lost, whereas only one in the modified course did.

Twelve of the thirteen students in the modified course correctly sketched the wave function, while only one student in the traditional course did so. The most common incorrect answer, labeled the “axis shift response,” was given by eight of eleven students in the traditional course, and only one student in the modified course. In the “axis shift response,” students sketch the wave function as sinusoidal in both regions I and III and exponential in region II, but draw the wave function in region I as oscillating about a higher imaginary axis than the wave function in region III. Six of these students also give energy loss answers. Our corroboration of this result is discussed in a later part of this dissertation.

### **3.4.2. Earlier tunneling interview results**

Bao applied his theoretical development of model analysis to three quantum scenarios given to students in interviews and as exam questions.<sup>38</sup> In the first scenario, students are asked to reason about a beam of electrons with energy  $E > 0$  incident on a potential step in two related scenarios: the first a step down from 0 to  $-U_I$ , and the other a step up from  $-U_I$  to 0. In the

second scenario, students are asked to reason about two beams of electrons incident upon a non-symmetric potential energy barrier, where the potential energy on the far side of the barrier is lower than that on the incident side. The first beam of electrons has energy lower than that of the barrier, the second has energy greater than that of the barrier. In the third scenario, students are given a stepped potential well; that is, there are two regions of the well, each with different potential energies. As it is most relevant to this dissertation, I will focus on Bao's findings about student understanding of the tunneling scenario.

Three of ten students interviewed over two semesters could not find the correct kinetic energy in different regions of the potential energy diagram. When this question was administered on exams, eleven of nineteen students in traditional courses gave the correct kinetic energies, while thirteen of sixteen students in a modified course (where one hour of lecture each week was replaced with tutorials, as was previously described) also gave the correct kinetic energies.

Three of five students from the traditional course that were interviewed had difficulties sketching the correct shape for the wave function. All five students from the modified course that were interviewed were able to sketch a qualitatively correct wave function for the tunneling scenario.

Ten of the eleven interviewed students said that the squared amplitude of the wave function represents the probability density, but most could not

describe the meaning of that phrase. Many students reverted to a classical interpretation of the amplitude of the wave function, linking it instead to the kinetic energy of the particle. Eight of the eleven stated that the amplitude was reduced in region III of the tunneling scenario because energy was lost during tunneling. Another student, who correctly stated that the kinetic energy was the greatest in region III, drew a sinusoidal wave function with the largest amplitude in that region, explaining that it was larger because of the larger kinetic energy.

In his dissertation, Bao also noted student difficulties with understanding potential energy diagrams. When shown diagrams of potential wells and asked, given a wave function sketch, to reason about where a particle is most likely to be located, many students would answer with an energy level, indicating that they thought of energy levels as physical locations. Furthermore, in interviews, students would talk about particles “falling” into the well and “climbing” out of the well, again indicating that they might be thinking of the diagram as a representation of a two-dimensional system.

### **3.4.3. Probability investigations**

Domert, Linder, and Ingerman used a phenomenographic framework<sup>39</sup> in their analysis of interview sessions with twelve undergraduate physics students at two Swedish universities.<sup>40</sup> During the interviews, students worked with a computer simulation that allows the user to create potential wells or barriers, set the energy of an incoming wave packet, and observe the

time-evolution of the wave packet as it encounters the region(s) of increased or decreased potential energy.

From students' remarks about probability, the researchers identified four analytical phases of the tunneling scenario, and used these phases to characterize student thinking about different facets of probability. The first phase involves an understanding of the representation – what is being sent into the system. The second phase involves mathematically analyzing the Schrödinger equation. The third phase analyzes the outcomes of the mathematical analysis in terms of the derived quantities of transmission and reflection coefficients. The fourth phase involves interpreting the mathematical results, and realizing that the probabilistic interpretation of the coefficients is not applicable to individual events, but rather to an ensemble of identically prepared systems.

Examining the interview data in light of these four analytical phases, the researchers were able to see the variation of how probability was understood by the physics students, and use the student views to categorize the results. In the first category, the understanding of probability is in terms of reflection and transmission. That is, when a wave packet encounters a barrier, there is a certain probability for the packet to pass through the barrier, and another probability that it will be reflected. In descriptions given which were coded in this category, students often referred to the 'height' or 'largeness' of the wave packet as representing whether or not the packet could pass the barrier.

A second category involved linking the probability description to energy. While working with a wave packet, students described how the packet was a superposition of waves of different energies, so there were probabilities of the particle having various energies. Students reasoning in this category described how higher energy portions of the wave packet were able to pass the barrier because the energy of that portion exceeded that of the barrier. There was no discussion of probabilities of lower-energy portions tunneling.

The third category they identified is the understanding of probability in terms of finding a particle at a certain location. Here, students focus much more on the spatial aspect than they do the temporal aspect.

The final category they observed is the understanding of probability in terms of an ensemble. Here, probability was viewed by students similarly to the third category, but tied to the notion of repeated experiments. The probabilistic interpretation of the wave packet during the experiment is discussed in terms of a collection of experimental results, rather than the outcome of a single trial.

The authors argue that all four aspects of probability are important to understanding the tunneling scenario. They found, however, that most students used only two of the categories, with the most common combination being the description in terms of reflection and transmission coupled with discussion of the probability of finding a particle at a given location. Furthermore, they recommend that instruction should include an in-depth

investigation of all the facets of probability, and that tunneling should be presented by using wave packets, rather than with the plane wave approach common in many texts.

### **3.5. Summary of previous findings**

Previous research into student thinking about quantum topics has revealed a tendency to use classical ideas when addressing quantum problems, difficulties describing the probabilistic nature of systems, and trouble using the uncertainty principle. Students also have difficulty reasoning about the time-dependent nature of systems, often inappropriately attributing time-dependent characteristics to time-independent solutions, a theme we note in our observations described in subsequent chapters.

Modified forms of instruction, such as tutorials and computer simulations, have shown some effectiveness in teaching quantum concepts among populations studied. However, none of these investigations included thorough comparisons with control populations, so it is unknown if nontraditional forms of instruction are preferable to lecture-based teaching.

Previously described difficulties that students have in understanding graphical representations of waves are relevant to investigations that involve descriptions of wave functions. Students often misinterpret wave function sketches, describing particles as moving in a wave-like fashion or linking the wave function amplitude to energy, as we describe in future chapters.

A small number of researchers have investigated student understanding of quantum tunneling. The initial research at the University of Maryland involved pretest and posttest findings on a question similar to the question at the center of our interview and survey investigations. Their findings first noted the prevalence of the energy loss idea and the phenomenon of “axis shift.” However, the research was done with a population of junior engineering majors at a single institution, and involved no interviews. Our work involved physics and engineering physics majors at various stages of their undergraduate and graduate careers at three institutions. Additionally, the interviews we conducted reveal more depth of student reasoning than is possible to glean from a test question.

Bao interviewed students about quantum tunneling (among other scenarios), but his tunneling questions dealt exclusively with the scenario involving lower potential energy on one side of the barrier than the other. Though we included this question in a final round of interviews with a population of seniors, the majority of our work centered around the simpler problem of a symmetric square barrier. Also, we used the student responses to develop curriculum which addresses some common student difficulties, as we describe in Chapter 7.

Domert, Linder, and Ingerman investigated student reasoning about probability in the context of the quantum tunneling, and identified four analytical phases of the tunneling scenario. Although we asked students

questions about probability, our questions also included particle energy and sketches of the wave function. Additionally, their research utilized computer simulations and wave packet representations. We did not use any simulations in our interview sessions. Also, we note that only one of nineteen students we interviewed used wave packets to guide his reasoning. Because the wave packet representation is not stressed in the introductory quantum instruction at UMaine, we did not investigate student ideas about this representation, choosing instead to explore student ideas about the mathematical and graphical solutions we presented in Chapter 2.

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<sup>1</sup> A survey of the papers in the resource letter authored by McDermott and Redish reveals that of the 100+ papers listed dealing with student understanding of physics content, more than 90% deal with introductory college, high-school, elementary and middle school, or pre-service and in-service teacher populations. Less than 5% deal with student understanding of advanced physics topics. L. C. McDermott and E. F. Redish, "Resource letter PER-1: Physics education research," *American Journal of Physics* **67**, 755-767 (1999).

<sup>2</sup> Examples of work on classical mechanics, electricity and magnetism, and thermodynamics at Grand Valley State University, Iowa State University, The Ohio State University, and UMaine are given in references 3-5 in Chapter 1.

<sup>3</sup> Research on these topics is currently underway at Kansas State University, The Ohio State University, The University of Colorado, The University of Pittsburgh, and UMaine, among others. Findings published by these groups are discussed throughout this chapter.

<sup>4</sup> D. F. Styer, "Common misconceptions regarding quantum mechanics," *American Journal of Physics*, **64**(1) (1996) 31-34.

<sup>5</sup> K. Oliver and L. Bao, "Student resources in quantum mechanics, or why students need meta-resources," 2002 Physics Education Research Conference Proceedings.

<sup>6</sup> D. Hamner, "Student resources for learning introductory physics," *Am. J. Phys.* **69**(S1), S52-S59 (2000).

<sup>7</sup> R. Müller, and H. Wiesner, "Teaching quantum mechanics on an introductory level," *Am. J. Phys.* **70**, 200-209 (2002).

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- <sup>8</sup> R. N. Steinberg, G. E. Oberem, and L. C. McDermott, "Development of a computer-based tutorial on the photoelectric effect," *Am. J. Phys.* **64**, 1370-1379 (1996). R. N. Steinberg, M. C. Wittmann, L. Bao, and E. F. Redish, "The Influence of Student Understanding of Classical Physics When Learning Quantum Mechanics," *Proceedings from the National Association for Research in Science Teaching*, 41-44 (1999), available online at <<http://www.phys.ksu.edu/perg/papers/narst/>>.
- <sup>9</sup> R. Müller and H. Wiesner, "Students' conceptions of quantum physics," *Proceedings from the National Association for Research in Science Teaching*, 20-22 (1999), available online at <<http://www.phys.ksu.edu/perg/papers/narst/>>.
- <sup>10</sup> H. Fischler, "Introduction to Quantum Physics - Development and Evaluation of a New Course," *Proceedings from the National Association for Research in Science Teaching*, 32-40 (1999), available online at <<http://www.phys.ksu.edu/perg/papers/narst/>>.
- <sup>11</sup> See the second reference in reference 8.
- <sup>12</sup> See reference 9.
- <sup>13</sup> L. Bao and E. F. Redish, "Understanding probabilistic interpretations of physical systems: A prerequisite to learning quantum physics," *Am. J. Phys.* **70**, 210-217 (2002).
- <sup>14</sup> C. Singh, "Student understanding of quantum mechanics," *Am. J. Phys.* **69**, 885-895 (2001).
- <sup>15</sup> E. Cataloglu and R. W. Robinett, "Testing the development of student conceptual and visualization understanding in quantum mechanics through the undergraduate career," *Am. J. Phys.* **70**, 238-251 (2002).
- <sup>16</sup> P. Fletcher and I. Johnston, "Quantum mechanics: Exploring conceptual change," *Proceedings from the National Association for Research in Science Teaching*, 28-31(1999), available online at <<http://www.phys.ksu.edu/perg/papers/narst/>>.
- <sup>17</sup> D. Zollman, N. S. Rebello, K. Hogg, "Quantum mechanics for everyone: Hands-on activities integrated with technology," *Am. J. Phys.* **70**, 252-259 (2002).
- <sup>18</sup> N. S. Rebello and D. Zollman, "Conceptual understanding of quantum mechanics after using hands-on and visualization instructional materials," *Proceedings from the National Association for Research in Science Teaching*, 2-6 (1999) 2-6, available online at <<http://www.phys.ksu.edu/perg/papers/narst/>>.
- <sup>19</sup> See reference 4.
- <sup>20</sup> D. F. Styer, "Quantum mechanics: See it now," Contributed talk at the 2000 winter meeting of the American Association of Physics Teachers, available online at <<http://www.oberlin.edu/physics/dstyer/TeachQM/see.html>>.
- <sup>21</sup> R. P. Feynman, *QED: The Strange Theory of Light and Matter*, (Princeton University Press, Princeton, New Jersey, 1985).

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- <sup>22</sup> E. F. Taylor, S. Vokos, J. M. O'Meara, and N. S. Thornber, "Teaching Feynman's Sum Over Paths Quantum Theory," *Computers in Physics* **12**, 190-199 (1998).
- <sup>23</sup> K. M. Robblee, P. Garik, and G. Abegg, "Using computer visualization software to teach quantum science: The impact of pedagogical content knowledge," *Proceedings from the National Association for Research in Science Teaching*, 11-14 (1999), available online at <<http://www.phys.ksu.edu/perg/papers/narst/>>.
- <sup>24</sup> L. C. McDermott, P. S. Scaffer and the Physics Education Group, Tutorials in Introductory Physics, (Prentice-Hall, Upper Saddle River, NJ, 2002).
- <sup>25</sup> See the second reference in reference 8.
- <sup>26</sup> See reference 10.
- <sup>27</sup> M. Euler, M. Hanselmann, A. Müller, and D. Zollman, "Students' views of models and concepts in modern physics," *Proceedings from the National Association for Research in Science Teaching*, 15-19 (1999) 15-19, available online at <<http://www.phys.ksu.edu/perg/papers/narst/>>.
- <sup>28</sup> H. Niedderer and S. Deylitz, "Evaluation of a new approach in quantum atomic physics in high school," *Proceedings from the National Association for Research in Science Teaching*, 23-27 (1999) available online at <<http://www.phys.ksu.edu/perg/papers/narst/>>.
- <sup>29</sup> Additional evidence of tutorial learning is presented in the fourth section of this chapter, dealing with previous research on tunneling.
- <sup>30</sup> M. C. Wittmann, "Making sense of how student come to an understanding of physics: An example from mechanical waves," Ph.D. dissertation, University of Maryland (1998).
- <sup>31</sup> M. C. Wittmann, "The object coordination class applied to wavepulses: Analysing student reasoning in wave physics," *International Journal of Science Education* **24**, 97-118 (2002).
- <sup>32</sup> R. N. Steinberg, M. C. Wittmann, and E. F. Redish, "Mathematical tutorials in introductory physics," *AIP Conf. Proceedings* **399**, 1075-1092 (1997).
- <sup>33</sup> B. Sherin, "The symbolic basis of physical intuition: A study of two symbol systems in physics instruction," Ph.D. dissertation, University of California, Berkeley (1996).
- <sup>34</sup> B. S. Ambrose, P. R. L. Heron, S. Vokos, and L. C. McDermott, "Student understanding of light as an electromagnetic wave: Relating the formalism to physical phenomena," *Am. J. Phys.* **67**, 891-898 (1999).
- <sup>35</sup> S. Vokos, P. S. Shaffer, B. A. Ambrose, and L. C. McDermott, "Student understanding of the wave nature of matter: Diffraction and interference of particles," *Am. J. Phys.* **68**(S1), S42-S51 (2000).
- <sup>36</sup> E. F. Redish, M. C. Wittmann, and R. N. Steinberg, "Affecting Student Reasoning in the Context of Quantum Mechanical Tunneling," *Contributed Talk CF14*, 2000 summer meeting of the American Association of Physics

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Teachers. M. C. Wittmann, J. T. Morgan, and L. Bao, "Students' descriptions of particle energy loss in quantum tunneling," *Eur. J. Phys.* **26**, 1-12 (2005).

<sup>37</sup> E. F. Redish, *Activity Based Tutorials, Module 2 (Modern Physics)*, (Wiley, New York, 2005).

<sup>38</sup> L. Bao, "Dynamics of student modeling: A theory, algorithms, and application to quantum mechanics," Ph.D. dissertation, University of Maryland (1999).

<sup>39</sup> A phenomenographic framework uses the student responses to define the categorization, rather than using established expert responses.

<sup>40</sup> D. Domert, C. Linder, and Å. Ingeman, "Probability as a conceptual hurdle to understanding one-dimensional quantum scattering and tunneling," *Eur. J. Phys.* **26**, 47-59 (2005).

## Chapter 4

### INTERVIEWS

Classroom responses and analysis of homework and exam problems can shed some light on the numbers of students who write down correct responses, or answer questions in particular ways, or make certain errors in calculations. However, they are limited in their ability to reveal the depth of student understanding. Clinical interviews allow the researcher to ask questions that are difficult or impossible to pose on standard classroom instruments. In addition, the interview allows the researcher to probe student conceptual reasoning about a topic, and adjust the line of questioning in a dynamic way not possible with standard assessments.

We chose to begin our investigations into student understanding of quantum mechanical tunneling with a series of interviews, in hopes of revealing common ideas presented by students, as well as observing common difficulties. The interview results informed the development of a survey on tunneling, used to probe the extent of common difficulties in a wider sample of students, as we describe in Chapter 5. The interview results also played a key role in the curriculum development for an introductory quantum physics course, described in Chapter 7.

We designed an initial protocol to use in interviews with undergraduate physics students. The protocol was modified three times throughout the

course of our investigations, either to make the series of questions more pointed or to focus on a different aspect of the scenario.

Each of the interview tasks used in our research study centers around asking students to reason about tunneling through a square potential energy barrier. However, the order of presentation and the applications discussed within the interviews have varied. In the following sections, we describe the evolution of the protocol, discussing the student responses to each version of the protocol, and how those responses led to adjustments in the next version. In addition, we discuss the major ideas that were evident in our results.

#### **4.1. Parallel-Plate Protocol**

While our aim was to ask students about the standard square potential energy barrier tunneling scenario seen in most textbooks and discussed by many lecturers, we did not want to begin with this rather abstract and theoretical situation. Instead, our initial goal was to first provide some context for the problem and have students reason about a physical situation involving charged particles, then shift the questioning to the realm of quantum mechanics.

##### **4.1.1. Design of the Parallel-Plate Protocol**

Our initial task asked students to reason about a parallel-plate capacitor. They were read the following description at the beginning of the interview:

“Suppose you have a system like this. There are two thin metal plates; one of them has a net positive charge, and the other has a net negative charge. Assume the plates are very large, and the holes are ideal, so we

won't worry about field effects around edges. Also, each plate is hooked up to a charge source, so the charge on the plates remains constant. Each plate has a small hole drilled in the middle. Some distance away, you have a device that shoots electrons. This device allows you to control the initial energy of the electrons, and directs them towards the plates. While many electrons will hit the plate, occasionally an electron will pass through the hole. It is these electrons that pass through the hole that we'll focus on."

As the statement was read, the students were shown the picture in

Figure 4-1(a). They were then asked to describe the electric field, the force on an electron moving through this system, and the potential energy in three areas: (i) to the left of the positive plate, (ii) in between the plates, and (iii) to the right of the negative plate.

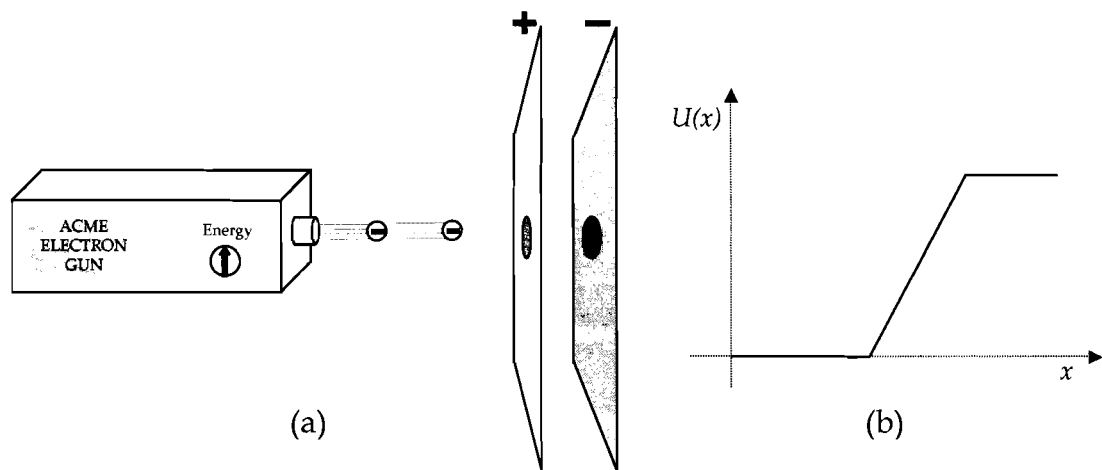


Figure 4-1: System from the parallel-plate interview protocol.  
 (a) The electron gun and parallel plate picture shown to students.  
 (b) The potential energy graph for this system.

Because the electric field outside of a parallel plate capacitor is zero (by superposition, the fields of the positive and negative plates cancel outside of the system), there is no force on the electrons to the left of the positive plate or to the right of the negative plate. Therefore, the potential energies in these

regions are constant. Because the electron feels an attractive force from the positive plate and a repulsive force from the negative plate inside of the capacitor, its potential energy increases. Thus, the potential energy graph of the system looks like Figure 4-1(b).

Next, students were asked to reason about the effects of (i) moving the plates closer together, and (ii) introducing a second set of plates to the system, as shown in Figure 4-2(a). The potential energy of an electron in such a system, shown in Figure 4-2(b), should be similar to a standard square potential energy barrier.

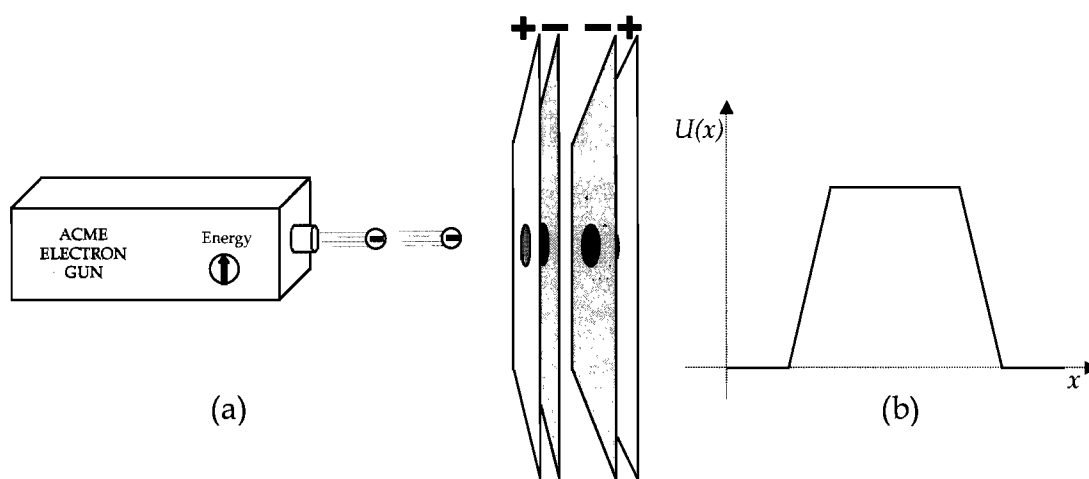


Figure 4-2: (a) Modifications to the initial system included moving the plates closer together and introducing a second pair of charged plates. (b) The accompanying potential energy graph.

The next set of questions focused on the energy of the electrons incident on the plates. Participants were first asked to reason about the behavior of electrons encountering the parallel-plate system with energy twice that of the maximum potential energy in between the sets of plates. They were then

asked how that behavior changed if the initial energy of the incident electrons was reduced to 90% that of the maximum potential energy.

The latter portions of the interview were designed to ask subjects whether they had heard of tunneling, whether or not tunneling was appropriate to think about in this situation, and to describe their ideas about the wave function and its utility. The students were to be asked to sketch the wave function corresponding to this system, describe what information about the system could be gleaned from the sketch, and sketch and/or describe changes to the wave function corresponding to changes to the physical system, such as moving the sets of plates further apart. In reality, as we describe in the subsequent sections, the difficulties in reasoning about the initial physical system left little time for questions about tunneling.

#### **4.1.2. Results: Eric**

Two students were interviewed using the parallel-plate protocol. I first interviewed “Eric”<sup>i</sup> in November of 2002. At the time, Eric was a senior physics major enrolled in the senior-level quantum physics course. As he mentioned during the interview, the class had recently discussed writing wave function solutions for various potential energy situations.

##### **4.1.2.1. Ideas about system potential energy**

When I asked Eric to reason about the potential energy of electrons to the left of the initial parallel-plate arrangement, he began talking about electric

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<sup>i</sup> All names used throughout this thesis to reference students are aliases.

fields. He struggled with whether or not an electric field existed to the left of the plates, eventually deciding that the field to the left of the plate was uniform and pointing to the left, because the field contribution from the negative plate was shielded by the positive plate. Even when I asked him to first think about each plate individually, then to reason about putting the plates together (intending for Eric to use superposition to rationalize the field cancellation outside of the plates), he stuck with his “shielding” idea. This led him to rationalize that the electrons felt forces in all parts of the system, creating non-constant potentials in all regions.

Eric initially tried to use ideas about the force felt by the electrons as a way to sketch the potential energy diagram, claiming that moving in the direction of a force increased the electron’s potential. This led him to sketch potential energy as increasing until the positive plate position, and decreasing beyond it. When the interview questioning shifted to focus on the speed and kinetic energy of the electrons, and Eric correctly reasoned that the electrons would speed up when approaching the positive plate, and slow down when approaching the negative plate. He used conservation

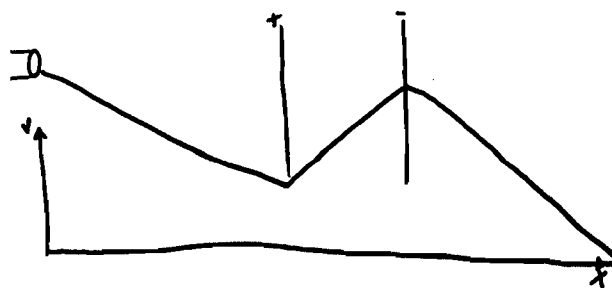


Figure 4-3: Eric’s revised sketch of system potential energy, showing energy decreasing, then increasing between plates, then decreasing again.

of energy to deduce that his earlier responses about the potential were incorrect. This caused him to change his earlier potential graphs, but to still leave the potential as increasing or decreasing in all regions, as shown in Figure 4-3.

Twenty minutes had elapsed in the interview at this point, and Eric had only reasoned (incorrectly) about the potential versus position graph for a single pair of plates. I decided to show the correct potential energy diagram to him. Given this information, he was able to correctly describe the changes to the potential versus position

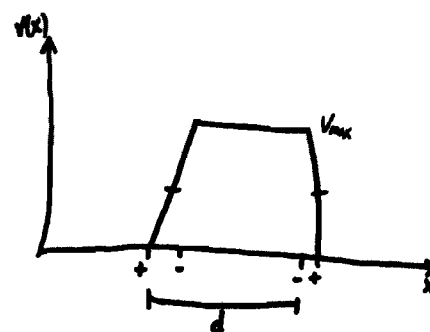


Figure 4-4: Eric's sketch of system potential energy, following intervention.

graph that occurred when (i) the plates were moved closer together, and (ii) a second pair of plates was added to the system, as is shown in Figure 4-4.

#### 4.1.2.2. Ideas about electron behavior

I then asked Eric about the behavior of electrons sent into the system with initial kinetic energy twice that of the maximum potential energy of the system. He reasoned that the velocity would remain constant outside of the system in the absence of an electric field. When asked about the behavior between the two plates, he again turned to force reasoning, stating "...force is  $-\nabla V$ , so as the potential increases, force is getting more negative..." Although he correctly stated the force-potential relationship, he directly compared the

force to the potential, rather than, for example, the slope of a graph of the potential as a function of position. This led him to reason that the velocity “should decrease exponentially.” While reasoning about the velocity between the pairs of plates, he again seemed to rely on an incorrect analysis of the force-potential relationship, stating that particles “should slow down some more, but linearly now, because the potential is constant.” He quickly stated that the velocity would increase exponentially in between the second negative and positive plate.

Next, I asked Eric about electrons sent into the system with initial kinetic energy equal to 90% of the maximum potential energy of the system, and whether or not any of them would make it through the system. Initially, he said no, reasoning that “the potential is larger than the energy that the electrons have, so the force would be greater than any... would be great enough to slow it down to zero, or push it back the other way.”

#### **4.1.2.3. Describing tunneling**

I asked Eric if he had ever heard of tunneling, and he admitted he had, briefly described tunneling as “a probability in quantum mechanics that allows a quantum particle to tunnel through a potential that classically it wouldn’t be able to make it through.” He also stated that there was a small probability that an electron could tunnel through this system, because an electron “is a quantum particle.” When probed on this description, Eric replied that quantum particles have “very small mass and large speeds.”

#### 4.1.2.4. Ideas about wave functions

I asked Eric if he had ever heard of wave functions, and whether he could describe and sketch the wave function describing the electrons in this system. Eric said that describing wave functions was very difficult, but that he could tell me what the square of the wave function is: a “probability of finding the particle... within a given region.” Before sketching the wave function, Eric wrote down  $\psi_A = Ae^{ikx} + Be^{-ikx}$ , remarking that he just had this idea in class that morning. I asked him what  $k$  represented, and he wrote down  $k = \frac{\sqrt{2mE}}{\hbar}$ , stating that it “is basically a parameter that is dependent on mass and the energy of the particle.”

Eric sketched the wave function as sinusoidal in all three regions, as is shown in Figure 4-5. He paused after drawing a sinusoidal wave function in region A (shifted off the position axis), and took some time to decide how the function would be different in region B. He finally concluded

that “the addition of the potential would increase this number (the constant  $k$ ), which would increase the amplitude.” He drew a sinusoidal function with a decreased amplitude in region C, but said that the amplitude in region C was probably greater than the amplitude in region A, because “you should still see the effect of the addition [of the energy] on the amplitude.”

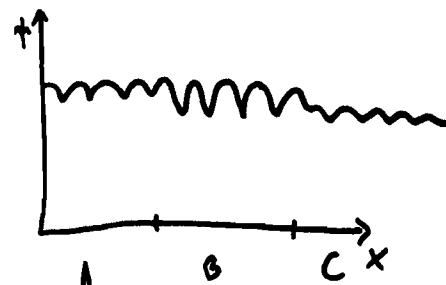


Figure 4-5: Eric's sketch of the wave function for the parallel-plate problem.

Eric was then asked to reason about a beam of electrons with initial kinetic energy equal to 50% of the maximum potential energy of the system, and whether or not that would affect the wave function in any of the regions. He replied that this would decrease the amplitude in each region, while leaving the frequency unchanged. He sketched a wave function with correspondingly lower amplitudes on the same set of axes, drawing the functions as oscillating about an imaginary axis lower than the initial function. When asked to think about the effect of moving the pairs of plates farther apart (essentially creating a wider barrier), Eric stated that the wave function characteristics would remain the same in all three regions, except that it was “spread over a larger area” in region B. Specifically, he said that the amplitude in region C would remain the same.

#### **4.1.2.5. Ideas about probability**

The questioning was then shifted to focus on the probabilistic aspects of the system. I asked Eric if the probability of finding an electron in region C had changed when the barrier was made wider. He responded affirmatively – “yes, it’s less likely, since the... barrier... it’s going through is larger. The potential is stronger.” This led him to realize that the wave function could not remain the same in region C when the barrier width was increased. After contending that the potential was now “higher” (in addition to spatially “wider”), he stated that the amplitude in both regions B and C would be reduced.

To see if these responses would influence his earlier ideas, I asked him to revisit his earlier answers. Eric revised his sketch to now say that the amplitude of the wave function in region C (in all scenarios) was less than the amplitude of the wave function in region B. He was initially convinced that the wave function in region B was both sinusoidal and possessed a higher amplitude than the function in regions A and C, because “the potential is... adding to the energy,” but changed his mind and reversed his answer, stating that the “potential subtracts from the energy, which decreases the amplitude.”

#### **4.1.2.6. Ideas about particle energy**

At the end of the interview, Eric was asked about the energy of an electron found in region C, compared with its energy in region A. He stated that the energy should be less, because the “effect of the potential decreased the energy.” Because an hour had passed, there was not time to ask him any follow-up questions about this idea.

#### **4.1.2.7. Summarizing Eric’s ideas**

In summary, we find that Eric seemed to favor reasoning about forces and fields rather than energies, and often used the former to deduce the latter. He was aware of quantum tunneling, and could write down partially correct solutions to Schrödinger’s equation for the square barrier system. When he sketched the wave function, it was sinusoidal in all regions. Although he initially made changes to the amplitude of the wave function in connection to system energy, he later connected amplitude to the idea of probability,

arguing that the probability of tunneling is reduced if the barrier is widened. He reasoned that particles lose energy when they tunnel.

#### **4.1.3. Results: Michelle**

“Michelle” was interviewed a few days later, using the same protocol. Also a senior, Michelle and Eric were enrolled in the same quantum physics course.

##### **4.1.3.1. Ideas about system potential energy**

Shown the same initial picture, Michelle focused on the effect the hole had on the electric field. Told not to worry about edge effects around the hole, she surmised that there would be a cylindrical region with no field that the electrons passed through, therefore experiencing no change in potential. I quickly modified the questions, asking her to think about holes in plates that would instantaneously open to allow electron passage, but at all other times exert forces on the electrons.

Michelle correctly deduced the direction of the electric field inside the pair of plates. I then asked her to sketch a graph of potential energy versus position for the system. She had difficulty in deciding how to connect ideas of electric field and potential energy, and how to separate ideas about the kinetic and potential energies of electrons. In addition, twice in her verbal decision-making she referred to “one over r-squared” ideas, perhaps recalling formulas for the force or electric field from a point charge. She finally settled on a potential that decreased as the electrons approached the positive plate,

remained constant in between the positive and negative plates, then increased to the right of the negative plate, as is shown in Figure 4-6.

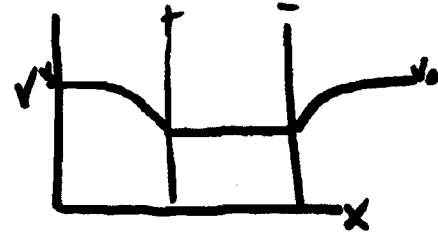


Figure 4-6: Michelle's initial sketch of the potential energy of the system.

I asked her to imagine a situation where somehow there was no field outside of the plates. She initially had trouble with this idea, struggling with both wanting a force on the electrons outside of the plates, and whether her graph best represented force or potential energy. She changed her mind, however, to say that there would be no force outside of the plates, based on the given idea that the field outside the plates was zero.

Michelle continued to have difficulty differentiating the ideas of force and potential energy, and finally went with her ideas about force – the electrons experience a force inside of the two plates that causes an acceleration – to decide that the potential energy increased between the plates. It is unclear how solid the ideas were to her, though, as it took her an additional minute to decide which graphical representation for the potential energy was correct, and included a reminder of the “zero field outside the plates” idea.

With this reasoning, Michelle was able to deduce that moving the plates closer together would increase the slope of the increasing potential energy portion of the potential energy graph, and that introducing a second set of reversed plates would create a nearly square potential energy barrier.

#### 4.1.3.2. Ideas about electron behavior

Following the protocol, I asked Michelle to think about sending a beam of electrons into this system, having given the electrons an initial total energy equal to twice the potential energy of the barrier. She reasoned that the velocity would remain constant to the left of the system, but would increase in between the first set of plates because “we’re adding energy to it, and the energy has to go somewhere.”

When questioned about the increasing velocity, Michelle re-admitted her earlier energy struggles: “Maybe I’m not clear on the difference between a potential energy and kinetic energy of an electron.” At this point, however, she seemed convinced that the electron, entering a region of increasing potential, “gets more energy”, and that “I want to put the energy someplace. And the place where I think I want to put the energy is in kinetic energy.” Her responses seemed to indicate that she viewed the plate system as possessing its own potential energy that was somehow passed off to the electrons, rather than viewing potential energy as an interaction between the electrons and the system.

Michelle carried the idea of increased kinetic energy forward, reasoning that the electrons would have “constant velocity” between the two negative plates, as the “energy would stay the same.” Symmetrically, she reasoned that the electrons would slow down in between the second set of plates, and emerge at “constant velocity equal to the initial.”

Next, I asked Michelle to think about electrons sent in with initial total energy equal to 90% of the potential energy of the barrier, and whether or not the reduced energy affected the motion of the electrons. She stated that it would not, saying "it's going to increase it by the same amount..." again indicative of a model in which the parallel plate system adds energy to the electrons.

Attempting to change lines of questioning, I asked Michelle if energy was conserved for an electron in the system. Although she initially responded affirmatively, she soon changed her mind, again insisting that energy was conserved for the system (plates and electrons), but that the plates "gave" energy to the electrons. Furthermore, she stated that energy conservation for the system was only an artifact of the gaps between plate pairs being equal, "If those weren't equal, then the energy wouldn't be conserved. It's just by symmetry that the energy is. It's not any fundamental law."

I then asked Michelle to think about potential and kinetic energy in the context of a falling ball. She correctly described gravitational potential energy being transferred into kinetic energy as the ball fell, but stated that she couldn't see how this was related to the electron problem. I stated that the falling ball was in a gravitational field, while the electrons were in an electric field, and asked her what was fundamentally different about the objects' interactions with their respective fields. She again said that she was having trouble "differentiating between potential and kinetic energy in an electron."

Her answers again seem to suggest that she may have been thinking about potential energy as a property of the plate system, rather than an interaction between the electrons and the system.

Changing courses once again, I asked her if it made intuitive sense that the electrons sped up as they approached the first negative plate, consistent with her previously sketched velocity graph. She responded “no,” then amended her velocity ideas to say that the velocity would “increase there (to the left of the system), decrease (between the first set of plates), increase (between the two inside plates), increase (between the second set of plates), decrease (to the right of the system).” After several additional prompts, including reminders to think about a scenario with no field outside of the system, Michelle agreed that the velocity would initially remain constant, decrease between the first pair of plates, remain constant between negative plates, increase between the second pair of plates, and remain constant to the right of the positive plate. She also agreed that this agreed with the principle of conservation of energy. She produced the sketch shown in Figure 4-7 to represent her ideas.

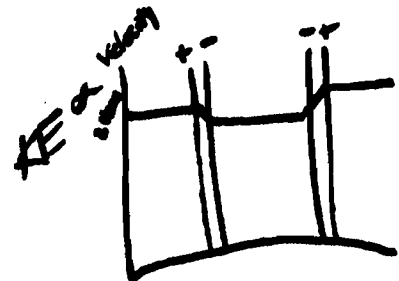


Figure 4-7: Michelle's sketch of the electron velocity in the parallel-plate system.

We then revisited the scenario where electrons are incident on the system with initial total energy equal to twice the potential energy of the barrier, and then reduced to 90% of the barrier

potential energy. She reasoned that the electrons would slow down in the first case, but still pass through, but was troubled with what would happen in the second case, as using conservation of energy principles would suggest a negative kinetic energy in between the plate pairs. She stated, "it would probably... not be able to go through the whole system. It would be... bounced back? Or, because of quantum, it may somehow tunnel..."

#### **4.1.3.3. Ideas about tunneling**

Asked to define tunneling, Michelle said, "tunneling is a chance that an electron or some other object, atom, or even table, will defy, um, classical laws and interpretations that we try to impose upon it, and tunnel through a potential that is classically not be able to overcome." She stated that tunneling would be appropriate to think about in the aforementioned scenario because "an electron is small enough that it becomes a definite probability." I asked her how one would decide if the electron would be able to pass through the system. She replied that one would need to use Schrödinger's equation, and that "it's an equation that the square of it magically gives us the probability that something will happen to the electron - that either it will pass through, or that it will be reflected."

#### **4.1.3.4. Ideas about wave functions**

When I asked her what the wave function is, Michelle gave a two-and-a-half-minute response in which she admitted having great difficulty trying to understand wave functions. She restated that the square of the wave function

gives us a “probability amount.” She also touched on the collapse of the wave function upon measurement, and the subsequent expansion if a system is left alone. She said that she had been reading a lot of outside material, and wasn’t at present happy with any of the interpretations. She concluded her monologue by saying that the whole idea was “so counterintuitive.”

When I asked her what the wave function for this system looked like, Michelle initially wrote down a sum of exponentials. I asked her what the graph of the wave function would look like, and she stated that “it’s gonna be some periodic wave form” because “this is basically the addition of sines and cosines.” Questioned about the wave function

in all three regions individually, she stated that the wave function in region A would be sinusoidal. In region B she sketched a sinusoidal. In region B she sketched a decaying exponential function on top of a square barrier, as is shown in Figure 4-8.



Figure 4-8: Michelle’s sketch of the wave function in the barrier region.

She reasoned that the probability would decay exponentially, stating “I don’t really know how to draw the wave function.” In region C she described the wave function as similar to that in region A.

Asked to sketch the wave function over all three regions, she sketched an oscillating function, reduced the amplitude in the

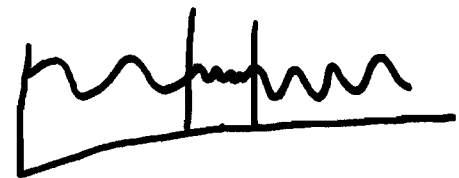


Figure 4-9: Michelle’s sketch of the wave function in all regions.

middle, and then increased it again, as shown in Figure 4-9. She stated that in the middle “it decays exponentially,” but it was not clear whether she meant the amplitude of the sinusoidal function, or something different. At this point, over an hour had passed, and the interview was ended.

#### **4.1.3.5. Summarizing Michelle’s ideas**

In summary, we find that Michelle had difficulty reasoning about the energy of electrons, and deciding when conservation of energy was appropriate to use. With sufficient prompting, she was able to construct the potential energy diagram for the system. In subsequent responses, it seems that she viewed the potential energy as a property of the plate system, independent of the electrons, which caused her to have difficulty reasoning about the behavior of the electrons in the system. In a limited amount of time, she vaguely described sinusoidal wave functions, and sketched wave functions superimposed on a potential energy barrier.

#### **4.1.4. Summary of parallel-plate interview results**

While it was anticipated that the initial phase of the interview – reasoning about the charged plates and sketching the corresponding potential energy diagrams – would take the first 10-15 minutes of the interview, in reality this was not the case. Both interview subjects took at least twice the expected length of time to reason about the system, and were at some point shown the correct potential energy diagrams after several minutes of struggling to build a coherent system model. Although not a goal of our session, we find that

both students have difficulty with connections between the ideas of electric field, force, and potential energy in the context of discussing a parallel-plate capacitor.

Little time was available at the end of the interview session for reasoning about quantum mechanical tunneling, and both participants seemed mentally fatigued when answering the tunneling questions. Both had heard of tunneling. When asked to sketch wave functions, both sketched sinusoidal shapes, with Michelle initially sketching a decaying exponential form in the barrier region. Only Eric was asked about particle energy, and he stated that energy was lost in tunneling.

In reviewing the results, we felt that the amount of time students spent reasoning about electric fields made it nearly impossible to spend any amount of time probing their ideas about tunneling. We decided to revise the protocol to begin with another (and in our view simpler) physical scenario.

#### **4.2. Modification of the Protocol – Bead on a Wire**

Following the struggles both participants encountered using the initial protocol, the interview task was revised. We adapted a problem stated in a tutorial on quantum tunneling developed by researchers at the University of Maryland.<sup>1</sup>

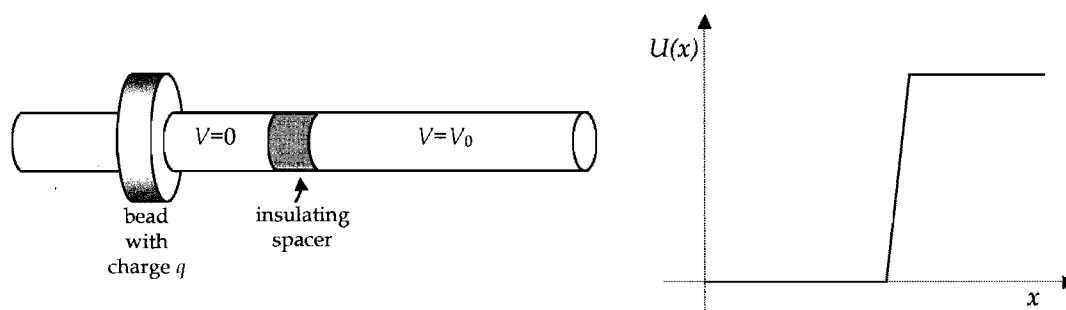


Figure 4-10: Representations from the sliding bead problem.  
 (a) Picture of a charged bead on a wire held at different electric potentials. (b) Potential energy graph for the single spacer scenario.

#### 4.2.1. Design of the bead on a wire protocol

The task asks students to reason about a charged bead sliding along a wire, and to ignore friction. The left-hand portion of the wire is held at an electric potential of 0, while the right-hand portion of the wire is held at an electric potential  $V_0$ . A small insulating spacer separates the two sides, as shown in Figure 4-10(a). Because the electric potential energy is related to the electric potential by the charge of the object,  $U = qV$ , the bead's potential would be zero in the left-hand region, and some non-zero constant value in the right-hand region. The graph of the potential energy would resemble Figure 4-10(b).

A second insulating spacer is introduced, as shown in Figure 4-11(a). Now, the region of higher electric potential is surrounded by two regions of zero potential, causing the potential energy of the bead as it moves throughout the system to look like Figure 4-11(b).

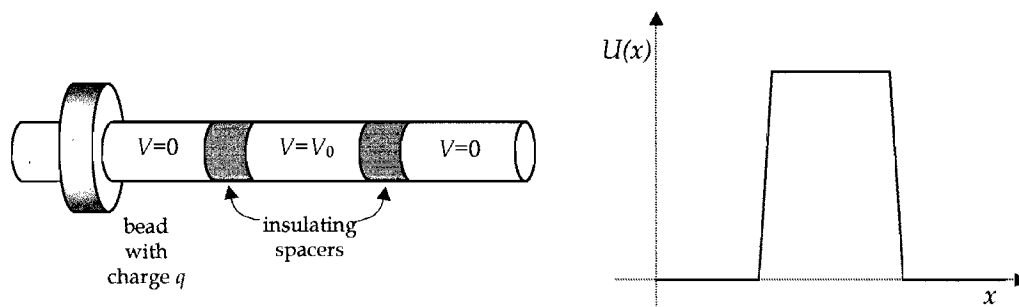


Figure 4-11: Representations from the second part of the sliding bead problem. (a) Picture of a charged bead on a wire held at different electric potentials. (b) Potential energy graph for the double spacer scenario.

The protocol was designed to have students initially construct the potential energy graphs given the diagram and description of the sliding bead, and reason about the behavior of a sliding bead (i) with initial total energy greater than  $qV_0$ , and (ii) with initial total energy less than  $qV_0$ . They were then asked to compare and contrast the behavior of the sliding bead with a particle incident on a square potential energy barrier using the ideas of quantum mechanics. The same two senior physics majors, Eric and Michelle, who completed the senior-level quantum physics course during the fall of 2002 were interviewed in February of 2003.

#### 4.2.2. Results: Eric

##### 4.2.2.1. Constructing the potential energy diagrams

When asked to sketch a graph of potential energy as a function of position, Eric admitted that he was “trying to remember the difference between potential and potential energy.” He then went on to share his initial idea that

the potential energy would increase until it got to the insulating region, and then be constant after that.

I asked him why the potential energy would increase in the first region, and he said that it was because the bead was getting closer to the nonzero potential region. When he could not recall the exact relationship between electric potential and electric potential energy, I supplied him with the definition “electric potential is electric potential energy per charge.” Given this information, he was quickly able to sketch a correct potential energy diagram for the scenario.

#### 4.2.2.2. Reasoning about bead behavior

I next asked Eric to describe the motion of the bead sliding along this wire, given that the bead had an initial total energy greater than  $qV_0$ . He stated that the bead would continue to move to the right, with no change to its speed. When I asked him to consider a bead with initial total energy less than  $qV_0$ , he stated that once it crossed the insulating region, it would start to slow down.



Figure 4-12: Eric's graph of the bead's velocity.

He stated that the “potential on the wire is creating... a force on the charge  $q$ .” He sketched the graph of bead velocity shown in Figure 4-12.

Next, I gave Eric the scenario with the two insulating spacers. He quickly sketched a qualitatively correct potential energy graph. Considering a bead with energy greater than  $qV_0$ , he stuck with his previous assertion that the

bead's motion would be unaffected, but seemed less sure of his response. Analyzing a bead with energy less than  $qV_0$ , he revised an earlier description of a bead slowing down then moving back in the opposite direction to now say that the bead would merely slow down and come to a stop at some location before it reached the second insulating spacer. I asked him if it would be possible to move the spacers close enough together so that the bead would make it past the second spacer, and he agreed that it probably would.

It seems that Eric had trouble differentiating between the concepts of "potential" and "force." Given that the right hand side of the wire has constant, non-zero electric potential, he states that the bead slows down, which is consistent with the idea of a constant acceleration (and therefore constant force). It's not clear why he later changed his mind and stated that the bead stops, but this may indicate difficulty with the concept of the acceleration of an object at rest.

#### **4.2.2.3. Reasoning about quantum particles**

Because the interview to this point had remained mired in classical physics issues and had not touched on issues of quantum tunneling, as desired, Eric was shown a "step" potential energy diagram, similar to what he had sketched for the first scenario. I asked him to think about a particle incident on this system, using ideas he'd learned in his quantum courses. I first asked him to sketch the wave function for a particle in this scenario. He sketched an oscillating function in the region of zero potential, and wrote down a sum of

complex exponentials,  $\psi = Ae^{ikx} + Be^{-ikx}$ . When discussing his answers, he mentioned that the “square of  $\psi$ ” is equal to the probability density, but could not clearly describe the physical significance of the amplitudes. He stated that the  $k$  terms in his complex exponentials were “related to the energy of the particle, and the potential energy... and the mass of the particle.”

Asked specifically to reason about the wave function in the second region, Eric wrote down a second sum of complex exponentials, this time replacing  $k$  with  $l$ , and stated that the  $l$  term required taking into account the potential energy in the second region. He was unable to reason how the new amplitude terms,  $C$  and  $D$ , related to the previous amplitude terms  $A$  and  $B$  he had given for the wave function in the first region.

I then asked Eric to think about the situation where the incoming particle had energy less than  $U_0$ , the maximum potential energy of the step.

He sketched a sinusoidal function in the first region, oscillating about some axis perhaps coincident with the energy level, and a decaying

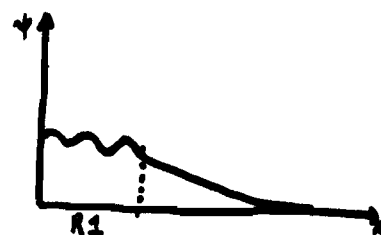


Figure 4-13: Eric's wave function sketch for the potential step scenario.

exponential function in the second region that approached the axis, as is shown in Figure 4-13.

I asked Eric what was fundamentally different about the quantum potential step, compared with the bead sliding on the wire. Specifically, I reminded him that he had said a bead with initial energy greater than the

potential energy of the step would be unaffected in its motion, yet he'd also said that a quantum particle would have a different wave function in the two regions. I asked him to explain why there is a quantum effect but not a classical effect. He replied,

"I guess the whole theory, the whole groundwork of quantum mechanics. Um, in our classical world, um, you know, a plane flying high above the ground doesn't see, you know, doesn't care about elevation changes on the earth. Um, but, for quantum particles, um, these potential energy changes do cause effects. How much, and that, you know, that, I don't think I fully understand that myself, but I think that's sort of the whole idea behind quantum mechanics, is that these wave functions "see" ... potential energy, even though in a classical sense it would not."

We see evidence in this statement that Eric uses elevation to reason about potential energy, and thus is likely reasoning about gravitational potential energy. Furthermore, the unaffected plane is spatially above the "barriers" on the earth's surface, which may indicate lack of distinction between spatial height and energy levels. Finally, we note that the language suggests Eric is thinking of the wave function as an object, reminiscent of Wittmann's observation that students assign particle properties to waves, which we discuss in the previous chapter.

#### **4.2.2.4. Reasoning about quantum tunneling**

The final situation Eric was asked to reason about involved a quantum particle and a square potential energy barrier. I first asked him to sketch a wave function corresponding to particle energy greater than the maximum potential energy of the barrier, and he sketched an oscillating function in all three regions, as shown in Figure 4-14(a). He stated that "something changes"

in the center region, though he was unsure of what, and said that the function in the far right region was “pretty much similar.” When I pressed him on this, he revised his statement to “exactly the same.” I asked him further questions about how the wave function changed in the middle region, and he stated that he was relatively sure the amplitude changed, though he could not reason whether it would increase or decrease. He also said that the exponential terms would contain the

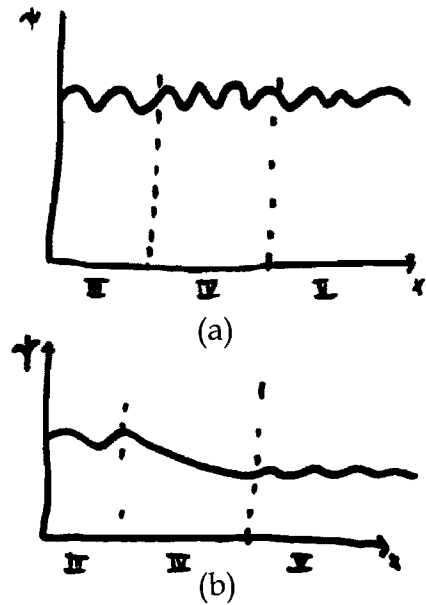


Figure 4-14: Eric's wave function sketches for (a)  $E > U_0$ , and (b)  $E < U_0$ .

same  $l$ 's (in the  $\psi \propto e^{ilx} + e^{-ilx}$  term) that were present in the right-hand region of the potential step. I asked him to think further about whether an increased amplitude or decreased amplitude in the center region made more sense, but he was unable to choose one. He mentioned offhand that the amplitude might relate to the energy, but quickly changed his mind.

Next, I asked Eric to sketch the wave function for the situation where the particle energy was less than  $U_0$ . He sketched a sinusoidal waveform in the left-hand region, a decaying exponential in the center region, and a sinusoidal waveform in the right-hand region, as shown in Figure 4-14(b). In the right-hand region, the wave function had both decreased amplitude and wavelength, relative to that in the left-hand region, and was oscillating about

an imaginary axis lower than the “axis” in the left-hand region, consistent with the behavior first noted by Redish, Wittmann, and Steinberg that we described in the previous chapter.

When I asked him how the wave function was different in the two outside regions, he replied that in the right-hand region, “the particle has lost some energy... due to the... interaction with the potential... the particle has lost some energy, so the amplitude has gotten a little bit smaller. So I guess maybe the amplitude does have something to do with the energy of the particle.”

#### **4.2.2.5. Describing quantum behavior**

I asked Eric what was meant when it is said that a particle can tunnel through a barrier. He replied,

“...classically where a particle couldn’t, you know, if it didn’t have sufficient energy to make it over a potential barrier, for example, it would reach the, at the edge of the barrier it would stop, but quantum mechanics says that it can actually tunnel through, its wave, its wave function doesn’t immediately go to zero at that point. It can leak through the barrier. So there’s a finite probability of finding it inside the barrier, and even outside, to the, into the, uh, outside the barrier.”

I asked him if he could think of any “real-life” examples of tunneling, and though he thought that some existed, he could think of none at the time.

#### **4.2.2.6. Revisiting particle energy**

I re-questioned Eric about the energy of particles in both regions. He reiterated that the particle’s energy in the right-hand region had decreased. I asked him where the energy had gone, and he replied that “it interacted with the potential, and lost some energy because of it,” but could not reason about

what form (heat, sound, etc.) the energy loss took. I asked him to again examine his sketch of the wave function and discuss whether or not it was consistent with his energy-loss idea, and he stated that it was, since he had drawn a decreased amplitude in the far region.

#### **4.2.2.7. Effects of increased mass**

In the final minutes of the interview, I asked Eric to reason about what would change if he increased the mass of the particle. He said that the increased mass would decrease the wavelength of the wave functions, and that there would be less probability of tunneling. I then asked him what would happen if we could continue to increase the mass, at which point he said that he thought an infinite mass would have no probability of tunneling, but that having an infinite mass would be impossible. I asked him then to return to the bead scenario, which certainly didn't have infinite mass. He stated that the bead would behave classically, and there was no chance the bead would make it past the region of higher potential energy.

#### **4.2.2.8. Summarizing Eric's ideas**

In summary, we find that Eric still seems to have difficulty differentiating between force and potential, and using energy ideas to reason about the motion of particles. He also seems to not differentiate between spatial height and energy levels, as evidenced by his discussion of objects moving "over" barriers, and by the fact that he sketches wave functions spatially higher when describing higher energy particles.

We note that Eric's wave function sketches improved since the first interview, as he is now aware that the wave function is exponential when the particle energy is less than the potential energy. However, his sketches exhibit "axis shift" characteristics, and he remains convinced that tunneling particles lose energy.

#### **4.2.3. Results: Michelle**

##### **4.2.3.1. Constructing the potential energy diagrams**

When I asked Michelle to describe the potential energy of the bead on the wire in the single spacer scenario, she stated that the potential was a constant. Even when questioned about moving the bead to different parts of the wire, where the electric potentials were different, she insisted that the bead's potential would remain the same.

I asked her about whether she was thinking about potential or potential energy. When she said "potential," I asked her to reason about potential energy. She admitted that she always confused the two ideas, adding that she also had trouble distinguishing force and field.

I then gave her the relationship between potential and potential energy – "potential is defined as the ratio of electric potential energy per charge." She was still unclear on how to describe the potential energy.

Switching focus, I asked her to think about how the two halves of the wire could have different electric potentials. She said that the left half, at zero potential, could be connected to ground, but that the right half, at a positive potential, would have to contain more positive



Figure 4-15: Michelle's sketch of charge distribution in each region of the wire.

charges than negative. She produced the sketch shown in Figure 4-15 to illustrate her reasoning. Using her illustration of charge distribution in either half of the wire, she concluded that the bead would move slower in the right half of the system than it did in the left half, though she said she was unsure if "speed" were the right term, preferring "ease."

Next, I asked her about the kinetic energy of the bead in this system, and she stated that the bead likely had more kinetic energy in the left half of the system than in the right. Furthermore, this led her to deduce that the potential energy was greater in the right half of the system, using conservation of energy.

#### 4.2.3.2. Reasoning about bead behavior

I then asked Michelle to reason about the behavior of a bead, sliding from left to right, given an initial energy greater than  $qV_0$ . She asked me whether she should assume classical or quantum, and I stated classical. She reasoned that the bead would start out fast, slow down when it hit the "wall", but keep going. Furthermore, she clarified that the speed of the bead in each region

would be constant. When asked about the situation with energy less than  $qV_0$ , she said that it would bounce back, since it lacked enough energy to keep going.

Michelle was next asked about the effect of inserting an additional insulating spacer into the system. She correctly sketched the potential energy diagram, shown in Figure 4-16, then reasoned that the bead with sufficient initial energy would “come in with some speed, hit the first boundary, slow down, when it hits the second boundary, it’ll speed up to the same speed there.” When asked about a bead with initial energy less than  $qV_0$ , she said that it would bounce back, since “you can’t get through the barrier,” because there wasn’t sufficient energy.

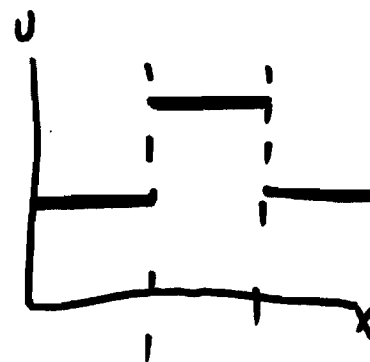


Figure 4-16: Michelle’s potential energy diagram for the double spacer scenario.

#### 4.2.3.3. Reasoning about quantum particles

The interview shifted to asking about quantum mechanical situations with similar potential energy diagrams. When I asked her the wave function for this system, Michelle first drew a sinusoidal wave form in the left-hand region that she described as “a kind of wave format” that could be described mathematically by “sines and cosines, or  $e^{ikt}$ .” Asked to think about the situation where the particle described by the wave function had energy greater than the potential energy of the step, Michelle sketched an oscillating

wave form that was reduced in amplitude and wavelength in the right-hand region, as is shown in Figure 4-17. When I asked her what the amplitude represented, Michelle linked it to the difference between the potential energy of the particle and the potential energy of the region. Asked about other differences, she noted that she had drawn it with “frequency increased,” but was unsure of whether or not that was correct.

We next turned to the case with an incident particle with less energy than the maximum potential energy of the step. Michelle sketched a similar waveform in the left-hand region, noting that the amplitude was smaller, but that the wavelength and frequency would remain the same. In the potential step region, Michelle

described it as a “decaying function” that would “decay eventually down to zero.” Her sketch for this situation is shown in Figure 4-17. She added that the rate of decay would depend upon “the difference between the... potential energy of the region, and the potential energy of the particle.” When I checked on her description of an “exponential decay,” she said that she thought it would really be an exponentially decaying sine or cosine function.

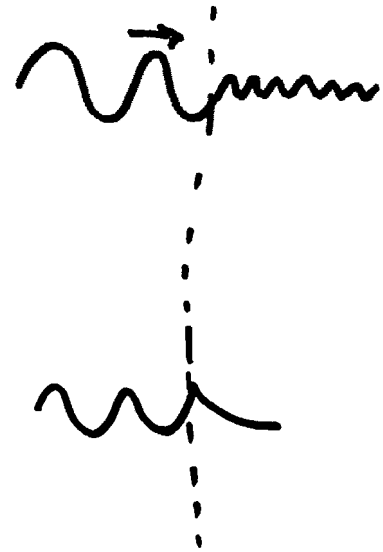


Figure 4-17: Michelle's wave function sketches for the potential step, when  $E > qV_0$  (top), and  $E < qV_0$  (bottom).

#### 4.2.3.4. Reasoning about quantum tunneling

When shown the potential barrier scenario and asked again about the greater energy situation, Michelle said that the wave function would look similar to what she had sketched for the potential step, and that the wave function in the right-hand region “will go back to what this one (referencing the left-hand region) looks like.” She again struggled with whether or not the frequency changed in the middle region, but couldn’t recall any way of deciding if it should be changed. Her sketch is shown in Figure 4-18.

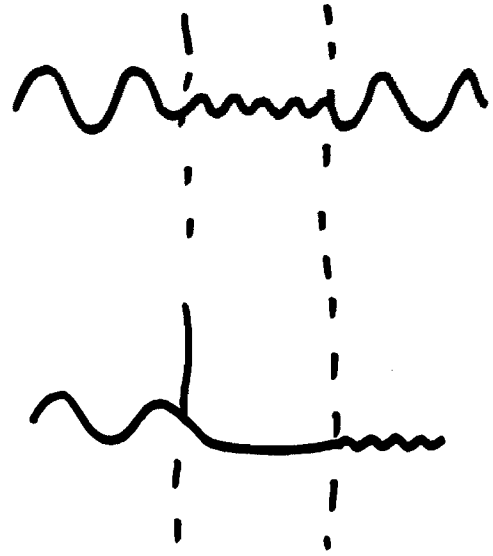


Figure 4-18: Michelle’s wave function sketches for the potential barrier scenario, for the cases when  $E > qV_0$  (top), and  $E < qV_0$  (bottom).

For the situation with energy less than that of the potential barrier, Michelle sketched a similar shaped wave function as she did for the potential step, noting that the amplitude should be reduced for the particle with less energy. She drew a sinusoidal wave function in the right-hand region, noting that if it “made it through” it would have less amplitude. Her sketch for this situation is also shown in Figure 4-18.

#### 4.2.3.5. Defining tunneling

Asked to describe what tunneling (in the context of quantum mechanics) meant, Michelle replied that

“there’s a chance that it [the particle] will behave non-classically... classically we have a barrier, and if we try to throw a classical object over it, and it doesn’t have enough energy to do so, it won’t. In quantum mechanics, if there’s a barrier that the particle should not be able to cross over, sometimes it does, and that is referred to as tunneling.”

Her description here that tunneling is a quantum effect is similar to her response during the parallel-plate interview. However, she now attaches spatial terms – “throw a classical object *over* it,” “not be able to cross *over*” – that may indicate she has similar trouble to Eric in differentiating energy levels and spatial heights.

#### 4.2.3.6. Comparing particle energies

I asked her to focus on the last scenario we had discussed, and compare the particle’s energy on either side of the barrier. She said that the energy was less in Region V (to the right of the barrier), because “to get over the barrier... it has to sacrifice some of its energy for some reason.” I asked her what happened to the energy the particles lost, but she replied that she didn’t know. She described this phenomenon as a “little black box”, which sometimes particles can go through, and other times they cannot.

#### 4.2.3.7. Effects of increased mass

I concluded the interview with asking her about the effect of increasing the particle mass. She stated that this would decrease the wavelength, but that the

amplitude remained the same in Region III (the incident region, to the left of the barrier). In Region IV (the barrier region), she stated that the wave function would decay faster, and therefore there would be “less chance that it could get through.” The amplitude of the wave function, according to Michelle, would be even smaller in Region V. I asked her if the quantum results for very heavy particles were consistent with expected results in classical mechanics, and she said it was. Asked if the bead could ever make it through the wire system, she said, “technically, yes.” I ended by asking her if classical and quantum mechanics were truly separate ideas. She replied

“I think they’re kind of the same idea, it’s just on a different scale. Um, classically you’re just, classically is a simpler model, like taking the solar system with circular orbits instead of elliptical – that sort of thing. It’s functional for a certain set of problems, and once you get out of that certain set of problems, you have to take other factors into account. And that’s where quantum mechanics comes in.”

#### **4.2.3.8. Summarizing Michelle’s ideas**

In summary, we find that Michelle has difficulty distinguishing between the concepts of electric potential and electric potential energy. However, she is able to reason about a microscopic model of the system that provides qualitatively correct explanations.

Michelle seems more comfortable using conservation of energy ideas to reason about the system than Eric was. She is able to correctly reason that an increase in potential energy will result in a decrease in kinetic energy, and a subsequent slowing of the object under consideration.

Although Michelle draws qualitatively correct wave functions, there is some evidence that she ties increased energy to a higher spatial position for the wave function. In addition, she links wave function amplitude and energy, and discusses the “frequency” of the wave function, although the graphs she is using to reason with are sketched as functions of position.

#### **4.2.4. Summarizing results from the sliding bead protocol**

The physical context of the problem – a bead sliding on wire sections held at various potentials – presented difficulties to both students. Neither was confident of their understanding of electric potential, and had to be provided a definition to be able to sketch the potential energy step and barrier required for answering the rest of the questions. Once they had correctly described the potential energy, both had additional difficulty describing the motion of the charged bead in the system.

We note that both subjects seemed to improve in their ability to sketch wave functions; for example, both sketched decaying exponential functions in step and barrier regions of the given problems. Both linked particle energy to wave function amplitude, however, and stated that energy was lost in the tunneling process.

Once again, the difficulty reasoning about the physical scenario we presented them took up more time in the interview session than was anticipated. As a result, we did not have the opportunity to ask them all of the questions about tunneling that we had intended.

### 4.3. Further Modifications – The Square Barrier Protocol

Although both prior versions of the interview protocol were fruitful for eliciting student ideas on electric fields, forces, and potentials, we found both required too much of the interview time in setting up the real topic of interest: student ideas about tunneling through a square potential energy barrier. After reviewing the results of the parallel-plate and bead-on-a-wire protocols, we decided to drop the leading component of a physical context for the problem, and instead begin with the theoretical scenario, returning at the end of the interview session, if time permitted, to experimental parallels to the theoretical description.

#### 4.3.1. Design of the square barrier protocol

At the beginning of the interview, students were shown the potential energy diagram pictured in Figure 4-19. They were told that some region of space, labeled “Region B,” the potential energy,  $E_2$ , was higher than it was in the surrounding areas, held at  $E_1$ . The outer regions were labeled “A” and “C.”

During the first four interviews using this protocol, I asked students to reason about a single electron incident on this system from the left. We designed this question to probe whether or not

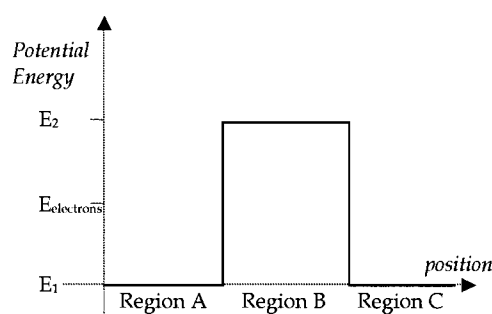


Figure 4-19: Potential energy barrier diagram used in the square barrier interviews.

they had encountered the concept of wave packets to describe a single particle. When none of the four subjects expressed any concern over using simple sinusoidal wave functions to describe the single particle, rather than the wave packet representation, the question was changed to ask students to reason about a beam of electrons, which the solutions described in Chapter 2 more accurately model.

#### **4.3.1.1. Description of the spring 2003 protocol**

During the first round of interviews conducted during the spring of 2003, I first asked students to describe and sketch the wave function that could be used to describe the interaction of this beam of electrons with the system. Following their sketch, I asked them questions about the features of their wave function (for example, to describe the information conveyed by the amplitude and wavelength for sinusoidal wave forms), to discuss the probability of finding electrons in Region C, and to compare the average energy (per particle) in Region C to that in Region A.

Using their responses as a springboard, subjects were then asked to describe how the average energy and probability of detection changed (if at all) with changes to the energy of the barrier, the width of the barrier, or the energy of the incoming electron beam. We also included questions asking students to describe the behavior of an incoming electron beam with energy greater than that of the barrier.

In the second half of the interview, students were shown a model of a scanning-tunneling microscope (STM), as shown in Figure 4-20(a), where a tip passes over a surface being studied. I asked subjects if they had ever heard of a scanning-tunneling microscope, and if so, to briefly describe its utility and operation. They were asked about each part of the "scanning-tunneling microscope" name, specifically to probe how they connected the concept of tunneling to the behavior of the instrument, and to see if any interpreted the word "microscope" to mean that one could visually observe the atomic structure of a surface.

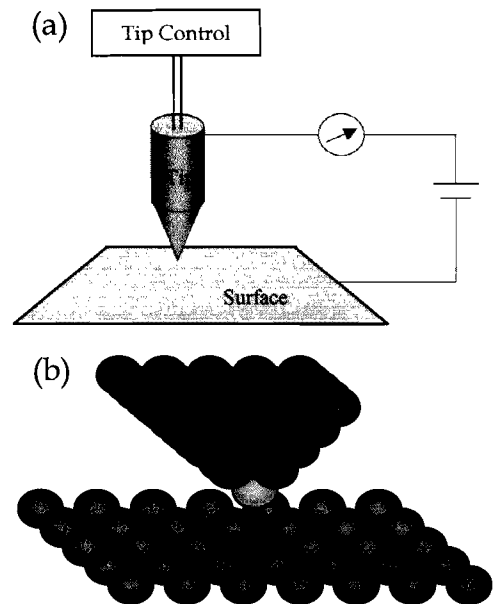


Figure 4-20: (a) Model of a scanning-tunneling microscope. (b) Atomic-level model of the tip and surface being studied.

Following that, they were shown the model pictured in Figure 4-20(b), and told that each sphere represented an individual atom, where the top collection represented the tip, and the bottom sheet the surface being studied. They were asked to focus on the interaction between the atom on the end of the tip and the closest surface atom (colored differently than their surroundings). I then asked whether or not the electron behavior in an STM could be modeled with the square potential energy barriers we previously discussed. In this

section of the questions, we were interested in seeing whether students would account for the potential difference between the tip and surface in the diagram, and whether this would make them modify any of their sketches.

#### **4.3.1.2. Fall 2003 revisions to the protocol**

We reworked the protocol slightly for the fall 2003 round of interviews. Following discussion at a research committee meeting, the wave function sketching task was moved to later in the interview, and initial questions instead focused on student ideas about probability of tunneling and energy of tunneled particles. Students were asked to imagine a beam of electrons with given energy incident on the system, and to think about what observations could be made if an electron detector was set up in Region C. They were asked how the number of detected electrons was affected by changes to the barrier energy, width, or electron energy. Next, students were asked about the energy of electrons that had tunneled, and probed about how the energy of the detected particles changed, if at all, with changes to the barrier or the electron beam.

Following the initial questions, students were asked to sketch the wave function for the scenario. Additionally, I asked them and to describe how their sketch was consistent with their previous reasoning. If they stated that it wasn't, I asked them to modify their reasoning or sketch so that they were consistent. They were also asked how the picture of the wave function would

change given the previously described changes to the barrier and/or electron beam.

#### **4.3.1.3. Further revisions in spring 2004**

For the spring 2004 interviews, the scanning-tunneling microscope questions were dropped. Rather than provide a specific physical scenario for students to reason about, most subjects interviewed using the revised square-barrier protocol were asked if the square-barrier scenario was just a theoretical idea discussed in textbooks, or if tunneling through a barrier was an actual physical phenomena. If they responded with the latter, I asked them to describe some situation that could be modeled with the square potential energy barrier they had previously reasoned about. In a few of the interviews, I asked respondents to compare the quantum behavior they had just described to a classical counterpart, such as bowling balls rolling at a hill.

#### **4.3.2. Participation in square-barrier protocol interviews**

The square barrier protocol was by far used the most extensively in our investigations. From the spring of 2003 to the spring of 2005, I interviewed a total of 16 students using this set of questions. Four sophomores, Selena, Nicole, Jeanette, and Jack, who had all completed the introductory quantum physics course the previous fall, were interviewed during the spring 2003 semester (and were asked the initial order of questions, including the scanning-tunneling microscope portion).

Analysis of the results led to slight revisions of the protocol (as previously described), and a second round of interviews with two seniors – Christine and Curtis – were conducted in the fall of 2003. At the time, both were enrolled in the senior-level quantum physics course.

The pool of subjects during the spring of 2004 included two students who had just completed the sophomore-level course, Adam and Steven. In addition, I interviewed three graduate students, Tony, Jimmy, and Mark. We gave these students the same series of questions, as we were interested in observing how, if at all, their answers differed from those of the undergraduates.

Finally, five additional students who completed the sophomore-level course, Kara, Madeline, Patrick, Paula, and Luis, were interviewed during the spring of 2005. During one of these interviews – Luis – there were audio problems with both the video camera and tape recorder, and so no transcript of the interview is available for analysis, and only responses gathered from the field notes are included in the analysis.

I interviewed four seniors who had completed the senior quantum course in the same semester, Adam, Bart, Jack, and Selena. Since three of the four had previously participated in square-barrier interviews, the protocol was revised to be more mathematical in nature. The general results of those interviews are discussed in a later section in this chapter. Specific responses

for the three re-interviewed subjects are analyzed in the context of the evolution of their ideas about tunneling, the subject of Chapter 6.

Rather than discuss individual results for these interviews, we instead discuss overall responses to the various phases of the interview, noting, where applicable, differences between populations. Because more students were interviewed using the revised protocol, I will use that question order to discuss student responses.

#### **4.3.3. Results from the square-barrier protocol**

While we note common difficulties in student reasoning in this section, we do not spend much time discussing themes in the student reasoning, leaving this analysis for the case study examination presented in Chapter 6.

##### **4.3.3.1. Awareness of tunneling**

All 16 students interviewed stated that a portion of the electrons would be detected in region C, and many called this phenomena “tunneling.” Typical phrases were:

- “I’m pretty sure that I’ll observe a portion of the electrons that were fired here coming out through here (indicating the right-hand region).”  
– Steven
- “We will find a fraction of the electrons tunneling through the barrier.”  
– Mark

All also stated that other electrons would be reflected by the barrier, with two suggesting that reflection happened at both the A-B and B-C interfaces. A

few suggested that some of the electrons might be trapped in region B, a response I did not probe further in the interview sessions, but was observed again in responses from the introductory audience we describe in Chapter 7.

#### **4.3.3.2. Reasoning about probability**

*Barrier Width.* Asked about how changes to the barrier width affected the number of electrons that tunneled, 13 correctly said that fewer electrons would tunnel through a wider barrier. Adam stated, “you see fewer and fewer electrons until you see none at all.”

Two argued that the number was unaffected by the barrier width. For example, asked if the chance an electron would make it to the right-hand region was affected by barrier width, Nicole stated, “I don’t believe it will, cause I think it just has to do with the energy level of the barrier, as compared with the electron.” One student was unsure of the influence of barrier width.

*Barrier Energy.* When I asked about the effect of increasing the potential energy of the barrier on the number of detected electrons, 11 correctly stated that fewer electrons would tunnel, though they often struggled with precise descriptions of why. For example, Kara stated, “The difference between the height of, the energy of the barrier and the energy of the electrons is greater... so there’s a bigger, not a gradient, but something like that...” Three thought there would be no effect. Paula argued, “that [the “height” of the barrier] just seems kind of arbitrary. Just the fact that it’s greater than that [the particle

energy], I would think that's all that matters." Two were unsure of the influence of barrier potential energy.

*Beam Energy.* Thirteen of the students said that increasing the energy of the beam of electrons would increase the number detected in region C. For example, Jack stated "as the particle's or an electron's energy increases, its chance of going through a potential barrier is increased." Two said this would not affect the number of particles detected; both had previously argued that the energy of the potential barrier had no effect. Most students connected the two sets of questions and used consistent reasoning. Only one student who initially argued that barrier energy didn't affect probability of detection in the right-hand region now stated that increased particle energy did increase the probability of tunneling.

Asked about the scenario when the electron energy exceeded the potential barrier energy, five of the students correctly stated that a small portion of the beam was still reflected by the barrier – a non-classical effect. Three of the five were the graduate students. Tony said, "classically, if you had this as an energy-position, things would just all go through. But in this case there's still some probability of bouncing."

Six of the subjects thought all electrons would be detected in region C, unaffected by the potential barrier. For example, Madeline thought that the electrons "wouldn't even feel the barrier...nothing would change... if you're assuming a perfect place."

#### 4.3.3.3. Reasoning about particle energy

*Energy Loss.* Nine of the 16 students had an energy loss model – that is, tunneling requires energy, so the average energy of individual electrons detected in region C must be less than the average energy in region A. We note that all nine in this group had only completed the sophomore-level quantum physics course. Below is a sampling of their discussions:

- “Well, whatever energy it had in region A is going to be dissipated when it tries to penetrate the potential well, and when it comes out of region C, it’s going to have whatever’s remaining.” – Patrick
- “It’s (the particle’s energy in region C) probably less... cause... I think it depends on the energy; if the electron doesn’t have enough energy, it won’t even make it through B...” – Jeannette
- “The average energy would be smaller, because the amplitude has decreased, but I think the period of the wave would remain the same.” – Steven

Some, like Steven, explicitly referred to their previously sketched wave function when reasoning about the energy, and linked the wave function amplitude with energy.

For this group of nine, the perceived effects of changes to the system were mixed. Seven thought more energy would be lost in a wider barrier; two were not sure. For the situation where the energy of the barrier was increased, four

thought more energy would be lost, four thought the same amount of energy would be lost, while one was not sure.

The results are more revealing, however, when the two responses are looked at together. Three of the students thought more energy was lost when either change was made to the barrier (both barrier energy and width affect the amount of energy loss), while three thought that the same amount of energy was lost in a higher energy barrier, and greater loss only occurred in wider barriers. (Three of the students did not address one or both of the scenarios.)

It is possible that students using their classical physics resources would reason the energy loss was unaffected by barrier energy; the spatial distance the particle must travel and the corresponding work done on the particle are only increased when the width is adjusted. Selena admitted as much, stating “I’m thinking in terms of classical physics where, you know, work is energy, uh, force over distance...”

*Energy Ensemble.* Two of the graduate students were the only subjects that discussed the idea of the electron beam containing an ensemble of energies. Neither reasoned that this would increase the average energy post-barrier, an issue we discuss in the next chapter describing the results of our tunneling survey.

*Energy Conservation.* All of the students who correctly argued that no energy loss occurred stuck with this idea regardless of barrier and/or electron

energy changes. They differed on their reasoning and level of surety, however. Some seemed to recite a memorized idea, while others reasoned from equations:

- “Energy’s the same, you’re just going to see a smaller intensity, once again equating amplitude with intensity and frequency with energy. So the same energy, less of it.” – Adam
- “If they do tunnel though the barrier, I don’t think they lose any energy doing it... there’s just a probability they’d continue on their path, even though the barrier’s there.” – Kara
- “ [The particle’s energy] would be the same... because A and C are the same potential, so the kinetic energy would just be the total energy of the electron minus the potential energy of the region where you measure it.” – Mark

When asked about increasing the energy of the electron beam, students viewed this as analogous to decreasing the barrier energy, and gave answers consistent with their previous reasoning. That is, if they held an energy-loss model and had previously reasoned that more energy was lost when the barrier energy was increased, they now argued that less was lost, since the particle’s energy was closer to the barrier’s energy. Those with energy conservation ideas said that the measured energy would match that of the incident particles.

#### **4.3.3.4. Population differences**

Most of the errors regarding energy or probability were made in the sophomore-level population. Both seniors, with the exception of one who thought all electrons would pass the barrier if the electron energy exceeded that of the barrier, answered all probability questions correctly. One of the seniors correctly cited energy conservation, while the other debated both ideas, and opted for energy loss about halfway through the interview. All three of the graduate students responded correctly to all questions regarding the probability of detection and the electron energies.

#### **4.3.3.5. Wave function sketches**

More variation was observed in the student sketches of the wave function(s) corresponding to this scenario. Ten of the sixteen (including all three graduate students and both seniors) sketched the wave function as sinusoidal in regions A and C, and a decaying exponential in region B, as shown in Figure 4-21(a). Half of this group, however, showed the “axis-shift” tendency, where the wave function in the region to the right of the barrier is sketched as oscillating about an imaginary axis that is spatially lower than the imaginary axis its counterpart in the region to the left of the barrier oscillates about,<sup>2</sup> as is shown in Figure 4-21(b).

Three of the students sketched the wave function as sinusoidal in all three regions. One showed the wave function’s amplitude exponentially decreasing in region B, as shown in Figure 4-21(c). The other two indicated that the

amplitude would change in region B, due to the higher potential energy, though one argued it would increase, the other that it would decrease.

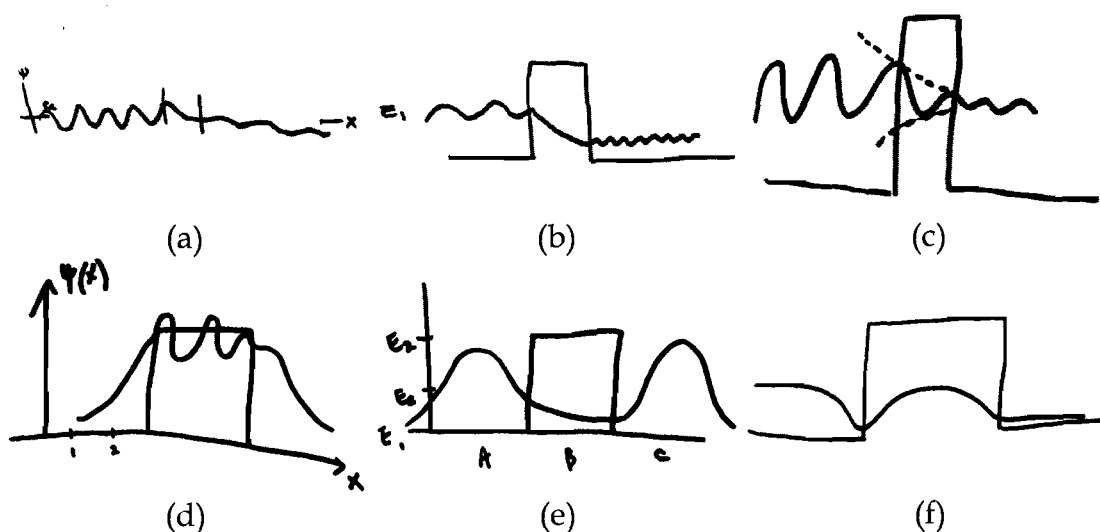


Figure 4-21: Variations of student wave function sketches for the square barrier problem: (a) correct solution; (b) mostly correct solution, exhibiting axis-shift; (c) sinusoidal everywhere; (d) reversed sinusoidal and exponential characteristics; (e) connected Gaussians; (f) solutions that resemble bound state solutions in the barrier region.

One student sketched the wave function as exponential-sinusoidal-exponential, as shown in Figure 4-21(d). Another sketched to Gaussian-like shapes in regions A and C, connected through region B, as shown in Figure 4-21(e), resembling the solution for two adjacent square wells. A third sketched and discussed square-well bound state solutions in the square barrier, with somewhat flat functions outside the barrier, as shown in Figure 4-21(f).

#### 4.3.3.6. Sketching wave functions on potential barriers

We note that 14 of 16 students, excepting one senior and one graduate student, first sketched the potential energy barrier when asked to provide a

sketch of the wave function for the system, although they were given a blank paper on which to do so. Of these 14, nine sketched the oscillating portion of the wave function in region A above the horizontal axis, perhaps coincident with the given energy level of the incoming electron beam. All six of the “axis shift” students were in this group. Many students in this group labeled their axes “ $V$ ” and “ $x$ ,” indicating that they represented “potential energy” and “position.”

#### **4.3.3.7. Energy connections**

All but two students who sketched a sinusoidal-shaped wave function in Region C reduced its amplitude relative to what they sketched in Region A. Seven students – including six who sketched the sinusoidal-exponential-sinusoidal pattern, and the one who sketched Gaussian-like shapes in Regions A and C, explicitly discussed their perception of a connection between amplitude and the energy of the particles.

Eight of the 16 students made a connection between the wavelength and energy, arguing either that the wavelength was longer in Region C (for those with an energy-loss model, since  $E \propto 1/\lambda$ ), or that it remained the same. Many of the students talked about the wave function’s “period” or “frequency,” neither of which is directly accessible on a graph of  $\psi(x)$ .

#### **4.3.3.8. Mathematical reasoning**

Many of the students wrote down  $c = f\lambda$  or  $v = f\lambda$ , coupled with  $E = hf$ , to reason about the energy-wavelength connection. We note that these

relationships describe light waves, not electrons. Few wrote down other equations to guide their reasoning. Though several mentioned “Schrödinger’s equation” in discussing the problem, only two wrote down the equation, while three others wrote down forms of a solution (*i.e.*  $\psi(x) = Ae^{ikx} + Be^{-ikx}$ ) to inform their reasoning. Several students made reference to their inability to answer certain questions due to their failure to “remember that formula.”

#### **4.3.3.9. Reasoning about a scanning-tunneling microscope**

Of the five students shown the diagrams of the scanning-tunneling microscope, four thought that the square potential energy barrier they had just discussed served as an adequate model of what was going on, while one claimed to have no memory of discussion of an STM, and offered an alternative scenario. The four matched the tip of the STM to either region A or region C, with the surface the opposite, and said that the gap between tip and surface was represented by region B. Three described how variations in the tip to surface distance would create barriers of varying widths, thus creating changes in the tunneling current to allow for surface mapping. Two were sure that a computer was needed to produce a map of the surface and that it was not directly visible to the human eye; the other two were unsure.

#### **4.3.3.10. Other physical examples**

Two of the students interviewed using the later version of the protocol suggested the scanning-tunneling microscope as a “real-life” example of tunneling. Other experimental apparatus suggestions included a single

positive charge (described as exerting a repulsive force on the approaching electron), a “wall” or “stack” of atoms, the nucleus of an atom, a gap in a wire in a circuit, and sheets of metal and/or paraffin inserted between an electron source and detector, modeled after an experiment performed in another class. One graduate student suggested a circuit containing a forward-biased and a reverse-biased diode in series, or two parallel-plate capacitors in a circuit, oriented opposite to each other – precisely the setup given in our initial protocol.

#### **4.3.3.11. Mixing quantum language and ideas**

Several students mixed references to potential “barriers” and “wells.” One student was insistent the increased potential energy region was a potential well, and proceeded to sketch bound state solutions for region B. Others discussed needing to know the energy “levels” or talked about stationary states in region A (often with reference to a second barrier or wall coincident with the given vertical axis). Perhaps this is indicative of more time spent in their introductory course on the square-well problem, and/or an inability to distinguish bound state solutions from the description of a free particle.

#### **4.3.3.12. Probability density and complex wave functions**

Many students talked about the probability of finding an electron in a certain location as being related to the “square of the wave function,” or occasionally  $\psi^*\psi$ . Some would sketch the square of a sinusoidal wave, including the associated zeros. When questioned about whether this meant

electrons could never be found there, students often referred to the strangeness of quantum mechanics or the Heisenberg uncertainty principle. Only two graduate students used the explanation of a complex wave function to explain that the probability in a given region has no zeros. Three students talked about “common sense” or “logic” not applying in quantum mechanics, while one talked about classical mechanics being about everyday phenomena, things she’d previously thought about, whereas quantum was about totally new, strange things.

#### **4.3.4. Summarizing square barrier results**

We find that the revised protocol was more successful in allowing us to probe student understanding of a theoretical tunneling problem, the standard square barrier. Students did not need to spend the beginning portion of the session deducing a potential energy diagram from some physical scenario.

We note that more than half of the students, predominantly those who had only taken a single quantum physics course, believe energy is lost in tunneling. Among that group, there is no coherent model, as some attribute the amount of energy lost to barrier width, others to the barrier energy, and others to a combination of the two. Several students seem to use classical notions of work to reason about energy loss in this situation. We also observed students connecting particle energy to wave function amplitude.

A higher portion of students answered questions about the probability of tunneling correctly. Nearly three-quarters of this population correctly reason that tunneling probability is affected by barrier width and barrier energy.

We observed a variety of wave function sketches. Though many produce a generally correct sinusoidal-exponential-sinusoidal pattern, several of these exhibit the axis-shift characteristic, and often sketch a shorter wavelength to correspond with reduced amplitude. Other variations include sketches that resemble bound state solutions for particle-in-a-box problems. We see little evidence that students use mathematical solutions to the Schrödinger equation to decide how to sketch the wave function.

Unless presented with some physical scenario, such as the scanning-tunneling microscope, most undergraduates are unable to connect the theoretical tunneling problem to any physical system.

Perhaps the most notable result is the lack of distinct categories within which to group student ideas. We observed wide variances in the student responses. For example, although it is possible to note the number of students with the energy-loss idea, we do not observe that all students with this idea sketch the wave function a certain way, or that they all answer the probability questions in the same manner. We do observe, however, some common themes in the evolution of a single student's ideas over time, as is discussed in Chapter 6.

#### 4.4. The Schrödinger Equation Protocol

Discussions in research advisor and committee meetings occasionally brought up the concern that the interviews asked students to reason about quantum systems in a way that was likely unfamiliar. Many quantum mechanics classes and books rely heavily on mathematical solutions to potential energy diagrams, and discuss (if at all) the conceptual ideas following the mathematical manipulations. When four senior physics and engineering physics majors, all who had completed the senior-level quantum physics course the previous fall, volunteered to be interviewed during the spring 2005 semester, this provided an opportunity to create a more mathematically oriented set of questions, to see if using equations could help guide student reasoning towards correct conceptual reasoning.

##### 4.4.1. Design of the Schrödinger equation protocol

To begin each interview session, students were given the Schrödinger equation in two forms, the latter being an algebraic rearrangement of the former, to try and account for varying presentations in different texts:

$$-\frac{\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2} + V(x)\Psi(x) = E\Psi(x) \quad (4-1)$$

$$\frac{d^2\Psi(x)}{dx^2} + \frac{2m}{\hbar^2} [E - V(x)]\Psi(x) = 0 \quad (4-2)$$

Participants were asked to identify each of the symbols in the equation to ensure they were familiar with the representations used. They were then

asked to reason about, in turn, each of the following three potential energy scenarios:

$$\begin{array}{ccc}
 V(x) = \begin{cases} 0 & x < 0 \\ V_0 & x > 0 \end{cases} & V(x) = \begin{cases} 0 & x < -a \\ V_0 & -a < x < a \\ 0 & a < x \end{cases} & V(x) = \begin{cases} 0 & x < -a \\ V_0 & -a < x < a \\ -V_0 & a < x \end{cases} \\
 \text{(a)} & \text{(b)} & \text{(c)}
 \end{array}$$

Figure 4-22: Three potential energy scenarios used: (a) potential step, (b) potential barrier, and (c) potential barrier with reduced potential energy on one side.

Subjects were provided with a graph of each of the three energy scenarios. For the latter two, the region of higher potential energy was specifically chosen to range from  $-a$  to  $a$  to avoid possible confusion of the vertical axis on the plot with a barrier of some sort, perhaps leading students to reason about bound state solutions in the left-hand region, as was seen in previous interviews.

For each scenario, students were asked to reason about solutions for two cases, (i)  $E > V_0$ , and (ii)  $E < V_0$ . For each case they were asked what the general solutions to the Schrödinger equation were. Then, they were asked to sketch the wave function solutions, and to discuss what information could be gleaned from the solution, specifically with regards to the choice of a sinusoidal or exponential solution. Next, they were asked to discuss the meaning of the amplitude and wavelength in sinusoidal solutions, and the probability of finding particles in various regions. Finally, they were asked to compare the energy of particles in various regions.

#### **4.4.2. Results – Schrödinger equation protocol**

Four senior physics or engineering physics majors, Adam, Bart, Jack, and Selena, participated in the Schrödinger equation interview. Three of the group – Adam, Jack, and Selena – had all been previously interviewed. Additional results from the interviews are presented in Chapter 6, where we examine the evolution of their ideas over time. Here, we present general results for all four students.

##### **4.4.2.1. Finding and sketching solutions for a potential step**

All of the four participants had no trouble identifying any of the symbols presented in the given Schrödinger equation.

When asked to consider the first scenario – the potential energy step – and to describe the solutions to the Schrödinger equation in each of the regions for the case where  $E > V_0$ , they invariably described solutions as “sinusoidal” waves. I asked each to be more specific, and to both sketch the solutions in each region and to write down the general form of the solutions.

Adam, Bart, and Selena drew their wave functions on potential energy graphs, either on diagrams I had given them, or by first reproducing the potential energy graph on the blank paper provided. (One verbalized struggling with the significance of drawing a wave function on a potential energy “coordinate system.”) All three drew sinusoidal functions in both regions, reducing the amplitude in the right-hand region. Two of the three drew functions oscillating about some imaginary axis perhaps coincident with

the given particle energy, as shown in Figure 4-24(a). The third, Bart, drew a sinusoidal function that oscillated between some level higher than the step and the horizontal axis; when it reached the potential step, the function then oscillated between the same high level and the top of the step, as shown in Figure 4-24(b). We refer to this as the “filling in the space” response. Two of these students said that the wavelength of the wave function in the right-hand region would be the same, referencing the fact that the particles had the “same energy.”

Jack drew a sinusoidal wave function that was identical in both regions, as shown in Figure 4-24 (c). The axes were labeled “ $\psi^*\psi$ ” and “ $x$ .” Later in the interview, after considering the third case, this subject came back and revised his drawing to indicate a longer wavelength in the right-hand region.

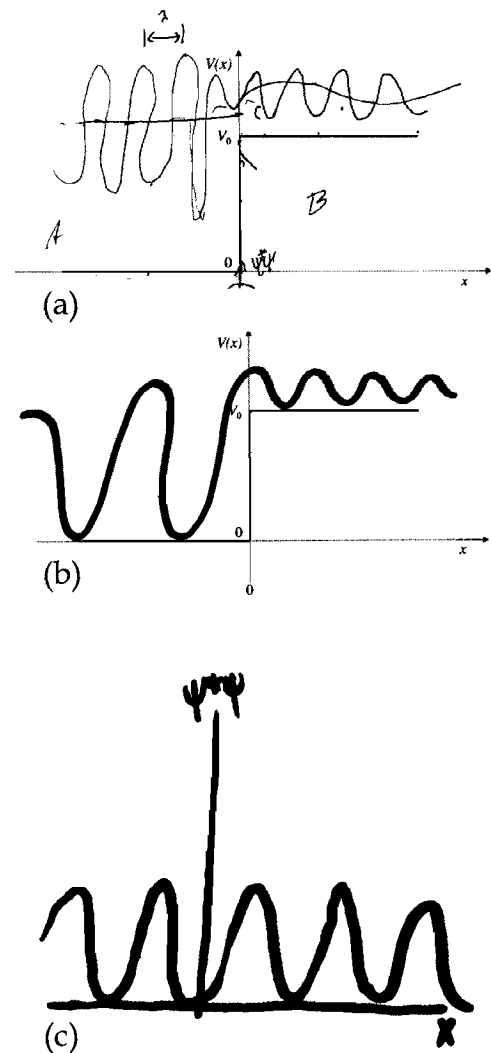


Figure 4-23: Wave function sketches for the potential step scenario: (a) sinusoidal waveforms oscillating about an axis coincident with the particle energy, (b) a “filling in the space” solution, and (c) sketched graph of the square of the wave function, identical in both regions.

This axes labeling was not unique, a second student labeled the axes similarly – “ $\psi^2(x)$ ” – and a third described his sketches as “the square of the wave function.”

#### 4.4.2.2. Mathematical solutions

Although they were initially asked to “describe the solutions to the Schrödinger equation,” none of the four initially wrote down algebraic expressions to guide their reasoning, and had to be prompted to do so. When they were asked, they wrote the following expressions:

<p><u>Jack's solution:</u></p> $\psi = \sin(yx) + \cos(yx)$ $y = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$	<p><u>Selena's solution:</u></p> $\psi(x) = Ae^{\frac{\sqrt{2m(E-V)}}{\hbar}ix}$
<p><u>Adam's solution:</u></p> $\psi(x) = A \sin \omega_1 x + B \cos \omega_1 x$ $\psi(x) = Ce^{-\omega x} + De^{\omega x}$ $\omega = \sqrt{\frac{2m(E - V)}{\hbar^2}}$	<p><u>Bart's solution:</u></p> $\psi(x) = A \sin \left[ \left( \frac{2m}{\hbar^2} (E - V) \right)^{1/2} x \right]$

Figure 4-24: Student solutions to the Schrödinger equation.

While all four are essentially correct, they were not produced without some effort. For example, Selena initially wrote a similar expression without the  $i$  or  $x$  and had reversed the  $V$  and  $E$ . She took a couple of minutes to work out that the former had to be included and the latter reversed. When I first asked Adam to write down the general form of the solutions, he replied, “no, I

don't think I can," and only produced the shown equation after being asked to use the Schrödinger equation to try and figure out the general form.

None of the four students went first to the mathematics to guide their reasoning about any of the scenarios, and only two of the four used the equations they had written to reason about the changing wavelengths in various regions. In addition, two of the four students seemed convinced at various points in the interview that the amplitude of the wave function was somehow dependent on the particle energy and the potential energy of a given region.

#### **4.4.2.3. Considering $E < V_0$**

Next, the participants were asked about the case where  $E < V_0$  for the same energy diagram, and all described the wave function as some sort of exponential decay in the right-hand region. Of the two who did not claim to be sketching the square of the wave function (although one had previously labeled the vertical axis as " $\psi^2(x)$ "), one sketched the exponential decay as falling below the average "oscillation height" of the function in the left-hand region, while the other drew an exponential decay approaching this imaginary axis. Bart said that for the exponential decay function, one or both amplitude constants would become complex, and remarked that this "kind of supports" the amplitude constants depending on the energies.

#### **4.4.2.4. Reasoning about solutions for a potential barrier**

For the potential energy barrier scenario, all of the participants sketched a sinusoidal – decaying exponential – sinusoidal pattern for the wave function (or square of the wave function, as described previously), with three continuing to sketch on top of potential energy diagrams. One of the “barrier” sketches exhibited the axis-shift characteristic previously described in this dissertation. Though three of the four said that the wavelength of the wave function in the right-hand region would be equal to the wavelength in the left-hand region, citing energy conservation, Bart drew a longer wavelength, stating that there was “some characteristic” that required the wavelength to increase as the amplitude decreased, and that he thought it had “something to do with conservation of energy.”

#### **4.4.2.5. Reasoning about a new situation**

There was some concern that the answers to the standard square potential energy barrier might have been memorized, as similar questions were presented on numerous previous occasions (as described later in the Chapter 6). The participants were given a third scenario, where the potential energy in the right-hand region was lower than either of the other regions.<sup>3</sup> We were interested in seeing how students thought about this scenario, both for the insight into their reasoning about a new situation, and for comparison to the findings of Bao. The four students each had different characteristics to their answers.

Jack drew a similar wave function sketch as he had for the previous scenario, saying that they were “mathematically very similar.” He drew a shorter wavelength in the third region (and at this point returned and corrected his the wavelength on his first sketch), stating that “ $y$ ” was greater. He stated that the amplitude should be the same, and that you were roughly equally likely to find the particle beyond the barrier in this situation or the previous situation.

Selena drew a wave function sketch similar to her previous sketch, but thought that the amplitude to the right of the barrier region should now be greater, because the energy difference was larger. She thought that the wavelength was “definitely changing,” but couldn’t settle on whether it was longer or shorter, and did not use her mathematical solutions to help her decide. She was adamant, however, that the energy was the same on both sides of the barrier.

Adam sketched an identical pattern in the first two regions, but increased the amplitude in the third region. As he had linked amplitude to probability of detection, I asked him about this choice. He provided the analogy of a classical potential well, and the fact that a particle wants to stay in the area of lower potential energy, reasoning that now, with lowered potential energy, one would be more likely to observe particles there. He thought the wavelength should be the same in both regions, and when I asked him if this

was consistent mathematically, he was unsure, saying that he didn't recall whether the wavelength was related to the kinetic energy or the total energy.

Bart, who had previously "filled in" the wave function above the potential step, drew an identical wave function in the first two regions, but now drew an oscillating function in the third region that varied

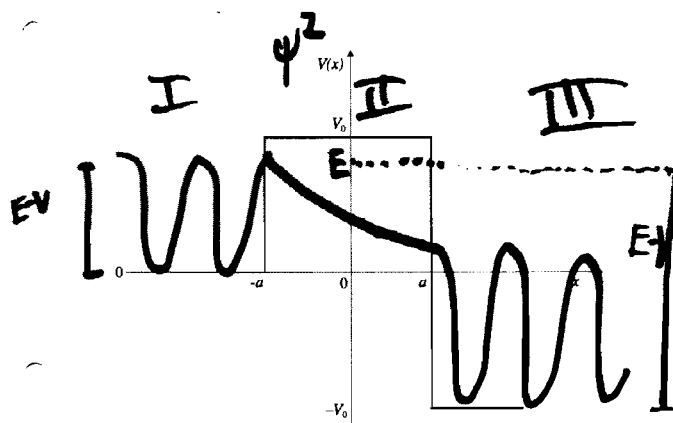


Figure 4-25: Bart's wave function sketch for the final scenario.

between the height of the decaying exponential and the graphed potential energy of the region, as shown in Figure 4-25. This resulted in a higher amplitude in that region, and I asked him about it. He said that he believed this was correct, because the amplitude was somehow tied to the difference in the energies. He was unsure of whether or not the wavelength should be the same.

#### 4.4.3. Summary of Schrödinger equation interview findings

In summary, we found that all four of the students were able to qualitatively reason about and sketch wave function solutions for various scenarios. Additionally, they were able (with varying amounts of difficulty) to write down mathematical solutions to the Schrödinger equation for various problems. However, we see little evidence that they check for coherence

between the mathematical solutions and the graphical solutions, and rarely use one to inform the other.

We also see evidence that the energy-loss idea has disappeared in this population, perhaps due to the increased emphasis on this problem in their senior quantum physics course. For some, though, this seems to be a memorized idea rather than an idea from a coherent model of the physics.

#### **4.5. A Summary of Interview Results**

Although four different interview tasks were used, and the square-barrier protocol was modified after an initial round of interviews, several ideas emerge from across all interviews.

For most students, tunneling is a difficult topic to reason coherently about, especially after a single course on quantum phenomena. There are many ideas – including probability (or probability density), kinetic and potential energies, and wave functions (including type of wave function and its characteristics, such as amplitude and wavelength) – that must be carefully pieced together in a non-contradictory model. Students with incomplete or incorrect models answer differently, depending on the context of the question. When inconsistencies are pointed out to them, they struggle to create a logically connected model.

A significant number of students believe that energy is lost when particles tunnel. A number of factors may contribute to this idea. The interviews have shown that some students want to use everyday ideas about “tunneling”

(passing through a mountain) and “barriers” (physical obstructions). Some of the students refer to notions of work, force, and distance. Other times students used ideas from mechanical waves to connect wave function amplitude and energy. The population of students with the aforementioned ideas were not mutually exclusive. We note, however, the absence of this idea in the graduate student population we studied, as well as students who had received explicit instruction on the idea.

It seems that most students were more successful reasoning about our questions on probability of particle detection, and often relate probability (or probability density) to the amplitude of the wave function. Many recite a connection to  $\psi^*\psi$ , but can have difficulty sketching this. Often, they simply square a sinusoidal wave form, then struggle with the interpretation of suggested points of zero probability (when in fact the product of a sinusoidal wave function and its complex conjugate is non-zero everywhere). We also observe that many students connect both energy and probability to the amplitude of the wave function.

Though many students are able to draw relatively correct graphs of a wave function as a function of position, three issues have emerged. First, many students superimpose their wave function sketches on potential energy graphs. This leads many to struggle with interpreting the wave function, and they often relate wave function amplitude with energy. A second related issue is that many students sketch the wave function oscillating about an

imaginary axis coincident with a given energy level, which can again contribute to misunderstanding the connection between particle energy and wave function. Third, many of these students exhibit the previously described “axis-shift” phenomenon, where the wave function oscillates about a lower imaginary axis on the far side of a barrier. These students always describe the particle as losing energy as it tunnels.

There seems to be difficulty with understanding and interpreting a potential energy diagram, both in relating it to some physical scenario, and in failing to relate potential energy to an interaction between the particle and the system. Responses suggest that a significant number of students see the barrier as somewhat independent of the particle, possessing its own energy, which may or may not be “given” to the particle during the tunneling process.

To see how widespread these issues are among wider populations of students at UMaine, and also to try and determine whether the issues were isolated or regional, we developed a survey probing student understanding of several of the ideas explored in the interviews. The development of the survey, and data from the various survey administrations are discussed in the following chapter.

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<sup>1</sup> E. F. Redish, Activity Based Tutorials, Module 2 (Modern Physics), (Wiley, New York, 2005).

<sup>2</sup> This result, first documented by Redish *et. al.*, is discussed in Chapter 3.

<sup>3</sup> In this situation, one can use energy conservation ideas to reason that the kinetic energy of the particle will be greater, thus lowering the probability of finding it within this region, thus reducing the amplitude of the wave

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function. The energy difference ( $E - V$ ) is greater, increasing the wavelength of the sketched wave function.

## Chapter 5

### SURVEYS

Though the initial round of interviews revealed some notable results about student ideas on quantum mechanical tunneling, such as the physical barrier/energy loss idea and the difficulties reasoning about potential energy diagrams, we were interested in seeing how widespread various views about the phenomenon of tunneling were. Generally, in physics education research we find that student volunteers for interviews are not entirely representative of the course population, and are often the top students in a course. We wanted to see if there were any differences in responses from a larger pool of respondents. Additionally, we were interested in seeing whether or not students at other institutions gave similar answers to our questions. To that end, we developed a survey, designed to be completed in 15-20 minutes, to probe student understanding of quantum tunneling.

#### **5.1. Overview of survey evolution and administration**

The initial survey was developed in the spring of 2003, and administered that same semester to students at The University of Maine. Following the analysis of the results from the initial administration of the survey, it was modified during the 2003-2004 academic year, and administered once again at UMaine in the spring of 2004. Modified versions of the survey were administered twice as part of exams in the senior-level quantum physics

course during the fall of 2004. The overall results from that class are presented in this chapter; individual results from three students are examined in Chapter 6. Finally, instructors at two outside institutions volunteered to give the survey to their students during the fall 2005 semester as part of their quantum physics courses.

## **5.2. Goals**

We have two goals in presenting the survey and its data in this chapter. First, as described above, we wish to know what students believe about quantum tunneling in situations similar to those described in the interviews. Second, we want to describe the process by which surveys are developed so as to help others develop similar surveys or revise this survey to meet their own needs.

To satisfy the second goal of this chapter, I first discuss the development of the initial version of the survey. Next, we present the results of the initial administration of the survey, and how the analysis of the results led to the refinements to the document. We discuss the results gathered from the revised survey, comparing the responses of different populations at the University of Maine, and comparing those results to those gathered from outside institutions. We conclude by summarizing the overall findings of the survey, and comment on possible contributions to some of the population differences we observed.

### 5.3. Initial Survey

#### 5.3.1. Content of the survey

We titled the survey the “*Quantum Energy & Probability Survey*” (QEPS), specifically leaving the term “tunneling” out of the title to try and avoid triggering any possible misconceptions that students linked to the term.

The 2003 version of the survey is presented in Appendix A.

The survey shows a potential energy “barrier” (Figure 5-1), and describes the situation:

A stream of charged particles with energy  $E_{\text{particle}} = 0.5 U_{\text{barrier}}$  is incident on this potential barrier in Region A. A detector set up in Region C indicates that some of the charged particles are found in Region C.

The first three questions ask students to compare the energy of the detected particles with  $E_{\text{particle}}$ , and to explain their reasoning for their choice of energy responses.

Additionally, students are asked to describe the factors that

determine the

probability of a particle being detected in region C.

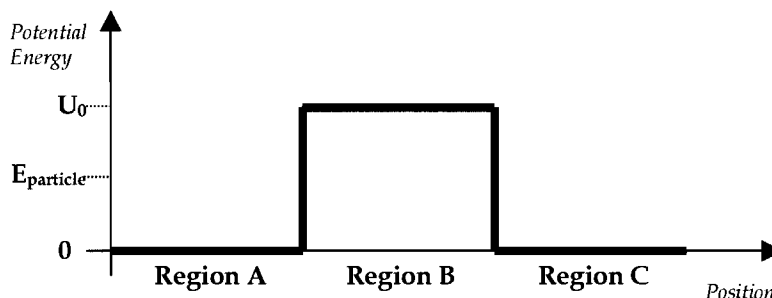


Figure 5-1: Potential energy barrier diagram from the original version of the survey.

The next section of the survey presents five scenarios, asking students to consider (i) a wider barrier, (ii) a barrier with twice the potential energy, (iii) the original barrier, with the particle energy increased to  $0.9U_0$ , (iv) the original barrier, with the particle energy decreased to  $0.1U_0$ , and (v) the

original barrier, with the particle energy now  $1.25U_0$ . For each scenario, five multiple-choice answers regarding the probability of detecting particle in region C are given; that there are fewer, more, or the same number of particles, that there are now no particles detected, or that it is impossible to know the relative number of particles detected.

The same five scenarios are shown on the final two pages, again in multiple-choice format. On these pages, students are asked to think about the energy of the particles detected in region C, and to compare it to the energy of detected particles in the first scenario. A total of nine options are provided – that the particles *lose* more, less, or the same amount of energy as in the first scenario, that the particles *gain* more, less, or the same amount of energy as in the first scenario, that the energy in region C equals  $E_{particle}$ , that there are no particles in region C (and thus no particle energy), or that it is impossible to determine the energy of the particles detected in region C. We recognized that choosing between nine options on a multiple choice test was possibly very difficult for students, and revised the format as we describe in a future section.

### **5.3.2. Results from the initial survey**

The initial survey was developed in the spring of 2003, and administered that same semester to students in a sophomore-level classical mechanics course (most of whom had completed the introductory quantum physics course the previous semester), and students in a senior-level statistical mechanics course. Fifteen students in the sophomore-level classical mechanics

course took the survey in the spring of 2003; all but two of them had completed the introductory quantum physics course the previous semester. Four students in the statistical mechanics course took the survey; three were seniors, while the fourth was a graduate student. Three of the four (including the graduate student) had completed both undergraduate quantum courses (or equivalent).

### 5.3.2.1. Comparing particle energies

On the initial set of questions, only four students from the sophomore population stated that the detected particles in Region C would have the same energy, while the other 11 said that the energy would be less. Three of the four advanced students stated that the particles would have the same energy. Some of the explanations for both groups are given in Table 5-1.

Table 5-1: Student explanations of particle energy.

<u>Energy Loss Model</u>	<u>Energy Conservation Model</u>
<ul style="list-style-type: none"> <li>• "The potential barrier Region B lessens the energy of the particles."</li> <li>• "Energy is 'lost' in getting through the barrier."</li> <li>• "Transmitted pulse &lt; reflected pulse."</li> <li>• "It will take some energy for the particles to penetrate the barrier in region B."</li> <li>• "Some energy is dissipated as the particle tunnels through the potential barrier."</li> <li>• "Exponential decay through B due to tunneling leads to lower energy function in C."</li> <li>• "Particle should lose energy tunneling through the barrier."</li> </ul>	<ul style="list-style-type: none"> <li>• "Particles tunneling don't lose energy."</li> <li>• "Well, if all the particles have energy <math>E_{\text{particle}}</math>, then that's that."</li> <li>• "When objects tunnel through potential barriers, I don't believe there is any energy lost."</li> <li>• "When tunneling through the potential barrier energy the particle will not gain or lose energy, because energy must be conserved."</li> <li>• "The particles don't lose energy as they tunnel."</li> </ul>

A few students gave explanations that did not seem to discuss energy, while two left the explanation section blank.

### 5.3.2.2. Factors affecting probability of tunneling

The most frequent student responses regarding the factors that affect the probability of a particle being detected on the far side of the barrier are summarized in Table 5-2.

Table 5-2: Common responses regarding factors that affect tunneling probability

<u>Factor affecting probability</u>	<u>Number of students</u>
• energy of the particle	9
• width of the barrier	7
• “height”/energy of the barrier	6
• “size” or “area” of Region B	3
• wave function	2
• $U_0 - E_{\text{particle}}$	2

Single responses observed include “number of particle,” “shape of the barrier,” “velocity,” “transmission coefficient,” and “the ability of the particle to tunnel.” One student (who had taken neither quantum physics course) gave no answer to this question.

### 5.3.2.3. Probability answers for the five scenarios

Respondents were generally successful at answering questions dealing with the probability of detecting particles in region C. The responses for each of the five scenarios are shown in Table 5-3. Correct responses are shaded.

Table 5-3: Probability answers given for each of the five scenario changes. ( $n = 19$ )

Scenario Change	Fewer Particles	Same Number	More Particles	Can't Tell
Wider Barrier	15	3	1	--
Increased Barrier Energy	14	5	--	--
Increased Particle Energy	--	5	14	--
Decreased Particle Energy	16	3	--	--
Particle Energy > Barrier Energy	1	--	17	1

Although the right response for each question was chosen by a majority of the students, only 7 of the students answered all probability questions correctly.

#### 5.3.2.4. Energy answers for the five scenarios

Students were not so successful on the energy questions, however, as might be surmised by the numbers of students who indicated energy loss on the first question. Their responses are shown in Table 5-4.

Table 5-4: Energy answers given for each of the five scenario changes. ( $n = 19$ )

Scenario Change	Energy loss is...			Energy is the same	Energy gain is...	
	less	same	greater		same	greater
Wider Barrier	--	2	9	7	1	--
Increased Barrier Energy	2	6	4	6	--	1
Increased Particle Energy	3	7	--	7	1	1
Decreased Particle Energy	1	7	5	6	--	--
Particle Energy > Barrier Energy	4	3	--	11	1	--

We note that of the population with energy loss ideas, four said that more energy would be lost for both a wider and “taller” barrier, which perhaps is linked to a realization that both affect the decay of the wave function in the barrier region. Five said that more energy would be lost when encountering a wider barrier, but that the energy loss would be the same when encountering a barrier with increased energy. This is consistent with descriptions we observed in interview sessions, including the student with the “snowball through a snowbank” model, where the height of a physical hill has relatively little influence on the amount of energy required to tunnel through it.

We note that in the pair of questions dealing with increased and decreased particle energy, seven students answered that the energy loss was the same in both of these situations as it was in the reference scenario, and three of these students were ones who previously said more energy was lost when encountering a wider barrier but not when encountering a taller barrier. It is possible that a subset of students view energy loss as a function of the barrier, and the same regardless of the “level” the particles “hit” the barrier at. Three of the “energy loss” students said that less energy would be lost by higher energy particles, and less by lower energy particles, suggesting that perhaps they view energy loss as a function of “energy distance” of the particles from the barrier.

Six students kept energy conservation ideas throughout all questions.

## **5.4. Redesigning the Survey**

In this section, we describe the many choices that were made to revise the original version of the survey. These choices include: modifying the format away from multiple choice and to free response questions, dropping some questions because their data was redundant, adding new questions which asked for sketches of the wave function, and re-ordering the questions.

### **5.4.1. Format and question modifications**

The original version of the survey yielded some useful insights into student reasoning about the phenomena of tunneling through a square potential energy barrier, but we felt there were several areas in need of improvement. The first survey had been designed with a significant portion of the questions in multiple-choice format, which had originally been chosen for ease of analysis. In the case of the energy questions, however, this required nine options, to account for all possibilities. (For example, if students had an energy-loss model, options had to be available for greater, less, and the same amount of energy loss for each given change.) We realized that in the scope of the work for this thesis it was unlikely that we would have hundreds of surveys to process. As a result, the multiple choice format was dropped in favor of a free-response, “explain your reasoning” style of questioning that we hoped would provide additional clues to student reasoning.

The opening questions of the survey, asking about the energy of particles detected on the far side of the barrier and factors that affected tunneling

probability, were left relatively unchanged, though the regions were now labeled with Roman numerals. Additionally, the word “average” was inserted into the description of the particle energy, “a stream of charged particles with *average* energy  $E_{particles} = 0.5 U_0$  is incident...” (emphasis not included in the survey document). This turned out to not be insignificant, and led to a new trend in energy answers that will be discussed later in this chapter.

#### 5.4.2. Reducing redundancy

We saw students use consistent reasoning for given pairs of barrier changes in our initial round of interviews. For example, when asked about the effect of the width of the potential energy barrier, students who reasoned that fewer particles tunneled through a wider barrier would always say that more particles tunneled through a narrower barrier. Therefore, only the wider barrier and increased barrier energy scenarios were included in the first version of the survey. However, we had not always seen consistent results when reasoning about increases and decreases in the energy of the incoming particles, so questions of increased and decreased particle energy were included in the initial design. However, when the great majority of survey respondents reasoned consistently about the two situations, we removed the decreased energy situation to allow for the inclusion of additional questions.

### 5.4.3. Adding new questions

The initial series of interviews showed wide variations in student sketches of the wave function for this potential energy scenario, as well as disparate views on its interpretation and utility. To attempt to gather a wider pool of results in this area, we inserted three questions, asking students to sketch the wave function in each of the three regions, discuss how the wave function could be used to compare the average energies of particles in Regions I and III, and how the wave function could be used to compare the number of particles in the same two regions.

The scenarios of increased barrier energy and increased barrier width were left in, but in the new format questions about the number of detected particles and the average energy of detected particles were placed sequentially following each scenario description, rather than placing all probability questions in one section, and all energy questions in the next. The two situations were placed side-by-side on a single page, in hopes that students would compare both when answering, and might yield insight into the prevalence of certain types of reasoning. For example, students using a style of reasoning consistent with energy loss due to a hard barrier will often state that more energy is lost in a wider barrier, because of the increased spatial distance to tunnel through, but that increasing the barrier “height” does not have the same increased loss, since the width of the region remains unchanged.

The final two scenarios were also carryovers from the previous survey, asking respondents to reason about increased particle energy at two different levels,  $E_{particles} = 0.75 U_0$ , and  $E_{particles} = 1.25 U_0$ . Each energy scenario was presented symbolically and graphically, and followed by two questions. The first asked how the number of detected particles compared with the number detected in the original scenario. The second asked how, if at all, the wave function had changed for this new system, to be answered with a sketch and explanation. We dropped questions about the energy, reasoning that the previous scenarios would reveal the students with energy loss ideas, and instead included the questions about the wave function in hopes that the sketches and reasoning would provide clues to the connections students made between wave function representations and their ideas about energy and the probability of tunneling. In addition, we felt that the final scenario,  $E_{particles} = 1.25 U_0$ , would provide insight into whether students grasped the perhaps subtle idea that quantum mechanics predicts some beam reflection even in this situation, and give us clues into how they decided to sketch the wave function.

## **5.5. Results from the Second Version**

### **5.5.1. Participation in the survey**

Five groups answered the second version of the survey. The modified version of the survey was administered during the spring of 2004 to two groups at the University of Maine – sophomores in a classical mechanics

course, and juniors and seniors in a seminar course. All of the population had at some point completed the sophomore-level introductory quantum physics course (many, but not all, during the previous semester), while six had also completed the senior-level quantum physics course the previous semester. These populations were analyzed separately in the results presented later in this section. It should be noted that eight students in this pool had taken the previous version of the survey the year before, and two had been interviewed the previous year as well.

In the fall of 2004, the instructor for the senior-level quantum physics course at UMaine spent a portion of the class time discussing the square barrier problem in class. He included a portion of the survey questions on the first preliminary exam in that course. Following the first prelim, additional class time was spent working out the fact that the average particle energy on the far side of the barrier is actually higher if one assumes the beam contains particles of different energies. The entire survey was included on the final exam. Results from the final exam are presented in the following section.

A colleague at Rice University in Houston, Texas, volunteered to administer the survey to his students, within *Introduction to Quantum Physics I*, a 300-level class. The class consisted of 3 hours of lecture per week, and utilized the text by Townsend.<sup>1</sup> The survey was administered during November of 2005. Students were given about 25 minutes in class to complete the survey, which approximately half did, while half turned it in later. The

instructor “took a traditional approach to the tunneling problem and intentionally did not emphasize how the energy of the particle would behave during the tunneling process.”<sup>2</sup>

Another colleague at Laurentian University in Sudbury, Ontario also volunteered to give the survey to her students in an introductory quantum physics course. The survey was used twice within that course, both as a pretest two weeks before covering tunneling in class, and as a posttest two weeks after class discussion of the phenomenon.<sup>3</sup> Though comparison of the two sets yields interesting results and apparent improvement, many of the students did not complete large portions of the pretest survey, so only posttest results are presented as comparative data.

### **5.5.2. Comparing survey responses**

Below, the results are discussed in a question-by-question format. Following the comparative data, additional discussion unique to each population is presented.

Not all students responded to all questions. Frequently, questions were answered without giving accompanying reasoning. Where appropriate, numbers of the population who answered a given question are listed in parentheses following the number with that particular answer. For example, 9 (15) indicates that nine students gave a particular response of the 15 students who gave some response to the question. The absence of parentheses indicates all respondents in that particular population answered the question.

In the tables, coding is not mutually exclusive. For example, a student who cited barrier energy and width as factors that affected the probability of tunneling was scored in both categories.

#### **5.5.2.1. Particle energy and probability**

The first section of the survey asks how the energy of tunneled particles compares to the energy of incident particles, and what factors influence the probability of detecting a particle in region III. The results are presented in Table 5-5.

We note that several students (most notably from the UMaine advanced quantum and Rice University populations) interpreted the phrase “average energy” to mean that the beam contained particles with many different energies, and correctly discussed that higher energy particles have higher probabilities of transmission, thus making the average energy of the particles in region III greater than the average energy of the particles incident in region I.

Table 5-5: Responses to initial questions on particle energy and factors affecting probability of tunneling.

	UMaine Sophomores ( $n = 17$ )	UMaine Juniors/ Seniors ( $n = 6$ )	UMaine Advanced Quantum ( $n = 11$ )	Rice University ( $n = 20$ )	Laurentian University ( $n = 8$ )
<b>How does the average energy of the particles detected in Region III compare with the average energy of the incident particles in Region I?</b>					
Lower/less	11	4		7	3
Same	5	1	9*	7	2
Higher/more	1	1	5*	6	3
<b>Explain your reasoning.</b>					
tunneling takes energy	7 (12)**	1 (3)	0	3	0 (5)
no energy lost /energy conservation	2 (12)	1 (3)	9*	7	1 (5)
higher energy particles have higher probability	1 (12)	1 (3)	5*	6	1 (5)
<b>What factors determine the probability of a particle being detected in Region III?</b>					
barrier width /value of $a$	11 (15)	5	11	19	3 (7)
barrier height /value of $U_0$	10 (15)	2	11	16	4 (7)
$E_{\text{particles}}$	6 (15)	4	10	14	2 (7)
$E_{\text{particles}} - U_0$	1 (15)	0	1	1	1 (7)
particle mass	1 (15)	0	0	3	1 (7)

\* Three of the UMaine advanced quantum students discussed both the scenario of all particles having the same energy, as well as an ensemble of energies.

\*\* Numbers in parentheses indicate the number of students in a given group who answered the question. The absence of parentheses indicates that all students in the group answered the question.

### 5.5.2.2. Sketching the wave function

The next portion of the survey asked students to sketch the wave function that described the particle beam. A wide variety of shapes were drawn. The most common sketch was sinusoidal in region I, a decaying exponential in region II, and sinusoidal in region III, as shown in Figure 5-2(a). Occasionally, we saw sketches exhibiting the previously described "axis shift," as in Figure 5-2(b). A small minority of students drew the function as sinusoidal in the

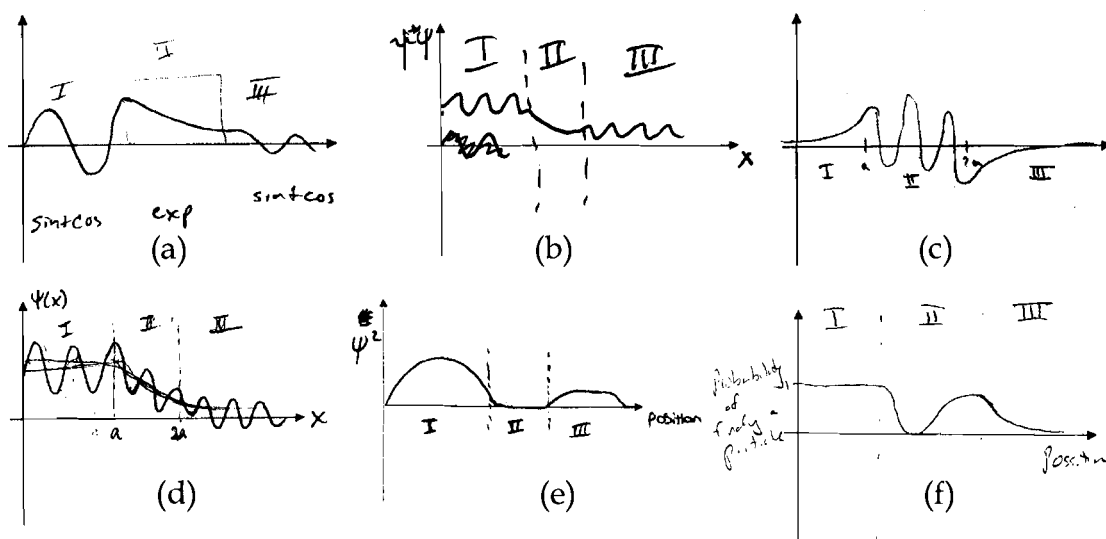


Figure 5-2: A sampling of the variety of wave function sketches from the surveys: (a) sinusoidal in regions I and III, decaying exponential in region II; (b) sketches with "axis shift;" (c) sinusoidal in barrier region and exponential in regions I and III; (d) sinusoidal in all regions; (e) bound state solutions on either side of the barrier; (f) sketch labeled "probability" and "position."

barrier region, and exponential in the surrounding regions, as shown in Figure 5-2(c). Some students drew the wave function as sinusoidal in all regions, as in Figure 5-2(d). A number of students drew shapes that resembled ground state solutions for a square "well" potential in regions I and

III, connecting them with some function through region II, as in Figure 5-2(e). Still others sketched horizontal lines in one or more regions, which often coincided with labeling the vertical axis “probability,” as shown in Figure 5-2(f).

We summarize the characteristics of the wave function sketches for the different populations in Table 5-6.

Table 5-6: Wave function sketch characteristics for all populations.

	UMaine Sophomores ( <i>n</i> = 17)	UMaine Juniors/ Seniors ( <i>n</i> = 6)	UMaine Advanced Quantum ( <i>n</i> = 11)	Rice University ( <i>n</i> = 20)	Laurentian University ( <i>n</i> = 8)
<b>Sketch the wave function that can be used to describe the particles in Regions I, II, and III. Label your axes.</b>					
<i>Shape sketched</i>					
sin-exp-sin	8 (15)	3	10	13	5
exp-sin-exp	2 (15)	0	0	1	0
sin-sin-sin	2 (15)	0	1	3	1
<i>Amplitude</i>					
reduced	9 (15)	2	11	17	3
same as region I	1 (15)	1	0	1	2
<i>Axis characteristics</i>					
imaginary axis	10 (15)	4	2	7	2
axis shift	2 (15)	1	0	5	0
<i>Vertical axis labels</i>					
$\psi$ or $\psi(x)$	1 (15)	2	9	10	3
$E$ or $U$	2 (15)	0	0	4	0
$ \psi ^2$ or $\psi^*\psi$	3 (15)	1	0	0	1
no label	7 (15)	1	1	5	4

In the correct solution, the amplitude of the sinusoidal function is reduced in region III, indicating a reduced probability of detecting particles in this portion of the system. The vast majority of students who drew sinusoidal waveforms reduced the amplitude in the right-hand region.

We also noted the vertical position of the wave function sketch. As has been previously noted,<sup>4</sup> many students begin their sketch of the wave function in the incident region coincident with the energy level of the incoming particles. They may, for example, draw an oscillating wave form about an imaginary axis vertically displaced from the given horizontal axis, or begin their sketch at a level they label " $E_{\text{particle}}$ ." In addition, many of these students' sketches exhibit the "axis shift" characteristic.

Finally, as we discuss in the interview findings, students are often confused as to what to label the vertical axis of a wave function, particularly when they have seen wave functions sketched on top of potential energy diagrams.

It is interesting to note that only two of the 62 respondents to this survey drew a potential energy barrier, as opposed to numerous such instances during the interview sessions, as described in the previous chapters. While we have not investigated the reasons for this observation, it may be that students do not sketch a potential energy barrier in this situation because it is shown on the same page in a previous question, and they are clearly being asked to sketch the wave function on a blank set of axes. On interview tasks, by contrast, when they are asked to sketch wave function solutions, they are not given a blank set of axes on which to do so.

### 5.5.2.3. Utility of the wave function

The next two questions dealt with how the wave function could be used

- (i) to compare the average energy of the particles in regions I and III, and
- (ii) to compare the number of particles in regions I and III. The rationale for the first question was two-fold: to see if students would correctly relate energy to the wavelength of the sinusoidal function, and to elicit the number of students who were linking wave function amplitude with energy, a carryover from classical waves. The results to these questions are shown in Table 5-7.

Table 5-7: Relating wave functions to average energy and probability of tunneling.

	UMaine Sophomores (n = 17)	UMaine Juniors/ Seniors (n = 6)	UMaine Advanced Quantum (n = 11)	Rice University (n = 20)	Laurentian University (n = 8)
<b>How can the wave function be used to compare the average energy of the particles detected in Region III with the average energy of the incident particles in Region I?</b>					
wavelength	0 (10)	0 (3)	2	0 (18)	1 (6)
frequency	2 (10)	0 (3)	0	4 (18)	0 (6)
$\langle \psi   H   \psi \rangle$ , $\langle E \rangle$ , or $\langle \psi   E   \psi \rangle$	0 (10)	0 (3)	8	6 (18)	0 (6)
Schrödinger equation or $H   \psi \rangle = E   \psi \rangle$	1 (10)	1 (3)	0	1 (18)	2 (6)
amplitude	4 (10)	0 (3)	0	1 (18)	2 (6)
<b>How can the wave function be used to compare the number of particles detected in Region III with the number of particles incident in Region I?</b>					
amplitude	4 (10)	1 (4)	1	0	2 (6)
$\psi^* \psi$ or $ \psi ^2$	3 (10)	1 (4)	1	0	2 (6)
reflection or transmission coefficients	0 (10)	0 (4)	9	0	1 (6)

The students who stated that the wavelength could be used to determine the average energy of the particles were consistent in their reasoning. The two UMaine students previously answered that the energy was the same in region III, and sketched a waveform with the same wavelength. The Laurentian student who mentioned wavelength drew a decreased wavelength and stated that energy was lost.

#### **5.5.2.4. Barrier changes and probability**

The next section of the survey probes student reasoning about the effect of changing some barrier attribute. Two changes are given: an increase in the barrier width, and an increase in the barrier energy. Students are asked how these changes affect the number of particles detected on the far side of the barrier, as well as the average energy of these detected particles. We first discuss the responses to the probability questions, followed by the energy questions. The results of the two questions dealing with numbers of detected particles are presented in Table 5-8.

As we observed in the interviews, students were generally successful with questions dealing with the probability of tunneling.

Table 5-8: Student responses dealing with the probability of tunneling when the barrier width or energy is increased.

	UMaine Sophomores ( <i>n</i> = 17)	UMaine Juniors/ Seniors ( <i>n</i> = 6)	UMaine Advanced Quantum ( <i>n</i> = 11)	Rice University ( <i>n</i> = 20)	Laurentian University ( <i>n</i> = 8)
<b>Increased Barrier Energy: How does the <u>number</u> of charged particles detected in Region III in this system compare with the number of charged particles detected in Region III in the original scenario?</b>					
fewer	12 (16)	4 (5)	11	20	7 (7)
same amount	4 (16)	1 (5)	0	0	0 (7)
<b>Reasoning</b>					
“higher” barrier/ $U_0$	10 (16)	4 (4)	11	14 (19)	3 (6)
$E-U_0$	0 (16)	0 (4)	0	1 (19)	2 (6)
lower probability	4 (16)	0 (4)	4	0 (19)	0 (6)
transmission coefficient	0 (16)	0 (4)	1	6 (19)	0 (6)
increased exponential decay	2 (16)	0 (4)	2	0 (19)	2 (6)
same barrier width	2 (16)	0 (4)	1	0 (19)	0 (6)
<b>Increased Barrier Width: How does the <u>number</u> of charged particles detected in Region III in this system compare with the number of charged particles detected in Region III in the original scenario?</b>					
fewer	11 (16)	5 (5)	11	18	7 (7)
same amount	5 (16)	0 (5)	0	1	0 (7)
<b>Reasoning</b>					
wider barrier/ $a$	9 (15)	2 (2)	10	13	5 (6)
increased exponential decay	2 (15)	0 (2)	3	3	2 (6)
lower probability	3 (15)	0 (2)	2	1	0 (6)
transmission coefficient	0 (15)	0 (2)	2	6	0 (6)

When reasoning about the increased barrier energy scenario, most students cite this increase as the cause of fewer particles being detected. A small

number discuss the difference between particle energy and barrier energy, while others mention a lower probability of finding particles in Region III or greater exponential decay of the wave function. A small subset, nearly all from the Rice University sample, mentions the changes to the transmission coefficient. Two students from the UMaine sophomore population said the number of particles detected remains the same, citing the unchanged barrier width as their reasoning.

The wider barrier scenario had similar results, with most students correctly answering that fewer particles are detected, and most citing the increased width of the region as support of that claim. The same six students in the Rice University sample discuss it again in terms of the transmission coefficient, as well as two students from the UMaine advanced quantum population.

#### **5.5.2.5. Barrier changes and energy**

In this section, we discuss student responses dealing with the average energy of particles that have tunneled for the same two scenarios. While analysis of individual results may yield clues into student reasoning about the role of the barrier, often their reasoning is incomplete or even absent. Insight into possible ideas about barrier affects on particle energy can be seen, however, by examining the answers to all three questions that deal with the energy of particles that have tunneled, including the original energy question.

Students were divided into three main categories based on their responses to the first question; those with *energy loss* models, those with *energy*

*conservation* models, and those who used *selectivity* ideas (stating that the average energy was higher, since higher energy particles are more likely to tunnel). We then looked at their energy answers to the two barrier change scenarios, and categorized them by their descriptions. For example, “less-less” in the energy loss group indicates students who said that the average energy would be lower for both an increase in barrier energy and an increased barrier width. The results are presented in Table 5-9.

Table 5-9: Energy question results from the survey, divided by type of reasoning. The first response is for the increased barrier energy scenario, and the second is for the wider barrier scenario.

	UMaine Sophomores (n = 17)	UMaine Juniors/ Seniors (n = 6)	UMaine Advanced Quantum (n = 11)	Rice University (n = 20)	Laurentian University (n = 8)
<b>Energy Loss Models</b>					
less-less	4 (14)	1 (4)	0	2 (19)	2 (7)
less-same	1 (14)	2 (4)	0	0 (19)	0 (7)
same-less	3 (14)	0 (4)	0	2 (19)	0 (7)
greater-same	1 (14)	0 (4)	0	1 (19)	0 (7)
<b>Energy Conservation Models</b>					
same-same	4 (14)	0 (4)	7	7 (19)	2 (7)
greater-greater	0 (14)	0 (4)	2	0 (19)	0 (7)
<b>Selectivity Models</b>					
greater-greater	0 (14)	0 (4)	1	0 (19)	2 (7)
greater-less	0 (14)	0 (4)	0	1 (19)	0 (7)
greater-same	1 (14)	0 (4)	0	1 (19)	1 (7)
less-same	0 (14)	0 (4)	1	0 (19)	0 (7)
same-greater	0 (14)	0 (4)	0	2 (19)	0 (7)
same-same	0 (14)	1 (4)	0	0 (19)	0 (7)

#### 5.5.2.6. Student energy models

We propose, often based on interview results, possible models that students could be using to reason about the phenomena in each of the categories.

Some students with energy-loss ideas propose that additional energy is lost in both wider barriers and barriers of increased energy (“less-less”). Many of these students link energy to the amplitude of the wave function. Having often correctly sketched the wave function for each of these scenarios, where the amplitude is indeed decreased in region III, they then reason that the energy is reduced in each of these situations.

Other students with energy loss ideas state that more energy is lost in “taller” barriers, but the same amount of energy is lost in the wider barrier scenario (“less-same”). Perhaps this set of students links energy loss to the difference between particle energy and barrier energy (not unreasonable, as the decaying exponential term contains this difference), and neglect to reason about the effects of the increased width on the same decay. (Another twist to this, seen in two responses on the survey, is that the particles lose *less* energy in a “taller” barrier, but the same amount in the wider barrier. Perhaps these students reason that the loss is less dramatic the farther removed the particle energy is from the barrier energy. It is also possible that they misunderstand the mathematics.)

The reverse of the “less-same” reasoning, that more energy is lost in a wider barrier, but that the same amount of loss occurs in the “taller” barrier (“same-less”), is often exhibited by students who view the potential energy graph as a physical barrier, one the particles must fight through. The energy loss, perhaps linked to the classical notion of work, is a function of the “tunneling distance” only, and is unaffected by increasing the energy of the barrier.

The energy conservation model (“same-same”), expressed by the largest subset of students on the survey, can be rationalized if one thinks about a beam of particles, all with energy equal to the given average energy. Many students who give this set of answers give explanations such as, “energy is not lost in tunneling,” or “the barrier affects the amplitude, not the frequency.”

Two of the students in the UMaine advanced quantum class stated that the energy would be the same on the first question, but later argued that the average energy would be greater for both an increase in barrier energy and barrier width. Examining their explanations, it seems likely that they recalled, yet not fully understood, class discussions on higher probability of tunneling for higher energy particles.

The selectivity models are interesting in that student explanations suggest two schools of thought, one focused on transmission probabilities for different energies, and the other perhaps viewing the energy barrier spatially. Virtually all students in the selectivity category talk about a range of energies in the

beam. For students who reason that the post-barrier energy for tunneled particles is greater in both the wider barrier and increased barrier energy scenarios, they are perhaps relating the transmission coefficient to the particle energy, where higher energy particles indeed have higher probabilities of tunneling, and reasoning that increasing the energy or width of the barrier favors the higher energy particles.

A small group of students responded that the average energy in region III was even greater for the greater potential energy barrier scenario, but unchanged for the wider barrier scenario. Responses on two of these surveys suggest that these students may not be reasoning about tunneling “through” the barrier at all, but rather arguing that in a beam with particles having a range of energies, the particles with energy higher than the barrier will make it “over” the barrier, while others are reflected. Thus, a greater barrier potential energy will indeed be more selective.

Perhaps the most noticeable result in this category is the lack of uniformity of answers, suggesting that this idea is difficult for the students to reason about. We also note that the correct answer, “less-less,” is not given by any student.<sup>ii</sup>

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<sup>ii</sup> The following explanation is from the exam key authored by W. Unertl: “The transmission coefficient increases superlinearly as  $E/U_{\max}$  increases from 0→1. Since  $E/2U_0 < E/U_0$ , this new case falls in a flatter region of T (the transmission coefficient) vs.  $E/U_{\max}$ . Thus, the change in transmission from lower energy particles to higher energy particles will be relatively smaller (for increased barrier energy) than in the original scenario.  $\langle E \rangle$  is less than in the original scenario.” For a wider barrier: “The transmission function is more

### 5.5.2.7. Changing particle energy

The third page of the survey contains questions regarding the effects of changing the particle energy. In the first scenario, students are asked to compare the amount of charged particles detected in region III when the average energy of the particles is increased, but still less than the maximum potential energy of the barrier. Additionally, they are asked to sketch the wave function for this scenario, and to explain any changes relative to their first wave function sketch. The results are summarized in Table 5-10.

Table 5-10: Student responses for the increased particle energy scenario.

	UMaine Sophomores ( <i>n</i> = 17)	UMaine Juniors/ Seniors ( <i>n</i> = 6)	UMaine Advanced Quantum ( <i>n</i> = 11)	Rice University ( <i>n</i> = 20)	Laurentian University ( <i>n</i> = 8)
<b>Increased Particle Energy: How does the <u>number</u> of charged particles detected in Region III in this system compare with the number of charged particles detected in Region III in the original scenario?</b>					
fewer	0 (16)	0 (5)	0	1	1 (7)
same amount	4 (16)	0 (5)	1*	0	0 (7)
more	12 (16)	5 (5)	11*	19	6 (7)
<b>Reasoning</b>					
"higher" $E_{\text{particle}}$	10 (16)	4 (4)	11	14 (19)	3 (6)
$E - U_0$	0 (16)	0 (4)	0	1 (19)	2 (6)
higher prob.	4 (16)	0 (4)	4	0 (19)	0 (6)
trans. coeff.	0 (16)	0 (4)	1	6 (19)	0 (6)
<b>Wave function sketch accompanying increased particle energy scenario.</b>					
higher amp. in region III	3 (14)	1 (5)	7	6 (18)	2 (7)
imaginary axis	9 (14)	3 (5)	2	6 (18)	3 (7)
axis shift	2 (14)	1 (5)	0	5 (18)	0 (7)

\* One UMaine advanced quantum student could not decide whether the number of particles would increase or remain the same.

strongly attenuated for higher values of  $E/U_0$  compared to the original scenario. Thus, the difference in transmission from low to high energies is smaller. The average energy still increases over that of the incident particles but is smaller than in Region III of the original scenario."

Most of the students reason that more particles will be detected in region III. The four who indicated that the number would not change, all from the UMaine introductory quantum population, all had previously indicated that increasing the barrier energy did not affect the number that tunneled, either. The predominant reasoning given for an increase in detected particles was the increase in the average energy of the particle beam.

Not all students provided new wave function sketches. (All 11 of the advanced quantum students did, however, likely because this was a graded examination problem.) Of those who responded to this question, most provided pictures similar to their initial sketches on question three. It was often difficult to interpret the changes students had made without explicit written comments; the “higher amplitude” row in the table notes students who either provided an obvious increase in amplitude, relative to their initial sketch, or wrote that the amplitude had increased. Significant numbers of students shifted their wave functions higher relative to the given axes, often marking a new particle energy level on the vertical axis. In addition, many of those students exhibited the previously described “axis shift” phenomenon.

#### **5.5.2.8. Particle energy exceeds barrier energy**

The final question pair dealt with the case where the average energy of the beam of particles was greater than the potential energy of the barrier. Like the previous pair, the first question dealt with the number of detected particles, while the second asked for a sketch of the wave function.

Rather than asking students to compare the number of detected particles to the previous scenarios, they were asked to compare the number to the number of incident particles. We intended this question to probe whether they would use principles of quantum mechanics in this situation, and realize that a probability of reflection remains at each boundary, or view this as a classical case with sufficient energy for all particles to pass. The results of the probability question are tabulated in Table 5-11.

Table 5-11: Student responses regarding number of detected particles when the particle energy exceeds the barrier energy.

	UMaine Sophomores ( <i>n</i> = 17)	UMaine Juniors/ Seniors ( <i>n</i> = 6)	UMaine Advanced Quantum ( <i>n</i> = 11)	Rice University ( <i>n</i> = 20)	Laurentian University ( <i>n</i> = 8)
<b>Particle energy exceeds barrier energy: How does the <u>number</u> of charged particles detected in Region III in this system compare with the number of charged particles incident in Region I?</b>					
fewer	3 (15)	1 (5)	9	13	5 (7)
same amount	12 (15)	4 (5)	2	7	2 (7)
<b>Reasoning</b>					
$E > U_0$	2 (13)	0 (2)	1	3	2 (7)
all particles go "over" / no barrier	5 (13)	1 (2)	0	4	1 (7)
probability of reflection	2 (13)	1 (2)	9	8	4 (7)
transmission coefficient	0 (13)	0 (2)	0	4	0 (7)

The population was relatively evenly divided between the two ideas, with 31 (53%) realizing that some particles are still reflected, and 27 (47%) thinking that all particles would pass the barrier. The divide is not equal on a group-by-group basis, however. The largest fractions that stated that all particles

pass came from both UMaine populations months after instruction; the three groups who were currently taking a quantum physics course did much better.

Students were again asked to explain their reasoning. The most common responses dealt with the relative energy difference, the probability of reflection, and the transmission coefficient. There were also a significant number of students discussing the idea that the barrier no longer affected the particle beam, since the beam now had sufficient energy. A few referred to the barrier spatially, saying that the particles “go over” the barrier.

Characteristics of accompanying wave function sketches are listed in Table 5-12. Again, not all students answered this question, and wave function characteristics were only coded if a clear difference existed or an explanation was given.

The majority of wave function sketches were sinusoidal in all regions. Most students who previously began their sketch coincident with the average energy of the particles continued to do so, shifting the sketch even higher relative to the given axes.

Table 5-12: Characteristics of student wave function sketches for the scenario when the particle energy exceeds the barrier energy.

	UMaine Soph. ( <i>n</i> = 17)	UMaine Juniors/ Seniors ( <i>n</i> = 6)	UMaine Advanced Quantum ( <i>n</i> = 11)	Rice University ( <i>n</i> = 20)	Laurentian University ( <i>n</i> = 8)
<b>Wave function sketch accompanying increased particle energy scenario.</b>					
sin-sin-sin	7 (13)	3 (4)	9	14 (17)	4 (7)
sin-exp-sin	1 (13)	0 (4)	2	2 (17)	1 (7)
<b>Region II characteristics (for students who sketched a sinusoidal waveform in the region)</b>					
shorter wavelength	2	1	1	3	2
same wavelength	2	1	5	4	0
longer wavelength	2	1	2	6	1
lower amplitude	3	1	8	6	1
same amplitude	2	2	2	6	1
higher amplitude	2	0	0	1	1
<b>Region III characteristics</b>					
shorter wavelength	2	1	1	3	0
same wavelength	6	2	5	11	2
longer wavelength	0	0	2	0	1
lower amplitude	4	1	5	8	1
same amplitude	4	2	2	7	0
higher amplitude	0	0	0	1	2

## 5.6. Summary of Survey Findings

There were some notable differences between answers given by the different populations. Students from Rice University gave much more mathematically or equation-oriented responses to many of the questions than did the groups from the other two institutions. They were the only population to specifically mention and reason about the transmission coefficient. With the exception of the UMaine advanced quantum population (who were

answering in a high-stakes test environment), the Rice students were also the most thorough, with most students answering all or almost all of the questions, usually with accompanying reasoning. Since the Rice students could finish the survey on their own time, this may be evidence that the survey takes longer to complete than the 20 minutes we had intended.

Laurentian University students had the highest incidence of sketching wave functions that resembled bound state solutions in Regions I and III, and connecting them in some fashion through Region II. This was particularly noticeable on the pretest results, and is not surprising, as they had previously worked with solutions in potential “wells,” and they may have been attempting to use those tools in this new situation.

The first two groups of University of Maine students, surveyed several months after quantum instruction, had the highest incidences of the energy-loss idea, though this was present in other populations. They also were the population that most frequently sketched the wave function coincident with energy level, and had the highest rate of labeling the vertical axis of the sketch of the wave function “ $|\psi|^2$ ” or “ $\psi^*\psi$ .” This trend may be indicative of some facet of the quantum physics instruction at UMaine; many of the interview subjects also stated that they could only sketch and interpret a sketch of the square of the wave function, not the wave function itself.

Overall, the UMaine advanced quantum population performed the best on the survey questions. We are not surprised by this result for a number of

reasons. First, they answered the questions on a graded exam, with pressure to perform well, while all other populations were answering an ungraded survey. Second, significant instructional time was spent on this problem in the course; we see this as evidence that direct instruction can improve student understanding of tunneling. Finally, several in this population had taken previous versions of the survey, and/or been interviewed previously, both of which would increase their familiarity with our questions.

It is difficult to draw direct comparisons between the populations, though, due to the differences in the administration of the survey. The two outside schools administered the survey to students currently taking a quantum physics course, while the population at UMaine was surveyed months (or years) after their last quantum class. The students at Laurentian University saw the questions twice, four weeks apart, with the pretest experience potentially providing them with details to pay attention to in the course discussion. The students at Rice University were allowed to complete the survey outside of class if needed, and could conceivably consult outside sources or at minimum have additional time to review the coherence of their responses.

Themes common to the interview findings are still present, however. Though differing in prevalence by institution, all populations have some fraction of students who believe energy is lost in the tunneling process. Within that umbrella are varying models – using classical ideas of work, for

example, to reason that more energy is lost in a wider barrier, or relating energy loss to the difference between particle total energy and barrier potential energy.

Another recurring issue seems to be student difficulty understanding the terms “barrier” (often confused with “well”) and “tunneling,” and becoming confused with their everyday meanings. Many students seem to view the barrier as a physical object that must somehow be overcome, or gotten “over.” The latter may be tied to difficulty with spatial interpretations of energy levels, evidenced by the wave functions sketched above the axis, seen in the interview sessions as well.

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<sup>1</sup> J. S. Townsend, A Modern Approach to Quantum Mechanics, University Science Books, 2000.

<sup>2</sup> Private communication with H. Pu, November 16, 2005.

<sup>3</sup> Private communication with M. Ouellette, December 1, 2005.

<sup>4</sup> E. F. Redish, M. C. Wittmann, and R. N. Steinberg, “Affecting Student Reasoning in the Context of Quantum Mechanical Tunneling,” Contributed Talk CF14, 2000 summer meeting of the American Association of Physics Teachers. M. C. Wittmann, J. T. Morgan, and L. Bao, “Students' descriptions of particle energy loss in quantum tunneling,” *Eur. J. Phys.* **26**, 1-12 (2005).

## Chapter 6

### CASE STUDIES

Many investigations in the field of Physics Education Research involve one-time measurements of a population, looking at how students answer questions during a course or on a survey. Though the research may be carried out over many years, the student population in a particular class is new or mostly new each term. Few studies contain any long-term analysis of an individual student's reasoning, or examine how one student's ideas change (or do not change) over time.

#### 6.1. Description of the Population

Studying a relatively small population of advanced undergraduate physics students over a three-year period provided us with a unique opportunity. Several of the undergraduate physics majors at UMaine between 2002 and 2005 were surveyed twice, or completed the survey and answered the survey questions later on an exam in their quantum physics course. Six students were interviewed twice.

Three students in the physics or engineering physics programs, Adam, Jack, and Selena were interviewed twice (once following instruction in the introductory quantum physics course, and again following instruction in the senior-level quantum physics course), answered tunneling survey questions as part of their senior-level quantum physics course exams, and completed the

previously described survey at least once. These multiple data points of student responses on similar tasks provides a unique view of the evolution (or lack thereof) of their ideas about quantum tunneling during their undergraduate physics career.

Jack was studying for a Bachelor of Science in Physics; Selena and Adam were in the Bachelor of Science in Engineering Physics program. Selena and Jack both completed the introductory quantum physics course in the fall of 2002. I interviewed both the following spring. Later that semester they both took the initial version of the tunneling survey. Adam took the introductory quantum course a year later, in the fall of 2003. I first interviewed Adam during February of 2004. All three subjects took the revised version of the survey near the end of the spring 2004 semester.

In the fall of 2004, all enrolled in the senior-level quantum physics course. The instructor of the course was familiar with our previous findings, and modified his lecture-based instruction to emphasize the idea of energy conservation. Class discussion on square-barrier tunneling included the idea of an ensemble of particles with different energies and the calculated proof that higher energy particles have a greater transmission probability, thus making the average energy of the particles that have tunneled higher than the average energy of the ensemble of incident particles. The instructor included questions from our tunneling survey on one of the preliminary exams and on

the final exam. (The discussion about an ensemble of energies followed the first exam where tunneling questions were given.)

Finally, I interviewed the three students again during April of 2005. The protocol was revised to first emphasize the mathematical solutions to the Schrödinger equation. Students were then asked to use their mathematical solutions to reason about probability, energy, and the shape and characteristics of the wave function, as was discussed in Chapter 4.

Though one would expect that repeated exposure to the same question might yield identical results over and over, we find that students generally changed both their answers to certain questions and their explanations as time passed. Additionally, the way they changed their answers and reasoning differed from student to student, so no easily generalizable claims can be made. There is some evidence that when students learned correct answers to certain questions, though, these answers were remembered. However, each student struggled in some way to put all of the necessary pieces together into a coherent model.

Much of the data presented in this chapter has been already given in Chapters 4 and 5. To allow for easier examination of a single student's ideas, we present the student responses to various tasks and questions in three separate sections, analyzing how their thinking about various sub-topics changed, or remained static, over time. In addition, we include information about the development of their attitudes to and beliefs about quantum

physics. In the summary section of this chapter, we discuss overall themes that arise from analysis of the three.

## **6.2. Case Study – Adam**

Adam was one of the most verbally responsive students interviewed, often giving lengthy answers to questions. He was rather good at verbalizing his viewpoints, and often would self-debate, out loud, two competing ideas before settling on one. Occasionally, in the course of a soliloquy, he forgot the original question and had to be reminded what he was being asked. Unlike the other two subjects discussed in this chapter, Adam took the two quantum mechanics courses only one year apart, and the multiple data points collected are more closely spaced.

Several portions of the interviews with Adam are quoted throughout this section. The full transcript of the 2004 interview is in Appendix C; the 2005 interview transcript is in Appendix D.

### **6.2.1. Ideas about energy**

When asked in the initial interview about the energy of particles that were detected on the far side of a potential energy barrier, Adam reasoned that it would be the same, linking energy to the “frequency” of the wave function. He did, however, discuss conflicting models. Though his first response (which he ultimately settled on) was that energy remains the same, he also discussed the ideas of electrons colliding with each other, and thus losing

energy.<sup>iii</sup> Also, he was unsure of whether the link between energy and frequency, which he recalled from learning about classical, electromagnetic waves, was applicable when talking about wave functions. Once he had made up his mind, he stuck with the energy conservation idea when asked about the scenarios involving changes to the potential barrier or the energy of the particles.

His reasoning remained consistent when he took the second version of the tunneling survey a few weeks later. In all scenarios, he stated that the energy of the particles was the same on either side of the barrier, again explaining that the energy is connected to the frequency.

On the first exam in the senior-level quantum physics course the following fall, he again stated that the energy would be the same. His reasoning changed, though, as mention of the “frequency of the wave” was now absent. Rather, he now argued that the total energy was the sum of the kinetic and potential, and the potential energy of the system was the same in both regions I and III.

Subsequent class discussion evidently refined his reasoning further. On the final exam in the senior-level quantum course, he stated that the average energy either “remains constant” or “increases slightly,” though he did not explicitly discuss the ensemble energy idea to support the latter answer. He stated that we measure “the kinetic energy, given by  $(E - V)$ , which returns to

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<sup>iii</sup> The mechanism of colliding electrons causing energy loss was rare among interview subjects.

its original value in region III.” He gave identical answers on the increased barrier energy scenario, but stated only that “the average energy remains constant” on the wider potential energy barrier scenario.

In the second interview, conducted the following spring, Adam again stated that the energy would be the same on both sides of the barrier. He related the energy of the particles to the wavelength of the wave function, and subsequently argued that the wavelength was the same in two regions of unequal potential energies (when particle energy exceeded the potential energy in both), when in reality the wavelength of the wave function is determined by the energy difference.

#### **6.2.2. Ideas about probability**

While we were discussing a beam of particles incident on a region of increased potential energy in the initial interview, I asked Adam what would be observed in region C. He replied that there would be fewer electrons than were incident in region A. He also reasoned that fewer electrons would be detected for a wider potential energy barrier.

When first asked about a barrier with increased potential energy, he said fewer particles would make it past, though he spoke of it as the “height of the well” and a “particle in a box,” perhaps confusing the case under examination with other standard scenarios presented in introductory quantum courses. He digressed into discussion of a cart on a track, talking about where the cart could and could not be found, relative to its energy, and lost his train of

thought. When I restated the question, he changed his mind, and said that the barrier energy would not affect the number detected, and that only the width of the barrier mattered.

His reasoning remained consistent on the survey. In the increased barrier energy scenario, Adam stated that there would be “no difference” in the number of detected particles, relative to the original scenario, and that “you might say the particles are going through, not over the barrier.” For the wider barrier scenario, he stated that “exponentially fewer particles are in III” because of “extended exponential decay,” though it was not clear whether he was referring to the wave function of the particle(s). In the final scenario, where the particle energy is greater than the potential energy of the barrier, he wrote that the same number of particles should be detected in both regions I and III, but did not explain his reasoning.

Adam was again consistent with the “height doesn’t matter” reasoning on the first exam in his senior-level quantum course. He stated that the “intensity” of particles is the same, unaffected by an increase in barrier potential energy. This was graded as incorrect.

By the final exam, he had corrected his response, now stating that in the increased barrier energy scenario “the number of detected particles decreases from the original scenario to this. The exponential relates to the quantity  $(E-V)$ .” He said that the number of detected particles would be reduced for a wider potential energy barrier as well. For the scenario where particle energy

exceeded barrier energy, he now said “the number of particles is somewhat less in region III than in region I, because a certain fraction of incident particles deflect (a strictly quantum mechanical phenomenon).”

In the second interview, Adam once again related wave function amplitude to “intensity,” stating that the amplitude of the wave function on the far side of the potential barrier would be smaller. The increased barrier energy and increased barrier width scenarios were not part of this interview, but he was asked at the end to talk about a scenario where the potential energy was lower on the transmitted side than on the incident side, and he struggled with how to reason about this situation. He finally settled on the idea that particles want to be in regions of lower potential energy, so the amplitude in the region to the right of the barrier would be greater than it was in the original scenario, since the potential energy had been lowered on that side.

### **6.2.3. Sketching the wave function**

When he was asked to sketch the wave function corresponding to the square potential energy barrier scenario during the initial interview, Adam discussed for several minutes standing wave patterns, the particle in the box, and the quantization of both energy and distance – all seemingly fragmented ideas he was recalling from his quantum course. He finally drew a waveform superimposed on a potential energy barrier; it remained sinusoidal in the barrier region, but decreased in amplitude as limited by an exponential

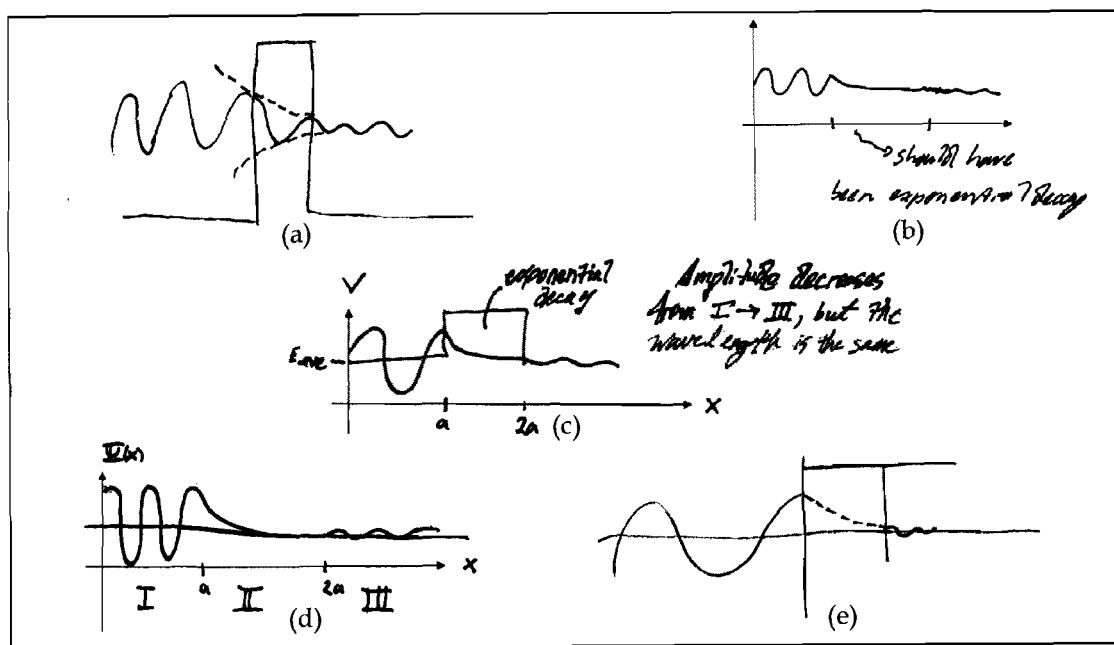


Figure 6-1: Adam's sketches of the wave function for the square barrier problem: (a) during the initial interview; (b) on the 2004 survey; (c) on the first preliminary exam in his senior quantum course; (d) on the final exam; (e) during the final interview.

envelope like in damped harmonic motion, as shown in Figure 6-1(a). He stated that in a sufficiently wide barrier, "this (the wave function) tapers out to zero," while for a thinner barrier "it does its decay thing, and whatever amplitude it had coming out of here, you're going to have in region C." He stated that the amplitude would be smaller, while the frequency remained the same.

On the survey, Adam sketched the sinusoidal-exponential-sinusoidal pattern shown in Figure 6-1(b). The amplitude is clearly reduced in the  $x > 2a$  region, and it appears that the wavelength is as well, though in a subsequent response he stated that the frequency is the same. There are no axes labels,

and we note that the wave function oscillates about some imaginary axis, perhaps coincident with the given particle energy level.

Adam kept the same general shape on his wave function sketch on the preliminary exam in his senior quantum course, shown in Figure 6-1(c), but provides more details. He superimposes the sketch on a potential energy barrier, and labels the axes " $V$ " and " $x$ ." It is now clear that his wave function oscillates about an axis coincident with the given particle energy level. He explicitly states that the amplitude decreases, but the wavelength stays the same.

On the final exam, he has changed his vertical axis label to " $\psi(x)$ ," and no longer sketches the potential energy barrier, as shown in Figure 6-1(d). However, the wave function still oscillates about some imaginary axis.

During the second interview, Adam sketched all wave functions on top of either given potential energy diagrams, as shown in Figure 6-1(e), or first drew potential energy diagrams before superimposing the wave function on the paper provided. For the square barrier scenario, the wave function was again sinusoidal-exponential-sinusoidal, and he indicated that the wavelength in regions on either side of the barrier was the same, despite his sketch that clearly shows otherwise.

#### **6.2.4. Discussing the wave function**

Even though the square barrier protocol contains series of questions on both the probability of detecting particles and the energy of detected particles

prior to any discussion of the wave function, Adam discussed wave functions immediately:

I: What would you observe, or what would you be able to detect in Region C?

A: Depending on how wide this is, I may detect nothing. If it's very thin, I'm going to detect... I'm going by the wave function, I'm guessing... Um, if it's very thin, I'm going to see the same wave function with a smaller amplitude. It's going to hit this thing and exponentially decay really fast. But if it's good and wide, nothing's going to make it through.

I asked him how he would detect a wave function. He replied:

A: How would you detect a wave function? You can't really detect a wave function – as soon as you detect it, it's no longer a wave function, it collapses.

When I again asked what would be detected, he revised his answer to “electrons, if they made it through.”

Later in the interview, we returned to the topic of the wave function, what it looked like, and its utility:

A: They talk all the time about a wave hitting the barrier and its amplitude just exponentially decreasing. So, what I'm fighting with specifically is what this is representing.

I: OK. What the, the... region B is representing, or...?

A: What the whole crap is representing. And this is purely a matter of forgetting it. So this is not... mmm... the wave function is not necce- is not really... Oh, I'm remembering this now. That matter has both a material and uh... wave nature, as demonstrated by the two-slit experiment, combining the light, where photons were talked about as a particle but also demonstrated to act like a wave.

Twice in the subsequent discussion, Adam alluded to the difficulty he was having reasoning about wave functions:

A: So, once again, I don't know how to correctly interpret this wave. If this were a physical wave, higher energy would mean higher

frequency, which... yeah, this isn't working. I'm pretty sure I'm interpreting the meaning of this wave wrong. I'll finish the sentence. In a physical wave you have the same intensity beam, the same physical beam... so in a physical wave you'd have the same amplitude, but the frequency would be greater. In which case, the amplitude would... so if you increase the energy... I gotta stick with one model here. All right. So I'm going to stick with the interpretation as I remember it from physical waves, which I don't believe is actually what's going on here. Oh, forget it.

We note Adam's admission that he didn't know how to interpret the representations he had seen. We see evidence of sophistication in his reasoning, where he is able to consider two ideas before choosing one. In fact, he is able to return and finish thoughts he had started explaining, even if he had moved on to consider alternatives. Additionally, he uses classical wave ideas to try and make sense of the properties of the wave function.

Because of the altered focus of the questions in the second interview, Adam did not talk extensively about his views on the wave function or the difficulties he had understanding the representation. He did, at times, discuss the utility of the "square of the wave function":

A: ... it's the square of the, of the wave function, that has physical significance.

I: OK. What is, what is the significance of the square of the wave function?

A: Well, as I recall, from Schrödinger's equation, um... (talks to self)... that gives you the probability of... OK, now that's coming back to me. That's all related to the probability of finding a particle at a location.

I: What is related to the probability?

A: The, in - OK, the intensity of the squared wave function...

A bit later he grappled with the probability interpretation again:

I: ...but the interpretation of the wave function?

A: So what I'm trying, trying to work out in my head, is how this whole probabil – the probability interpretation of the wave function, which is, you'd get it by squaring the wave function...

I: OK.

A: ...essentially the psi star psi thing...

I: OK.

A: ...and how, just a wave function drawn on the  $x$  and potential energy, form a system, how that is drawn, what the significance of how it's drawn is... I don't recall.

A year later, he seems to still struggle with how to interpret wave function representations.

Because of Adam's propensity to sketch the wave function superimposed on potential energy diagrams, and the possible reasoning difficulties this was causing, I asked him about whether or not this was appropriate:

I: And so, my question is, are those appropriate labels for a graph of the wave function? Or would the wave function have different labels?

A: Potential energy. Well, this... huh. There's a good question? That, the axes you've labeled are appropriate for the solid line you've drawn.

I: OK.

A: That is indicating the shape of the potential.

I: OK.

A: The, the wave function, no the wave function should be on a different set of axes, now that I think about it. It should just be psi.

I: And what does psi represent?

A: By itself?

I: Yeah.

A: Without thing, well, without the psi star psi thing? Physically, not much.

I: If it was psi star psi, it would represent what?

A: Probability. If you, the probability of detection in a location. For every situation where I've seen it applied.

Once again, Adam had returned to the idea of the "square of the wave function," which he also described as "psi star psi." We note that this re-

labeling of his wave function sketches as " $\psi^*\psi$ " led him to difficulty with zeros in the function, as we describe in the next section.

### 6.2.5. Ideas about quantum

At the very end of Adam's initial interview, I asked him about when quantum principles apply:

- I: You said earlier that you were having some conflict because you were thinking of a physical concrete wall, but no, this is a quantum scenario, right?
- A: Oh, I was, I was interpreting the, the diagram, and, see before this, said potential energy... So, because there was no scale on here, I was saying, well, if this is a, a macroscale thing versus a microscale thing.
- I: So, yeah, when, when does quantum work? In your mind, when, when can we use the quantum rules, versus using the classical rules?
- A: Oh, when you are talking about very small things, on the order of magnitude of light waves.
- I: OK. How big is a light wave?
- A: Uh, ten to the negative nine-ish.
- I: Ten to the negative nine what?
- A: Oh, meters.
- I: For... wavelength, or?
- A: Oh, yeah, wavelength.

Seemingly, Adam saw quantum physics as describing reality for small objects.

He then clarified that the "small things" referred to the object's wavelength:

- I: So, in order to use the quantum rules, you need objects to have, relatively short wavelengths. Is that what you're saying?
- A: Yooo, short wavelengths.
- I: Or, forget short, because that means comparison, but you were saying somewhere in the, in the ballpark of light, ten to the minus nine meters.
- A: Yeah, that's when... I understand this starts to really kick in. And where you have to acc - uh, it happens everywhere, that's where you have to start to account for it.

Rather than view quantum mechanics as a refinement of classical physics, it seems that Adam viewed the two realms as separate, used to address different problems.

In the second interview, the final series of questions dealt with whether or not it was appropriate to sketch a wave function on top of a potential energy diagram, as discussed in the previous section. Adam talked about how a graph of " $\psi$ " had little meaning, but " $\psi^* \psi$ " related to probability density. When he revised his sketch labels, stating that he was drawing graphs of " $\psi^* \psi$ ," I asked him about the zero points:

I: The final thing I wanted to ask was back here when you just sketched a sample graph for  $\psi^* \psi$ , and we're talking about probability, you said take an infinite amount of measurements, or measurements for a long time, and you're very likely to find particles here, and sort of likely to find them here, and never here.

A: Mmm-hmm.

I: So, if you were likely to find a particle here, and here, but not in between, how does that work?

A: (laughs) You tell me. I don't know. I don't think that is known, um, I don't think quantum wave - I've been told anyway that quantum mechanics doesn't consider that a legitimate question. That, a fact is it, it seems to work that way. You have one particle, take a million measurements of it after letting it go back to whatever initial conditions, you'll be it here, you'll be it here, you'll be it here. That you will not find it in those locations? It does not have a, I understand it does not have a classical, um, doesn't lend itself to a classical explanation. I don't know how it does that.

I: So if we were thinking back in the well, if I'm, if I'm in this position, and I'm in this position, I must have gone, you know, straight line from here to here. We can't use that kind of thinking in this realm?

A: No.

Two things are of note in that exchange. First, Adam never mentions the Heisenberg uncertainty principle, which limits the precision with which

positions, velocities, or energies may be determined. Second, although he repeatedly makes mention of “psi star psi,” we see little evidence that this is any different in his mind from simply squaring a sinusoidal function. For a plane wave, there are no areas of zero probability, as a complex exponential multiplied by its complex conjugate is constant.

#### **6.2.6. Discussion – Adam**

Though his reasoning changed slightly over the three semesters we studied Adam, he remained convinced that energy was not lost in the tunneling process. Typically, he related the energy to the frequency and/or wavelength of the wave function. Although there is evidence that he reasoned about the issue of an ensemble of particle energies on the survey questions given on the final exam in his last quantum course, we saw no evidence of that reasoning lingering on the second interview several months later, although it is possible that the focus of the questions was sufficiently different that other issues were being considered, and precluded this argument.

It is possible that in some fashion Adam initially thought about the given scenario as an actual physical barrier. In addition to his comment about a “concrete barrier” in the first interview, his reasoning that the “height” of the barrier didn’t affect the probability of tunneling is consistent with others who describe a physical barrier.

Though the general shape of his sketches of the wave function in the barrier region are improved by the second data point – the survey – we note

that Adam continued to sketch wave functions on top of potential energy barriers, even after explicit instruction that included sketching separate graphs. This led him to some uncertainty during the final interview as to whether or not this was appropriate, and whether or not the amplitude of the wave function was connected to energy.

Adam also preferred to talk about the “square of the wave function,” or “ $\psi^*\psi$ .” There seems to be some thread of reasoning consistent throughout many of the students we studied; perhaps this is an interpretation of some remark made by a professor in one of the quantum mechanics courses Adam and others took.

### 6.3. Case Study – Jack

Jack responded rather quickly to questions during interviews, rarely pausing for more than a few seconds to think over the questions. He was also quick to state when he didn’t know something, or couldn’t recall it, rather than to try and piece an answer together from the things he did know. He remained rather logically consistent throughout both interviews.

In addition, though he gave many wrong responses during the initial interview, it seems as though the interview was a learning opportunity for him. Following that first session, he asked whether or not he gave the correct answers. This part of the session was not videotaped, coming after the end of the formal interview period. When told, for example, that energy was not lost in tunneling, he seemed to remember this fact, stating it on all subsequent

surveys, exams, and interviews (though his discussion of energy was further refined by class discussions in his senior quantum class).

The transcripts of the two interviews with Jack can be found in Appendices E and F.

### **6.3.1. Ideas about energy**

During the first interview, Jack mentioned energy loss early on in his description of what happens when a particle encounters a region of increased potential energy:

J: Well I know because I was taught that... when the particle of some certain potential energy, or of some energy, encounters a potential barrier, there is a possibility, calculated through, well, wave equations and their integrals, that a particle will actually just go straight on through, losing energy as it does so, and come out the other side of the potential barrier at a lower energy and continue on its path.

When I asked him about the scenario with increased barrier energy, Jack reasoned that more energy would be lost. It seems that his model at the time was that energy loss is determined by the relationship between the total energy of the particle and the potential energy in the barrier region. I asked him if there was some threshold energy required for tunneling, and he replied that the particle had no real minimum, and that “as long as it has energy, it can still lose energy to go through.” He was not asked about the relative amount of energy loss in a wider barrier during this interview session.

Shortly thereafter he took the first version of the survey. His energy responses reveal that he recalled the discussion of the correct responses

following the interview. On all scenarios, he used the principle of energy conservation, stating that the energy of the particles was the same on both sides of the barrier.

A year later, his energy reasoning remained consistent on the revised version of the survey. He stated that the average energy of the particles to the far side of the barrier was the “same, the particle doesn’t lose energy in traveling through the potential barrier, just probability in being there.” He kept this reasoning in all subsequent scenarios.

His reasoning remained nearly identical the following fall, when he responded to survey questions included on the first preliminary exam in the senior-level quantum physics course: “Energy is the same, the energy of the particles remains constant, only the probability of finding them is lower.” The subsequent class discussion on ensemble energies seemed to alter his response, as evidenced on the final exam. Asked to compare the average energies of particles in both regions, he now stated that it was “higher, because the particles in region I that have a higher than average energy have a higher probability of tunneling to region III; on average, the particles in region III will have a higher energy (the particles with lower energy reflect more often).” He later argued that the average energy of particles found in region III would be even greater for both the increased barrier energy and wider barrier scenarios.

During the second interview, Jack stated that the energy on either side of the barrier remained the same regardless of the “type of graph,” referring to the various scenarios regarding changing barrier characteristics or particle energies. There was no mention of the ensemble of energies idea during this session, though we are not surprised, as the first task during this interview was to solve the Schrödinger equation, not reason about a “beam of particles.”

### **6.3.2. Ideas about probability**

During the initial interview, Jack stated that two things could happen when a particle with less energy than the maximum potential energy of the barrier encountered the barrier region: “Because the potential barrier is higher than the electron, it can either reflect back off and go back the way it came, it can tunnel through...” He alluded to the fact that there might be other possibilities, but said he couldn’t remember them. Jack thought that the chances of observing an electron on the far side of the barrier were reduced by either increasing the barrier energy (“the difference between... the electron’s potential energy and the potential energy of the barrier is a much greater distance”), or by making the barrier wider. I asked him if there was some limiting width, to which he replied, “theoretically, no matter how wide you put the barrier, there is a chance the particle will be in region C...”

We note evidence of linking energy levels with spatial position, as he relates an energy difference to a distance.

On the first survey, his reasoning about the probability of tunneling remained consistent. He said fewer particles would be detected if the barrier energy was increased, barrier width was increased, or particle energy was decreased; more would be detected if the particle energy was increased.

A year later, the answers remained the same on the revised version of the survey. The rewritten question regarding a stream of particles with energy greater than the barrier energy yielded the argument that the number of particles in the regions on either side of the barrier “should be about the same. The particles have a higher energy than the barrier so the particles aren’t really affect(ed).”

The first preliminary exam in the senior quantum mechanics course contained a portion of the survey questions, only one on probability of tunneling. For the scenario with increased barrier energy, Jack stated that the number of detected particles should decrease, but also stated that it was “probably by a factor of 2.” No additional explanation for this choice of scaling factor is given.

On the final exam containing all the survey questions, Jack remained consistent with his previous reasoning – fewer particles were detected when barrier energy or width was increased, more were detected when particle energy was increased. For the scenario where particle energy exceeds barrier energy, he reasons that the number “should be about the same, but still a little

smaller; some of the particles will still reflect at the potential barrier but most will go through to region III."

During the final interview, Jack stated that the probability of finding a particle drops off as a decaying exponential in regions where the potential of the system is greater than the energy of the particle. Later, he referred to his wave function graphs (which he called graphs of " $\psi$ -star- $\psi$ ") as showing the possibilities for either a stream of particles or a single particle incident on some system. We note that he does not seem aware of the use of wave packets to describe single particles.

Probed on the sinusoidal nature of his sketches, he struggled with what the zero points meant:

- I: Then is that true, that there would be regions where there's no probability of finding particles? If you were to do these measurements?
- J: It seems that way, it's really kind of strange, if I were to think about it, yes.
- I: How, then, does a particle, you know, get from one place to another, if there's no probability of being in between?
- J: I've been wondering that for a couple of years now.
- I: No good thoughts on that?
- J: Well, a lot of it has to do with... mathematically it's kind of explained using Heisenberg's uncertainty principle, sort of, kind of.
- I: Which says what?
- J: Um, well, there's, there's various versions, uh, that's one of them... t... I think that's another one... Basically, you can only know so much between the position, or where a particle is, and where it's going. If you're aiming a stream of particles you'd know, more or less, quite a bit about where it's going, and you can know less and less about where the particle actually is, and that kind of comes along into the zero points there.

Later, he explicitly linked the probability to the amplitude of the wave function sketch, reasoning that the probability of detecting a particle on the far side of any barrier was lower than on the incident side.

### **6.3.3. Sketching the wave function**

When Jack was asked to sketch the wave function accompanying the square-barrier scenario on the first interview, he first sketched the potential energy diagram. When he started to sketch the waveform, he remarked “that’s the electron potential that’s roughly the midpoint of its sinusoidal.” He also refined his sketch to include the idea that the wave function in the first region had to be near a maximum at the boundary with the middle region, though he could not reason why that was the case. He described the shape as a “log decay” in the middle region, and sinusoidal once again in the third region. His sketch is shown in Figure 6-2(a). He also reasoned that the wavelength had to be longer in the post-barrier region, to match the idea that energy was lost in tunneling. He sketched the opposite, though, making the wavelength shorter.

The first version of the survey did not ask for a sketch of the wave function. On the second survey, which Jack took about a year after first being interviewed, his sketch once again exhibited the sinusoidal-exponential-sinusoidal characteristics, as shown in Figure 6-2(b). We note two points – first, he once again begins the wave function at a level labeled with reference to the particle energy, and second, though the amplitude is clearly reduced in

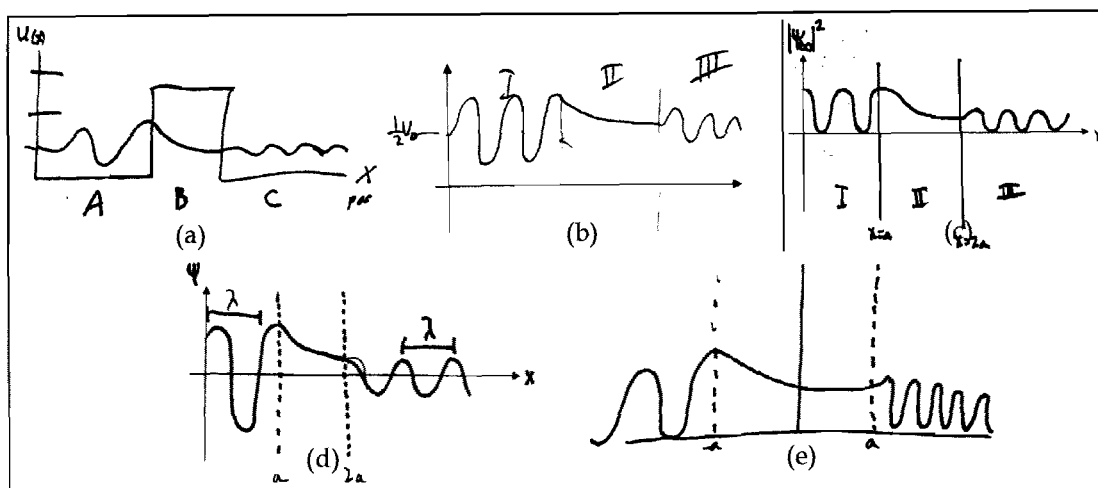


Figure 6-2: Jack's sketches of the wave function for the square barrier problem: (a) during the initial interview; (b) on the 2004 survey; (c) on the first preliminary exam in his senior quantum course; (d) on the final exam; (e) during the final interview.

region III, it exceeds the height of the exponential function at the boundary between regions II and III. (This is possible, but not for the boundary conditions as he has drawn them.) Additionally, on the two additional cases asking for wave function sketches for situations with increased particle energy, he sketches the wave function spatially higher on the axes and again uses energy labels.

On his wave function sketch on the first exam during his senior quantum course, shown in Figure 6-2(c), the axes were labeled " $|\psi(x)|^2$ " and " $x$ ." The wave function was sinusoidal in both regions I and III, and generally decaying in region II. There seem to be points where the value of the plotted function is zero.

On the final exam at the end of the semester, his sketch, shown in Figure 6-2(d), continued to have the same shape characteristics, but two improvements were evident. First, he labeled the vertical axis now as " $\psi$ ," and the sinusoidal portions of the wave function now oscillate about the given zero axis. He also indicated that the wavelengths of both sinusoidal portions were equal.

During the second interview, Jack's sketch seemed to most closely match his previous sketch on the first prelim. His drawing is shown in Figure 6-2(e). He labels his axes " $\psi^*\psi$ " and " $x$ ." Jack indicated verbally that the decreased amplitude related to the probability of finding a particle. As with Adam, improvements in understanding made between the prelim and the final have largely disappeared.

#### **6.3.4. Describing the wave function**

When I asked Jack what the wave function for the square barrier scenario looked like during the initial interview, he replied that "wave functions are generally sinusoidal in shape, they look... sinusoidal-ish," and when the potential energy was greater than the particle energy "it looks more like a log decay equation as opposed to a sinusoidal equation." There was no mention of how these shapes came out of solutions to the Schrödinger equation, and since the focus of the interview was on qualitative reasoning about the situation, I did not ask him for the connection.

He struggled a bit when I asked him to describe the meanings of the characteristics of his wave function. Asked about amplitude, he initially replied, "I'm thinking it's related to energy, but the energy is where it is. I don't remember what the amplitude is related to, but it's related to something." A short while later, he reasoned that, "the equation is for potential energy... energy is not going to be on the inside of that sinusoidal function." Clearly, at this point, Jack was not connecting particle energy to the wavelength of his sketch. We note further evidence of a possible connection between energy levels and spatial positions from Jack's statement "energy is where it is."

When discussing the scanning tunneling microscope during the second half of the interview, Jack made another statement about the wave function:

"I believe  $\psi$  is the potential function, of its energy, and  $\psi^* \psi$  is the probability, the integral of  $\psi^* \psi$  dot, position vector, the integral of the  $\psi^2$  squared equation is the probability of finding it at some point in the graph..."

Once again, Jack seems to be connecting the wave function to energy, though it's not clear what the exact link is.

At the end of the first interview, I asked Jack about how one would locate an electron. Though his answer involved discussion of the wave function, it is included in the following section on Jack's more general ideas about the applicability of quantum mechanics.

Two years later, Jack discussed the meaning of the wave function sketches he had drawn:

- I: ... do these pictures, then equations, represent what would happen to the particle?
- J: Well, they don't necessarily represent what actually happens to the particle, they just represent the possibility, if we shot a whole stream of particles at say, said barrier, and we were measuring, particles as they went flying through, or if we measured where they are... stream of particles, right.
- I: So you think this is a good model for a stream of particles?
- J: Well, if you're...
- I: Or a, a model for a stream of particles?
- J: It's a model... well, it, it works for both a single particle and a stream of particles. If you measure a stream of particles, you can actually graph, perhaps, like the number of counts you find, on where particles are, and it should look something like that. Or if you, it's kind of hard to graph a single particle, cause a soon as you make a measurement on that particle, you destroy its wave function, as for what it used to be, and it kind of spikes here at where you measured it at...
- I: OK.
- J: ...and it doesn't work anymore.
- I: Wouldn't, then, making a measurement on a stream of particles destroy that wave function as well?
- J: Well, you'd just, well, you'd destroy the wave function of the single particle that you measured, not the wave function of a stream of particles, cause each particle, assuming you'd make each particle in the stream have the same wave function, if you measured each particle, then it just ruins that one particle from the stream, but you still have the rest of the stream to measure from. And so you can measure at different points, and get, you know, something like that.

Jack seems convinced that the wave function sketch is similar for both a stream of particles and a single particle, but that the ensemble is preferable, because a measurement of a single particle's wave function will not "collapse" the wave functions of the rest of the particles. There seems to be no evidence that Jack has any intuition about wave packet descriptions of single particles.

When Jack drew his solution for the square-barrier scenario, and discussed how it matched his algebraic solutions, I pressed him on why the increasing exponential term was discarded. In his response, it is evident he is matching

his graphical and algebraic solutions to some understanding of their physical significance:

- I: Is it possible that we could draw an increasing exponential here, or...?
- J: No, you're not gonna draw an increasing exponential, cause physically that would mean that you're more likely to find it over here than you are over here, and since you're – the energy of the particle is less than the potential barrier, the probability of finding the particle farther and farther and farther along inside the potential barrier is going to be smaller and smaller and smaller. And so you're not going to have an increasing exponential there.

Further on, I asked Jack about the specific terms in his written solution to the Schrödinger equation. He had changed his mind from his responses two years prior, and now included energy into the “wavelength” term in the sinusoidal function:

- J: The wavelength should still be the same.
- I: And why is that?
- J: That's determined by the constants that are in front of  $x$ , and it's the same constant. Still, still the same sort of equation...

### **6.3.5. Ideas about quantum**

Jack made a few references to his beliefs about the use of quantum mechanics during the first interview. Previously, he stated that energy was lost in tunneling. When I asked him to describe the minimum amount of energy that particles could have and still be able to tunnel, he replied that negative energies seemed unlikely, but “it's quantum mechanics – common sense doesn't apply.”

Asked to reason about any “real applications” of tunneling, Jack stated:

“Well, if you could get enough electrons going through a potential barrier, and enough protons, theoretically you could just fall through your

chair and land on the floor, or you could walk through a door. It's extremely improbable, but it's still possible."

It seems evident, then, that Jack has embraced the idea that quantum mechanics describes physical phenomena in terms of probabilities, that nothing is truly "impossible," but rather has an extremely small chance of occurring.

In discussing a model of a scanning-tunneling microscope, Jack stated that "there's potential it's (the electron) there, it could be there, it could be there.

You don't know." I pressed him to describe this a bit further:

- I: Is the electron at a definite location, we just have the inability to find it, or does it not have a definite location?
- J: I know there are places because of when you look at... psi squared...
- I: What's psi?
- J: Psi is the, how do I say this? I'm trying, I believe psi is the potential function, of its energy, and psi star psi is the probability, the integral of psi dot, position vector, the integral of the psi squared equation is the probability of finding it at some point in the graph, and there are points where, since the sinusoidal function equals zero or something or a negative number, if it goes there, when you square it, it becomes zero and there are points where you actually can't find the particle, no matter how often you look.
- I: OK.
- J: Exactly where is probably there, there, there,...
- I: So, which of those would you lean towards, then? The electron has a definite position, we just can't find it? Versus the electron has no definite position, therefore we can't find it.
- J: It doesn't have a definite position, because there's no way of knowing which direction it's going to go, because, oh, we do have an uncertainty principle where... I guess like the better we know momentum, the less we know about its position, the more we know about its position, the less we know about where it's going, so where it's going next, we don't know. It's impossible to know, if we know where it is at one point, it's impossible to know where it's going to be at some other point in the future. All we can look at is probability, so there is no definite position as to where it is.

We see evidence that Jack has begun to adopt some fundamental ideas in quantum mechanics, such as non-locality. At this point, however, he is again struggling with how the wave function is connected to particle energy.

Jack made few references to his ideas about the applicability of quantum ideas in the second interview, and focused primarily on providing a coherent set of solutions to the given scenarios. He did make an offhand reference to when quantum tunneling ideas are relevant (speaking in the context of beam energy higher than the potential barrier energy):

J: I don't know if you could say. Cause the energy is greater than... and so, you probably (have a) wave-particle kind of thing, and usually in quantum mechanics when it reaches a barrier, there actually is a potential for it to, just reflect back.

I: OK.

J: Even though it actually has a higher energy, but it, since it has a higher energy, you don't have to worry about quantum tunneling, and its wave function should just be the same in both places.

It seems possible that Jack has abandoned his ideas from the exam responses a semester ago, where he argued that reflection is a possibility, even if the particle energy exceeds the barrier potential. Previous students who describe the wave function as the same in both regions consistently reason that all particles pass the barrier.

Additional ideas about the collapse of the wave function and the relevance of the Heisenberg uncertainty principle were discussed in previous sections.

#### **6.3.6. Discussion – Jack**

Of the three, it appears that Jack learned the most from the initial interview, where some of his incorrect ideas were addressed following the

interview sessions. Although he originally expressed an energy-loss model, this idea was absent in all subsequent discussions, where he either argued energy conservation or discussed the idea of higher transmission probabilities for higher-energy particles.

Jack also seemed to have a reasonable functional understanding of the ideas tied to the tunneling scenario. He described everything in terms of probability, reasoned that this probabilistic interpretation would prevent one from knowing exact positions, and that the probabilities would exist for strange behavior in even in unlikely situations.

Like Adam, Jack seemed unsure about sketching the wave function itself, and seemed unsure of the meaning of the wave function. Note that he described the wave function in terms of energy while also describing energy loss in tunneling. Vestiges of this response remained as his axis of oscillation remained an indicator of the particle energy. Throughout the interviews, he preferred to reason about and draw  $\psi^*\psi$ . He stated on numerous occasions that it was what he was “used to thinking about.”

#### **6.4. Case Study – Selena**

Like Jack, Selena was also generally quick to respond to questions, often leading her to state answers that were inconsistent with previous responses she had given. When inconsistencies were pointed out to her, she generally chose one idea over another based on instinct, rather than reasoning. Of the three case study students described in this chapter, she arguably was affected

the least by instruction, returning to many of her initial ideas following all undergraduate instruction, and giving what could be interpreted as memorized responses to questions where her reasoning had changed. She also was quite open in her views about the nature of quantum mechanics, and how it was difficult for her to adopt some of the strange ideas.

The transcripts from the two interviews with Selena may be found in Appendices G and H.

#### **6.4.1. Ideas about energy**

As a sophomore, Selena stated in her initial interview that she believed energy was lost when particles tunneled through a square potential barrier. Asked to explain, she discussed the classical notion of work, but admitted that she didn't know if that was appropriate to use or not. For the increased barrier energy scenario, she initially stated that more energy would be lost, but later revised her answer to say the amount of energy loss would be the same. She stated that more energy was lost if barrier width increased. These two responses are consistent with other students who have discussed a physical barrier; within reason, it does not take more energy to tunnel through a taller physical barrier, but it does if the barrier is widened.

The same answers were also given on the quantum tunneling survey responses in her sophomore and junior years. We are not surprised by these results, as she had no additional quantum physics courses in the interim that might have influenced her understanding of the scenario.

Selena showed improvement on both senior-year exams. She indicated that the energy of tunneled particles remains the same as that of incident particles, since “energy is not lost in travel.”

On the final exam, she also stated that the particles with the highest energy are more likely to be found on the far side of the barrier, though she did not say that this would make the average energy higher. When interviewed during her senior year, Selena stated that “the average energy of the particles from one side to another, here (pointing to the post-barrier region) will be higher because “only higher energy particles are able to tunnel.” We note the incomplete interpretation of the previous semester’s classroom discussion, suggesting that she may not have developed complete reasoning about this argument, and has rather memorized an answer.

#### **6.4.2. Ideas about probability**

Early in her first interview, Selena states that “when it (the electron) hits the barrier, because it is a finite barrier, there is a small probability – you gotta do the math – of the electron tunneling through...” I then asked her what determines the probability a particle would be found in Region C:

S: That is the kinetic energy of the particle, the mass of the particle, and the size of the barrier... thrown in to the square root and some other stuff, do a little magic, and poof, there’s your probability.

I: So you’re saying, throw them into the square root; are you...

S: Well there’s a square root function in the, there’s a square root in, it was the potential minus the kinetic, and something with the mass, and... that’s the part that I remember.

Though for a time after she had sketched the wave function Selena discussed the amplitude as connected to energy, she revised that opinion, saying that it was actually “the probability description, the probability of where it will be found.” I asked her where the particle was most likely to be found, and she indicated the peaks of the sinusoidal waveform she had sketched. She also argued that there was zero probability of finding the particle where the sketched wave function touched the horizontal axis.

Asked about doubling the energy of the potential barrier, Selena thought that particles would be much less likely to make it through. She also stated that, “if it’s a wider barrier, there’s going to be less chance it’ll get all the way through.” Since she was reasoning that energy was lost, and more energy was lost in a wider barrier, she agreed that it was likely that there was some barrier width where the particle would not make it through.

On the first version of the survey, which she also took that spring, Selena was consistent with her interview responses. She said that fewer particles would be detected on the far side of the barrier when barrier energy increased, barrier width increased, or particle energy decreased. She said more particles would be detected when particle energy was increased.

On the revised version of the survey, which she took about a year later, she stated that “height, width, and energy difference” affected the probability of tunneling. However, this was not consistent with her later responses. On the increased barrier energy scenario, Selena stated that fewer particles would

tunnel, but on the wider barrier scenario, she said one would detect the same number, since the barrier height was what influences transmission.

On the survey portion included in the senior-level quantum physics course the following fall, Selena stated that the width and height of the barrier and the energy of incident particles determined the probability of detection. On the increased barrier energy scenario, the only one included on this exam, Selena stated that the number of particles detected drops to zero, since  $E < U_0$ .

She correctly answered all probability questions on the final exam, stating that fewer particles are detected if barrier energy or width is increased, and more are detected when particle energy is increased. In addition, she discussed the possibility of reflection when the particle energy exceeds the barrier energy, and thus reasoned that fewer particles would be detected in the far region.

The same scenarios were not part of the final interview, which focused more on reasoning about the mathematical solutions to the Schrödinger equation. She did, however, discuss some of her views on probability.

- I: Is it OK to talk about probability if you're talking about single things? Single particles, single things?
- S: You can, it's not necessarily effective, because probability assumes a large sample. Otherwise, you know, you have to take a large sample and, of a lot of particles in the same setup before you can get an idea of what's going to happen. And individual particles can go do whatever the heck they want.

Later, she talked about how one could measure probabilities, but not wave functions:

- I: OK. Um, you, you mentioned probability quite a few times, and how the fact that we could measure probability, but not a wave function, I think...
- S: Right.
- I: ...I'm trying to remember back 10 or 15 minutes here. What do you mean by that, when we say we, we, you know, we can measure probability, we can't measure a wave function?
- S: Um, you can't measure  $\psi$  of  $x$ . You can't say at this time this particle is in this spot.
- I: OK.
- S: Um, cause it's not allowed. (Laughs.)
- I: OK.
- S: The uncertainty principle says we're not allowed to do that.
- I: All right.
- S: You can measure the square of a wave function, which gives you an average of where a whole lot of stuff would be over time.
- I: OK.
- S: And the, you can measure the probability of something being somewhere at some point, but you can't actually nail down one particle at some time and say it's in this spot.
- I: You said the square of the wave function is probability?
- S: Right.
- I: So, could we, uh, you say we can't, we drew these wave functions, we could never measure these wave functions, but we could measure probabilities that correspond to them?
- S: Mmm-hmm.

#### 6.4.3. Sketching the wave function

During her sophomore year interview, Selena first sketched a potential barrier, superimposing her sketch of the wave function on top, as shown in Figure 6-3(a). Though the wave function exhibited generally correct characteristics in each region (sinusoidal-exponential-sinusoidal), problems existed. First, the superposition led her to first describe the vertical axis for the wave function as representing energy, later crossing out that axis label in favor of " $y$ ," which she referred to as "probability." Second, her sketch

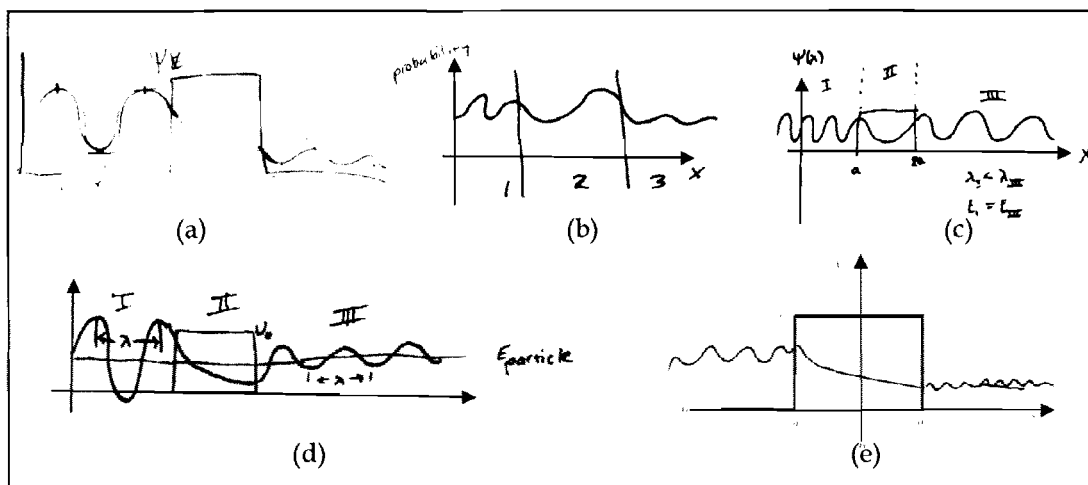


Figure 6-3: Selena's sketches of the wave function for the square barrier problem: (a) during the initial interview; (b) on the 2004 survey; (c) on the first preliminary exam in her senior quantum course; (d) on the final exam; (e) during the final interview.

exhibited an "axis-shift," consistent with the sketches of many students who believe energy is lost in the tunneling process.

On the junior-year survey, Selena no longer superimposed her sketch on top of an energy barrier. However, she drew the wave function as sinusoidal in all regions, shown in Figure 6-3(b), with what appears to be an increased wavelength in the barrier region. Survey questions asked the respondents to show by sketching how the graph of the wave function is different when incident particle energy is increased. On both of these sketches (not shown), Selena drew a similar shape function, but shifted it, so as to make the function oscillate about a higher imaginary axis that is coincident with the given energy level of the particles.

On the first senior-year exam, Selena returned to superimposing her sketch of the wave function on a potential barrier shape, though she does label the

axes " $\psi(x)$ " and " $x$ ," shown in Figure 6-3(c). It is unclear whether her function is sinusoidal or a sum of increasing and decreasing exponentials in the barrier region. It appears that she keeps the amplitude of the wave function the same in regions I and III. Though she stated that the energies in both of those regions are equal, she indicated incorrectly that the wavelength of the function is greater in region III. Her sketch, though sinusoidal, once again oscillates about some imaginary positive axis, perhaps coincident again with the energy level of the particles.

On the final exam, the wave function is more clearly a decaying exponential in the barrier region, though it decays far below the amplitude of the sinusoidal portion of the wave function in region III, shown in Figure 6-3(d). She indicated that the wavelength and energy are the same on both sides of the barrier, but she failed to label the given axes. She also sketched an axis of oscillation, labeling it " $E_{particle}$ ." She labeled sketches of the wave function on further questions involving increased incident particle energy similarly.

During the second interview, Selena sketched her representations of the wave function on the provided scenario sheets (which described the potential energy symbolically and graphically), once again placing the wave function on top of the potential steps and barriers, as seen in Figure 6-3(e). Though the general shape of the wave function solution was correct for the rectangular barrier, her sketch once again includes the "axis shift" problem. When asked

about the vertical axis label for the plot of a wave function, Selena replied that it is “potential energy.” She admitted that this caused conflicts, however: “...which makes half of what I’ve said wrong, because I didn’t keep the amplitude the same. But this is the part that I always get messed up with when I’m thinking about it, cause I remember pictures that look like this, but not necessarily where the axes were...”

#### **6.4.4. Describing the wave function**

There is some evidence that Selena may think of the wave function as describing the physical path of a particle. In her first interview, when asked to describe the behavior of electrons in the system, she says, “well, you have your nice wave-particle duality, so it moves along in a wave form...”

Later, Selena sketched the wave function corresponding to tunneling through a rectangular potential energy barrier. She described the “waves” on either side as being the same, and was questioned on exactly what she meant:

- S: ... uh, the wave function is dependent on the nature of the particle, not external conditions, so it’s, it has to be the same wave on either side, but it’s lost energy.
- I: Would you describe these (pointing to the sinusoidal portions of the wave function on either side of the barrier, see Fig. 1a) as being the same wave?
- S: Yeah, I draw badly, but sure, you know...
- I: Is everything the same about them?
- S: Well, there’s less energy here, and depending on the size of this area the, um, wait a minute, that doesn’t make any sense – this has less energy, the wavelength should be the same... I think.
- I: Why do you think the wavelengths should be the same?
- S: Because it’s the same particle, and the wave function is describing the particle. And the particle, the only thing that’s changed about the particle by going through the barrier is the amount of energy that it has, which is indicated by the height of the waves.

Two years later, discussion again addressed Selena's ideas about the wave function. She was questioned about labeling the vertical axis of a graph showing wave function as a function of position, and stated that the label was "potential energy." She noted that this caused problems, since she had previously said energy is conserved. Though she stated, "if you just said these waves are  $\psi$  of  $x$ , we have no problem," when pressed on the two models, she stuck to energy:

- I: Would it be OK to write  $\psi$  of  $x$  equals  $V$  of  $x$ ? I mean, if it's energy, could we...
- S: Yeah, it would, you could say... the  $\psi$  of  $x$  for the potential is a constant starting at  $x$  equals zero.

Though she stated a qualitatively correct relationship between energy and wavelength earlier in the interview and also realized that several contradictions are solved by not equating the vertical axis of a wave function plot with energy, she remained determined to equate the two. In doing so, she changed many of her correct ideas discussed earlier in the interview, abandoning the careful logical connections in an attempt to match an explanation to her sketch.

#### **6.4.5. Ideas about quantum**

In both interviews, Selena made references to her beliefs about the nature of reality and quantum theory's ability to describe it. She consistently spoke of quantum mechanics as an imprecise science. In her sophomore year, we

were discussing the zero points on her wave function graph, and I asked her if an electron could ever be located at those points:

- I: So the electron will never be in that region...
- S: Uh, whether or not it actually is there we don't know, but we will never find it there.
- I: Could you describe that a little bit further?
- S: Uh, the math we have for describing these things is crappy, um, we don't actually know what's going on, we're assuming a whole lot of things, and, uh, according to the equations we have that work with observed stuff, we will not find it here.
- I: So we could in principle take a measurement, you know, every second for the rest of our professional lives on this system, and never find the electron in this position.
- S: Not if we're using these equations and the apparatus we've got, no.

At the end of the interview, Selena seemed surprised that the questions were done, and discussed further her feelings about quantum mechanics:

- S: Here I thought I'd get to gripe about Schrödinger.
- I: Oh! Please! Tell us what you think about Schrödinger.
- S: OK, that whole probability of the cat being half dead and half alive, and that being dependent on who's looking - it's just a big lie. That equation is for the cat, not the observer.
- I: So...
- S: Somebody finds out the cat's alive or dead, the cat's alive or dead, it doesn't matter whether Joe in the next room knows or not.
- I: Is the cat, then, in the box, either definitely alive or definitely dead, we just can't know?
- S: Yeah. But once somebody finds out, that cat has a definite state, period end. It's not dependent on who finds out who else knows, cause it's an equation for the cat, not for me looking at the cat.
- I: What about the electron here in this situation?
- S: Yeah.
- I: Is the electron definitely in region A, or in region B, or in region C, we just can't maybe know which region for sure it's in?
- S: Uh, we know that we will find it here and we'll find it here, and we'll never find it here; we don't know what the hell is going on in here, and we don't know how it gets from here to there.
- I: Does at any instant in time, it, does the electron have a definite location? That is, it's either in region A, or in region B, or in region C?
- S: Maybe.

- I: Maybe?  
 S: Maybe.  
 I: Not sure?  
 S: Don't know. You have to go find out to know for sure, and in order to go find out, you have to mess with it, and that changes where it's going to go.  
 I: In a hundred years, will physics be able to tell you where an electron is for sure?  
 S: Maybe, I don't know.

Two years later, she had not abandoned that feeling. Her views came up again in a discussion of whether or not a particle could be detected at a given location:

- S: It can, it can be here, it's most likely to be here...  
 I: All right.  
 S: ...it will never, ever be found here. Which is not to say if you measured this same setup, one million times, you know over the rest of your life that you would never, ever, ever find one here, the math says you won't, but, you know, if you did it enough times, you might.  
 I: In other words, the math might not be accurate.  
 S: I don't buy that Schrödinger's equation is a hundred percent right. Yeah, it works, but, you know, derivative twice with respect to position, derivative once with respect to time, it works, I think we, we're missing something.  
 I: So, in other words, in, is it something along, akin to the fact that, you know, here comes Newton in whenever writing down  $F_{\text{net}} = ma$ ...  
 S: Mmm-hmm.  
 I: ...and this works great for a couple hundred years, until we find very small applications where it doesn't apply anymore.  
 S: Yeah.  
 I: Along comes quantum, we have all these equations, so they seem to work for a, a different set of problems, but they're not entirely accurate either?  
 S: Right.

#### 6.4.6. Discussion – Selena

Selena showed improvement on the energy-loss misconception, but little change on other concepts and beliefs about the validity of quantum physics.

Selena held the idea that energy is lost for most of her undergraduate physics career but abandoned it, at least in verbal and written responses, in the presence of specific instruction on the square-barrier tunneling scenario during a senior-level quantum mechanics course. It is not clear whether Selena's new answers arose from a solid conceptual understanding of the phenomena or merely memorization of phrases and ideas repeated by the instructor on multiple occasions. The phrase "only higher energy particles are able to tunnel," given during the second interview, suggests the latter. In reality particles of all energies possess a non-zero probability of tunneling; it is merely greater for higher-energy particles. Also, it may be that knowing "the answer" to the energy loss question introduced new confusions and created a willingness to abandon good ideas in an attempt justify her sketch.

As we previously discussed, Selena superimposes the wave function on the potential energy barrier in four of five sketches of the wave function, a representation not uncommon in quantum mechanics texts and computer simulations. While there is arguably some merit to this approach, we believe that representations of this form can cause unnecessary confusion and may have led to Selena's struggle with labeling the vertical axis of wave function sketches.

We note that Selena's sketches of the wave function are most correct during or shortly after instruction. While many features of her sophomore interview sketch are correct (function type in each region, axes label), these are

not present in the sketch from the junior year survey. Her sketches on exams during the senior-level course show improvements in function shape, amplitude, and wavelength from the preliminary exam to the final. The final interview sketch returns, however, to the “axis shift” response given on the initial interview, and no apparent effort was made to match wavelengths for the portions of the wave function on either side of the barrier. This may suggest that without meaningful conceptual change, persistent, incorrect ideas return in the absence of instruction. It may also indicate that questions tied to grades, such as the examination questions, yield more careful and reasoned answers than responses in volunteer interview sessions.

Selena uses the terms “wave” and “wave function” interchangeably, suggesting that the two are not clearly distinguished in her mind. This may lead her to link energy with amplitude, an idea from classical waves. If she then remembers the general shape of the wave function for this scenario, which her sketches suggest she does, we can understand why she is adamant, during her first interview, that tunneling particles lose energy, and why she struggles, during her second interview, to reason about the wave function in when she knows that energy is conserved.

Selena seems to characterize the particle by its wave function and wavelength. Thus, if the particle has not changed, neither can its characteristic features. Her insistence during the first interview that the wavelengths of the two portions of the wave function are the same because “it’s the same

particle” may be an artifact of her quantum physics training, as many books and lecturers discuss an object’s deBroglie wavelength as a means of deciding whether to analyze it with classical or quantum physics.

Finally, we see little evidence that Selena’s belief in quantum physics as an accurate model of the world has changed over her undergraduate experience.

## **6.5. Themes**

Though each of the students studied had unique perspectives on the tunneling scenario, faced different challenges in reasoning about our questions, and used different explanations, we note that there are similar patterns that are observed in all three. In the absence of instruction on quantum physics, there was a tendency to gravitate back towards earlier ideas, even if a change had been present during course instruction. Sketches of the wave function often led to confusion over the representation of the wave function, leading students to make improper connections. Finally, in all three students, there is a lack of coherence between various representations.

### **6.5.1. Reversion**

The first interviews with these students were scheduled in the semester following their first quantum physics course. Since this was likely their first exposure to some of the strange conclusions of quantum mechanics, we wanted to let them synthesize their ideas from the entire course before being asked to use the ideas to explain phenomena. The second interviews were all several months after the senior-level quantum physics course. Because there

had been an increased emphasis on the discussion of the tunneling scenario in their class, we wanted to see how many of the new ideas remained some time later.

We observe a tendency to revert back to earlier, more established ideas, even if newer ideas were presented on exams during the term of instruction. In the first round of interviews with these students (and others), there was a tendency to use ideas from classical physics to answer our questions, something that is not surprising given the level of experience they have with Newtonian ideas versus quantum ideas.

Reversion was evident in different ways in each individual. Adam never seemed to struggle with the energy-loss misconception, though he did discuss it as an alternate idea during the first interview. However, in both early probes of Adam's understanding, he used the somewhat circular reasoning that energy was the same because the "frequency" was the same. In the presence of direct instruction on the topic in the senior quantum course, his reasoning evolved to include discussion of energy conservation principles, comparing the kinetic, potential, and total energies, and reason that the wavelengths of wave functions would be the same because of energy conservation. In the final interview, though, he had gone back to connecting energy and frequency.

Jack seemed to learn the answer to the energy questions, and stick with that reasoning on subsequent questions, even developing a coherent

explanation in connection to the wave function. His wave function sketches, however, showed evidence of reversion. Though he was initially unsure of whether “energy” was an appropriate label for the vertical axis of a wave function sketch (no doubt aided by the fact that he superimposed his wave function sketch on a potential energy barrier), he moved on to discussing and labeling his sketches as the square of the wave function, or “ $|\psi(x)|^2$ .” During the final exam, he labeled the axis “ $\psi$ .” Later, during the final interview, he once again described his drawings as showing the “square of the wave function,” claiming that was what he was used to discussing and sketching.

Selena’s reversion is most evident in her drawings of the wave function. Her first interview sketch exhibited the “axis-shift” characteristic, and was superimposed on a potential energy barrier. During direct instruction in her senior year, the sketches improved as have been previously described. However, in the final interview, Selena once again drew her wave function on top of a potential energy barrier, and the sketch again exhibited axis-shift properties, though she did not in that session indicate energy loss.

### **6.5.2. Connections**

All three also seemed to struggle with connecting ideas in appropriate ways. At one point or another, all three wrestled with whether or not it was appropriate to connect the vertical axis of a wave function sketch with energy. Though Adam and Jack eventually said this was not appropriate, Selena seemed to think it was all right.

There was also evidence of connecting the term “barrier” to a physical object, and mixing ideas of position and energy. Adam talked about an actual concrete barrier, while Selena used work ideas to reason that wider barriers take more energy. At some point, all sketched wave functions oscillating about some level coincident with the given energy level (often in conjunction with superposition of sketches of wave functions and potential energy diagrams). Jack explicitly talked about the “distance” between energies, where movement to a higher energy is spatially higher as well.

There seems to be a fundamental lack of understanding potential energy as an interaction between two or more objects. Often, the subjects talked about the barrier’s energy and the particle’s energy, as if the barrier were the object, rather than a graphical representation of the increase in potential energy of the particle and some outside system. This perhaps stems from difficulties and/or inexperience in interpreting energy diagrams.

### **6.5.3. Coherence**

Adam, Jack, and Selena all seem to have compartmentalized their knowledge in regards to the square-barrier tunneling scenario. Though they answer questions about the probability of tunneling, the energy of tunneled particles, and what the wave function looks like in various situations, they have difficulty connecting the concepts. As was previously mentioned, all at one point or another struggled with whether the wave function had units of energy.

We wrote the Schrödinger equation protocol to try and address this lack of coherence. The style of questions in previous surveys and interviews was perhaps sufficiently different from “textbook” problems that the students did not automatically use mathematical reasoning in addressing the presented problem, and tried to reason about situations conceptually, without using any mathematical tools. Because this conceptual understanding was undeveloped, they had difficulty making coherent sense about the situation. We thought that by starting an interview with the mathematical solutions to the Schrödinger equation for various scenarios, students might use the equations to reason about the probability density, energy, or shape of the wave function.

There is little evidence that they made this connection. Although he’d written down a solution to the Schrödinger equation containing sine and cosine terms, including reference to the energy difference ( $E - V$ ), Adam could not determine whether the wave function’s wavelength should change when the potential energy on the far side of the barrier was lowered, even when I asked him explicitly to refer back to his solutions:

I: And how would the wavelength, uh, compare in regions A and C?

A: No, that was, that was about something back here in the very beginning. The wavelength should be the same.

I: It should be the same in both?

A: Yep.

I: Is that consistent mathematically with, I’m assuming, again in C, it would be this, um, form right here (indicating the mathematical solution he’d previously written)?

A: Um, yeah. Is that consistent? The quantity you’re looking at is  $E$  minus  $V$ . I think what I’m concerned, with, though, is that, this total energy is the same. So that’s the question. Does the wavelength come from the kinetic energy? Or does it come from the total energy? And that’s

what I don't recall. If the wavelength is connected, or married to the total energy, then the wavelength is going to be the same in every scenario. If it's related to the kinetic energy, it's gonna change dramatically depending on where, what the level of potential energy is.

I: And not sure which one it's related to?

A: I'm not sure, no.

Though Jack was eventually able to figure out the change in wavelength for the same scenario, he struggled with whether or not the probability of detecting a particle in this region would change:

I: OK. How does the amplitude now in this, you know, post-barrier, um, compare with the amplitude that we had over here? The same amplitude, or might be different, or...?

J: Should be the same amplitude, roughly. Actually, it could be a little different, but... I'm guessing it would be the same amplitude.

I: So, equally likely to find the particle here as here, in these two scenarios.

J: Like, in the whole region...

I: Yeah, right, right, right. Not at a specific point.

J: Yeah.

Though he had correctly reasoned about the energy difference, he did not connect this to the idea that a particle with greater kinetic energy will be less likely to be observed in a region.

Even though Selena seems to know that the energy difference (in reality the kinetic energy) is related to the probability density, and hence the amplitude, it's not evident that she's putting all of those pieces together:

I: ...are these the same amplitudes? I mean, should they be, or should they be different?

S: They should be different because the potential is different. And that's pretty much what determines the amplitude there, the relative energy difference between the particles and the barrier.

I: All right. In this case with the lower potential over here, is the amplitude going to be greater than or less than what it was over here?

S: I think it would be greater than.

- I: And why so? Greater in this case?
- S: Mmm-hmm.
- I: OK, and why would it be greater?
- S: Because the difference between the energy of the particles and the energy of the barrier is, is so much larger in this case.
- I: So, is amplitude somehow then tied to that difference?
- S: Yeah, cause the, the number, the actual number of particles that are going to get through, which is the amplitude, is determined by the barrier size, in relation to the energy of the particles.

## 6.6. Summary

The standard scenario of tunneling through a square potential energy barrier is difficult for students to reason coherently about. There is some evidence that direct instruction can correct some misconceptions (*i.e.* energy loss), but two courses seems insufficient time to build a solid conceptual understanding, and tie that understanding into mathematical solutions. Additionally, there is evidence that in many areas, students revert back to previous ideas after instruction.

## Chapter 7

### TEACHING TUNNELING TO NON-SCIENCE MAJORS

One goal of physics education research is to apply results of investigations into student learning to curriculum modifications that help students learn the physics better. As discussed in Chapter 6, initial findings on student understanding of tunneling were used to modify the instructional emphasis of one professor in one semester of the undergraduate quantum physics course at UMaine. However, departmental factors did not allow for introduction of quantum physics tutorials or other significant curriculum modifications in the two quantum physics courses taken by physics majors. Instead, we were presented an opportunity to work with a population of general education students. This chapter details the work done in that venue.

In the first section of this chapter, we describe the course and population of the course where tunneling tutorials were introduced. In the second section, we summarize the goals of the course activities that precede the tunneling tutorials, so that the reader may make her or his own judgment on the feasibility of the tasks presented, as well as have some understanding of the specialized course language. In the third section, we describe in detail the iterative process of writing and revising the tutorials that deal specifically with tunneling. Finally, in the fourth section, we analyze the pretest and posttest results from this population.

## 7.1. Overview of Descriptive Physics

### 7.1.1. Original course structure

In the fall of 2003, members of the Physics Education Research Laboratory began instruction in one of the department's service courses, *PHY 105*:

*Descriptive Physics*. Our goal was to use a non-traditional approach to try to teach a population of students with non-science majors ideas about quantum and modern physics, in contrast to most introductory courses that survey basic physics ideas that are hundreds of years old. A central tenet of the course was that learning about the nature of science would be more interesting in content areas that included counter-intuitive and contradictory material. The course was designed to fit into the existing PHY 105 format.<sup>1</sup> Rather than the lecture driving the development of topics within the course, the laboratory was the main venue for introducing new ideas, with lecture instead focusing on critical thinking skills, model-building, and discussing ideas from the lab.

The course consists of three one-hour lectures each week, and a single three-hour tutorial-laboratory period. Though some published and non-published curriculum from other institutions was modified and used in our tutorial-labs, current members of the research group wrote much of the course material.

In the fall of 2003, two sub-groups worked on authoring alternating tutorials. Each week, the entire development group undertook a trial run of

the tutorials, and edited and modified them as necessary. Finally, the tutorials were given to the students.

A single graduate student in the research group served as the instructor for each tutorial-laboratory section. Most of the tutorials were 18-20 pages long. We generally felt the tutorials were too long, but we found that students were finishing the tutorials in less than the allotted time. The instructors found it challenging to keep up with all groups and facilitate their progress in a useful way.

#### **7.1.2. 2004 modifications**

We revised the structure of the tutorial-laboratory periods in 2004. In addition to re-ordering the presentation of topics within each week, we shortened the tutorials, and introduced “board meetings.” During these sessions, groups answer one or more assigned questions, then gather as an entire class to discuss their responses and critically analyze those of the other groups. There were usually two board meetings per laboratory period. Students submitted weekly reflective essays in addition to the homework.

#### **7.1.3. 2005 modifications**

We undertook additional minor changes in the fall of 2005. We kept the board meeting sessions, but re-wrote or revised many of the questions for these sessions. The tutorials underwent minor revisions, most notably in an attempt to replace series of redundant questions with tables useful for explicit side-by-side comparison, and introducing additional sense-making tools. We

give examples of each change later in this chapter. In addition, a second volunteer instructor was added to each section to increase the level of interaction between student groups and instructors. The weekly reflective essays were replaced with posttests for most topics to provide the instructional staff with immediate feedback on the level of understanding students gained in the weekly tutorial.

## **7.2. Description of Concepts and Topics in Tutorials**

While the focus of our work in the context of this research project was the tunneling tutorials that are taught near the end of the course, the teaching materials on tunneling cannot be approached as stand-alone exercises for this population of students. Rather, the sequence of ideas introduced in the course is carefully crafted to give students the resources needed to address several quantum mechanical problems, including tunneling. In the following sections, we briefly discuss the content of each tutorial, and in the process introduce the course-specific language and tools that students have available to try to address the problem of quantum tunneling. We will refrain from pointing out which tutorials were developed by specific group members. Instead, we wish to emphasize the entire curriculum that we have developed.

### **7.2.1. Tutorial 1: Seeing the same thing as other people**

Our first tutorial starts off with activities adapted from *Light and Shadow*, a tutorial authored by the Physics Education Group at the University of Washington.<sup>2</sup> Students use masks with various shaped holes and different-

shaped bulbs to build a model that predicts shapes observed on a screen for various bulb and mask combinations. The key ideas students develop in this section are that light travels in a straight line, light waves can move through each other, and one can model complex light sources as a series of point sources, then use superposition to determine a final observed shape.

The first board meeting asks students to explicitly state the components of their light and shadow model, and explain what evidence they have for various parts of the model. They also critically analyze common misrepresentations of the model.

The second half of the tutorial continues with the idea that light moves in a straight line, utilizing slits cut in masks along with mirrors to develop ideas about the law of reflection and range of sight. The tutorial introduces two image location techniques. The first, called the “Mel and Taylor” technique,<sup>3</sup> develops the idea that an object is found at the intersection of two distinct lines of sight. The second, the method of parallax, uses the observation of the relative position shift of two aligned objects to reason about the relative position of the unknown object.

The final board meeting asks students two questions. In the first, they’re asked to think about how their ideas change, and what advice they could give fellow students who have incorrect physical ideas. In the second, students are asked to detail the few core principles that govern the behavior of light, based on observations they have made during the tutorial period.

### 7.2.2. Tutorial 2: Superposition and interference of waves

The second tutorial adapts materials taken from the *Activity-Based Tutorials*.<sup>4</sup> It begins with class-wide demonstrations of wave pulses traveling on springs. Students then return to their tables and view several video clips of wave pulses on springs that allow for frame-by-frame analysis. The exercises lead them to the conclusion that wave speed is independent of pulse shape or amplitude, but can be affected by the tension on the spring and/or the spring density.

Next, they are introduced to the superposition of two wave pulses. Motivated by another video clip showing two pulses overlapping, students figure out that wave heights of overlapping pulses add either constructively or destructively, and practice this model with a theoretical, ideal representation.

The first board meeting asks groups to discuss both points. In one question, they are asked to respond to a student that argues that more force by the hand will produce a faster moving pulse. In the second, they must figure out possible wave interference scenarios for given pulses.

In the second half of the tutorial, adapted from the *Tutorials in Introductory Physics*,<sup>5</sup> students view the wave pattern created by a point source in a wave tank, then reason about circular wave fronts. They are introduced to wave vocabulary such as *period*, *wavelength*, *crest*, and *trough*. Using diagrams of circular wave fronts, they then work with the idea of two overlapping wave

patterns, and learn about constructive and destructive interference and the patterns created by those lines.

The final board meeting again has two tasks. In the first, students imagine they are a “blind floater” in a large wave tank, and reason about what their motion or the lack thereof could tell them about the number of wave sources in the tank. In the second, they compare and contrast the behavior of water waves to light, which can be described as a wave.

### **7.2.3. Tutorial 3: Analogies connecting light and waves**

The third tutorial again contains elements from the *Tutorials in Introductory Physics*.<sup>6</sup> Students are introduced to the consequences of a wave model of light. After reviewing the key ideas from the previous tutorial about constructive interference and nodal points/lines, students work with creating parallel wave fronts in small wave tanks at their tables. Various width barriers are used to allow construction of narrower and narrower openings, and students observe that the narrower the opening, the more the waves post-barrier behave like they are emanating from a point source.

In the first board meeting, students discuss ray diagram representations of water waves. Additionally, they once again compare and contrast light waves and water waves.

The second half of the tutorial begins with having students observe what happens to parallel wave fronts that pass through two narrow slits using their table wave tanks. Because careful observations are difficult to make due to the

reflections from the side of the tank, students then work with a diagram of the overlapping pattern of crests and troughs to identify maximum constructive interference lines and nodal lines.

They then shift from a top-down representation to a perspective view, and the “gray barrier” is introduced into the far end of the tank. Students reason

about  
the  
pattern  
created  
on this  
gray  
barrier,

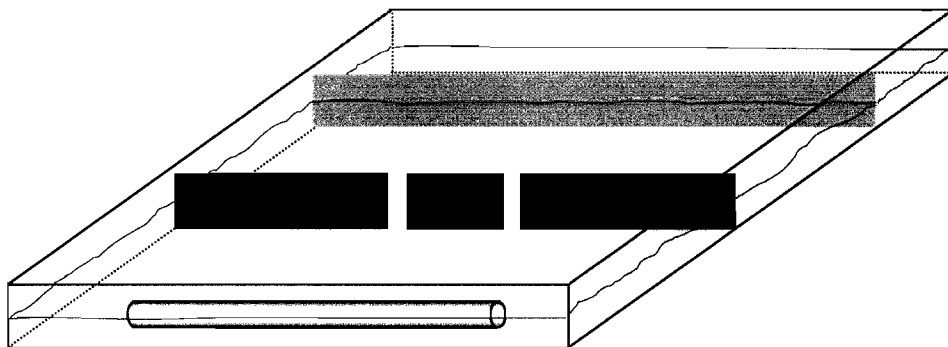


Figure 7-1: Perspective-view diagram of the wave tank and gray barrier.

shown in Figure 7-1, by inked water over time. They share their predictions in a board meeting with their peers. At the end of this process, they ideally come to a consensus that the gray barrier will show “humps” at intersection of regions of constructive interference with the barrier, while the intersections of nodal lines will leave the water surface undisturbed.

Students are then asked to reason about the affect of shining a laser through two slits in a mask. Predictably, given their ideas from the first tutorial, many predict that two lines of light will be viewed on the screen, and are surprised when they observe an interference pattern on the far wall demonstrated by passing a laser beam through two narrow slits. The

subsequent questions lead them to use their water wave ideas and the gray barrier analogy to reason that light must be a wave, and is interfering with light waves from the other slit. They then predict the effects of moving the laser closer to the slits, moving the screen closer to the slits, and changing the slit spacing, each of which they then check experimentally.

The tutorial concludes by asking students to consider how the pattern they observed would change if one of the slits were covered. They do not experimentally check this, as we do not wish to further introduce the idea of single-slit diffraction. Rather, the goal is to have students realize that the interference pattern they have observed is a result of two sources of light interfering. The final board meeting asks students to discuss the “bending” of light, as well as to compare and contrast their water wave model with their light interference model.

#### **7.2.4. Tutorial 4: Doing impossible things**

This and subsequent tutorials were developed at the University of Maine without reference to previous instructional materials. This tutorial begins with a brainstorming board meeting, asking students to free associate ideas that come to mind connected with the terms “photon” and “electron.” They then reason about what would be observed in four scenarios. In the first, a paintball hits a wall. In the second, paintballs are fired through a mask with two rectangular holes held in front of a wall. In the third and fourth, students revisit the previous week’s gray barrier scenario, recalling what would be

observed on the barrier for an unrestricted series of waves, and the same waves that first pass through two narrow openings. Additionally, they review the two-slit interference pattern that they observed the previous week.

In the next section, the idea of a photon as a small “particle” of light is introduced. Students reason about the pattern shown in Figure 7-2, which is described as the pattern created by an extremely low-intensity laser beam that passes through two narrow slits. They are asked to identify which part of the

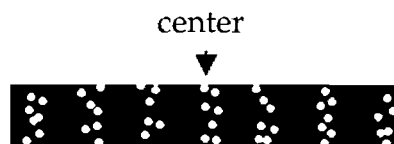


Figure 7-2: Photon pattern for low intensity light passing through two narrow slits.

pattern is consistent with their water wave model, and which part is consistent with their “paintball” model of particles. Additionally, they reason about how the pattern would be different if one of the slits were covered.

In the next section, students reason about patterns of electrons. They are told that an electron beam is incident on a two-slit apparatus, and that the top picture in Figure 7-3 is observed. They are asked whether or not they identify a pattern in the picture. Most see this as simply a random distribution.

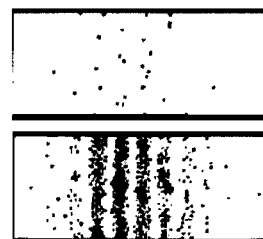


Figure 7-3: Electron interference pattern after a short time (top) and later (bottom).

A board meeting is placed in the middle of this section, asking students to consider three questions. First, they are asked whether their ideas about electrons were similar to their ideas about photons in the previous section. Second, they discuss whether or not paint balls

would behave as the electrons did. Third, they are asked to think about and try to explain a situation involving a laser that emits only a single photon at a time, yet the photons are observed to never land in the dark bands. The last question is used to try and gauge how facile they are with the ideas up to this point, and whether they are seeing the strange predictions of these introductory quantum ideas.

Following the board meeting, they are shown the electron pattern shown in the bottom picture of Figure 7-3. They are again asked to identify wave-like and particle-like aspects of this pattern.

A computer simulation of two-slit electron interference<sup>7</sup> is used in the next section to illustrate how the pattern forms over time. For the first time in our course, they use a histogram to illustrate regions of the observed pattern where many, few, or no electrons are hitting. They then compare and contrast the histogram they have sketched to the picture of the ink on the gray barrier from the previous tutorial.

After asking how the electron pattern would change if one of the slits was covered up, the tutorial concludes with another board meeting. The goal of this session is to have students consider where their previous models apply and where they break down, as well as again consider the fundamental strangeness of their observations. They compare and contrast electrons with water waves and paint balls. Additionally, they are asked whether the wave-particle nature of electrons bothers them. Since we have observed that many

aren't troubled by these ideas, they are also asked to suggest why it might bother other people.

#### **7.2.5. Tutorial 5: Probability<sup>8</sup>**

The first part of this tutorial deals with three classic probability experiments; drawing various types of balls out of a bin, flipping coins, and rolling dice. The first establishes the various ways probability can be represented. With the coin experiments, students begin to compare theoretical and actual outcomes. They also discover, by combining their individual results with those of first their tablemates, and later the rest of the class, that the actual outcome of the experiment moves closer to the theoretical prediction. The two dice experiments involve analyzing most probable and least probable outcomes for a system.

The first board meeting contains three questions. The first reintroduces the histogram from the previous tutorial's two-slit electron interference pattern, and asks students to discuss where the next electron is likely to land. The other questions deal with the two dice experiments, asking students to discuss the gambler's fallacy (the idea that previous rolls somehow influence future outcomes), and whether the theoretical or actual histogram is a better predictor of future rolls.

The next section of the tutorial links probability ideas with physical systems. Students begin by addressing the case of a man tossing a ball in the air, and predicting in which of three vertically stacked regions of equal height

the ball is most likely to be observed.

For the first time, we introduce the idea of analyzing multiple photographs, asking the students to think about taking 100 photographs of this system over equally spaced intervals, then sorting them in piles by the region in which the ball is found.

After comparing their results with their tablemates, they then analyze a video of a ball being tossed in the air (shown in Figure 7-4), and advance the video frame-by-frame to count the number of times the ball is found in each region. By dividing each number by the total number of frames, they determine the percentage of the time that the ball is found in regions A, B, and C.

Next, the students reason about a cart on an air track, connected with springs on each side, oscillating back and forth between five regions of equal width. The multiple photographs idea is revisited, but this time in the context of a series of 100 photographs taken at *random* times. After discussing their individual predictions, the students view another

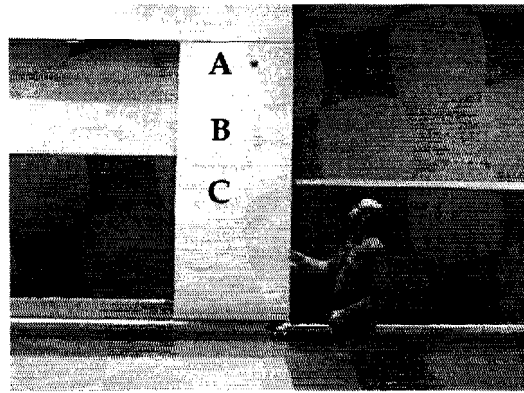


Figure 7-4: Frame from a video activity where a ball is tossed in the air, and students observe the number of frames the ball is found in each region.

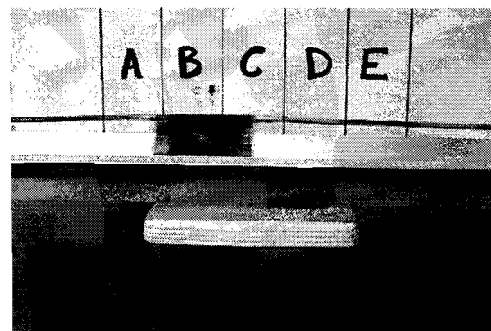


Figure 7-5: Frame from a video activity of an oscillating cart on an air track.

video clip of the system, and count the number of times the center of the cart is observed in regions A-E, as shown in Figure 7-5.

In the final board meeting, students discuss the similarities between the histograms they have created for coin flips and dice rolling and the histogram from the previous tutorial's activities involving electron interference patterns. For the first time, they are asked to think about one of the touchstone models for later parts of the course; a cart on a flat surface between two rigid walls that travels with constant speed in a given direction until it elastically collides with a wall and is sent back in the opposite direction. In this meeting, students construct a histogram for this system and compare it with the histogram for the oscillating cart. Finally, they are asked to think about the tossed ball experiment, this time with photographs taken at random times, and to discuss how, if at all, this changes their previous observations.

#### **7.2.6. Tutorial 6: The energy of motion and the potential for energy**

This tutorial begins with another brainstorming session, asking students to discuss what energy is, and how one would measure energy. Once these ideas are shared, students return to their table groups to discuss energy in the context of situations where the mass of one object is larger than another, though they have equal speeds, and vice-versa. They are then presented the formula for calculating kinetic energy and practice calculating the kinetic energy of a baseball and basketball.

Students return to the idea of wave pulses first studied in the second tutorial. Students have previously reasoned that higher amplitude but equal width pulses are the result of faster hand movements on the part of the person creating the waves. They thus link the amplitude of wave pulses on springs to energy, and revisit the idea that the pulse speed is independent of the amplitude.

In the context of analyzing wave trains (a series of wave pulses on a spring), students are first asked to work with graphical representations for ideas. We introduce “picture graphs” (“snapshots” of the system that show the behavior of every position in the system the system at a single instant of time) and “story graphs” (graphs that tell the “story” of what is happening at a single location in the system over time). Students sketch picture graphs for wave pulses on a spring, as well as story graphs for a single point on the spring.

In the second board meeting, students analyze how the speed of the wave train, motion of a point on the spring, and the required hand motion used to create the waves change if the tension of the spring is increased.

In the following section, we provide students with the energy-frequency relationship,  $E_{\text{photon}} = hf$ , as well as the wave speed relation  $v = f\lambda$ . They use these ideas to deduce whether red or green photons have greater energies.

We next introduce a first-order definition of potential energy as related to the ability of an object to increase its kinetic energy. When students reason

about a falling ball, this seems to work reasonably well. However, the definition is not so satisfactory in the context of a falling feather, which for most of its fall is moving at constant speed. We then introduce the definition for gravitational potential energy,  $PE = mgH$ , as a refinement to our first definition. Students calculate the potential energy for six balls at various heights and moving with various speeds.

In the final board meeting, students use the new definition of gravitational potential energy to further analyze the falling feather case. Additionally, they are given a picture graph of linearly increasing gravitational potential energy, and asked to create three distinct story graphs that could match this picture graph.

### **7.2.7. Tutorial 7: Combining ideas about probability and energy**

The tutorial begins with several activities involving a cart on a ramp. First, students think about how the speed changes as the cart accelerates down the ramp, and how the height above the table changes. Using these ideas, they sketch story graphs for the speed, kinetic energy, height, and gravitational potential energy of this system. Motion sensors and software (we use PASCO motion detectors and DataStudio™ software) are used to check their predictions, and students make any necessary corrections. They then predict what the picture graphs look like for this system (the definition of “picture graph” is modified from the “snapshot in time” idea to determining the speed,

energy, etc. at every position along the ramp), and once again use the computer to check their answers.

Next, they repeat similar experiments with a cart going up and down the same ramp. Through these activities, they observe that the story graphs for all quantities have changed, but the picture graphs look identical to those that described a cart going down the ramp only. Additional questions revisit the idea of taking a series of pictures of the system at random times, and whether the car would be more likely to be observed in the top half or bottom half of the ramp.

The students then are asked to reason about a system where the same cart now travels horizontally on a track. This time, however, the cart is holding a set of bar magnets, and for part of its journey travels through a region where bar magnets at the sides are oriented in such a way that they repel the magnets on the cart,<sup>9</sup> as shown in Figure 7-6. They describe what the motion of the cart will be like, and sketch picture graphs of kinetic and potential energies. They then use the motion detectors to check their predictions. Our goal with this activity is to

introduce a system with a region of increased potential energy that is not gravitational potential energy; we revisit this idea in the tunneling tutorials, described later in this chapter.

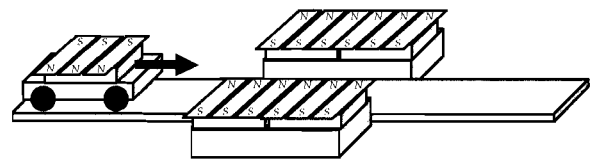


Figure 7-6: Diagram of a magnet cart traveling through region of increased potential energy.

The first board meeting asks students to compare the picture graphs and story graphs for the systems they have studied thus far. The goal is to get them to discuss the differences, and further differentiate between the two types of graphs, as for many systems the two graphs have similar characteristics. In a second question, they reason about how their pictures would change if the cart traveling up and down the ramp kept repeating that motion.

In an effort to have students apply their ideas to new systems, and again focus on a form of potential energy other than gravitational, we next introduce a vertical harmonic oscillator. The students observe the oscillation, and then predict what picture graphs of the kinetic and potential energies for this system look like. We do not expect them to sketch the parabolic shapes, but rather to reason that the kinetic energy is zero at the end points, and maximum at the middle, with the reverse for the spring potential energy. Using a harmonic oscillator system with a motion detector and force probes, students check their predictions. The potential energy graph is used to introduce the idea of a potential well. Additionally, we ask students to think back to the horizontal analog of this system - the cart oscillating on an air track in the probability tutorial.

The second board meeting asks everyone to analyze a student statement that the oscillating cart “likes to be in the bottom of the well,” so it is most likely observed in the center of the track. Students should recall that the cart

is actually most likely observed in the end regions, where the potential energy is greatest, and kinetic energy the least.

Next, we use the same physical system to introduce the “probability density” of finding an object in a certain segment of its motion as the probability of finding the object in a segment divided by the length of that segment. Students are asked to sketch a picture graph of the probability density of this system.

To further work with the definition, students are asked to think about a system where a series of balls travels on two different horizontal levels as they pass through a system,<sup>10</sup> as shown in Figure 7-7. They reason about the gravitational potential energy and kinetic energy of balls that pass through the system, and use those ideas to analyze and correct a student’s graph of the probability density of this system.

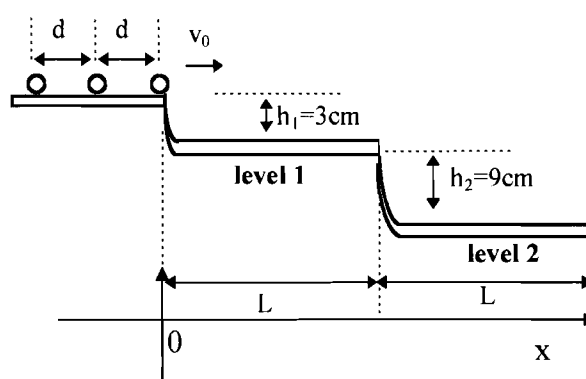


Figure 7-7: Diagram of the “balls on tracks” system.

### 7.2.8. Tutorial 8: Curviness

The goal of the curviness tutorial is to develop tools students can use to reason about functions that satisfy the Schrödinger equation. To do this, we

begin by asking students to reason about “Ed”<sup>iv</sup> driving down a curving road constructed of a series of semicircles. For various parts of the journey, they analyze whether the car is (from the point of view of an outside observer) to the right or left of various pylons in the middle of the road, and how the steering wheel of the car is oriented. We introduce the term “driving down the function” as a tool we then use to talk about various types of functions.

Students then work with a sinusoidal function. We call sinusoidal functions “s-functions” to help students begin their study of curves isolated from any previous ideas they have about sine curves (for example, that cosine curves are not sine curves and therefore not sinusoidal.) The language becomes more important when dealing with exponential curves, as discussed below. Students reason about the curviness of s-functions at various places on the function. We define positive and negative curviness as whether the steering wheel is pointing to the right or left as one drives from left to right down a function. Thus, for an s-function, the curviness is negative for all positive values of  $\sin(x)$ , and vice-versa. Students contrast the curviness of the s-function, where greater values of the function have greater curviness, to the initial road problem, where the curviness of the road is the same everywhere.

The first board meeting asks students to draw a variety of s-functions and non-s-functions, and comment on what characteristics define both categories.

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<sup>iv</sup> Ed is named for Ed Prather, the first Ph.D. student in physics education research from the University of Maine, and also a race car driver.

Following the board meeting, students work with a transparency sheet imprinted with various concentric circles to determine the value of the curviness at various points on enlarged pictures of (i) Ed's road, and (ii) an s-function. They are asked to return to their previous statements contrasting the two, and refine as necessary.

The second half of the tutorial introduces exponential functions, or "e-functions," defined as functions that always curve away from the horizontal axis. We use the term e-function to avoid a discussion of differences between curves with  $e^x$  and  $x^n$  behavior, a distinction students often fail to make. After answering questions about e-function characteristics, such as whether or not they can cross the axis, students use their transparency tool to determine the curviness at various points on an enlarged graph, and deduce that the function has greater curviness at higher function values, similar to the s-function.

Next, students fill out a reference table that contrasts s-functions and e-functions, specifically whether each function can cross the axis, is or isn't periodic, and whether it curves towards or away from the axis. Students use these ideas in the second board meeting, where they analyze ten graphs and determine whether they can be described as s-functions, e-functions, or neither.

The final section of the tutorial deals with the concept of "stitching" functions together. We state that functions must be connected "smoothly," so

that there are no “kinks” in the final stitched function. Students stitch a function together, and discuss regions where it behaves as an s-function, e-function, or neither, based on the previously described characteristics of each. Finally, they return to several of the graphs from the board meeting, and describe portions of each as s-functions, e-functions, or neither.

### 7.2.9. Tutorial 9: Bound states and more impossible things

This tutorial begins with stating our version of the Schrödinger equation, which was previously introduced in lecture:

$$-k \cdot (TE_{particle} - PE_{system}) \cdot \Psi(x) = Curv \Psi(x) \quad (7-1)$$

Students are told that  $k$  is some positive constant. They are asked what happens to the curviness of the wave function when the value of the function increases, and also what happens to the curviness of the function if the total energy of the particle is doubled while the potential energy of the system is kept constant.

An opening board meeting gives students an s-function graph, and asks them to sketch a corresponding graph of a new function with increased total energy. They also compare ideas on what will happen to the wavelength and amplitude of the wave function in this situation. If the curviness increases, the wavelength and amplitude should both decrease, consistent with a decrease in the probability of observing a particle with greater kinetic energy in a given region.

The next section connects the ideas of the wave function, introduced previously in lecture, and probability density. For various s- and e-functions, students sketch the corresponding probability density graph by determining the square of the absolute value of the wave function. Additional questions ask where a particle is most likely to be observed in a system based on analysis of the probability density graph.

Later, students fill in the table shown in Table 7-1 using the Schrödinger equation:

Table 7-1: Fill-in reference table for deciding whether a wave function solution is an s-function or e-function

sign of $-k$	sign of $(TE_{\text{particle}} - PE_{\text{particle}})$	sign of $\Psi$	sign of $\text{Curv } \Psi$	Does it curve toward or away from the axis?	is it an s- or e-function?
-	+	+	-	toward	s
-	+	-			
-	-	+			
-	-	-			

Thus, they determine that when  $TE_{\text{particle}} - PE_{\text{particle}}$  is positive, the wave function is an s-function; when it is negative, the wave function is an e-function.

After determining which of a set of functions corresponds to a given energy scenario, students move on to their first description of the potential energy of a system – in this case, a potential step. They analyze a situation

where the total energy of the particle is greater than the potential energy in both regions, and must figure out that the appropriate wave function is an s-function, but that its amplitude and wavelength change in the step region.

The last section of the tutorial introduces students to the case of the finite square potential energy well. From this point forward, we use a one-dimensional finite square well as a model of an atom. Though the system is obviously incorrect, many of the results from a 1-d situation are qualitatively applicable to the far more complicated 3-d situation. Piece-by-piece, students analyze the system in all three regions – to the left and right of the well, as well as inside the well. For each, they must figure out what type of function solves the Schrödinger equation in each region, then stitch the functions together to create an appropriate wave function graph for the lowest energy state. Homework questions ask students to sketch the solutions for the first and second excited states.

The final board meeting discusses the difference between this situation, where there is some probability of finding the particle outside of the well, and the example of the cart bouncing between walls, where we never expect to find the cart outside of the system. They are asked to summarize the rules we used to figure out that the particle could be found in unexpected places.

#### **7.2.10. Tutorial 10: Excited states and quantum physics**

The tutorial begins with reviewing the ground state solution from the last tutorial, as well as the first and second excited state solutions. Students then

analyze a new set of wave function graphs, deciding whether or not each could describe a physical system based on criteria established in lecture: the function must be an s- or e-function, and the wave function must go to zero as position goes to positive or negative infinity.

Next, students work with a computer simulation of a finite square well that sketches the wave function given an input energy.<sup>11</sup> They reason about whether or not the ground state energy can be equal to the potential energy of the well, then use the computer to narrow in on the value of the ground state energy.

Briefly, students step back from the computer analysis and compare their results to the touchstone problem of a cart between two walls. They reason that the energy ideas from this model are still applicable – the energy is still the same everywhere in the system – but that the probability density ideas are different. Thus, the cart between two walls is limited in its ability to model a particle in a finite square well.

Students then return to the computer simulation and determine the energy values for the first and second excited states. Additional states are analyzed in the accompanying homework. We also introduce the term “energy level” at this point.

In the next section, students use diffraction gratings to observe fluorescent light bulbs, and discuss which color bands are seen. They are then given energy values for the ground and first excited states in a system, and asked to

calculate the energy lost by an electron transitioning from the first excited state to the ground state. That value is then used to determine the wavelength and color of the emitted photon.

The second board meeting is again of the brainstorming variety, asking students how neon signs work. Following the board meeting, students observe sodium and mercury lamps with the diffraction gratings and identify the spectral colors. Additional questions develop the idea that the colors correspond to transitions between energy levels, not the energy values of the levels themselves.

The final experiment involves again observing the spectral lines of a fluorescent light bulb. Students calculate the energy of an ultraviolet photon, and then work through a series of questions involving the absorption of ultraviolet photons and the emission of red and teal photons. Using this reasoning, students are able to deduce which color band, red or teal, should be more intense, and verify this with observations.

#### **7.2.11. Tutorial 11: Molecules**

This tutorial uses the square-well ideas from the previous tutorial to model a two-atom covalent molecule (for example,  $\text{H}_2^+$  where two atoms share a single electron). Students sketch the wave function for a single potential energy well, and are told that introducing another potential energy well into the system, if it is sufficiently far away, has no effect on the wave function. A predictive board meeting follows, where students guess what will happen to

the energy of the ground state and the wave function as the wells are moved closer together.

A computer simulation<sup>12</sup> is next introduced, and used to observe the changes to the system as the wells are moved together. Students record the values of the ground state energies and the most likely observation location as the wells move closer. Additionally, they reason about the link between the curviness of the observed wave function and the value of the particle energy.

Next, students use the simulation to investigate excited energy levels. They observe that for two wells close together, the energy of the first excited state is near to the value for the ground state, and the wave function is merely inverted in one of the wells. Thus, they have an introductory model to explain energy level splitting. Subsequent observations of the second and third, and then fourth and fifth excited states reinforces this idea.

Though the two-well simulation sketches the shapes of the various wave functions, it does so by superimposing the wave function on top of the potential energy well, coincident with the energy value. We admit in the tutorial that this representation is less than ideal, and include several questions that remind the students of the problems with this practice.

### **7.3. Tunneling Tutorials**

In both 2004 and 2005, tunneling was the last idea discussed in the course. (An additional tutorial on applications of quantum ideas was originally last, but was incorporated into Tutorials 9 through 11 beginning in 2004.) In this

section, we describe the initial version of the tunneling tutorials authored in 2003, then how the tutorials were modified during the next two iterations of the course.

### **7.3.1. Design of the original set of tunneling tutorials**

Our overall goals in designing the tunneling tutorials were to have students use their quantum “toolbox” to analyze a new situation – a square potential energy barrier, then use these ideas to reason about physical phenomena. We chose to introduce students to the idea of a scanning-tunneling microscope, and later use tunneling ideas to address the possibility of alpha decay and to develop an understanding of widely disparate half-lives for various elements.

#### **7.3.1.1. First tutorial – theory of tunneling**

The first tunneling tutorial begins by asking students to recall the definitions they previously used for kinetic and gravitational potential energies, and use these ideas to analyze a bowling ball rolling up a hill. They are told that the bowling ball has 0 J of gravitational potential energy at the lower level, and 30 J when it is atop the hill. Initially reasoning about a ball incident with 40 J of kinetic energy, they use energy conservation to determine the speed of the ball at various locations in the system, and then sketch a picture graph of probability density. The tasks are repeated for the situation where the bowling ball has only 20 J of initial kinetic energy, and is thus unable to make it up the hill.

They then transition to thinking about an electron, but are told to use their ideas from classical physics. A system is described by:

$$PE_{system} = \begin{cases} 0 \text{ eV} & x < 1 & \text{Region A} \\ 30 \text{ eV} & x > 1 & \text{Region B} \end{cases} \quad (7-2)$$

Students sketch the potential energy diagram, and then draw a total energy line for a 40-eV electron on the same graph. Using conservation of energy, they determine the kinetic energy in each region, and reason that the electron can indeed be observed in region B. They sketch a probability density graph for this situation as well. They then repeat the process, analyzing a 20-eV electron instead, and determining that it cannot be observed in region B.

The tutorial then reminds students of the previously constructed Schrödinger equation, and asks them to now focus on quantum ideas in their analysis. They first consider an infinite wall where the potential energy is infinite for  $x > 0$ . After sketching the potential energy graph, they add a total energy line, then reason about whether or not it is possible to find an electron in either region, and if so, what type of wave function will exist in the given region. Students are then instructed to sketch the wave function in the appropriate region or regions. Because we want them to focus on reasoning about the wave function in the area where the potential energy changes, and because we do not introduce wave packets in this course, they are instructed to ignore how the wave function behaved to either side of the area of interest. Specifically, we don't want them to worry about how the wave function had to go to zero at either end, which our toolbox could not help them explain.

The next section repeats the same analysis procedure for a finite potential step, where the potential energy is 30 eV for  $x > 0$ , and the incident particle has an energy of 20 eV. Again, students sketch the wave function for this scenario, this time realizing that a solution does indeed exist in the  $x > 0$  region.

Next, the classical electron and quantum electron are compared. For both the infinite wall and potential step, students are asked to describe how the behavior of the classical and quantum electrons would be the same, and how they would be different.

We next introduce the potential energy barrier; students reason about a 20-eV electron incident on a system where the potential energy was 30 eV in the interval  $-1 < x < 1$ . The question sequence is similar to the previous cases, where the students sketch the total energy line, determine which type of function could exist in each region, and then sketch the wave function. Prior to sketching the wave function in the region to the right of the barrier, they are asked to analyze the following student dialogue:

Student 1: The function in Region O is going to be an s-function. Since the electron has the same total energy and the potential energy of the region is 0, just like it was Region M, we sketch the wave function exactly the same as we did in the first region – same amplitude, same wavelength, same curviness.

Student 2: No, you're wrong. Since the amplitude of the wave function decreased in Region N, there is less and less probability of finding the electron as the value of  $x$  increases. So the wave function in the last region will have a smaller amplitude and a smaller wavelength to reflect this decreased probability.

Students are asked whether they agreed with either student specifically in the context of the statements about amplitude and wavelength. Once they

decide on the correct elements of the wave function, they complete their wave function sketch. The section concludes by asking students whether they are more likely to observe electrons on the incident or transmitted sides of the potential barrier.

The final section in this tutorial deals with using deBroglie wavelength reasoning to determine when quantum applies. Students are given the relationship

$$\lambda_{deBroglie} = A \cdot \frac{1}{\sqrt{KE}} \quad (7-3)$$

and told that  $A$  is a constant that depends on the mass of an object. In a table, they are given values for  $A$  for three different objects; an electron, a bowling ball, and the earth. Provided with values of the kinetic energies of each, they are instructed to calculate the deBroglie wavelengths, and then asked whether or not quantum behavior could be observed in any of the three objects.

#### **7.3.1.2. Second tutorial – applying tunneling**

The second tutorial begins by asking students to review the square-barrier potential energy barrier from the previous week, and sketch the appropriate wave function for a 10-eV particle incident on a 30-eV barrier.

They are next asked to how the wave function changes in the three following situations; (i) if the particle energy were increased to 20 eV, (ii) if the barrier energy were increased to 50 eV, and (iii) if the barrier width were increased. For each scenario change, students sketch the potential energy diagram and added a total energy line, then answer questions about how (if at

all) the wave function changes in each of the regions. Finally, they sketch the new wave function.

To model the potential energy barrier of a nucleus, students next analyze a system of connected potential energy barriers, shown in Figure 7-8(a). For each region, they reason about what type of wave function would exist, then sketch the wave function for a 10-eV particle in this system. Next, they compare the results for this system to a single square barrier, shown in Figure 7-8(b), reasoning that there was less of a probability of tunneling in the latter case.

In the following section, the decay of uranium-238 is briefly described; the uranium decays to thorium-234, and an alpha particle is emitted. Students are then told that the potential energy for this system can be modeled by the graph shown in Figure 7-9. Students are asked how much energy a classical alpha

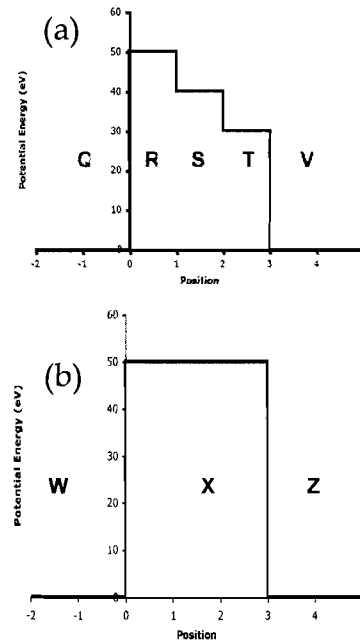


Figure 7-8: (a) Series of three square barriers of descending energy; (b) Single square barrier.

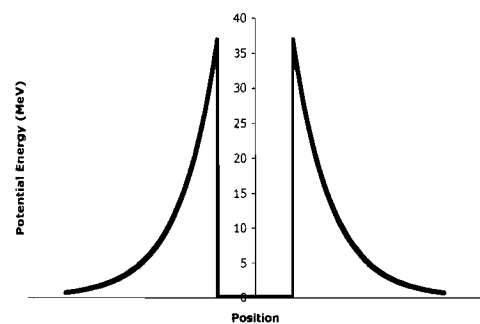


Figure 7-9: Model of the potential energy of a nucleus.

particle would need to escape from the nucleus, then told that it has been found that escaping alpha particles have energies ranging from 4 to 9 MeV, a fact that cannot be explained using classical physics ideas. Also, they are told to focus on analyzing just one side of the system, as the other is symmetrically identical.

Given this potential barrier, questions focus on the analysis of a 5-MeV particle. Students figure out the appropriate type of wave function in each region, then sketch the wave function for this system. Finally, they reason about how this sketch will change for a 9-MeV particle, which not only has a smaller ( $TE_{\text{particle}} - PE_{\text{system}}$ ) difference, but encounters a narrower barrier as well.

In the final section, students are given a table of half-life times, as shown in Table 7-2.

Table 7-2: Half-life values for various alpha particle emitting nuclei.

<b><math>\alpha</math>-emitting nucleus</b>	<b><math>\alpha</math>-particle energy</b>	<b>Average time to decay</b>
Polonium-212	8.8 MeV	$4.4 \times 10^{-7}$ s
Radon-220	6.3 MeV	79 seconds
Radium-224	5.7 MeV	5.3 days
Radium-226	4.8 MeV	2300 years
Uranium-238	4.3 MeV	$6.5 \times 10^9$ years

Using this table, students reason that the longer half-lives correspond to elements that emit lower energy alpha particles, which have a much lower probability of tunneling through the barrier.

### **7.3.2. Observations from initial use**

Following the initial use of the tutorial in the fall of 2003, we noted several problems with the tutorials:

- The instructors reported that the students were frustrated with the repetitive nature of the questions in the initial version, where they were asked to reason about the type of wave function in each region for a variety of different energy scenarios.
- To be answered completely and correctly, many of the wave function sketches and accompanying questions required students to refer back to their previous work. Anecdotally, the instructors reported that they rarely observed this.
- Although high numbers of students correctly answered written questions regarding energy and probability on the final exam, we observed relatively poor performance on problems requiring sketches of the wave function. These results are discussed in the next section of this chapter.

Because of these reasons, major revisions of the tunneling tutorials took place in 2004, which we describe next.

### **7.3.3. Revising the tunneling tutorials: 2004**

As was mentioned previously in this chapter, major portions of the tutorials were re-written in 2004, with the order of topics shifted slightly. Now, the tunneling tutorials were moved to be the final two tutorials in the

sequence. In addition, the tutorials were shortened, and board meetings were introduced into the format.

### 7.3.3.1. First tunneling tutorial: introducing applications sooner

The 2004 version of the first tunneling tutorial begins with a board meeting revisiting the double potential well scenario from the previous tutorial. Students sketch the wave function of the ground state, and then discuss the probability of finding an electron in either well.

To begin the tunneling activities, we kept the same opening activity regarding the bowling ball. The next section, where students analyze the predicted behavior of an electron using classical physics, was kept virtually the same as well, except that the model was now referred to as the “bowling ball” electron.

The next section received a large overhaul. Instead of asking students to sketch the wave function for various potential energy situations, we lead with a brief discussion of a scanning-tunneling microscope (STM),

accompanied by the diagram shown in Figure 7-10. Students are told that for

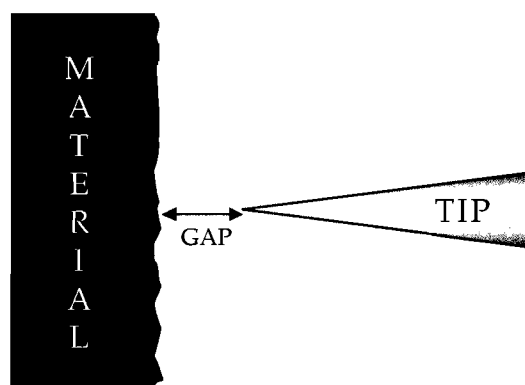


Figure 7-10: Diagram representing a scanning-tunneling microscope.

every 100 electrons that are on the surface of the material, 25 are subsequently observed in the tip. They first sketch a probability density picture graph for

this situation, and then reason about how that graph changes if the tip is moved closer to or farther away from the surface.

Next, the discussion turns to the energy of the electrons. Students are told that the system can be modeled with a 30-eV potential energy barrier, and asked what the electron energy will be in the tip. To further refine their ideas, they analyze the following student dialogue:

Student 1: The barrier's energy is higher than the electron's energy, so the electrons that make it through the barrier lose energy in the process. They'll probably lose about half, meaning the electrons that are in the tip will have about 10 eV of energy.

Student 2: No, energy is conserved, so the electrons in the tip will have the same energy, the same  $\Psi$  - everything about them is the same. That means the same probability density in the tip - wait, then shouldn't all the electrons make it to the tip? I don't get it.

With their groups, they analyze the components of each student's argument. It is only after reasoning about probability density and electron energy that the idea of sketching the wave function is introduced. Students address the ideas of changing the barrier energy or width in the context of changes to the STM system. Two questions at the end of this section introduce the common vocabulary of "tunneling" and "potential barrier," and ask students why these terms might lead to confusion or the idea that energy is lost.

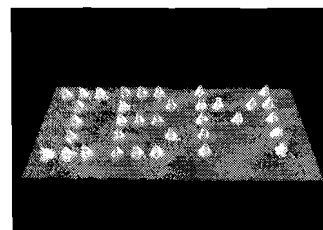


Figure 7-11:  
Representation of  
STM produced  
image.

In the second board meeting, students are shown a representation of an image produced by IBM engineers who moved atoms around on the surface of some material,<sup>13</sup> as shown in Figure 7-11. Students

are asked to describe how they could use a scanning tunneling microscope to probe this surface, and how the information gathered would allow them to deduce the pattern present.

Following a short section on the contrast between classical and quantum ideas, the tutorial concludes with the section on the deBroglie wavelength previously described. Two changes were made to this section. First, because of the mathematical difficulties we had observed in our student population the previous year, the steps required to calculate the deBroglie wavelengths of the electron, bowling ball, and earth were laid out in a table that led students through the calculation. Second, we added a graphical section at the end, where students first sketched the wave function corresponding to an electron tunneling through a potential energy barrier. Next, they used curvature arguments to reason how this sketch changed for an object with a deBroglie wavelength many orders of magnitude smaller, observing that the wave function in the barrier region decayed to near zero almost immediately.

#### **7.3.3.2. Second tunneling tutorial: reducing redundancy**

The initial board meeting in the second tunneling tutorial reviewed the square barrier reasoning from the previous tutorial.

Though the rest of the activities remained virtually the same, there were major changes to the format of the questions. Several pages of questions were replaced with two tables, one dealing with graphical representations, and the other with written representations.

The graphical representations portion of the reference sheet is divided into five columns: particle type, system potential energy, particle total energy, wave function, and probability comparison. The top portion of the table is filled in, with a standard tunneling situation. In subsequent rows, students are given changes in one or more of the columns, such as increasing the energy of the barrier or changing the electron to a proton, and are required to fill in the appropriate information in the remaining columns.

The written representations side of the reference sheet presents five changes to the system, such as changing the width of the potential energy barrier, or reducing the mass of the incident particle. For each, students are required to describe how, if at all, the wave function, energy of the detected particles, and probability of detection change.

The following section on alpha decay received minor edits. The energies of the three side-by-side potential barriers modeling the nuclear barrier were changed slightly, and students are now explicitly asked to consider the width of the barrier “seen” by the particle. After sketching the wave function and comparing to the model of a single square barrier, students analyze the width of the barrier “seen” by particles of two other energies. They are also asked why one would use the more complicated three-barrier model.

The final sections, asking students to sketch the wave functions for the alpha-decay scenario and reason about the tunneling probability for different

energy particles, was left intact. The tutorial concludes with the same half-life questions.

#### **7.3.4. Revising the tunneling tutorials: 2005**

Fewer changes were introduced in 2005. The format of the sessions, including board meetings, was kept the same. An additional instructor was present during the tutorials, as was previously noted.

Following the bowling ball scenario in the first tunneling tutorial, we re-visited the magnet cart activity previously described. After observing the demonstration and sketching the displayed kinetic energy picture graph, groups deduced the shape of the potential energy and probability density picture graphs. They then reason about what would happen if the magnet cart was initially pushed with half the energy, and whether there is any probability that the cart could make it through the center region.

The other change was to reformat the questions dealing with the probability densities for the scanning-tunneling microscope scenario. Now, the sketches are placed side by side with wave function sketches on another reference sheet.

Except for minor grammar or diagram changes, the second tunneling tutorial was kept in the same form as it was in 2004.

The 2005 versions of the tunneling tutorials can be found in Appendices J and L.

## **7.4. Measuring Effects of Tunneling Tutorials**

To help us gauge the efficacy of the materials we developed for the course, a pretest was administered each week (usually during a prior lecture). The pretests asked students to answer questions on the topics to be explored in the following tutorial. Posttest questions were usually in the form of exam questions on either one of the two prelims, or on the final exam. In 2005, we designed additional posttest activities to be administered during lecture immediately following the tutorial instruction, so that we would have more immediate feedback on student difficulties that persisted even after tutorial instruction. However, the tunneling posttest was designed to probe student understanding of curviness concepts, and is not discussed here.

In this section, we compare pretest results from the three years of instruction. We then present and discuss the results from a tunneling question that was included all three years on the final exam. Additionally, we discuss results from a 2005 posttest and an additional final exam question that shed insight on the level of student understanding of the situation.

### **7.4.1. Initial pretest: design and results**

The pretest (which can be found in Appendix I) that preceded the first tunneling tutorial dealt with two scenarios. In the first, students were shown a diagram of a small hill in an otherwise flat region, with a wall located at the right end of the system. Students were reminded of the definition of gravitational potential energy, and told that in this system, a ball would have

0 J of

gravitational

potential

energy on the

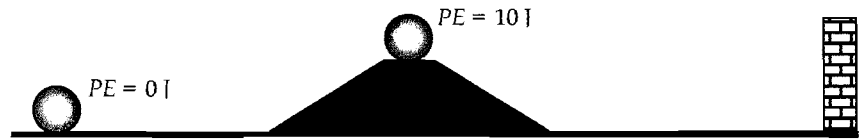


Figure 7-12: Diagram of ball-hill-wall system from the tunneling pretest.

flat surface, and 10 J of gravitational potential energy on top of the hill, as

shown in Figure 7-12. They were then told to think about a ball rolled into

this system from left to right with 5 J of kinetic energy, and asked whether or

not the ball would hit the wall on the far right.

Our expectation was that students would use the idea of conservation of energy to reason that the ball's kinetic energy would be converted into gravitational potential energy as it rolled up the hill, and because it only had 5 J of initial kinetic energy, it would not make it up the hill. Though they had not seen this particular problem before, they had dealt with the ideas of

kinetic and gravitational

potential energy in

several contexts,

including a cart sliding

up and down a ramp.

The percentages of

student responses are

shown in the chart in

Figure 7-13. We note

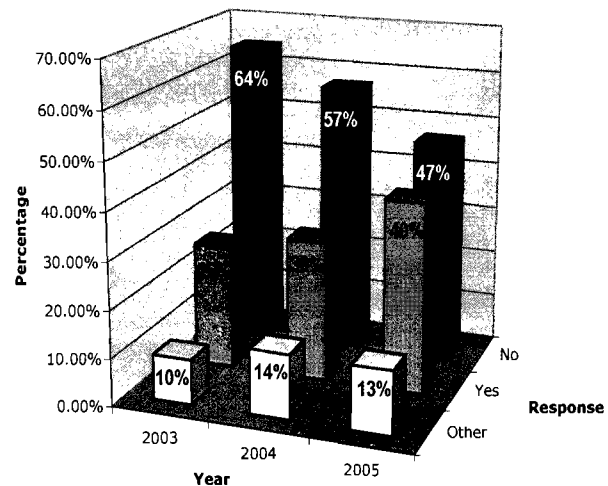


Figure 7-13: Results to the pretest question "Will the ball be able to hit the wall?"

that although changes were made in each successive iteration of the course to try to improve student understanding, we observed a decrease in the number of students who answer this pretest question correctly. Also, although the question was written to try and elicit a “yes” or “no” response, there is a small number of students each year who gave some other response or who left the question blank.

The question also asked students to explain their reasoning. Nearly all did so in terms of energy, which is not surprising, given that the problem statement contained several references to either kinetic or gravitational potential energy. For students who said the ball could not hit the wall, typical responses were “there’s not enough energy” or “the ball would need at least 10 J of kinetic energy to make it over the hill.” Students who said that the ball would hit the wall often stated that there was “enough energy,” and several talked about the ball’s energy would increase by the 10 J of gravitational potential energy shown in the diagram. This suggests that some of these students may view energy as a property of the hill to be “given to” the ball as it rolls past, reminiscent of some of the energy reasoning of advanced undergraduates during the interviews.

The second part of the pretest task asked students to consider a beam of electrons incident on a system where the potential energy was increased in some region, as shown in the diagram in Figure 7-14. Students were asked (i) to describe what happened to the electrons when they encountered this

region of increased potential energy, and (ii) whether any electrons would ever be observed in the region to the right of the barrier.

At this point in the course, students had been working with wave function solutions to various scenarios, most notably a finite square well. In addition, they had

used the Schrödinger equation to reason about the type of solution ("s-function," "e-function," or neither) in various regions, and had also seen some non-classical predictions of quantum physics, such as the idea that there was a non-zero probability of detecting electrons outside of the boundaries of the finite potential energy well.

There were a variety of responses as to what happens to the electrons in this situation. None of the students gave the right answer, that there is a probability that some particles will tunnel and be observed on the far side of the barrier. The leading responses are shown in Table 7-3.

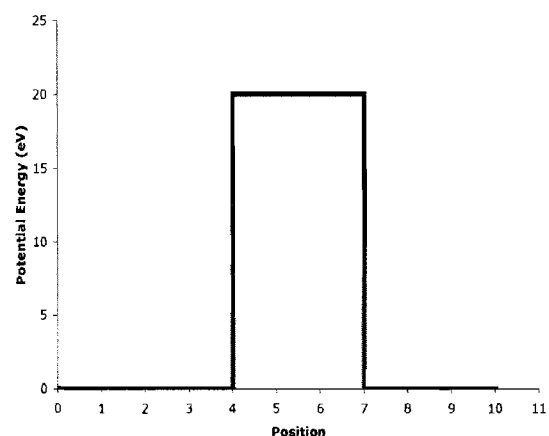


Figure 7-14: Potential energy diagram from the tunneling pretest.

Table 7-3: Common responses explaining what happens to electrons encountering a region of increased potential energy.

	2003 ( <i>n</i> = 42)	2004 ( <i>n</i> = 44)	2005 ( <i>n</i> = 30)
Energy increases	8	14	6
Energy decreases	7	11	3
Becomes excited	3	4	3
Speed increases	1	2	3
Speed decreases	1	9	2
Trapped/stuck	0	6	3

The leading answers were that the electrons' energy either increased or decreased; those who said the energy decreased used primarily energy conservation ideas. There was a small number of students each year who talked about the electrons becoming "excited" or moving "into an excited state." We believe this response comes from students trying to use ideas from previous tutorials where they worked on wave function solutions for excited states in the finite potential energy well. Other students talked about the speed of the electrons either increasing or decreasing. Another minority answer, given only in the last two years, was that the electrons were either trapped or stuck in the central region, which again may echo previous work they did involving bound and unbound states.

Student responses to the question of whether or not electrons could be detected on the far side of the barrier are shown on the chart in Figure 7-15. Again, 2003 had the highest percentage of students who answered this question correctly before any instruction on tunneling. Also, there continued

to be small numbers of students who answered in some other fashion, stated that they did not know, or left the question blank.

When explaining their reasoning, students again had a variety of responses. The most frequently seen ideas are shown in Table 7-4.

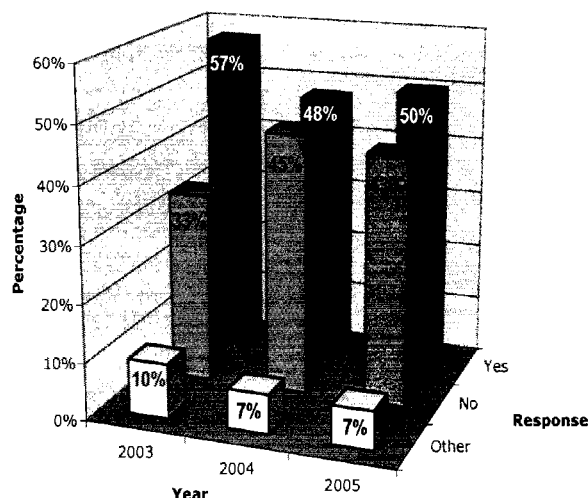


Figure 7-15: Student responses to the question "Will electrons be detected in Region C?"

Table 7-4: Common phrases from student explanations as to why electrons would or would not be detected on the far side of the barrier.

	2003 ( <i>n</i> = 42)	2004 ( <i>n</i> = 44)	2005 ( <i>n</i> = 30)
Energy	15	21	9
Probability	8	8	2
Wave function	4	5	2
"Well"	5	7	2

Students used energy both to explain why electrons could be detected, and why they could not; there was either sufficient or insufficient energy for this to occur. Others discussed the idea that there was a "probability" that the electrons could make it. Still others discussed the fact that the wave function could exist in that region, and therefore electrons would be found there. We saw several instances of students talking about the electrons going into or out

of a “well,” again tied to the scenarios that they had explored in previous weeks.

#### **7.4.2. Second pretest**

We wrote a second pretest to be administered following the first tunneling tutorial, but prior to the second tunneling tutorial. This served two purposes; first, as a check on the adaptation of the ideas from the first week’s activities, and second, to see how well they could extend those ideas to new situations, without the explicit instruction on how to do so. This pretest was administered in 2004 and 2005 only, and can be found in Appendix K.

The pretest shows the students a square potential energy barrier and an accompanying picture graph of the wave function. Students are told that the incident particle energy is less than the potential energy of the barrier region. They are then asked to think about a situation where the potential energy barrier is made narrower. For context, they are given the example of the STM tip moving closer to the surface from the previous week’s tutorial. They are asked to describe how, if at all, the wave function will change in any or all of the three regions, and to sketch the new wave function. Additionally, they are asked how the probability of detecting a particle on the far side of the barrier has changed, relative to the original situation.

The most notable finding in the analysis of student descriptions of changes to the wave function is the lack of persistent themes and the wide variety of explanations, covering such ideas as amplitude, energy, curvature (or

curviness), wavelength, the type of function, and other less definitive attributes. Several students remarked that the wave function in some region had gotten “shorter” or “smaller,” without being specific as to whether they were describing the amplitude, wavelength, or some other characteristic. The sketches of the wave function were often sloppily done.

Because of the less open-ended nature of the probability question, categorization of student responses is possible. Findings from that question, and comparison with students’ descriptions of the wave function in the third region are given in Table 7-5.

Table 7-5: Student responses regarding the probability of tunneling when the width of the potential energy barrier is increased.

<b>When the barrier width is decreased, the probability of detecting a particle on the far side of the barrier is...</b>	<b>2004 (<i>n</i> = 39)</b>	<b>2005 (<i>n</i> = 31)</b>
...increased.	54%	52%
...decreased.	15%	10%
...the same.	28%	35%
Wave function amplitude and probability reasoning mismatch	23%	23%

Although more than half of the students in both years correctly reasoned that the probability of tunneling increases when the barrier width has decreased, about a third of the students in each year reason that the probability remains the same, shortly after explicitly working with this identical situation in the tutorial. There also remain a significant number of students who are not consistent with their reasoning between questions on describing changes to the wave function and questions about the probability

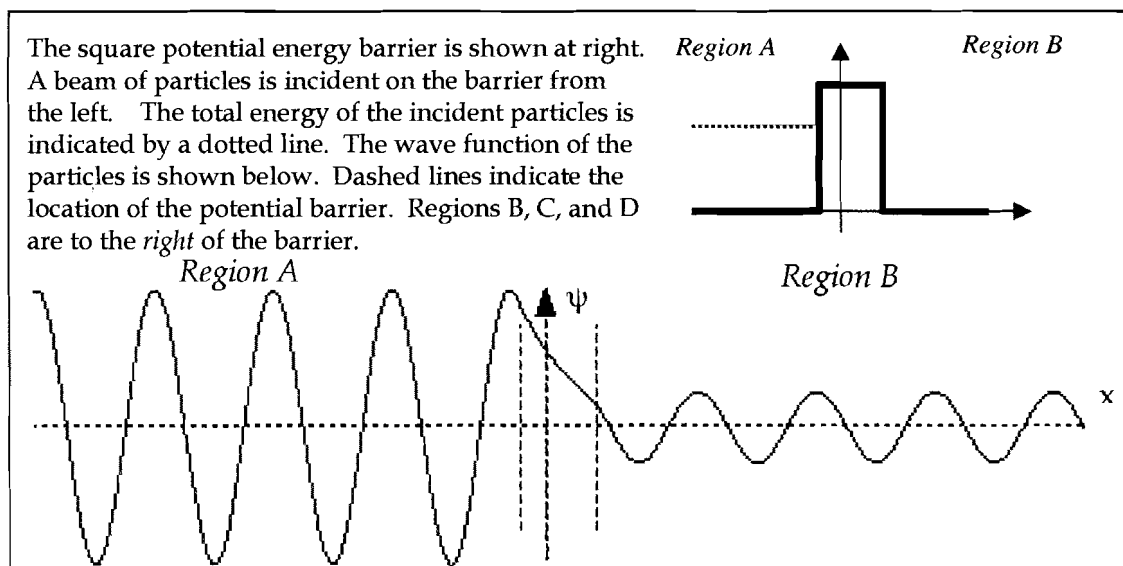


Figure 7-16: Scenario description for the final exam question on tunneling.

of tunneling. Students in this category did one of three things – stated that there was no change to the wave function, yet reasoned that the probability had either increased or decreased, stated that the amplitude of the wave function had decreased, yet the probability of tunneling remained the same or increased, or stated that the amplitude of the wave function had increased, but the probability had decreased.

### 7.4.3. Final exam questions

The first portion of the final exam question for all three years is shown in Figure 7-16. Students were given a potential energy diagram, and the sketch of the wave function that corresponded to this situation. Students were then asked to consider two changes to the system – the potential energy barrier made wider, and the potential energy of the barrier increased. For each, they were asked to sketch the new wave function, and explain their reasoning.

If the barrier is made wider, the exponential portion of the wave function in the barrier region decays further, so the amplitude of the wave function in the region to the right of the barrier is decreased further. Because the particles have the same total energy throughout, the kinetic energy is the same on both sides of the barrier, so the wavelengths of the sinusoidal portions of the wave function are identical. A similar answer is seen in the case where the potential energy of the barrier is increased, since the larger difference in potential energy and total energy yields an increased drop-off in the exponential portion of the wave function in the barrier region.

In analyzing the results, students were first coded in three categories – whether or not their answers were right (meaning that the sketch and explanation were both correct), whether the sketch was correct, but lacked sufficient reasoning, and whether or not their sketch and reasoning were consistent, regardless of correctness.

#### **7.4.3.1. Results – wider barrier question**

The results for the wider barrier question are shown in Figure 7-17. We note several things about these results. First, the numbers for answering correctly and sketching correctly are nearly identical. In each year there were only one or two students who sketched the correct wave function, and then provided incorrect or contradictory reasoning. Second, although in all years less than half of the students answered this question correctly, there was a 50% jump in the number of students answering this question correctly in 2004;

this result remained consistent in 2005. This is likely the result of a combination of factors, including

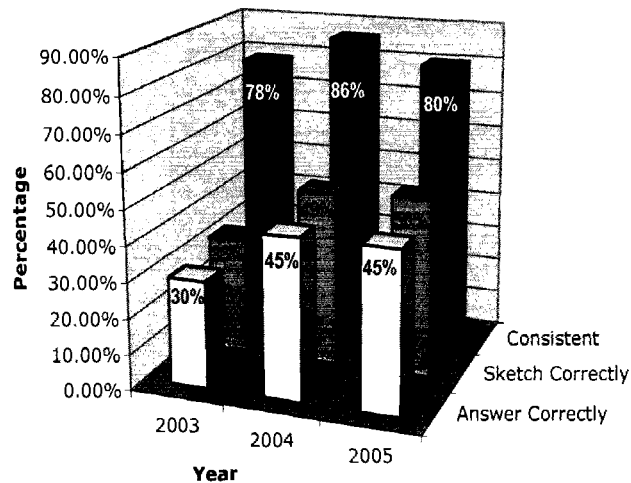


Figure 7-17: Student responses for the wider barrier scenario on the final exam tunneling problem.

rewriting the tutorial to reduce pages of similar questions and change the order of presentation, reformatting the same content into a reference table that allows for side-by-side comparisons, and an increase in dialogue about the ideas being explored, both with the introduction of board meetings in 2004 and the increase in the number of facilitators in 2005. Finally, we note that we achieve a consistently high percent of students who are consistent between their sketches and written explanations.

#### 7.4.3.2. Results – increased barrier energy question

The results for the increased potential energy question are shown in Figure 7-18. As evidenced by the lower numbers of students who answer correctly, this situation seems to be more difficult, as one must now reason how an increased energy difference affects the decay of the wave function. Again, we note an increase in the numbers of students who answer this

question correctly  
between 2003 and  
2004, when the  
tutorial was  
reformatted and  
portions rewritten,  
although there is a  
decline in 2005.  
Also, the numbers  
of students who  
answer consistently remain high.

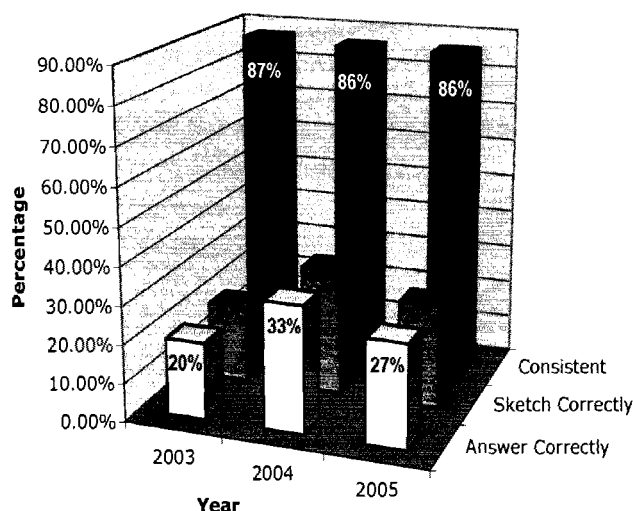


Figure 7-18: Student responses for the wider barrier scenario on the final exam tunneling problem.

Additional insight into the increase in the numbers of students who answer correctly can be seen by examining the ideas they use to explain their sketch. Table 7-6 contains the percentages of students who mention a particular characteristic, such as “amplitude” or “energy” in their explanations.

Table 7-6: Ideas students use to explain changes to the wave function.

Ideas	Wider Barrier Scenario			Increased Barrier Energy Scenario		
	2003	2004	2005	2003	2004	2005
curvature/curviness	9%	29%	14%	13%	45%	23%
amplitude	48%	65%	73%	41%	55%	70%
wavelength	26%	67%	66%	17%	61%	52%
energy	37%	12%	7%	28%	18%	9%

We note an increase in the number of students talking about the curviness of the wave function in 2004, though this number drops off again in 2005. Many more students in 2004 and 2005 discussed ideas useful for describing the wave function sketch, such as amplitude and wavelength, than they did in 2003. In addition, we observe a decrease in the numbers of students talking about the energy of the particles, which is a more difficult idea to tie to the sketch, as one must relate the particle energy to the sketched wavelength.

#### 7.4.3.3. Results – comparing energies

The second portion of the tunneling posttest question involved three short answer questions. The first probed for the prevalence of an energy-loss model, asking students to compare the particle energy in the regions on both sides of the barrier in the original scenario. The second asked students to compare the probability density to the right of the barrier in the wider barrier scenario to the original scenario. The third asked students to compare the particle energy on both sides of the barrier in the wider barrier scenario.

The results for the first energy question are shown in Table 7-7.

Table 7-7: Comparing particle energy on either side of the barrier.

<b>Energy is...</b>	<b>2003</b>	<b>2004</b>	<b>2005</b>
...lost.	28%	20%	9%
...the same.	72%	73%	84%
...gained.	--	6%	7%
<b>Because of...</b>			
...conservation.	67%	57%	80%
...effects of the barrier.	20%	8%	7%
...the amplitude of the wave function.	7%	16%	7%
...the wavelength of the wave function.	13%	27%	18%

We note that the majority of the students in all years state that the energy is the same. The number of students who say that energy is lost drops each year. There is no evidence that the students who say energy is gained are thinking of an ensemble of particle energies (as was discussed in the chapter detailing the results of the surveys); rather, their reasoning indicates that they are thinking of the barrier as possessing energy that may be “given” to the passing particles.

All students in all years who stated the idea of energy conservation in their reasoning stated that the energy would remain the same. All of the 2003 students who specifically mentioned the barrier or talked about the amplitude of the wave function had energy-loss models. In 2004 and 2005, barrier discussion was split between students with energy-loss ideas and those who stated that the barrier had no effect on the energy. The discussion of amplitude was divided as well between students who said energy is lost, energy is the same, and energy is gained. Students who stated that energy was conserved and discussed the amplitude of the wave function generally did so when stating that the energy is not connected to the amplitude of the wave function. Most of the students who discussed wavelength also invoked energy conservation, and linked the two ideas. For students with energy loss ideas who discussed wavelength, 4 of 5 (over all three years) were consistent, having previously sketched a longer wavelength for this situation, and reasoning that energy must be lost to match this increase in wavelength.

#### 7.4.3.4. Results – change in probability density

The results for the question comparing probability densities for a wider barrier and the reference barrier are given in Table 7-8.

Table 7-8: Comparing the probability densities for the wider barrier and the original barrier scenarios.

Probability density is...	2003	2004	2005
...less.	67%	67%	73%
...the same.	17%	18%	16%
...greater.	15%	14%	11%
<b>Because of...</b>			
...effects of energy.	11%	14%	14%
...effects of the barrier.	24%	24%	25%
...the amplitude of the wave function.	48%	43%	43%
...the wave function.	17%	27%	34%

Two-thirds of 2003 and 2004 students correctly stated that the probability density is reduced when the barrier is widened; this number went up slightly in 2005. The majority of these students in all three years linked their reasoning to the reduced amplitude of the wave function. Others, who had previously sketched the amplitude as the same, stated that widening the barrier had no effect on the probability density. Most of the students who discussed the barrier in their explanation stated that the probability density would be less, often because "the barrier is wider," "it takes more time for the particles to get through," or "the particles have to go farther."

#### 7.4.3.5. Results – energy for wider barrier scenario

Finally, the responses regarding the energy in the wider barrier scenario are presented in Table 7-9.

Table 7-9: Comparing particle energy on either side of the barrier for the wider barrier scenario.

Energy is...	2003	2004	2005
...lost.	33%	20%	16%
...the same.	67%	75%	77%
...gained.	--	2%	7%
<b>Because of...</b>			
...conservation.	63%	57%	73%
...effects of the barrier.	22%	10%	14%
...the amplitude of the wave function.	7%	16%	9%
...the wavelength of the wave function.	7%	25%	14%

These results closely mirror the findings in the first energy question. Most of the students who wrote that the energy was the same did so on the first question, though there are four students (over all three classes) who first stated that energy was lost in the reference scenario, and now stated that energy was the same here. Because the question did not ask students to compare the particle energy after tunneling to the particle energy after tunneling in the reference scenario, we have no evidence of the prevalence of classical ideas among students with energy-loss models.

## 7.5. Conclusions

Our work in the *Descriptive Physics* course has demonstrated the possibility of teaching some basic ideas from quantum mechanics to general education college students. Specifically in the context of ideas discussed in this chapter, we have developed a series of instructional materials that can be used to teach students strategies for solving a specialized form of the Schrödinger equation and sketching wave functions in various scenarios, such as a square potential

energy barrier. In addition, a significant number of these students can qualitatively reason about the effects of making changes to this situation, such as changing the energy or width of the barrier, or changing the energy of incident particles. Although not explicitly post-tested, we believe these tutorials are useful for students to use in developing beginning models of such phenomena as imaging with a scanning-tunneling microscope and alpha decay.

Although comparing this student population directly with undergraduate physics majors who have completed quantum physics courses is likely not appropriate, it is interesting to note that a smaller fraction of the students in this course believe energy is lost in quantum tunneling. Because our materials emphasize graphical solutions and qualitative comparisons, and teach students such practices as sketching a total energy line on potential energy diagrams to guide their reasoning, it may be that explicit attention to this problem including increased emphasis on energy reasoning is sufficient to lower the numbers of students who hold the energy-loss idea after instruction. This would agree with the findings from the survey questions included on the senior-level quantum physics course final discussed in Chapter 6, where students were far more successful on these questions after the instructor modified his instruction to address this difficulty.

We note that although our students performed better on some questions than the advanced undergraduates, the energy-loss idea was not absent from

this population. There are two possible explanations for this observation. First, we initially introduced this population to classical waves, including the idea that wave amplitude is related to the energy carried by the wave. It is possible that this classical idea was carried forward when they were reasoning about quantum wave functions. Second, it may be that the terms “barrier” and “tunneling” have deeply rooted connections for many people, based on their everyday experiences, and the thought of encountering some physical barrier or tunneling through a hill is strongly linked to energy expenditure.

Finally, we have shown that changes to class format and the structuring of activities within tutorials can produce increased understanding of physical phenomena. We introduced board meetings into the course during its second iteration in an effort to break up the long times students spent answering tutorial questions, and to try to strengthen the concept of building ideas as a scientific community. We restructured the order of presentation in the tunneling tutorials to lead with the more familiar ideas of energy and probability, followed by the more abstract reasoning about wave functions. Additionally, we replaced several pages of redundant questions with two tables that allowed for explicit side-by-side comparison of similar situations. A combination of these course and curriculum modifications likely led to the improved performance we observe on the graphical portion of the tunneling question included on the final exam, as well as the consistent performance on questions about energy and probability.

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- <sup>1</sup> R. R. Harrington and E. E. Prather, "Using an Introductory Physics Course to Train Pre-Service and In-Service Teachers," in *The Changing Role Of Physics Departments In Modern University*, AIP Conf. Proc. **399**, edited by E.F. Redish and J. Rigden, 797 (1997).
- <sup>2</sup> L. C. McDermott, P. S. Shaffer, and The Physics Education Group at the University of Washington, Tutorials in Introductory Physics. (Prentice Hall, Upper Saddle River, NJ, 2002).
- <sup>3</sup> M. C. Wittmann, R. N. Steinberg, and E. F. Redish, Activity-Based Tutorials Volume 1: Introductory Physics. (John Wiley & Sons, Inc., New York, 2004).
- <sup>4</sup> M.C. Wittmann, R.N. Steinberg, and E.F. Redish, "Making Sense of Students Making Sense of Mechanical Waves," *The Physics Teacher* **37**, 15 (1999). M. C. Wittmann, R. N. Steinberg, and E. F. Redish, Activity-Based Tutorials Volume 1: Introductory Physics. (John Wiley & Sons, Inc., New York, 2004).
- <sup>5</sup> See reference 2.
- <sup>6</sup> See reference 2.
- <sup>7</sup> From a CD of quantum simulations by M. Joffre, included in J. -L. Basdevant and J. Dalibard, Quantum Mechanics. (Springer-Verlag, Berlin, 2002).
- <sup>8</sup> M. C. Wittmann, J. T. Morgan, and R. E. Feeley, "Laboratory-Tutorial activities for teaching probability," to be published in *Physical Review Special Topics: Physics Education Research*.
- <sup>9</sup> This activity was suggested by Brant Hinrichs of Drury University, based on activities developed by the Kansas State University Physics Education Research Group.
- <sup>10</sup> Adapted from E. F. Redish, Activity Based Tutorials, Module 2 (Modern Physics), (Wiley, New York, 2005).
- <sup>11</sup> The simulation uses physlets (W. Christian and M. Belloni, Physlets: Teaching Physics with Interactive Curricular Material. (Prentice Hall, Upper Saddle River, NJ, 2001).) and PHP and HTML code written by Eleanor C. Sayre at the University of Maine.
- <sup>12</sup> See reference 11.
- <sup>13</sup> D. M. Eigler and E. K. Schweizer, "Positioning single atoms with a scanning tunneling microscope," *Nature* **344**, 524-526 (1990).

## Chapter 8

### CONCLUSIONS

In this thesis we describe research conducted over the past four years at The University of Maine on student understanding and learning of quantum mechanical tunneling. This research was motivated in part by our interest in studying physics students' learning of advanced topics in physics. Tunneling, although purely a quantum phenomenon, elicits a good deal of classical reasoning in many students and provides insight into the degree a student has adopted the ideas of quantum mechanics. In addition, we were interested in developing instructional materials and strategies and measuring their effectiveness to see if we could improve student understanding of tunneling.

Although (i) the investigation into student reasoning and (ii) the curriculum development and modification portions of this project took place in different student populations, we believe that several common themes exist. Additionally, we hope that the themes and ideas we have discussed provide useful insight for instructors at any level of introductory quantum instruction.

In this summary chapter, we first describe the ideas about tunneling that a majority of students seem to understand following traditional or interactive-engagement styles of instruction. We next note several of the ideas that still seem difficult for students after instruction. Finally, we comment on areas for future research suggested by our work.

## 8.1. Observed Successes

Much work has been done in physics education research documenting student difficulties.<sup>1</sup> Although we believe that this can be helpful, it is also important to note the ideas and techniques that students are successful with following instruction. Here, we briefly comment on three such successes from our observations on student learning of quantum tunneling.

### 8.1.1. Overcoming energy loss ideas

Both times we administered our tunneling survey at The University of Maine, we observed a high number of students who stated that energy is lost when particles tunnel. This idea was particularly prevalent in the sophomore population, but still present in a sizeable fraction of the senior-level students who had completed two quantum courses.

There is some evidence that this idea can be changed with instruction. Although the energy loss idea was still present, it was not so widespread in the two populations of physics students who took the survey at other institutions while they were taking a quantum physics course, as we have described in Chapter 5. We also note that all of the students who answered the survey questions on the 2004 senior quantum mechanics final stated that the average energy was either the same or greater, with all who discussed greater energy doing so in the context of reasoning about the particle beam as an ensemble of particles with different energies. This result followed an

increased emphasis on energy conservation and determining tunneling probability by the course instructor. Additionally, two-thirds to three-quarters of the students in our conceptual quantum physics course stated that the particle total energy was the same on both sides of a potential barrier. Also, all three of the graduate students we interviewed reasoned correctly about the energy questions. It may be that increased instructional emphasis on this idea and/or repeated exposure to this problem is sufficient to develop correct energy reasoning in a majority of students.

#### **8.1.2. Mathematical solutions to the Schrödinger equation**

We have limited evidence that students are able to determine mathematical solutions to the Schrödinger equation for various potential energy scenarios. In our final interviews with four senior physics majors, all could produce mathematically acceptable answers when asked to solve the Schrödinger equation first for a potential step, and then for a potential barrier.

Additionally, although it was not part of our study, we have studied a set of student mathematical solutions to the square barrier problem from an exam in the introductory quantum physics course from the fall of 2001. The majority of students could write down correct solutions after only one semester of modern physics instruction.

#### **8.1.3. Teaching basic quantum reasoning to introductory students**

As we detailed in the previous chapter, we have demonstrated that some basic ideas of quantum physics can be taught to non-science majors using a

specially designed series of tutorials involving observation and inference, and relying on the idea of building models to describe physical phenomena. In the context of tunneling, we have shown that these students can develop the ability to identify the type of wave function appropriate when the particle total energy is both greater than and less than the potential energy of the system, and can piece those ideas together to sketch a qualitatively correct wave function solution to the standard tunneling problem.

Additionally, there is some suggestion that following our method of tunneling instruction, where students qualitatively reason about various changes to the scenario, and are explicitly taught to use conservation of energy ideas, they can answer questions regarding energy and the changes to the wave function as successfully or more so than advanced undergraduate physics majors.

## **8.2. Observed Student Difficulties**

Even after instruction, several difficulties with understanding quantum tunneling remain in all of the populations we studied. Below, we comment on some of these.

### **8.2.1. Energy loss**

Although it appears that direct instruction on energy conservation in the context of quantum tunneling can greatly reduce the prevalence of the energy-loss idea, it still is present among a fraction of the students who study tunneling. Several of the students in our intuitive quantum physics course,

who did not deal with classical ideas of work and energy loss in our course, still stated that energy was lost for tunneling particles. This may be in part due to everyday understanding of the terms “tunneling” and “barrier,” or their limited exposure to classical waves. For other students in traditional physics courses, this may be due also to (i) observance of a decreased amplitude in the wave function, and recollection that amplitude and energy share a connection in classical waves, or (ii) superposing wave function sketches on top of potential energy barriers.

### **8.2.2. Understanding the wave function**

Although student ideas about the wave function were not the specific focus of our work, student responses, particularly in the context of the interview sessions, shed some insight into the difficulties students have with this concept. Among them:

- *Linking amplitude to probability density and energy.* Although most of the students we interviewed correctly relate the wave function amplitude with the probability density, many of these students also favor an amplitude-energy connection, and will reason that an observed decrease in amplitude signifies a reduced probability of detection and a loss in particle energy.
- *Identifying the labeling of the vertical axis on diagrams of wave functions.* Many students, particularly those who drew wave functions superimposed on potential energy barriers, linked the vertical axis of

wave function graphs to energy. Students with this difficulty often reasoned that energy was lost in tunneling. Others could not identify any meaning to " $\psi$ ," stating that they could only reason about the "square of the wave function." Still others directly linked  $\psi(x)$  to probability.

- *Reasoning about probability from the wave function sketch.* Although many of the students in the interviews tied probability reasoning to the "square of the wave function" or " $\psi^*\psi$ ," they seemed to not grasp the fact that the solution is complex. When asked to graphically reason about probability, for example, many would square the sinusoidal function they had sketched, and were troubled when questioned about the apparent areas of zero probability, often attributing it to the "strangeness of quantum."
- *Attributing time-dependent characteristics to time-independent solutions.* Closely related, we observed a number of students who used the time-independent solution and accompanying sketch to explain what would happen to particles over time. Some of these descriptions also suggest that students cling to deterministic ideas, and believe that the actual motion of particles can be described.

### **8.2.3. Matching mathematics with qualitative explanations**

Given the sophomore-level students' ability to mathematically solve the Schrödinger equation on the fall 2001 exam, it was suggested that perhaps the

difficulties we were observing in students in the interview sessions was due at least in part to the qualitative nature of our questions, something they were likely unfamiliar with from their undergraduate quantum courses.

Accordingly, we designed a new interview protocol that contained several questions first asking students to write down the form of the mathematical solutions to various scenarios before answering qualitative questions. This protocol was used with the four seniors we interviewed in the spring of 2005. Results from this protocol are discussed in Chapters 4 and 6.

Although all four wrote down reasonably correct solutions, including the energy difference term in the “wavelength” term in the sinusoidal function, there was little evidence that they used these mathematical solutions to make sense of our conceptual, qualitative questions. There seems to be fundamental difficulties connecting mathematical solutions of quantum physics problems with the qualitative description of the scenario.

#### **8.2.4. Potential energy diagrams**

Student ideas about potential energy diagrams were not the primary focus of our work, but we did observe several behaviors common to multiple students in the course of our investigation:

- *Viewing the potential energy diagram as separate entities.* Several of the students used language that suggested they viewed the potential steps and barriers as separate physical entities, possessing their own energy, which may or may not be given to the passing particles, rather than

viewing the potential energy as an outcome of the interaction between the particles and some physical system.

- *Linking potential energy barriers to physical barriers.* Several of the students admitted in the interview sessions that they reasoned about the diagram as a physical barrier, such as a hill or concrete barricade, which caused energy loss.
- *Spatial-energy confusion.* Several students used language in describing the particle's behavior that suggests they were thinking about the barrier as having physical height, and that higher energy particles would be spatially above (and often therefore unaffected by) the barrier.

#### **8.2.5. Linking theory to application**

In our initial interviews described in Chapter 4, the two students we questioned had difficulties in correctly describing the potential energy of our systems, whether it was the charged parallel plates or the sliding bead on the wire. After changing the interview protocol to begin with the theoretical situation, then later asking students to tie theory to application, most were unable to do so. They often stated that they were sure the situation did describe some physical behavior, but unsure of what type of system was modeled by a square barrier.

### 8.2.6. The intersection of classical and quantum worlds

Examining the overall difficulties students have with our questions suggests that students have the most difficulty with the tunneling ideas that have links to their previously established classical physics reasoning. Students seem to have little trouble with the idea that tunneling occurs (or at least accept it). They are also generally successful in reasoning about how tunneling probability

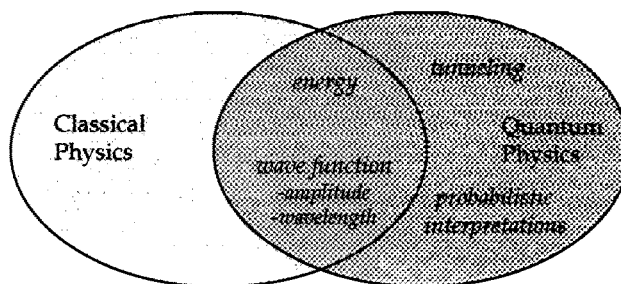


Figure 8-1: Model of the “zone of difficulty” for students reasoning about quantum tunneling.

changes with modifications to they system. We note that both of these are usually new ideas in a quantum course. Other questions regarding the nature of wave functions (which often resemble classical waves) and energy, with stronger ties to classical physics ideas, seem to be the areas where the students struggle the most. This zone of difficulty is illustrated in Figure 8-1.

### 8.3. Directions for Future Work

Our work has suggested several avenues for future research. Below, we discuss a few of these ideas.

- *Student understanding of potential energy diagrams.* Potential energy diagrams are widely used in advanced physics courses, but our study suggests that many advanced undergraduate students find

understanding exactly what is and is not represented on these diagrams difficult. We are aware of research at the University of Colorado that is beginning to look at this issue.

- *Student understanding of the wave function.* In most quantum physics courses, students are introduced to the idea of a wave function prior to dealing with wave function solutions to potential energy wells, steps, and barriers. Interviews with students that specifically focused on their ideas about the utility and meaning of wave functions might provide insight that would inform researchers studying student use of wave functions in various scenarios.
- *Effects of modified instruction on advanced undergraduate students.* The curriculum development portion of our work focused on developing tutorials for use in an introductory physics course for non-science majors. Though we believe we have demonstrated some success in this arena, it would also be useful to explore whether adapted versions of our materials were successful in more traditional settings, such as the quantum mechanics courses for physics majors.

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<sup>1</sup> For an overview of introductory physics difficulties, see L. C. McDermott and E.F. Redish, "Resource Letter: PER-1: Physics Education Research," *Am. J. Phys.* **67**, 755-767 (1999).

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## Appendix A

### QUANTUM ENERGY AND PROBABILITY SURVEY - 2003 VERSION

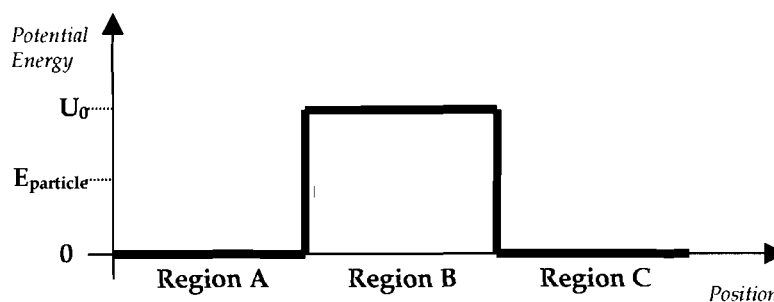
Name: \_\_\_\_\_ Major: \_\_\_\_\_

Have you completed PHY 236: *Introductory Quantum Physics*? ☐ yes ☐ no

Have you completed PHY 469: *Quantum & Atomic Physics*? ☐ yes ☐ no

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The potential energy of three regions, A, B, and C, is sketched below as a function of position:

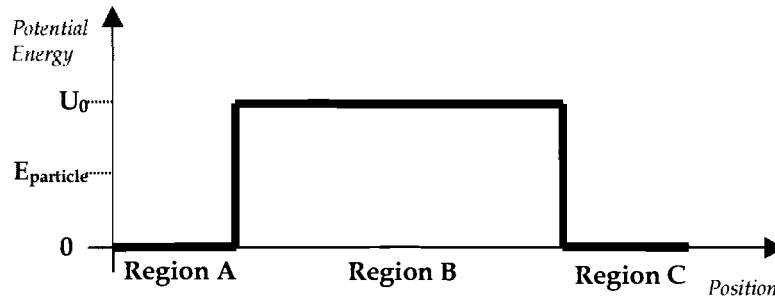


**Scenario I.** A stream of charged particles with energy  $E_{\text{particle}} = 0.5 U_0$  are incident on this potential barrier in Region A. A detector set up in Region C indicates that some of the charged particles are found in Region C.

1. What is the energy of the detected particles in Region C?
  - a. less than  $E_{\text{particle}}$
  - b. the same as the energy in Region A,  $E_{\text{particle}}$
  - c. greater than  $E_{\text{particle}}$
  - d. it is impossible to determine the energy of the particles in Region C
2. Explain the reasoning you used to determine your response to Question 1.
3. What factor(s) determine the probability of a particle being detected in Region C?

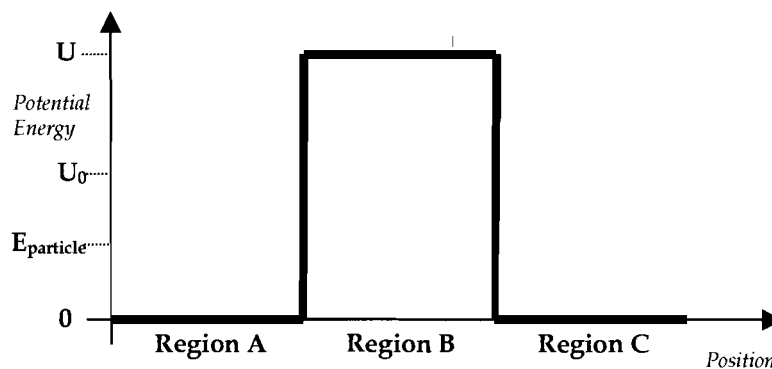
Use the following options to answer questions 4-8.

- There are no particles detected in Region C.
- There are now fewer particles detected in Region C.
- The number of particles detected in Region C stays the same.
- There are now more particles detected in Region C.
- It is impossible to know the relative number of detected particles in Region C.



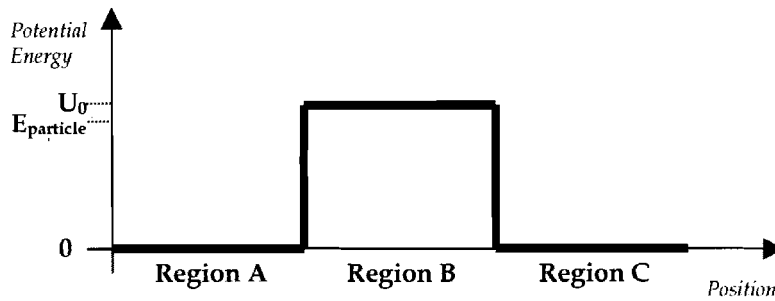
4. Which statement above best describes the number of particles detected in Region C, compared with Scenario I?

**Scenario II:** The width of the barrier is now doubled. The particles are at their original energy,  $E_{\text{particle}} = 0.5U_0$ .



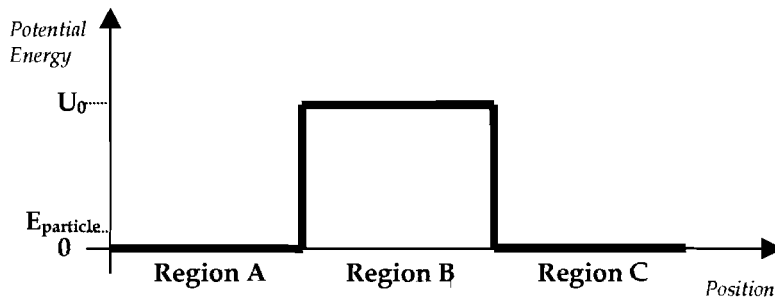
5. Which statement above best describes the number of particles detected in Region C, compared with Scenario I?

**Scenario III:** Region B is returned to its original width. The height of the barrier,  $U$ , is now doubled. The particles are at their original energy,  $E_{\text{particle}} = 0.5U_0 = 0.25U$ .



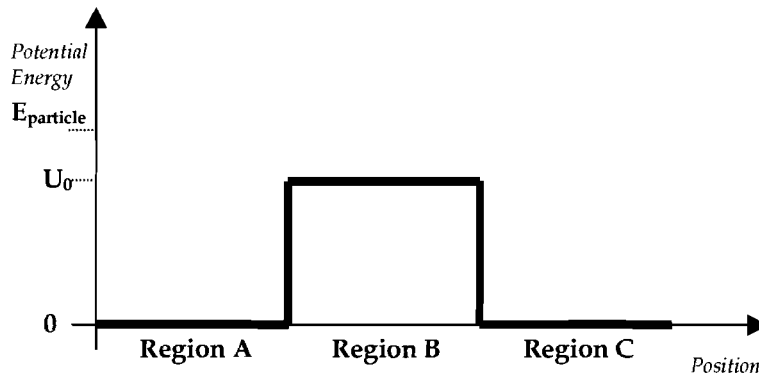
6. Which statement above best describes the number of particles detected in Region C, compared with Scenario I?

**Scenario IV:** The barrier is now returned to its original height,  $U_0$ . The particles' energy in Region A is increased to  $E_{particle} = 0.9U_0$ .



7. Which statement above best describes the number of particles detected in Region C, compared with Scenario I?

**Scenario V:** The barrier is now returned to its original height,  $U_0$ . The particles' energy in Region A is decreased to  $E_{particle} = 0.1U_0$ .



8. Which statement above best describes the number of particles detected in Region C, compared with Scenario I?

**Scenario VI:** The barrier is now returned to its original height,  $U_0$ . The particles' energy in Region A is increased to  $E_{particle} = 1.25U_0$ .

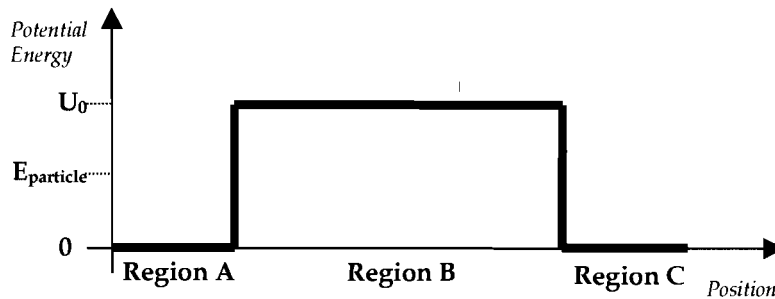
Use the following statements to answer questions 9-13.

The energy of the detected particles is **less** than  $E_{particle}$ , and...

- a. ...the amount of energy lost is the same as the amount of energy lost in Scenario I.
- b. ...the amount of energy lost is greater than the amount of energy lost in Scenario I.
- c. ...the amount of energy lost is less than the amount of energy lost in Scenario I.
- d. The energy of the detected particles will be the **same** as the energy in Region A,  $E_{particle}$ .

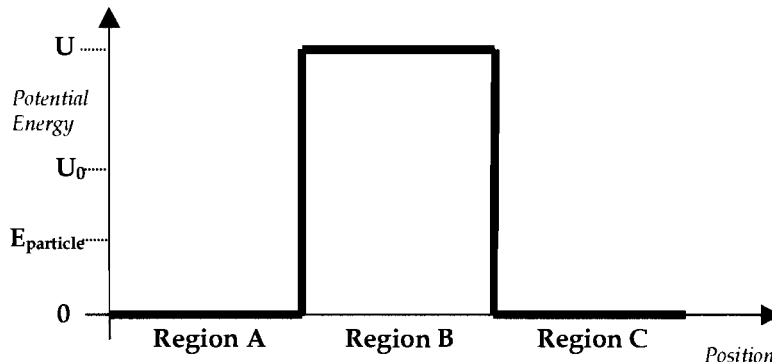
The energy of the detected particles is **greater** than  $E_{particle}$ , and...

- e. ...the amount of energy gained is the same as the amount gained in Scenario I.
- f. ...the amount of energy gained is greater than the amount gained in Scenario I.
- g. ...the amount of energy gained is less than the amount gained in Scenario I.
- h. There are no particles detected in Region C, and thus no particle energy.
- i. It is impossible to determine the energy of the particles in Region C.



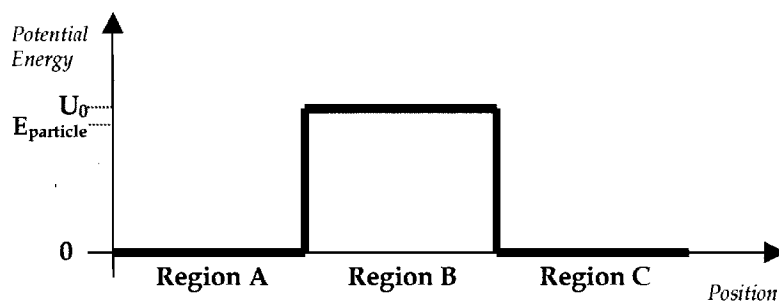
9. Which statement above best describes the energy of the particles detected in Region C, compared with Scenario I?

**Scenario II:** The width of the barrier is now doubled. The particles are at their original energy,  $E_{particle} = 0.5U_0$ .



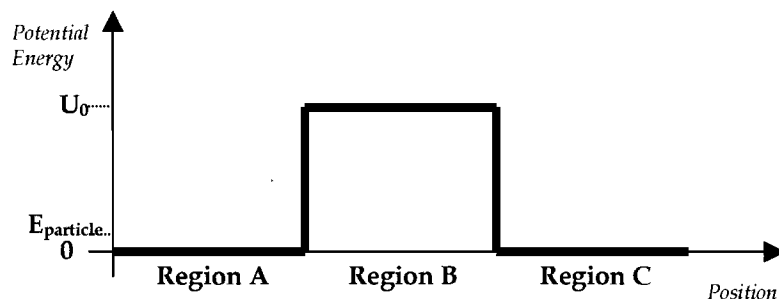
10. Which statement above best describes the energy of the particles detected in Region C, compared with Scenario I?

**Scenario III:** Region B is returned to its original width. The height of the barrier,  $U$ , is now doubled. The particles are at their original energy,  $E_{particle} = 0.5U_0 = 0.25U$ .



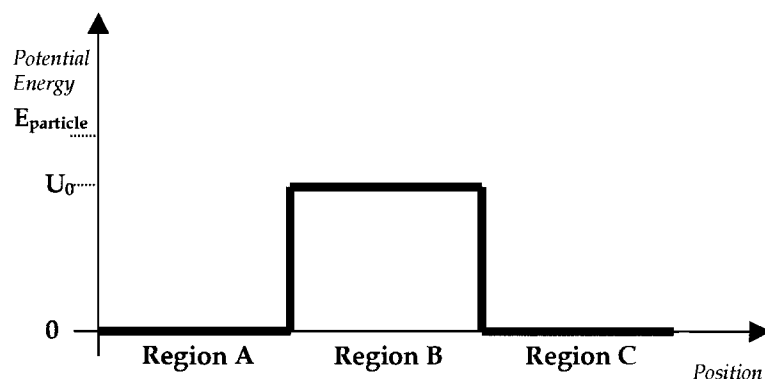
11. Which statement above best describes the energy of the particles detected in Region C, compared with Scenario I?

**Scenario IV:** The barrier is now returned to its original height,  $U_0$ . The particles' energy in Region A is increased to  $E_{particle} = 0.9U_0$ .



12. Which statement above best describes the energy of the particles detected in Region C, compared with Scenario I?

**Scenario V:** The barrier is now returned to its original height,  $U_0$ . The particles' energy in Region A is decreased to  $E_{particle} = 0.1U_0$ .



13. Which statement above best describes the energy of the particles detected in Region C, compared with Scenario I?

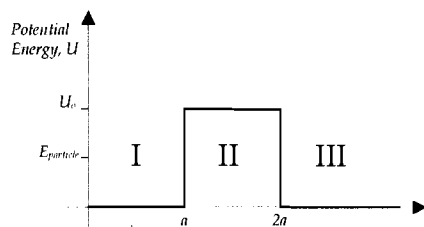
**Scenario VI:** The barrier is now returned to its original height,  $U_0$ . The particles' energy in Region A is increased to  $E_{particle} = 1.25U_0$ .

## Appendix B

### QUANTUM ENERGY AND PROBABILITY SURVEY – 2004 SURVEY

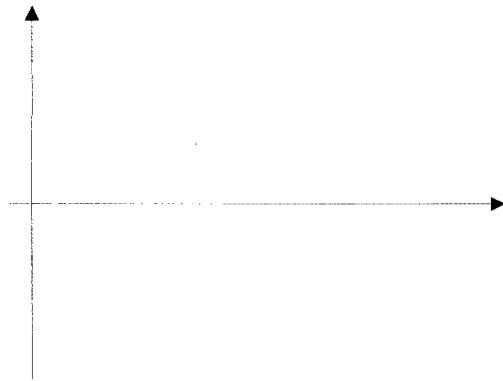
The potential energy of some system as a function of position is given by

$$U(x) = \begin{cases} 0 & x < a \\ U_0 & a < x < 2a \\ 0 & 2a < x \end{cases}$$



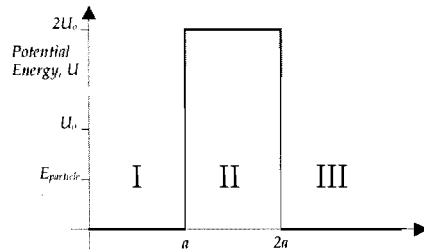
A stream of charged particles with average energy  $E_{particles} = 0.5 U_0$  is incident on this system from the left. A detector set up in Region III indicates that some of the charged particles are found there. (Hereafter, this system will be referred to as the 'original scenario'.)

1. How does the average energy of the particles detected in Region III compare with the average energy of the incident particles in Region I? Explain your reasoning.
2. What factors determine the probability of a particle being detected in Region III?
3. On the axes below, sketch the wave function that can be used to describe the particles in Regions I, II, and III. Label your axes.
4. How can the wave function be used to compare the average energy of the particles detected in Region III with the average energy of the incident particles in Region I?
5. How can the wave function be used to compare the number of particles detected in Region III with the number of particles incident in Region I?



The potential energy of a second system as a function of position is given by

$$U(x) = \begin{cases} 0 & x < a \\ 2U_0 & a < x < 2a \\ 0 & 2a < x \end{cases}$$



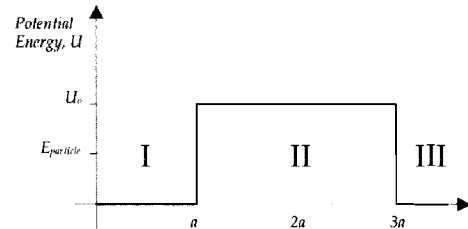
A stream of charged particles with average energy  $E_{\text{particles}} = 0.5 U_0$  is incident on this system from the left.

6. How does the number of charged particles detected in Region III in this system compare with the number of charged particles detected in Region III in the original scenario? Explain your reasoning.

7. How does the average energy of the charged particles detected in Region III in this system compare with the average energy of the charged particles detected in Region III in the original scenario? Explain your reasoning.

The potential energy of a third system as a function of position is given by

$$U(x) = \begin{cases} 0 & x < a \\ U_0 & a < x < 3a \\ 0 & 2a < x \end{cases}$$



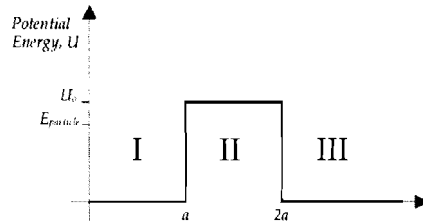
A stream of charged particles with average energy  $E_{\text{particles}} = 0.5 U_0$  is incident on this system from the left.

8. How does the number of charged particles detected in Region III in this system compare with the number of charged particles detected in Region III in the original scenario? Explain your reasoning.

9. How does the average energy of the charged particles detected in Region III in this system compare with the average energy of the charged particles detected in Region III in the original scenario? Explain your reasoning.

The potential energy of a fourth system as a function of position is given by

$$U(x) = \begin{cases} 0 & x < a \\ U_0 & a < x < 2a \\ 0 & 2a < x \end{cases}$$



A stream of charged particles with average energy  $E_{\text{particles}} = 0.75 U_0$  is incident on this system from the left. A detector set up in Region III indicates that some of the charged particles are found there.

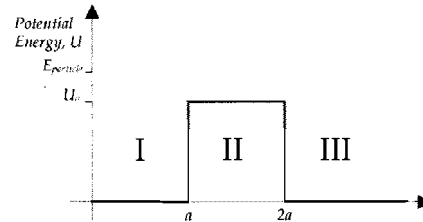
10. How does the number of charged particles detected in Region III in this system compare with the number of charged particles detected in Region III in the original scenario? Explain your reasoning.

11. How, if at all, is the wave function used to describe this scenario different than the wave function in the original scenario? Sketch and explain below.



The potential energy of a fifth system as a function of position is given by

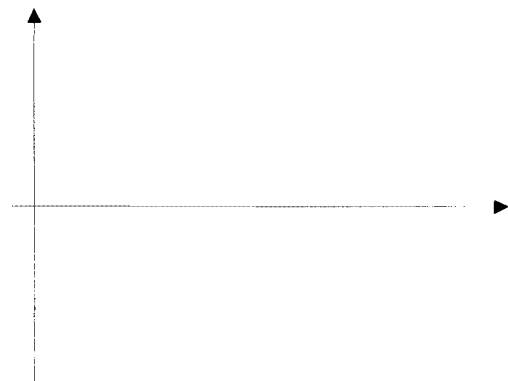
$$U(x) = \begin{cases} 0 & x < a \\ U_0 & a < x < 2a \\ 0 & 2a < x \end{cases}$$



A stream of charged particles with average energy  $E_{\text{particles}} = 1.25 U_0$  is incident on this system from the left. A detector set up in Region III indicates that some of the charged particles are found there.

12. How does the number of charged particles detected in Region III in this system compare with the number of charged particles incident in Region I? (Not Region III of the original scenario) Explain your reasoning.

13. How, if at all, is the wave function used to describe this scenario different than the wave function in the original scenario? Sketch and explain below.



## Appendix C

### TRANSCRIPT OF FIRST ADAM INTERVIEW

- I: Here's a picture to look at – oh, I should add that obviously the paper and markers are here that, if you feel you would like to draw anything to explain every – any of your answers, feel free to do so, that's...
- A: And I will.
- I: Good. All right, so here's a diagram of potential energy as a function of position, and I've arbitrarily divided into three regions – A, B, and C. Region A and Region C are at some potential energy  $E_1$ , and Region B is at some higher potential energy  $E_2$ . And what we're going to do is just shoot a beam of electrons at this system, so a beam of electrons with energy less than the energy in Region B is incident on the barrier in Region A, so it's coming in from the left.  
So imagine you were somehow an observer in this system that could only make observations in Region C.
- A: I'm not going to see very much.
- I: What would you observe, or what would you be able to detect in Region C?
- A: Depending on how wide this is, I may detect nothing. If it's very thin, I'm going to detect... I'm going by the wave function, I'm guessing... Um, if it's very thin, I'm going to see the same wave function with a smaller amplitude. It's going to hit this thing and exponentially decay really fast. But if it's good and wide, nothing's going to make it through.
- I: OK, so you may...
- A: This is, this is potential, yeah, potential energy, right...
- I: When you, when you say you would detect a wave function, how would you do that?
- A: How would you detect a wave function? You can't really detect a wave function – as soon as you detect it, it's no longer a wave function, it collapses.
- I: OK. So again, then, if I were to shoot a beam of electrons in here from the left, what could or would you observe in Region C?
- A: I would observe electrons, if they made it through at all.
- I: OK. And how would you know whether, I mean, how, how would you decide if they all made it through, if none made it through, if some of them made it through – what factors would influence that?
- A: As in how would I detect electrons?
- I: No, I... I'm saying you would detect electrons. Would you detect all the electrons that I originally sent, would you only detect some of them, would you detect none.

- A: So you're asking what happens to the electrons? All right. So what I'm thinking, let's do two scales, cause I'm not sure which one you're asking, so macroscale, this is a cement barrier, yea high, beam of electrons, they hit this thing, and they're not going through it, they're kind of running into it, and they're probably... I don't know exactly how, but I assume they're being absorbed to some extent, just kind of end up in the concrete, or they bounce off and fly away.
- I: OK.
- A: Now if this is quantum level, like the diagrams in class, it's gonna set up a wave, there'll be a node, no there will not necessarily be a node, at the barrier, and the wave function, once it gets inside, should I draw on this?
- I: Sure, yeah, if you - you can draw on this or on (unint), either one.
- A: Uh, here's the energy of, uh, call it a wave like that, no I don't want... So let's say, wait, how... I don't want it to be set up in a resonance cavity, not, not resonance, but um... See, now I want to ask questions.
- I: You can ask questions - I don't guarantee I'll answer them, but...
- A: General case scenario, has this little black box thing, ok, so for there to be a standing wave, I'm remembering via the quantum distance thing, for there to be a standing wave, the walls can only be at certain distances apart. And it will actually feel a force if they're not quite the right distance apart. All right, so but I'm switching back and forth between the representation of a wave function as I remember it and what an actual, I guess, uh, wave of light does. In... so I already did the, the macro case, the micro case where this is a wave representing part, um, no, just make it simple, a whole package of uh, waves that make up an electron...
- I: OK.
- A: ...so this is the representative wave. It hits here, there's a whole quantum tunneling thing, the wave will taper off very quickly, if I recall, so on this side, if this is a thick enough barrier, none of the wave function's going to make it through, and if it's thin enough, some of it will, and it will be of a smaller amplitude, meaning, so the question is what is a small wave function versus a big wave function? More energetic particle, perhaps?
- I: What do you think?
- A: Big amplitude means big energy in mechanical waves, no... big frequency means big energy. Ah, wait a second, amplitude... all right, this was the, the classical versus quantum model, ah, frequency determines energy, amplitude determines how much, no, how many electrons or whatever is incoming. Height, wavelength, amplitude, right. So amplitude is synonymous with intensity, frequency is synonymous with energy. Back to this. Welcome to the far side.

Lower intensity wave, same frequency, so there's less of it, so few electrons get through.

I: So the observation you would make in Region C is that you would detect less electrons than were incoming in Region A?

A: If any at all, yeah.

I: So what would happen, then, again lets think on the quantum level, not the cement wall, if we narrowed Region B...

A: No, you'd see...

I: ... in the original representation?

A: If it got thin enough, as it approached nonexistent, got really thin, uh, most of the electrons would make it through, and more and more would make it through.

I: OK, so as I narrow the, the region...

A: You see more and more electrons.

I: ...I, I detect more and more electrons in Region C? Uh, is the reverse true, then, if I make Region B wider, does that change anything about the observation?

A: You see fewer and fewer electrons until you see none at all.

I: What's gonna determine the width where I see none?

A: Ooh, there's an equation for that, but there is an... oh, I don't remember the equation anymore. But... what is going to determine the width?

I: Yeah, well you mentioned that it would get wider and wider until none would be observed...

A: That's a function of the frequency, as in the energy of the incoming beam - really energetic beam will go through a lot of things. Gamma rays will go right through you, and cosmic rays, and all that...

I: So, you...

A: Lower energy will be stopped.

I: You would need to know something numerically about the energy to be able to characterize the width of the region?

A: Yeah, I believe, yes...

I: We're not going to go into those calculations, I'm just trying to clarify what you're saying.

A: I would need to know a lot of things numerically. Seeing the equation again would help.

I: OK, um, what happens then in the original scenario if we increase  $E_2$ , so that, uh, it's in the original scenario, but Region B now has a greater potential energy. Does that change anything about what gets observed in Region C?

A: It does. I recall that it does. This is the infinite potential well scenario, which is, how does this go. This is particle in a box. Infinite walls... Oh, yes... How much gets through depends on the height of this as well, the potential.

I: Why is that?

A: Has an infinite potential, ok, check it out. It's the analogy that was kind of cool in class, and it could be right up your alley, is the, uh... you have many colors.

I: Yes.

A: Great. The usual it goes to one, the usual it goes to two, following the same thing, and going up like that and going up again over here. So, back to black, potential one (unint) in both cases, let me draw a little cart here. This is a cool analogy – cart drops from here...

I: Mmm-hmm.

A: High potential here, as a matter of fact, infinite potential, here's its... maximum energy, it's stuck in the well.

I: OK.

A: Go into the blue track, it can get over this hump, because it's maximum energy is up here, and... it's not going to get past here, so here's the width of the well. So how does this apply to this, cause that's obviously the next question. Or (unint) to this. Very – mmm – well, just said word for word. If the walls here are very high, the little cart is never gonna get out. Which means here... you would think, you would think to say that the, the wave function would be trapped between, well, in Region A. Would hit the walls, it would again decay – wait a second – see here is a good point of non-understanding. All right, so when a wave hits a wall, it decays – cool. How much decays presumably depends on the height of the wall, potential wall. Does it depend on that height, or not? No, no it shouldn't. We have potential, a low potential wave is not going to make it a high, over a high potential bump. So, restate the question, see if I've gotten it yet.

I: Sure. Uh, let's go back to what you originally answered, I think, and that may help us here. You, we'd said that Region B has some potential energy  $E_2$ . I shoot some electrons in with some energy that's somewhere between  $E_1$  and  $E_2$ , we can say half if we want to. And you originally said that...

A: Ohhh.

I: ...some would be detected in Region C. So my question now is if I increase  $E_2$ , but I leave the energy of the electrons the same, does that at all affect my observations in Region C?

A: I think I remember this. Mmm-hmm. No, it doesn't.

I: So the height has no affect.

A: The width of the wall has an effect. The height – I believe it tunnels in regardless, after it hits the wall and exponentially decays. Um... yeah, the height of the wall determines how much gets over. If you have high energy, it's gonna make it over that potential well, much like this, as a matter of fact, it's quite representative. High energy is gonna kind of make it over the potential...

I: Just so I understand, you're saying that if the electron energy is greater than  $E_2$ ? Is that what you're...

A: Yes.

I: ... in this scenario?

A: Yeah.

I: OK, OK.

A: Cause down here it's gonna hit the wall and decay, it's... I don't recall anything telling me that it's gonna decay more or less depending on the energy.

I: So, in this scenario...

A: Potential, potential energy.

I: ...in this scenario, then, you still believe you would detect electrons in Region C, but changing the height, making this taller, would not affect what you detect?

A: I believe that is correct.

I: Um, I'm gonna assume, then, that your answer is the same in reverse, in other words, if I make this a little bit shorter, but still greater than the energy of the electrons...

A: But still greater, by the same logic, yes.

I: OK, and then if I made it less than the energy of the electrons, that's the same scenario that you were just saying, with the electrons higher.

A: Something, something's gonna make it over.

I: OK.

A: I am gonna ask at the end of, the end of the interview if this is right.

I: That's totally fine.

A: Cool.

I: Um, what happens then, if I increase this energy of the electrons in the original case, so I make it greater than it originally is in this representation, but it's still less than  $E_2$ , does that affect anything about the observation over there in Region C?

A: Raising the electron energy is the exact same thing as lowering potential energy, well, qualitatively.

I: OK. So...

A: So, I'm gonna stick to my guns. I don't know whether my reasoning is right. If it is right, uh, based on the reasoning I've been using, the observation will not change.

I: OK. Uh, then if I lower it, if I lower the energy of the electrons, same thing?

A: If you lower the...

I: Still in between  $E_1$  and  $E_2$ .

A: (unint) Once again, by the same logic, which may or may not be correct, you're going to have the same  $n$  every time in Region C unless the energy of the electrons exceeds the potential, assuming a square thing like this.

I: OK. Um, lets return to our... uh, original scenario, so just like this, um, and we talked, you talked something about being able to detect electrons in Region C. Lets imagine – and don't worry about how we would set up this experiment – that we could measure the energy of those electrons over there in Region C.

A: Oh, (unint) potential.

I: So, in this original scenario, I send electrons in with a certain energy,  $E$  of electrons.

A: Mmm-hmm.

I: What energy will the electrons have that I'm able to detect over in Region C?

A: I have two conflicting recollections here.

I: OK, what are they?

A: First one is that a wave function in Region A will hit Region B, the amplitude will bottom out, get very small very quickly, but if it does make it through, it will keep – so it'll have a very small amplitude, but it will keep the same frequency, the wave will otherwise look the same.

I: OK.

A: Umm...

I: So, if it kept the same frequency, then what would you say about the energy?

A: Energy's the same, you're just going to see a smaller intensity, once again equating amplitude with intensity and frequency with energy. So the same energy, less of it.

I: OK. And the second of the conflicting ideas?

A: Umm... what was the second one? Could you restate the question?

I: I don't know, when, when I originally said the question I said what would the energy of the electrons that you were able to detect in Region C be?

A: Oh, yeah, yeah, yeah...

I: ...you said there were two...

A: Here's, here's the other thing, little... so, crystal lattice of some kind...

I: OK.

A: An actual electron comes in, hits this, particle one, particle one flies that way a little bit, electron gets reflected, bounce around, and it's gonna slow down, lose energy to the particles that it's departing energy to, which proceed to shake a little faster, whatever they do. So you would think, as I actually hit something, oh, here's where the problem is, this is a potential, this is a physical situation, which is... the graphs are not looking at, oh the pictures are not looking at the same thing. Never mind. I stick to my guns here, I stick to my guns here, independently, and they both reconcile.

I: So, in this scenario, the energy of the detected electrons over here in Region C is the same as it was in Region A.

A: Yes....

I: You're going with the first response. OK. Um, how is the... is that all affected by then changing the barrier; if I make the barrier narrower or wider, does that affect the energy that the electrons have that make it through to Region C?

A: The one thing I wish I could remember, or taken better note of... Wait a second! Hey, I'm remembering something. What I'm remembering - I can go back to this thing - is wave of energy up here. When it comes over, an area of higher potential, it changes. How does it change? Its... the... gets bigger, the, as in the amplitude increases. What'll the frequency do? I think the frequency decreases. OK, so there is a conflict of models.

I: So, yeah, lets, lets go with this for a little bit. You say the amplitude increases.

A: This is purely recollection. I'm trying to remember the logic behind it.

I: So you're remembering a picture that looked like this...

A: Yep.

I: ...and trying to reason why it might be.

A: It was not in my book but he, Dr. Lad, drew it up on the board, and I wrote it into the back of my book. How I drew a picture, of what the wave function does, in areas of different potential. How it reacts. If I recall, just, this is, pure recollection with no reason backing it up or no logic, when a wave function goes over an area of higher potential, the amplitude and frequency both increase, but that's completely, that's like a, uh, that's right, like rehearsing behinds...

I: So, do those, I mean, do those statements make any sense, or are you just saying its something you recall?

A: I recall that.

I: Is there any reasoning you could use to reason why? Let's just worry about the amplitude for now. What is the amplitude of the wave function telling you?

A: So once again we're defining the... the misunderstanding here. I guess I'm, I'm not sure if there are two or three models. Here's what it's coming down to. The classical model, versus the quantum mechanical model versus this. I'm sorry - thinking out loud, just...

I: No, that, that's exactly what we want you to do, this is great.

A: All right... classical model of a wave, the frequency means higher frequency is higher energy, so what I'm wondering is, in this depiction... do the, do the features of the wave, the amplitude and frequency, represent the same thing that they do when you're talk - when you're talking about an actual electromagnetic wave. If they do, so there's, so there are going to be two answers to every question then...

I: OK.

- A: If they do, then this makes no sense. Actually come to think of it... uh, potential, wait a second... (unint) this area, frequency, amplitude, same, and if the intensity is the same, so according to this, if I'm remembering this off the top of my head correctly, then that's saying when there's a smaller difference between the energy and the potential energy, then the intensity and the energy increase? That doesn't make sense. So I'm assuming this is, probably not quite right, or I'm misunderstanding the representation. And what was the question? Cause I completely lost it.
- I: The original question that we were talking about in this section was talking about what energy the electrons would have. The electrons that you're able to detect in Region C - how would their energy compare to the original energy they had in Region A?
- A: Instead of trying to reason this out, I'm going to stick with the idea that, mmm yeah, that amplitude corresponds, once again, amplitude will correspond to intensity, frequency corresponds to energy level...
- I: OK. So, what would your answer be?
- A: Oh, so the amplitude decreases, um, amplitude decreases, frequency stays the same. Energy stays the same, there's less of it.
- I: OK.
- A: And we've come full circle.
- I: So, any changes I might make to the barrier, would those affect your answer, or would you say the energy would still be the same? In other words, if I made barrier narrower, or wider, would that affect the energy of the electrons that were able to...
- A: It would...
- I: ...make their way through, get through?
- A: No, it would not. According to that, system of logic, which I'm not going to question right now, because I don't have time to think about it.
- I: And similarly making the barrier taller or shorter?
- A: The same energy, less of it. And so I'm (unint) how much you're gonna see, and, yeah, purely how much you're going to see as a frequent, as a function of the width of the barrier. The energy will not be changed.
- I: OK.
- A: Although, I still do remember something about this. This is once again going over a potential barrier.
- I: OK.
- A: And I remember that things change, specifically the amplitude changed. I think the frequency too. And that thought which I was specifically told in class stands in contradiction to what I'm saying now. But I'm ignoring this.

I: I mean, what would that mean? This that you recall, that the amplitude's higher there?

A: According to what I was just saying it would mean the intensity is greater.

I: OK...

A: ...which means...

I: ...and you, you'd said also the frequency was greater here? You seem to recall?

A: I seem to recall... the frequency was greater, which means the energy and the intensity is greater. What is the intensity of a wave represented in this scenario? I want to say how much of it there is, but that's not quite, it doesn't mean anything, not to me, anyway.

I: OK. I want to talk for a little bit about wave functions, and what they may or may not mean. Um, I noticed when we originally were just asking back here about detection that you started to draw some waves, and the first one you drew was this, which appears to me to be, uh, a couple of overlapping sinusoidal wave forms there.

A: Mmm-hmm.

I: Why did you draw two here, and then just one later on up here?

A: That's how I remember it being represented mostly in class, and there once again the whole point was showing what the amplitude and frequency did, I imagine. What this is about is the scenario with the black box, which was the argument of quantum distance; the distance is quantized, it, things in general on a very small level are quantized. What the argument for that... have I answered the question?

I: So, here you, you say black box, is that meaning that this side and this side are both barriers...

A: Yes.

I: ...for this wave?

A: And only nodes can hit there.

I: OK. Uh, what if there is no barrier here at the left end of region A...

A: Yep.

I: ...what will the wave function look like, for this beam of electrons that you're shooting in from region A?

A: There's no barrier on this side? On the left side?

I: Yeah.

A: Um, assume there is no barrier on the left side. They're just coming in from it, they have, I'm assuming they have a source...?

I: They're coming in from a source, but...

A: Anyways.

I: ...from what we can see, there's no barrier here.

A: What would the wave function look like?

I: Yeah, in region A.

A: So the question for me is what is going on right here... I do not know. What is the? What is going on in region A? I'm tempted just simply to... um... here's what I'm fighting with.

I: Mmm-hmm.

A: Once again, the physical interpretation, as in energy and electromagnetic waves in a sealed box. The nodes only can meet, oh there has to be a node at the barrier.

I: OK.

A: However, in this model here, same thing. They talk all the time about a wave hitting the barrier and its amplitude just exponentially decreasing. So, what I'm fighting with specifically is what this is representing.

I: OK. What the, the... region B is representing, or?

A: What the whole crap is representing. And this is purely a matter of forgettin it. So this is not... mmm... the wave function is not necce- is not really... Oh, I'm remembering this now. That matter has both a material and uh... wave nature, as demonstrated by the two slit experiment, combining the light, where photons were talked about as a particle but also demonstrated to act like a wave. All right. Try to recall too much at once, and I'm completely losing my train of thought. Better it would have been to simply review this. All right. Again, what is the question?

I: The original question in this section is, is asking you to talk about and/or sketch what the wave function looks like in region A, region B, and region C. Could you sketch what the wave function looks like as a function of position?

A: With what energy?

I: Uh, I'm sending electrons in here...

A: Below?

I: ...originally with, with, yeah, an energy that's halfway between  $E_1$  and  $E_2$ .

A: All right. Sure.

I: Do you wanna...?

A: Switch colors - I'm getting tired of black. I'm green now. So, what the wave looks like in part A? - actually there's no barrier here - is arbitrary, that's what you give me at this point, depending on the energy and intensity, and what that means...

I: All right, so, so draw an arbitrary wave.

A: Arbitrary wave, here we go. Oh, I remember something else now!

I: Which is what?

A: Which is... (unint) kinetic - this is what I'm trying to resolve. Now... wave has to have a node at the barriers. However, it can of course have... uh, certain, represented by n, what's it called. Whatever. A certain number of... peaks.

I: OK.

A: Trying to remember... the name of  $n$ . Anyways. So, depending – that's where the energy changes! That is the quantization of energy, that's where that come – that's what came out of the black box experiment. And I'm fighting with the same damn thing over and over again, as in representation at here...

I: OK.

A: What does it mean compared to representation, well, when you're talking about a black box and an actual physical standing wave inside (unint) between two things. This is a potential well, and I keep confusing it in my mind with a physical barrier.

I: All right.

A: So, I think I'm satisfied with the physical side. That's not what we're talking about. Talking about the interpretation of this thing, which I'm confusing with the physical side. All right, back to this.

I: So you've drawn a, you've drawn a, a wave function, and arbitrary wave function in region A. What would it look like in region B? What would it look like in region C?

A: Let me draw it smaller, just so I, kind of more, to work with. Here, um, scale doesn't really matter. Which is terrible. I'm going to start over.

I: Sure. You wanna, if you want... don't know if you want to refer to that paper...

A: Anyways. Oops, once again. There, that's an atom. Here's what I'm trying to draw. Nice big wave. Hits there. Here is your exponential, and... uh, wavelength stays the same. And this tapers out to... very small. I hope that now I'm getting somewhere. Scenario two... like this, I'll switch colors, just so I can do it again. Scenario two, the wall is thinner, so this tapers out to zero, its just, nothing gets through.

I: OK.

A: Um... same wave... same, oh, kind of... so basically it does not get to zero before getting out the other, other end of the wall, it does its decay thing, and whatever amplitude it had coming out of here, you're going to have in region C. So, that is what the wave looks like in region C. Same wave, smaller amplitude.

I: You say same wave, smaller amplitude – so what is the same about it?

A: Frequency.

I: Same frequency but smaller amplitude.

A: Yes.

I: OK.

A: We... hit this a couple of times.

I: OK. Is that sketch, then, that you've provided of the wave function here consistent with your previous reasoning about what you would observe in region C?

A: Yeah.

I: How, how is it? I think previously at the beginning of the interview you had said you would detect some electrons in region C, but not all of them.

A: Yes.

I: How is this sketch consistent in its description of that?

A: (unint) I'm breaking down is the interpretation of what this wave means. If I'm interpreting it as you do a physical wave where the amplitude represents the intensity, frequency represents the energy, in this case you have, a lower intensity beam of electrons with the same energy.

I: What does a lower intensity beam of electrons mean?

A: That's where it breaks down. Lower intensity means fewer electrons.

I: OK.

A: When you're talking about a beam.

I: So there's less electrons, but each electron in that has, on the average, the same...

A: The same.

I: ...energy as it had in region A?

A: Yes.

I: OK. Um, so, these two sketches were great in explaining what you were thinking about going from this barrier where nothing makes it through to this barrier that's now narrower where something makes it through.

A: Mmm.

I: Um, would anything change in this sketch, for example, where something does make it through, if we increase the barrier height?

A: (unint) using the same questions again and again. Um... once again, I had a ra, a recollection from class that something does change, but according to logic that I've used so far to make these, nothing changes. It goes right through the wall.

I: OK.

A: Potential barrier.

I: OK. Now, I - maybe this is even harder or worse, um - uh, actually no, let's - one more question in these I forgot. What happens then - we, we said OK, this is some arbitrary energy we start with.

A: Yes.

I: What happens to this picture of the wave function if I keep the blue barrier here, but I increase the energy of the electron. How does the wave function look different.

A: So, once again, I don't know how to correctly interpret this wave. If this were a physical wave, higher energy would mean higher frequency, which... yeah, this isn't working. I'm pretty sure I'm interpreting the meaning of this wave wrong. I'll finish the sentence. In a physical wave you have the same intensity beam, the same

- physical beam, so in a physical beam you'd have the same (unint), go back to wave, so in a physical wave you'd have the same amplitude, but the frequency would be greater. In which case, the amplitude would... so if you increase the energy... I gotta stick with one model here. All right. So I'm going to stick with the interpretation as I remember it from physical waves, which I don't believe is actually what's going on here. Oh, forget it. So in that case, which (unint) red...
- I: So now it's like sending higher energy...
- A: ...so higher energy, you'd have the same amplitude, the frequency is greater. Hits the barrier. Hits the same, cut you down lines, and, mmm, same, uh same amplitude of wave makes it through, only, higher energy. Same damped thing, I'm pretty sure.
- I: So, it would be the same picture – the only thing that would change now is the frequency of the wave.
- A: Yes. The, the same intensity would make it through. The energy coming out the other side would simply be higher.
- I: Now, you previously said that this is something that you're recalling from physical waves, and you're not sold on whether this is applicable here or not.
- A: The interpretation of this squiggle here, the sinusoidal wave, I'm not convinced that I understand it correctly. I'm not sure that this, uh, the amplitude of the wave in this representation does signify, um, intensity. Nor that the frequency does in fact represent the energy. As a matter of fact I want to, when I, when I draw this, I want to draw it the other way around. When you asked me what the, uh, intensity of the, of a higher energy wave would look like...
- I: Mmm-hmm.
- A: ... I want to, draw a bigger wave.
- I: With a higher amplitude.
- A: With a higher amplitude, yes.
- I: Why?
- A: That's a purely reactionary thing. Based on this being a potential, and not a phys, not a physical distance.
- I: OK.
- A: Um... And a way with higher energy, in the, in the model that, kind of need these things so we actually keep on the same page, I'll call this the physical model of the wave, I'll call this the other model of the wave.
- I: OK.
- A: So the other model of the wave, the intuitive one, based on a very unc, an uncertain interpretation of this thing. All right, so the other model, the way I want to think, that the higher intentio – intensity has a higher amplitude, bigger amplitude, greater amplitude, whatever. I want to say that, because a bigger amplitude would peak over the potential

- well. Going back to the cart. More energy, if you had more energy, you would make it over the well, just like the cart made it over the hump in the middle there.
- I: So, you could, you could give electrons in this model, uh, enough energy that the amplitude would go over the barrier? And what would that, how, how would that affect what the electrons did then?
- A: I do not know. Once again I'm not, I'm not feeling very sold on the interpretation of this wave.
- I: All right. You've obviously suggested two models, one where an increase in energy increases frequency, and another where an increase in energy increases amplitude.
- A: Yeah, let me just clarify. That is purely, um, an interpretation of the wave as depicted in this kind of diagram (video ends) - potential energy versus position.
- I: OK. Final question then. Um, we've been, you know, talking about this and it, it's quite evident to me that these are scenarios you've at least heard of and worked with a bit in class. Can you think, then, I mean, this is a potential energy diagram. What kind of physical system could it possibly represent. What kind of physical scenario might be modeled with this kind of diagram?
- A: What would make a potential well for an, for a particle? Give me a second. I bet I can come up with it. All right, so, mmm, turn the question around - how do you trap an electron? Huh! ... For example... I have a scaling problem here, but, and area of higher potential could have a... higher, um... um... positive electric field, so the, so the electrons which have a negative charge would be stuck between the positive areas. Good way to trap an electron.
- I: So, what would that look like?
- A: How do you mean, physically?
- I: You said there could be positive areas, and this would trap an electron between that?
- A: All right, yeah, so, an area, let's say... big positive area, mmm let me just think of a way to draw this, of representing this. So I'm trying to create an area where there's, where it's hard for an electron to go. It would need a lot of energy. OK, for example, check it out, two dimensions, keep this easy.
- I: OK.
- A: There's a positive charge sitting right here.
- I: OK.
- A: Generated however you please. Electron is sitting here... it's a negative thing. Going towards this area, this is not a particle, this is an area.
- I: OK.

A: It's going to feel a repulsive force, and, the intensity of the repulsive force would be representative of the height of the barrier, of the potential...

I: OK.

A: Uh, the potential barrier here. It has, if this has a high energy, a physical electron, going towards a positively charged area, if it's going fast enough, it will go right through it, it's gonna slow down a little, and be accelerated on the far side.

I: OK.

A: High enough energy, in this case, high enough on the y axis...

I: Is that greater than  $E_2$ ?

A: Uh, yeah.

I: OK.

A: That's gonna get by.

I: OK.

A: That is, yeah, that's it. That's what I'm trying to say. That would be an example.

I: OK. Um, last question, part b, no. You said earlier that you were having some conflict because you were thinking of a physical concrete wall, but no, this is a quantum scenario, right?

A: Oh, I was, I was interpreting the, the diagram, and, see before this, said potential energy and (unint). So, because there was no scale on here, I was saying, well, if this is a, a macroscale thing...

I: OK.

A: ...versus a microscale thing.

I: So, yeah, when, when does quantum work. In your mind, when, when can we use the quantum rules, versus using the classical rules.

A: Oh, when you are talking about very small things, on the order of magnitude of light waves.

I: OK. How big is a light wave?

A: Uh, ten to the negative nine-ish.

I: Ten to the negative nine what?

A: Oh, meters.

I: For... wavelength, or?

A: Oh, yeah, wavelength.

I: Oh, OK.

A: Wavelength of a beam of, of a piece of, a wave of light is somewhere, I, uh physical light is, where we're talking nanometers, so it's five hundred nanometers, so ten to the negative seven meters. So, red light, if I'm getting the number right, is somewhere like um, short wavelength high ampli - higher frequency. OK. Actually got to write this out. Frequency... and ampli - and, uh period... so we're talking about the frequency means shorter wavelength, so, it'd have to be in the high end, so seven hundred and seven fifty-ish would be red light,

if I'm not, if I'm not getting this backwards. So seven hundred fifty nanometers, which is somewhere ten to the negative seven meters.

I: So, in order to use the quantum rules, you need objects to have, relatively short wavelengths. Is that what you're saying?

A: Yooo, short wavelengths.

I: Or, forget short, cause that means comparison, but you were saying somewhere in the, in the ballpark of light, ten to the minus nine meters.

A: Yeah, that's when... I understand this starts to really kick in. And where you have to acc - uh, it happens everywhere, that's where you have to start to account for it.

I: OK. Good enough.

## Appendix D

### TRANSCRIPT OF SECOND ADAM INTERVIEW

- I: OK, um, at the top I've given you two forms of, uh, the time-independent Schrödinger equation, of which I hope you're familiar with. Oh - I haven't fixed my  $\hbar$  problem, but, uh, anyway, um, can you just run through - let's run through this top form, and let me know if you understand what all these symbols are. What's  $\hbar$ ?
- A: That's Planck's constant over two  $\pi$ .
- I: OK. M?
- A: Mass.
- I: Uh,  $\psi$  here?
- A: Is the wave function. It's the thing you put in as a function of whatever.
- I: All right.
- A: Position usually.
- I: V?
- A: Potential energy.
- I: And E?
- A: Is energy.
- I: OK, so we're good on that, and hopefully you can see these are algebraically equivalent - I've just done some rearranging so we can use whichever, uh, form we want to use here. So, I want you to think about, uh, the potential energy situation that I've given you here. The potential is zero to the left of, say,  $x$  equals 0, and it's some positive non-zero value to the right of  $x$  equals zero. Let's assume, uh, to start, that the energy is greater than  $V_0$ . Call that case 1.
- A: Sure.
- I: So, what do the solutions to the Schrödinger equation look like for that system?
- A: If this, if the energy is greater than  $V_0$ ?
- I: Yeah, and feel free to draw on this, whatever you wanted, uh, butcher paper, whatever.
- A: They're going to be a wave function - it's gonna be a wave, sine wave of some kind.
- I: OK.
- A: The amplitude is going to change at the boundary here.
- I: OK.
- A: Um, a wave incoming from the left, despite being greater than this  $V$ -not level, a portion of it will be reflected, a non, a non-classical happening.

I: OK.

A: So, chances are, there should... we call this, just trying to remember what the wave will look like over here. The, uh, total energy is the kinetic energy plus potential energy, so potential energy goes up, the kinetic energy goes down.

I: All right.

A: So, energy is related to frequency, so this part above the, uh, on the right, above the step, will have... a... slower, a lower frequency, (unint), it will have a longer wave over here. So the average is going to stay the same. Amplitude will change. And the frequency - will the frequency change? Ah, there's my answer for that.

I: So it looks something like this? You said the average stays the same? The average what?

A: Uh, sorry. The, um, the average between... the... the axis around which the wave rotate - uh, not rotates, but varies.

I: All right, so, you've sort of drawn - is that what you mean by the dashed line over here?

A: Yes, OK so this...

I: And then it would be the same.

A: ...this is... this (unint) here might be the energy...

I: OK.

A: ...that you're giving this thing.

I: OK. OK, and then it's the same level, then, over on this side. And, so as you've drawn this, you have less amplitude, but a longer wavelength over here?

A: See, I'm not sure about this. The amplitude has to do with the intensity. Um...

I: The intensity of what?

A: That's what I'm trying to remember? I have no idea even how much I've thought about quantum since quantum. So what are the axes, first of all? Axes are the potential, and position. So over here you have a lower potential. What was the question again?

I: The question was simply what the wave function looks like for this system, um...

A: Yeah, I think this... I would guess something like that.

I: OK. What does it mean, then, if you sketched a lower amplitude over here, relative to here. Does that have any physical meaning with this system?

A: The amplitude... yes! That's the intensity, that's how much of the wave, how much of the energy has passed this boundary. If I recall. I'm trying to remember what the, the wavelength is in particular. Um. Here's your step function... yeah. Let me just think out loud for a second.

I: Sure.

A: The question is, how much, if the ampli, if the amplitude relates to how much of the energy passes that boundary, the  $x$  equals zero boundary, or whether or not, how much, how much energy is related to the wavelength or frequency. Oh, this is related – this is... if I'm not mistaken, OK, now that I, just trying to think about this, the graph of the wave function is related to the kinetic energy. By this guy.

I: Can you expand a little bit more on that – what do you mean by that?

A: All right. A wave has energy.

I: OK.

A: The energy is divided up into the kinetic and the potential portions.

I: All right.

A: Here we have a function, of the potential, right here.

I: All right.

A: Drawn on there. The kinetic energy is going to be the difference between the total energy and the potential energy.

I: So, therefore the...?

A: So, you would, you would, you would think that at that, that does make sense, somehow. Potential energy plus the wave function has to add up to a constant.

I: The potential energy plus the wave function adds up to a constant?

A: That doesn't make sense...

I: Well, I just, I'm just saying, you drew a wave function that seems to vary in amplitude.

A: Mmm-hmm.

I: ...with position here. And so, and, and then you were given a constant potential. So it does not seem obvious to me that the potential plus the wave function would be a constant.

A: It's... taken over a long distance, it's per, it's the, it's the square of the, of the wave function, that has physical significance.

I: OK. What is, what is the significance of the square of the wave function?

A: Well, as I recall, from Schrödinger's equation, um... (talks to self)... that gives you the probability of... OK, now that's coming back to me. That's all related to the probability of finding a particle at a location.

I: What is related to the probability?

A: The, in – OK, the intensity of the squared wave function, (unint) the, the high intensity, the, uh, call it the y-axis coordinate.

I: OK.

A: Of the square of the function. Is going to give you the likelihood – OK, this is a, a picture of particle...

I: All right.

A: ...particle in a box type of scenario.

I: OK.

A: As a function of position. Say and the square of its wave function looks like so.

I: All right.

A: Um, that is to say, with infinite number of measurements, you would be most likely to find the particle in a number of locations. You would never find it at locations where the wave function says, the y coordinate is equal to zero.

I: All right.

A: You'd find it sometimes here, sometimes here, and more often here. These are not the same wave function.

I: OK. This, this is just talking about a, a different scenario...

A: Yeah.

I: ...but the interpretation of the wave function.

A: So what I'm trying, trying to work out in my head, is how this whole probabil - the probability interpretation of the wave function, which is, you'd get it by squaring the wave function...

I: OK.

A: ...essentially the psi star psi thing...

I: OK.

A: ...and how, just a wave function drawn on the x and potential energy, form a system, how that is drawn, what the significance of how it's drawn is... (unint) I don't recall.

I: So, back here you were seeming to suggest that, um, amplitude is related to the probability of detection.

A: Yes.

I: So, does that then mean as you've drawn it here that you are less likely to find a particle over here than here?

A: This is not a squared wave function.

I: OK.

A: Hum, I don't believe... let me see... I'm not really sure... You are less likely to find a particle over here.

I: And why is that?

A: Trying to put it in words, but, an incoming wave representing a particle...

I: OK.

A: ...um, part of the wave is reflected by this boundary. The intensity of the wave must be related to the, uh, probability of finding - of detecting a particle in a location.

I: All right.

A: So if part of the wave is reflected, you have a majority of the wave in the low potential area.

I: Mmm-hmm.

A: Less of it's over here. It would make sense, then, that you'd have lower chance of finding it in the, uh, higher potential area.

- I: OK. So are you satisfied, at least for now, of how you represented the amplitudes?
- A: Yes.
- I: OK.
- A: I'm not convinced about the, uh, the wavelength, though.
- I: OK. Um, so, do you think the wavelength should be different over here? Or, really not sure? Or, think it should be different, but don't know if it should be greater...?
- A: No, no, no - no, no, actually, I remember now. Over here we're looking, we're looking at the intensities, basically, and how much of the wave is there. The energy of that wave does not change. OK, so my, so this was a mistake. The wavelength, um, feeling more confident, should stay the same. The amplitude should decrease. I'm not convinced of that either. Hold on. The... the energy is related to Planck's constant times the frequency. The energy is the same, so the frequency should be the same. The speed hasn't changed, so the wavelength is the same. I'll stick with that.
- I: So, essentially you're arguing that the wavelength is related to the energy, and therefore, if I heard you right, because the energy is the same both places, the wavelength is the same?
- A: That's the argument I've made, yeah.
- I: So, all we've done is shrunk the amplitude, then.
- A: Yes.
- I: OK. Uh, what about if I gave you essentially the mirror image of this, so that you had a potential of  $V_0$  over here, and a potential of zero over here. Would, would that change anything about the sketch?
- A: Wave still has - this is  $V_0$ ... plus some initial energy... the potential energy would drop, the kinetic energy would go up. It s - seems like I'm trying to... um... anyway, I think it would be the same. Mirror image. No, actually no, it wouldn't. This case, if I recall, you have some wave coming in. Another non-classical effect. Hits this boundary - yeah, some of it's going to reflect. How is that related to the wave function? It comes from a, uh, boundary condition, and the part of the problem where you match the boundary conditions to the... wave function. So part of the wave is going to reflect. Wouldn't do that in classical mechanics. Our wave would not reflect off of this boundary.
- I: All right.
- A: In quantum it does, have some small - by some small probability. All right. So, the total energy of the wave is going to be again less over here. Smaller. So, if none were reflected, I think by this equals  $T$  plus  $V$  thing, you would have, same frequency for the same argument, um, greater amplitude, but some of this is lost... whatever. So this is the amplitude, that it would have, if nothing were reflected. Some is

reflected, so I would expect a greater amplitude in here – shoot, I’m not drawing this at all – there we go. OK. So I would expect a greater amplitude than in the high potential area. But not as great as it would be if there was no reflection.

I: All right – reflection aside, why would there be a greater amplitude over here than here?

A: Well, in the simplest scenario this is, there’s a lower potential energy here...

I: OK.

A: You’re likelier, more likely to find something in an area of low potential energy, like a marble at the bottom of a bowl, instead of halfway up the wall.

I: OK.

A: So, the amplitude, the square of the amplitude is related to the, likelihood, is related, like, related to the likelihood of finding something...

I: All right.

A: ... in a position.

I: All right.

A: So, you would expect a greater amplitude here. In the low potential area. (unint)

I: So, in essence, this really is the reverse of what you first drew. However, if I heard your argument right, the, the reflection, um, condition here lowers the amplitude of this portion of the wave function a little bit, and increases this one a little bit? And same logic here? This one gets increased a bit, and this one gets lowered a bit...

A: Yes.

I: ...because of the reflection?

A: Mmm-hmm.

I: OK. What, um, what would the solutions look like if you had to write them down. Could you write down just what the form of the solutions to the Schrödinger equation would be in these, uh, regions?

A: What would they look like? They’d be sine functions, or they could be exponentials, typically. Um... no, I don’t think I can.

I: Would it help at all if you looked at this, um, differential equation? Could you write down solutions using the differential equation, or not?

A: I would have to almost go for a guessing solution. It’s going to be one of two things, I’m guessing...

I: OK.

A: ...and that is, and this essentially comes to the form, um, the second derivative... OK, here we go. Yeah. Plus or minus some constant. More or less how it goes. No, there’s not even a second constant. Cause this is the constant, this is a function of  $\psi$ . All right. (unint).

- Yeah, so this is either going to be exponential solution, or a sine or cos - sine or cosine solution. The positive, uh... hmm.
- I: Yet, here, here we've, we've only given you a positive, and you've written down plus or minus. Can you talk a little about wh - why you did that?
- A: This is just more or less going back to the math I've seen a dozen times.
- I: OK.
- A: Um, this comes out of a separation of variables. Actually, let me see if I can derive this thing real quick. So the general form is, uh... right? Where it equals a constant.... So this is staying the same, say we're just doing it... x and y coordinate system. So, big function of x... just a function, I'm sorry... of... just doing x and y... I forgot how to do this. Where am I going with this anyway? I was trying, I was trying to even come back to why there's a plus or minus here?
- I: Yeah, yeah. Are you - you're just writing down a general form of a differential equation that you would have to solve.
- A: Yeah, second, second-order differential equation.
- I: OK.
- A: You go through separation of variables, and I remember it's possible to end up with a positive or negative here. One - the positive ends up, you end up with a... psi of, say x, is a sine of whatever... of... and the other one... gives you... C to the negative... or whatever. This would have to be omega.
- I: So, is omega just some other constant?
- A: Omega's just a constant, yeah.
- I: And T is...
- A: Time. This is for, a time-based system. Here we probably have... an x instead of a t, cause we're more concerned with position rather than time. And... our omega is going to be, can't remember if this gets inverted somewhere in the process. It could end up just being,  $2m \dots E \text{ minus } V \dots \text{ over } h\text{-bar squared}$ . I feel like - I know you end up with the square root of that somehow. Anyway, I'm not sure what question I'm answering anymore.
- I: OK. You'd said that solutions could either be sines or cosines or exponentials, and now you've proceeded to write down possible forms of those solutions - right?
- A: Mmm-hmm.
- I: Uh, is one form or the other what you would see in this scenario?
- A: I would expect to see the sines and cosines solution. Uh, there, you'd have different set of solutions, a different A and a B, for each area of this. Basically you, you'd, in this case you'd split this into two regions.
- I: OK.

A: And solve it, and solve the Schrödinger equation for A, then solve it for region B, then match the boundary conditions, saying at  $x$  equals 0, the solution for region A equals the solution of region B.

I: And that's the boundary conditions?

A: That is...

I: That the solutions have to match there at  $x$  equals zero?

A: The wave function has to be continuous, yeah.

I: OK. Um, all right. Let's, uh, think about the scenario now where the energy is less than  $V_0$ . Does anything change?

A: Aah, OK, that's, that's where you get the, that's where you get the exponential answers. Yeah. The majority's reflected, but you do have some transmission into, um, the potential barrier. Which decay exponentially.

I: OK. So what would that wave function look like? Maybe you can sketch it on the butcher paper here?

A: What the wave function looks like?

I: Yeah, I mean, you sketched this wave function for when energy was greater than  $V_0$ , right?

A: Oh.

I: What does the corresponding wave function look like for energy less than  $V_0$ ?

A: (sketches) So you have some incoming wave... gets to the boundary, um... then it decays exponentially.

I: So, it's still sinusoidal over here?

A: Mmm-hmm.

I: But now it's exponential in there?

A:

I: And you'd said that previously you could have both sines and cosines, it would just, there would be different coefficients in the different regions, right?

A: Mmm-hmm.

I: Can you have both of these pieces of the solution over here?

A: I seem to think yes, but... well, you're clearly only going to get a decaying  $x$ ... you're clearly getting ultimately a decaying exponential...

I: Why couldn't you get an increasing exponential there?

A: That'd be saying you have more probability of finding something inside brick wall instead of bouncing off a brick wall.

I: All right.

A: For example.

I: OK.

A: That'd be like saying the, the wave function would increase like that.

I: Yeah, I mean, I'm - this, this has both increasing and decreasing exponentials in it.

A: Mmm-hmm.

I: So, I'm asking why it couldn't be the increasing exponential solution.

A: That would – let's say there was some other side of the boundary, that would lead to a greater probability on this side, it seems like. It seems like there's a greater – it would be saying there's a greater probability of transmission through a barrier then, you'd have a greater probability of finding something on the far side of the barrier, or somewhere inside of a barrier.

I: OK. Then where you send it?

A: Mmm-hmm.

I: All right, well, um, let's talk about the square barrier then.

A: Here we are.

I: So, what do the solutions look like in, in, in this scenario? Uh, for  $E$  less than  $V_0$ .

A: OK. Have some wave... when it comes to here, not sure that's drawn correctly... the wave function will be a decaying exponential, decaying exponentially towards its average value. Let's say it does, there's some tiny amount that makes it out here, doesn't get totally killed by the decaying exponential, it's gonna be out here, will continue to do its periodic wave thing.

I: OK.

A: I'll call it the, I'll call C actually. This will be C, this will be B, this will be A.

I: So the wavelengths are equal in, in A and C?

A: It's just the probability of finding it over there is less.

I: OK. Um, how big, this amplitude is obviously less than this. Yeah? Um, and I don't know if this is an artifact of the drawing or whatever. How big does the amplitude of this function in C, relative to the amplitude at any point in B? I mean, is it greater than it is over here at, you know, almost to A, or...?

A: I'm not sure if I got the question. This is decay – a decaying exponential, doesn't really have an amp...

I: Amplitude...

A: ...amplitude.

I: ...right, an amplitude. Uh, height above the line you've drawn here – in other words, um, this is, obviously decaying back off. Can, can this function here in C, uh, go above whatever level that function is at, when it crosses this line, or is it determined by that height?

A: It's determined by the height of the decaying function when it reaches the boundary. Which is to say... so, encroaching wave... potential barrier... that's your decaying exponential. Now if the barrier were to end right here...

I: OK.

A: ...this would be the maximum wave height.

I: OK.

A: Around, uh, symmetric around that point.

I: OK.

A: Would it be? Wait – see, that’s the question, whether or not the height of the wave function corresponds to the average value, or the value around which the wave function oscillates?

I: I’m saying...

A: Or is...

I: ...does the amp, is the amplitude of the wave function over here in C related at all to the height, uh, that the wave function is above this line in region B?

A: Yeah.

I: OK. And I, I think you’re answering that with this sketch, in that you’re saying...

A: Yeah.

I: ...this, this amplitude is fixed by this level that you cross at.

A: Yes, so if the, if the barrier ends out here, you have a lower amplitude.

I: OK. Irregardless, the wavelengths match, for, for A and C?

A: Think so, yeah. It’s the same ener, same energy as an intensity, it’s the intensity that drops off.

I: OK.

A: The amplitude’s related to the probability of finding a particle. Or, it’s related to the probability...

I: All right. What would change in this scenario if we decreased the energy? What would change in the wave function in any of those regions? If, if anything would.

A: How do you represent this? The, if you decrease the energy, less would get through. So the result is that this... uh, exponential would drop to a lower level. Or, it wouldn’t start at such a high level. It wouldn’t have as far to drop. How does this work? Yeah, yeah, yeah, yeah. Call this  $E_1$ , and  $E_2$ . The  $E_2$  scenario. The intensity again is related to the amplitude, so let’s say we have the same amplitude as in the  $E_1$  scenario. I’m not sure I draw this right...

I: OK.

A: Yeah, OK, this is just going to drop off faster, is all. Why is that? How does that come out of the math? I’m thinking of Newton’s laws of heating and cooling – the larger the difference, the greater the rate of change.

I: Is that applicable here?

A: I don’t know if it is. That’s, that’s what the math looks like.

I: Does it come out of anywhere, anything you’ve written up here in the math, or?

A: I don’t see, um, although, Newton’s laws heating and cooling, that, that’s, that’s a first derivative. First, first order differential equation.

- And you come up with a, a solution that looks like that. Don't you?  
I'm not going to try to do it right now.
- I: OK. So, were the amplitude to be the same over here, you say it would drop off more quickly here, resulting in a lower amplitude over here in C?
- A: Mmm-hmm.
- I: How would the wavelength of this new function here in A compare to the wavelength of the old function? Would they be the same?
- A: Uh, no, they would not. The - oh, once again by  $E = h\nu$ , lower energy, constant times the frequency, frequency'd be lower, wavelength would be longer.
- I: OK, so you, this, this bottom one would have a longer wavelength, then.
- A: (unint)... I two which would, I'll show you,  $\lambda_1$ , greater,  $\lambda_2$  is greater than  $\lambda_1$ , yeah.
- I: All right. What about for the case, now, with this system, where the energy is greater than  $V_0$ ? What does the wave function look like then?
- A: Energy's greater?
- I: Yeah.
- A: So wavelength is shorter. Um... I know there is going to be some reflection... the degree of which is based on how high above, how high the energy is above this  $V_0$  energy...
- I: Mmm-hmm.
- A: If it's far above, there's gonna be - will it be less reflection, or less percentage of reflection of the wave? I'm not sure. Um, we've already done this scenario.
- I: Uh, well we did, we did this scenario.
- A: Yeah.
- I: So...
- A: It's going to be the same scenario, but it's gonna drop off.
- I: All right, well, then what does it do over here, once it gets beyond  $x = a$ .
- A: (unint) answer. What did I say before? The... amplitude decreases... there's gonna again be some reflection... well, the energy's the same, so the wavelength's gonna be the same... were it not for the ref, OK, ignoring the reflected energy for now, you'd expect it to go back to a  $I$ , to a nice large amplitude. Would you, I don't know? Yeah, I would say that the amplitude on, in this region C now, is going to be greater than in B, but less than in A. Which is beautifully depicted.
- I: So, why is it greater than it is in B?
- A: This is like the marble at the top of the, uh, the top of the upside down bowl.
- I: OK. Is that again back to your reasoning that you gave in the reverse of this, where you're more likely to be found in the lower energy areas?

A: Mmm-hmm. Yes.

I: OK. Um, one more wrinkle to all this, then. What would, what would change about your solutions here, and lets go back to thinking about  $E$  less than  $V_0$ ...

A: Aah...

I: ...if this barrier, if region C were at an even lower potential. Would that change anything about the solutions that you started off with here?

A: Still have an incoming wave on the left?

I: Uh, incoming wave from the left, energy less than  $V_0$ , yes.

A: Well, it doesn't seem like what the potential is over here would affect the incoming wave...

I: All right.

A: ...but once it's over here... I have two conflicting ideas. Idea one says this is the lower potential energy, a particle, the particle's gonna want to stay there.

I: OK.

A: Um... where's the energy again?

I: Uh, less than  $V_0$ . So, yeah, at some level there.

A: So, you have this scenario. Our, our true wave... who decays? A soft decay, just for... artistic convenience. Then, for the scenario where this is straight out here, we said, I've said, it doesn't, the potential does not drop below, uh,  $V$  equals zero. Went out that way. However, the potential is dropped now. The energy is the same. See, I, I still going to relate it to the, uh, the prob, probability to the, um, kinetic energy.

I: Is there a relation there?

A: (writes) In, well their, they are related. Like in the classical sc, scenario their relation is obvious. Um, if something's really going fast past a spot, if you're, say, the fast part of the swing of a pendulum.

I: OK.

A: OK, (unint) remember this now... OK, then your classical pendulum, if this is your probability... um, there's your equilibrium position, we have some scenario like that, it's going slowly near the end, fast in the middle, (unint) in the middle, I remember this particular sc - this particular case from quantum. And, as, as you look at the lower energy, the lowest energy level, you have some case like that. The second one you have some, this is, for the, uh, energy that comes into the equation. And when you end up taking high enough energies, you end up with something that approaches the classical scenario. Where is this going, anyway?

I: Uh, I think you were trying to reason if the wave function...

A: Oh.

I: ...was at all different over there in region C.

A: Yeah.

- I: Now that there's lower potential energy there.
- A: Oh, that's why I was trying to relate it to the kinetic energy. Obviously, that relationship is, not going, not necessarily going to be intuitive. All right, so... I still wanna say that, less the, less, energy is going to actually get through. OK, so I've already sketched this. That scenario where the potential bottom here is at  $V$  equals zero. Um... with a lower potential energy. I want to say that it's going to have a greater amplitude than it would have otherwise. I'm having trouble justifying that, though. That's the in, that's the...
- I: Why do you want to say that? Simply because lower potential energy is where things want to be found?
- A: Yeah.
- I: All right.
- A: But it seems like less of the wave, well not much of the wave would get through. It seems like the intensity should be, greatest... A, B, and C... in region A. Because an incoming wave is first of all reflected, and what's not reflected decays. It doesn't seem like a whole lot's gonna get through the barrier, compared to how much encroached in the first place.
- I: So, are you wanting to say then that the amplitude in C is greater than it was in this case? But less than A? Am I hearing that correctly?
- A: That would be my intuitive guess, yeah.
- I: All right. Uh, you, the other thing you've characterized on some of these sketches was wavelength. And how would the wavelength, uh, compare in regions A and C?
- A: No, that was, that was (unint) something back here in the very beginning. The wavelength should be the same.
- I: It should be the same in both?
- A: Yep.
- I: Is that consistent mathematically with, I'm assuming, again in C, it would be this, um, form right here?
- A: Um, yeah. Is that consistent? The quantity you're looking at is  $E$  minus  $V$ . I think what I'm concerned, with, though, is that, this total energy is the same. So that's the question. Does the wavelength come from the kinetic energy? Or does it come from the total energy? And that's what I don't recall. If the wavelength is connected, or married to the total energy, then the wavelength is going to be the same in every scenario. If it's related to the kinetic energy, it's gonna change dramatically depending on where, what the level of potential energy is.
- I: And not sure which one it's related to?
- A: I'm not sure, no.
- I: Uh, two quick questions here at the end. On all these, uh, scenarios, you drew the wave function on, um, these axes that are, uh, vertically potential energy and horizontally position.

A: Yep.

I: Does that mean that the vertical portion of the wave function represents energy?

A: Um, the vertical portion of it...

I: I mean, if you're plotting wave function, would you label the axes energy and position?

A: Um, no actually. Come to think of it, I'd usually be looking at  $\psi$ . If I was looking at a wave function, if I was looking at prob, the probability distribution...

I: Just...

A: ...I'd be looking at  $\psi^* \psi$ .

I: OK. Is that, equivalent to potential energy, or something different?

A: That's something different. What I've drawn down here is the  $\psi^* \psi$  graph.

I: OK. All right.

A: Um, the question is whether or not I'm plotting, I'm plotting the energy?

I: Well, I just noticed that you drew all of your wave functions on an existing axis that had labels of potential energy and position.

A: Yep.

I: And so, my question is, are those appropriate labels for a graph of the wave function? Or would the wave function have different labels?

A: Potential energy. Well, this... huh. There's a good question? That, the axes you've labeled are appropriate for the solid line you've drawn.

I: OK.

A: That is indicating the shape of the potential.

I: OK.

A: The, the wave function, no the wave function should be on a different set of axes, now that I think about it. It should just be  $\psi$ .

I: And what does  $\psi$  represent?

A: By itself?

I: Yeah.

A: Without thing, well, without the  $\psi^* \psi$  thing? Physically, not much.

I: If it was  $\psi^* \psi$ , it would represent what?

A: Probability. If you, the probability of detection in a location. For every situation where I've seen it applied.

I: OK. So, it wouldn't have units necessarily, it would just be a number.  $\psi^* \psi$ ?

A: What would it be? What are the units of it? I think it's one over meters. I could figure it out. Where's the easiest place here? So... can't remember the exact value of, uh, Planck's constant. It's 6.626, or something like that. 6 point something. Times ten to the negative... the Boltzmann constant is negative 34?

I: Frankly, I can't remember that one either.

A: (laughs)

I: Something. All right.

A: I'll try and remember the units. It's not coming back to me. I could answer the question if I remembered the units of Planck's constant. Where, oh, where else do I see it? Here we go. Um,  $E$  equals  $h \nu$ . This is gonna be kilograms meter... this would be Newtons, so it's kilograms... times distance meters squared seconds squared... this is going to be seconds, so that should be kilograms... um, meters squared per second... so, kilograms taken by  $U$ ... this doesn't seem likely... we've got put this units of  $h$  or  $h$ -bar, same units, squared, or one over  $h$ -bar squared meters squared per second... so that'd be units of  $h$ ... where was I going with this? There's, there's the question first... seconds per, seconds per meter... um, shoot, the way I've done this is not gonna, not tell me the units of  $\psi$ ... That doesn't tell me the units of  $\psi$ . I can't remember what the units of  $\psi$  are. Ah, here we go. In the exponential, this, this has to be unitless. (unint) but I've gotta assuming I've done this right. But  $\psi$ 's not in there... Oh, no, maybe  $\psi$  is in there... So, getting back to this, I'm not sure what the units of  $\psi$  are.

I: OK.

A: Which leaves me unsure of whether or not you can legitimately plot it on an energy graph. But I don't think it has units of energy, so I think you'd need a different graph...

I: OK.

A: ...that's the short answer.

I: OK.

A: And you have  $\psi$ .

I: That works. Uh, and the final thing I wanted to ask was back here when you just sketched a sample graph for  $\psi$  star  $\psi$ , and we're talking about probability, you said take an infinite amount of measurements, or measurements for a long time, and you're very likely to find particles here, and sort of likely to find them here, and never here.

A: Mmm-hmm.

I: So, if you were likely to find a particle here, and here, but not in between, how does that work?

A: (laughs) You tell me. I don't know. I don't think that is known, um, I don't think quantum wave - I've been told anyway that quantum mechanics doesn't consider that a legitimate question. That, a fact is it, it seems to work that way. You have one particle, take a million measurements of it after letting it go back to whatever initial conditions, you'll be it here, you'll be it here, you'll be it here(??) That you will not find it in those locations? It does not have a, I understand

it does not have a classical, um, doesn't lend itself to a classical explanation. I don't know how it does that.

I: So if we were thinking back in the well, if I'm, if I'm in this position, and I'm in this position, I must have gone, you know, straight line from here to here. We can't use that kind of thinking in this realm? Or...

A: No.

I: OK.

## Appendix E

### TRANSCRIPT OF FIRST JACK INTERVIEW

- I: Here is a diagram of something that's often referred to in classes as a potential barrier. That is, there's some region in space where the potential energy is higher than its surroundings. And so in this diagram Region B has a higher potential energy than Regions A and C. So an electron with an energy less than the energy of the barrier is incident upon the barrier in Region A, and we've indicated roughly the energy of the electron on the diagram. What's going to be the behavior of the electron as it encounters the, uh, potential barrier?
- J: Well, um, there's a couple possibilities. Because the potential barrier is higher than the electron, it can either reflect back off and go back the way it came, it can tunnel through... what else can I remember?
- I: Is that it – is that, is it going to do one of those two things, or are there other possibilities for it?
- J: Potentially there are other possibilities but I do not remember them at this point and time.
- I: So we're just gonna say that it – you said it could either bounce back, or it could go through the barrier. And so, if I said, uh, is there any chance the electron will ever be found in Region C, what would you say?
- J: Yes, there is.
- I: There is? OK. How do you know that?
- J: Well I know because I was taught that in like, Introduction to Quantum Physics class when the particle of some certain potential energy, or of some energy, encounters a potential barrier, there is a possibility, calculated through, well, wave equations and their integrals, that a particle will actually just go straight on through, losing energy as it does so, and come out the other side of the potential barrier at a lower energy and continue on its path.
- I: You mentioned the wave equation – have you ever heard of, uh, a wave function?...
- J: Yes.
- I: ...or heard it described in that way? What does the electron's wave function look like in Region A?
- J: Uh, let me see, wave functions are generally sinusoidal in shape, they look... sinusoidal-ish.
- I: OK. Can you sketch on the, on the, the, butcher paper here, what the, the, what the electron's wave function will look like in Region A? And

- while you're at it, if you want, what does it look like in Regions B and C?
- J: (sketches)... that's the electron potential that's roughly the midpoint of its sinusoidal, and it's going to look something like that. I think, unless I'm thinking of  $\psi$  instead of  $\psi^2$ , where it looks different than that, um, in the region there. It probably reaches actually roughly where its top is, instead, cause when it rea... when a particle goes through a potential which is higher than its parent energy, it looks more like a log decay equation as opposed to a sinusoidal equation.
- I: So what would it look like in Region B there?
- J: It looks something like that, but I think I... it goes more along the lines of something like that, it matches up for whatever reason, and then once you get to here, it has lower energy than it did over here, and its bumps are less, I guess (unint) I can call it.
- I: You mentioned that here it matches up for whatever reason – do you have any idea in your mind, do you know why that behaves that way, or...?
- J: I'm trying to think of why. The function needs to be continuous bo, on both sides of the potential barrier, and if it goes in down here and starts up there it's not continuous, so, mathematically I know one way if it starts up there and ends up there, and then it goes down.
- I: Any physical intuition about this? I mean, how would the electron back here know what the wave function needed to be right here?
- J: (Laughs). Mmm – how much do I remember? I know there's a reason, to be perfectly honest, I don't know.
- I: OK.
- J: I can't remember.
- I: So, just to recap, you said the wave function is somehow sinusoidal in here, it's uh, what was the word you used in here – the decaying log?
- J: Yes.
- I: OK.
- J: Log decay.
- I: Log decay, and over here it's sinusoidal again?
- J: Yes.
- I: Comparing the sinusoidal wave function in Regions A and C, uh, what's different about them?
- J: C will be a lower magnitude than A because it lost energy going through the potential barrier. Shape is smaller, it's shifted down a little bit more...
- I: How do their wavelengths compare?
- J: Wavelengths, ooh boy.
- I: What are you thinking about when... (unint)

- J: Good question. (Laughs). I kind of think since it has lower energy that means its frequency decreases, which means it's spanned out farther, which means it should have a longer wavelength.
- I: Why would you say that if it has a lower energy its frequency decreases?
- J: Because there's an equation which directly relates a particle's energy and its frequency, um, I could be wrong, but I think it's 'h' over, that's supposed to be 'v' or \_\_\_\_ I can never draw it right.
- I: (unint)
- J: (Laughs)
- I: ...there is a Greek letter nu.
- J: Because I know the speed of light equals its frequency times its wavelength, um, that's the frequency of the particle, and that's Planck's constant, or it's h-bar, I can't remember which. I haven't looked at it in a really long time, but I know its energy is directly related... I can't remember the exact equation, but I remember that the relationship between energy and frequency was that if frequency decreases, so does energy, and so the converse is going to be true - if its energy is lower, its frequency is less, which means there's further distance between a certain point in the wave which means a longer wavelength.
- I: OK. You've sketched the wave function in Regions A, B, and C - what would you label your axes as? Um, would you do so?
- J: (writes)
- I: So, x is what? You don't have to write it, you can just tell me.
- J: Position.
- I: And u of x is...
- J: Potential.
- I: OK. What happens if we take this same scenario, but, uh, make the original barrier narrower? That is, Region B takes up less spatial position.
- J: OK.
- I: Does that change anything about the electron's chances of being observed on the other side?
- J: Yes it does, because there's a smaller area that has a higher potential energy, then it has a greater chance of being able to tunnel through. Cause I know the equation for the probability of it going through is directly related to what, how far the distance is that it has to go through.
- I: So you would say narrowing the barrier increases the probability of observing it in Region C?
- J: Yes.
- I: If I widen the barrier, does the reverse logic work?
- J: Yes.

- I: Is there a limit to how wide I could make a barrier, uh, and still be able to observe an electron on the other side?
- J: Theoretically, no matter how wide you put the barrier, there is a chance the particle will be in Region C, but once you get to a certain point, it's so probable, improbable, trying to think of the word, that it will be on the other side that there's no point in widening it any more, because it won't decrease it significantly. But theoretically it should always be able to go to C.
- I: What happens if, uh, back to the original scenario, we now make the original barrier taller, so that  $E_{\text{barrier}}$  here is say twice what it originally was, but we leave the energy of the electron the same. Does this change anything about the electron's chances of being observed in Region C?
- J: Yes, that will actually decrease its probability of going through the potential energy barrier, because the difference between its, the electron's potential energy and the potential energy of the barrier is a much greater distance, and so it has a far smaller chance of being able to go through.
- I: So what would that sketch look like compared to this one? If you want to do another one over there on the side...
- J: (sketches). So the potential energy is higher?
- I: About twice what it was, sure. Higher.
- J: It's higher. If the ener, if the energy of the electron is the same, um, it will go through, it will actually decrease even more than it did there, and, I guess it actually has a longer wavelength than it did over here. It's not drawn to scale, exactly. Just roughly.
- I: So the decrease in Region B is more dramatic than, than it was in the original picture?
- J: Yes. (unint). It had to use more energy to get through a much higher potential energy barrier than it did for this one.
- I: How does the wave function in Region A now with this twice as tall barrier compare to the wave function in Region A in the original scenario.
- J: In Region A it should be the same no matter what, because that's the starting off potential energy - it hasn't reached the potential energy barrier yet. And it's not exactly drawn to scale.
- I: When you draw these wave functions as sinusoidal functions, what do you think the amplitude represents, of that sketch?
- J: Whew. Amplitude. The amplitude is related to how much... no, it's not. I'm trying to remember. Let me think, I took this class last semester.
- I: Any idea, or is nothing coming to mind?

- J: I'm thinking it's related to energy, but the energy is where it is. I don't remember what the amplitude is related to, but it's related to something. It's not just some arbitrary thing.
- I: You mentioned possibly energy. Is that still a candidate?
- J: I think so. I don't remember.
- I: If it were energy, what would that say about the energy here in Region A, of the electron?
- J: What do you mean?
- I: In other words, this wave function that you've drawn, does uh, does the wave have the same amplitude everywhere?
- J: Well it's a sinusoidal function, it's (unint) the amplitude constant in front of the sinusoidal function, it would be the same.
- I: All right, so the amplitude remains constant – let's talk about the wave height. Obviously, in what you've drawn the wave is not as tall here as it is here, for example. So does the, does the wave height represent anything, or...?
- J: Well, since, it's, it's a sinusoidal function so its value increases and decreases with respect to whatever variable is in the sine or cosine or tangent or cotangent – whatever the function is.
- I: Do you think it's related to energy?
- J: No.
- I: Why not?
- J: Cause when I'm thinking about it now, the equation is for potential energy,  $V$  equals some constant times a sinusoidal function. I don't... energy is not going to be on the inside of that sinusoidal function. Position and other variables needed to cancel out, uh, units, will be in there. So the change in height might not be related to energy, but the amplitude still possibly could be.
- I: OK. Uh, again returning to this original scenario then. What happens if we give the electron more energy, and say, you know, it's now instead of half it's 75% or 90% or whatever of the energy of the barrier, but it's still less than the energy of the barrier. Does that change anything about the electron's chances of being observed in Region C?
- J: As the particle's or an electron's potential energy increases, its chance of going through a potential barrier is increased. If you want to take it to an illogical extreme, you can say the electron's energy is way up here, and it's way much higher than the potential energy barrier's energy, so it just goes right over it – it doesn't matter. So it makes sense that increasing the energy increases its chance.
- I: What about the reverse, then? If I decrease the energy, does that decrease the chance of being observed in Region C?
- J: Yes.
- I: Is there, uh, some limit, then? I mean, how low can I go?

- J: I don't think you can have a negative energy, um, but I don't remember. Its quantum mechanics (laughs) – common sense doesn't apply.
- I: I guess my question was based on the fact that I believe, and correct me if I'm wrong, that earlier you had said that it starts out with some energy, it tunnels through, and it has less energy over here.
- J: Yes.
- I: So if some energy is lost in the process, I'm wondering is, is there some threshold or, or minimum level of energy that the electron has to have in Region A in order to have any chance of being observed over here in Region C?
- J: No. As long as it has energy, it can still lose energy to go through.
- I: So there's no minimum value it would need to be.
- J: No.
- I: So how do we decide, then, how much energy is lost in the barrier?
- J: That's based upon the difference between the potential energy of the barrier to the elec... the, the potential energy of the electron, maybe a change in height of the potential of the barrier, how wide the barrier is, and I think that's it.
- I: So I don't need any minimal level of energy in A...
- J: No.
- I: ... but whatever energy A has, it'll lose some fraction of that, and there's still a probability of observing it in Region C.
- J: Yes.
- I: OK. Obviously a very theoretical thing to think about, um, is there any real application of tunneling, or is it just some, you know, thought experiment discussed in quantum mechanics.
- J: Whew. Now by applications, you mean like, practical application in real life...
- I: Yeah, I mean, so we talk about this electron and there's a chance that it'll make it through this potential barrier. Is there any, you know, physical situation where this is true, or...?
- J: Well, if you could get enough electrons going through a potential barrier, and enough protons, (unint), theoretically you could just fall through your chair and land on the floor, or you could walk through a door. It's extremely improbable, but it's still possible. Um, as for practical applications of knowing it...
- I: Is there anything happening, you know, around us, where tunneling is occurring? Obviously you mentioned the probability of us walking through the door is very low. Anything that has a good enough probability that it happens on a regular basis?
- J: Possibly, but I can't think of it right now.
- I: OK. Um, here's a model of a scanning-tunneling microscope...
- J: Ah, yes.

- I: ... and you may have heard about this last semester in your course.
- J: A little bit.
- I: Uh, simplistically in my diagram, it's actually very, very simple. Uh, the scanning-tunneling microscope consists of a very sharp tip that's passed over the surface of some material, and it gathers information that obviously they can use to map the surface of the material. The tip and the surface are connected to some voltage source, and there's a meter that's used to read the amount of current that's passing through the circuit...
- J: Yep.
- I: ...and obviously there's some sort of...
- J: Control.
- I: ...control device that moves the tip around. So, what's going on, you know, on the atomic level? How is, uh, a scanning-tunneling microscope able to map the surface?
- J: Ah, yeah. From what I was able to understand of it, if you have some kind of surface here, it has atoms spread out, and each of those atoms has some kind of probabilistic electron cloud. And what the tunneling microscope does, it comes down, it has a tip - not exactly drawn to scale - and what it does is it comes down, and the voltage connected between the two, it measures the voltage between the tip and the surface, and when it comes near some kind of potential electron cloud, voltage will change, and that will change, and so what this thing does is it tries to move such that it keeps the voltage the same, so if the voltage increases it moves such that the voltage will decrease, if the voltage decreases, it will move such that it will increase, as it's moving along here. And so, if you've some kind of electron cloud, it goes around, it will move up or down or over or way up, as it needs to, to get the line of the equipotential lines of the surface.
- I: OK.
- J: And using that, you can map where atoms are and possibly, um, you get, where, you can't, um, you can't get an exact map, but you can get fairly close to what the potential looks like.
- I: So what, what's the limitations, then; you said you can't get an exact map.
- J: Depending on the accuracy of your voltage meter, and how well the tip can move up and down, um, it might look slightly different than what you might measure it to be, um... Possibly the tip could somehow affect the electron cloud, but I think they try and keep that to a minimum.
- I: What, uh, do each of the elements of the name refer to? That is, why do you think 'scanning' is in the name?

- J: Scanning. Um, hmm. Well, essentially it's scanning the surface where the potential lines are; it's figuring out how it's mapped out, sort of like a topography map, I suppose, would be almost a way to put it.
- I: And 'tunneling' - why would 'tunneling' be part of the name?
- J: It's measuring the voltage, which is a measure of potential energy, and tunneling is directly related to how much potential energy is within a region, and whether an electron or some other particle can just go right through the barrier, or through the surface.
- I: What would the barrier be in this situation, then?
- J: The equipotential lines of, the uh, hmm. Hmm. Hmm.
- I: Is charge, is - are electrons tunneling in this situation?
- J: Electrons are tunneling in between where the tip is and where the potential it's measuring for are. And by the number that it's tunneling at, I believe that's how it measures where the voltage is, or how high the voltage is, and so by measuring the number of tunnelings, it figures out what the voltage is, so it doesn't actually measure it directly; more of an indirect measure of voltage.
- I: What about the, uh, third word - 'microscope'. Why is it called a microscope?
- J: I think it's called a microscope cause it's looking at things that are on a very microscopic scale. The width of an atom is something, and, is measured in angstroms, so pretty small, you can't exactly look at it with the naked eye. The distances between the tip and whatever the potential you're looking for is, is extraordinarily small, there's no way you'd be able to get it by hand.
- I: So you said we can't see on the order of angstroms with the naked eye. Does this device allow us to see the surface?
- J: Sort of.
- I: What do you mean by sort of?
- J: From the measurements you make, you can use a computer program to map out sort of what it looks like, getting kind of like a 3-D diagram, (unint) swells... swell thing. It's not going to be an exact measurement, cause it's measuring indirectly.
- I: OK. Um, want to focus the conversation here at the end just a little bit on the interaction between the tip and the surface, and suggest perhaps that this could be a model of the tip and the surface, where the surface is the, the sheet of blue atoms there, and the tip is of course the red atoms, but I've colored the very tip atom yellow and the, the surface atom of interest green, and so we're just going to talk about the interaction between the yellow and the green there.
- J: Alrighty.
- I: If, um, if we represent these atoms on an energy diagram in one-dimension...
- J: One-dimension, OK.

- I: One dimension, turns out it might look something like this. What will the electron's wave function, uh, look like in the region corresponding to the tip of the scanning-tunneling microscope.
- J: It'll look like a sinusoidal function much like, uh, the one on the piece of paper that is so nicely hidden, but (unint)... the electron's there, it's energy is going to be some kind of sinusoidal function, going up like that.
- I: Is it possible that the electron will tunnel through this energy barrier?
- J: Yes.
- I: How do you know that?
- J: Well, we kind of went through that on the previous diagram. If the electron has energy and there's a potential barrier there, no matter how high the energy barrier is, there's still a chance that it can reach through, and go to the other side of the energy barrier where it'd be lower.
- I: All right, so what will the electron's wave function look like in each of the other two regions, then, in the gap region, and in the surface region.
- J: Well, if this is going through a potential barrier, it's some kind of log decay function, goes like that, and once it reaches here, it once again becomes a sinusoidal function, with a longer wavelength, small amplitude, lower energy.
- I: Is the log decay function here, uh, any different than the original log decay function?
- J: Hmm. It'll be different in that the potential between the electron's potential and the potential barrier is changing, it's not a constant (unint) that'll change exactly the amount that it decreases by, it'll change maybe its slope a little bit at the li... potential became, um...
- I: Um, steeper, less steep, any idea?
- J: It should be less steep, because the potential of the gap is decreasing.
- I: OK. And you already said that it's sinusoidal over here as well?
- I: If, if we now move the tip closer to the surface, does that change anything about the electron's wave function in any of the three regions?
- J: Well, if you get it closer, that means the gap is smaller, the potential barrier becomes shorter, as it were, but I'm not sure if you're shortening it from this end or this end, so I won't know if the potential barrier's still as high, or if it's lower, but regardless, it's smaller, so it decays, but it will not decay as much, because there's less distance for it to decay, and it'll come out at some higher value.
- I: So, there'd be more energy here than there previously was?
- J: Yes.
- I: Ah, if we move the tip farther away from the surface, is it the reverse then?

J: Yes.

I: So this makes the barrier now wider...

J: It'll decay longer, it'll come out with lower energy.

I: OK. What happens if I increase the potential difference between the tip and the surface? Looking back to this original...

J: Increases the height of the, uh, potential of the gap.

I: So, let's make this barrier taller?

J: If you're making the barrier taller, it will decay more, because of the larger difference between the two potentials, this will come out with smaller energy (unint)

I: OK, just to make sure, I don't know if you took that as a suggestion, but, increasing the potential will raise the height of the barrier? What will the new barrier look like, if you could use a different color, perhaps, for contrast.

J: If I use a different color, my (unint)

I: (unint)

J: Increasing the potential between...

I: The potential difference between here, so, you know, take this out and instead of five volts, it's now ten volts, or whatever.

J: So, they're ac..., they're not increasing this, that's increasing this right here, cause, or this is the potential of the tip, and this is the potential of the surface...

I: OK.

J: ...and what you're doing is actually increasing this energy, there.

I: Does, uh, anything change about the barrier, or is the barrier exactly the same?

J: I'm pretty certain the barrier remains the same.

I: Does anything change about the electron's wave function in any of the three regions?

J: I don't remember exactly, but it is possible that the relation between the decay of this is also related to how high this energy barrier is in relation to that, as well as the potential of the electron, and though change in both of these, cause if that one changes, obviously that one's going to change, but I do not remember exactly, or if it does, which way it goes.

I: Would it change it at all in the tip region, or no?

J: It's possible; I don't remember exactly.

I: If I, uh, bring the tip close to the surface, and somehow I have a device, and a short time later I'm able to take a measurement, uh, where will I find the electron, if we assume there's only one electron in the system?

J: You don't know.

I: No idea?

J: There's potential it's there, it could be there, it could be there. You don't know.

- I: Is the electron at a definite location, we just have the inability to find it, or does it not have a definite location?
- J: I know there are places because of when you look at...  $\psi^2$ ...
- I: What's  $\psi$ ?
- J:  $\psi$  is the, how do I say this? I'm trying, I believe  $\psi$  is the potential function, of its energy, and  $\psi^*$   $\psi$  is the probability, the integral of  $\psi$  dot, position vector, the integral of the  $\psi^2$  equation is the probability of finding it at some point in the graph, and there are points where, since the sinusoidal function equals zero or something or a negative number, if it goes there, when you square it, it becomes zero and there are points where you actually can't find the particle, no matter how often you look.
- I: OK.
- J: Exactly where is probably there, there, there,...
- I: So, which of those would you lean towards, then? The electron has a definite position, we just can't find it? Versus the electron has no definite position, therefore we can't find it.
- J: It doesn't have a definite position, because there's no way of knowing which direction it's going to go, because, oh, we do have an uncertainty principle where... I guess like the better we know momentum, the less we know about its position, the more we know about its position, the less we know about where it's going, so where it's going next, we don't know. It's impossible to know, if we know where it is at one point, it's impossible to know where it's going to be at some other point in the future. All we can look at is probability, so there is no definite position as to where it is.
- I: OK. That's it.
- J: OK.

## Appendix F

### TRANSCRIPT OF SECOND JACK INTERVIEW

- I: Here's the time-independent Schrödinger equation, equation written in two forms, and I don't know if, edit too, I couldn't figure out how to write an  $\hbar$ ...
- J: It's a...
- I: ...on the equation editor.
- J: ...I think it's a...
- I: Is it in there?
- J: ...it's a symbol somewhere, in Microsoft...
- I: Oh, OK.
- J: ...Office that has it, I thought.
- I: Um, anyway, time-independent Schrödinger equation, and so I just want to go through this, the equation first with you in two different forms. I don't know which one you're more familiar with working in. Obviously, algebraically, they're the same thing. Um, are you OK with what all the symbols are – so  $\hbar$  is what?
- J: It's Planck's constant divided by  $2\pi$ .
- I: OK. And  $m$  in this equation refers to?
- J: If we deal with the mass of the particle, whether or not if it has a mass is dependent on whether you're talking about photons or electrons or whatever.
- I: OK. Um,  $\psi$ ?
- J: It's a general wave function.  $\Psi$  is kind of irrelevant, we only really care about  $\psi$  star  $\psi$ , which is the probability function of a particle beam at any point in space if we go to make a measurement.
- I: OK. Uh,  $V$  of  $x$ ?
- J: Potential of – well, in this case, it would be a one-dimensional potential, dependent upon where you are.
- I: OK. And  $E$ ?
- J: Natural energy of a particle.
- I: OK. Good, so you're OK with all the symbols, and..
- J: Yeah.
- I: ...that, and I don't know, some books use  $U$  of  $x$ , others use  $V$  of  $x$ , et cetera. So, I want to talk about a system, uh, that has a potential energy as shown, so its zero for, uh,  $x$  less than zero, and some  $V_0$  for  $x$  greater than zero.
- J: Yep.
- I: What would the solution of the Schrödinger equation be like if we assumed, uh, the case, the first case we'll think about is when, uh,  $E$  is

greater than  $V_0$ , and (unint) draw on this, draw on, you know, the butcher paper, anything you want.

J: All right. Well, I'll just draw that again... Step function... I'm assuming that's what you're asking.

I: Yes. So what, what kind of solutions would you have, uh, to this, for this system?

J: Uh, let me think about it for a minute. If  $E$  is greater than  $V_0$ , it should have just a sinusoidal, well,  $\psi$  star  $\psi$  would have a sinusoidal form that stays above zero. Or at zero at certain points. It's gonna have... uh, boy, it's been a while... so if I solve the differential equation, I get sinusoidal forms... star  $\psi$ , star  $\psi$ ... um, square root of the inverse of that... so that's energy... I once wondered about that, why the units don't work out.

I: What do you mean when you say the units don't work out?

J: Uh, cause energy has units of Joules, mass has units of kilograms,  $\hbar$  squared - oh, right, has Joules-seconds, squared. That's squared. One of the Joules goes away - OK, equals here, cause the  $dx$  squared has units of per meters squared, OK... backwards.

I: So, you're satisfied that it works out.

J: Yeah. I know it works out, I just always forget the units work out, (unint) I know that they do. Cause the zero over here didn't have the units (unint)

I: OK. OK.

J: This is notation to myself (unint) that potential isn't zero.

I: OK.

J: Uh, the energy is greater than that, so it should just be... lets see,  $x$  equals zero... it should just look like that.

I: So, just to be clear, you're, you're graphing  $\psi$  star  $\psi$ ?

J: Yes.

I: OK. Um, is it possible to graph  $\psi$  versus  $x$ , or no?

J: You can graph  $\psi$  versus  $x$ .

I: Will, will it look any different than that, or?

J: Well, yeah, you'd have values that are less than zero at certain points.

I: All right. OK. Would the, would the solution be any different than for  $x$  less than zero, and  $x$  greater than zero. To me, it looks like what you've drawn is the same in both of those regions.

J: True.

I: Is that true in this case, or not?

J: I don't know if you could say. Cause the energy is greater than... and so, you probably (unint) wave-particle kind of thing, and usually in quantum mechanics when it reaches a barrier, there actually is a potential for it to, just reflect back.

I: OK.

J: Even though it actually has a higher energy, but it, since it has a higher energy, you don't have to worry about quantum tunneling, and it's wave function should just be the same in both places.

I: So you, you've, uh, sketched  $\psi$  star  $\psi$  graphically...

J: Yes.

I: ...what would the solutions look like if you wrote them out algebraically, I guess, in those regions. Was it, was it this, what you were writing before, or...?

J:  $\psi$  is kind of a function...

I: So, some sine of  $y \cdot x$  plus cosine of  $y \cdot x$ , and  $y$  is this stuff?

J: Yeah.

I: And that solves this differential equation.

J: I hope so, it should. It really should. I had it backwards.  $y$  is that now. Cause I was thinking of this in terms of...

I: OK, all right. So,  $y$  should be the inverse there? Uh, square... OK. Um...

J: It's a second-order differential, so the solutions gonna have the square root of the (unint) and stuff from the... I guess the function variable.

I: Previously you said the wave function would be the same in both of those regions.

J: Yeah.

I: As you've written it down algebraically, is it then the same in both of those regions?

J: Yeah.

I: OK. Um, let's think now then about, uh, case 2, where energy is less than  $V_0$ , but greater than zero, OK?

J: OK.

I: How does that change anything that we've done here?

J: So you want, the case with  $E$  less than  $V_0$ , and greater than zero.

I: Yep.

J: That's the case you want to now talk about. And I'm assuming its not (unint)

I: Sure. Why not?

J: Well, (unint) it down like that, at some future point, you're slowly (unint). So, the energy is less than  $V_0$ , the negative case is still gonna look the same... there, I drew, I drew that one wrong... I wasn't thinking -

I: Drew which one wrong? Over here?

J: Yeah.

I: What, what was wrong about it right there?

J: It should be a maximum right there.

I: It should be a maximum at  $x = 0$ ?

J: Yeah.

I: And why is that?

- J: I'm not really sure how describe it mathematically, I just remember abstractly, its always just a maximum right there, and I can drop that. Cause I know that the barrier between the potential difference, like for the case where I'm doing now, its going to be a maximum there, and the potential drops off as a log, as a natural log, well, it drops off as an exponential, according to... but in order to do that it has to be on a... well, not necessarily a maximum, it has to be at some common zero point right there, cause you can't start off at zero.
- I: All right, so again this is a graph of  $\psi$  star  $\psi$ ?
- J: That's what I'm used to drawing.
- I: OK. Um, something you said a couple of minutes ago, I just wanted to ask about - you said something about the potential drops off as  $x$  goes?
- J: Well, the probability of finding a particle at the distance, or at a position at  $x$  greater than zero drops off as the decaying exponential in the region where, the, I guess the potential of the system is greater than the energy of the particle.
- I: All right, so, um, but you used the word potential. Is that what you were really thinking, or is that different? To me, potential refers to the potential energy, and that is constant over here.
- J: Well, yeah.
- I: So you, you would rephrase the probability is...
- J: Yeah...
- I: OK.
- J: ...the probability of finding it.
- I: OK. Um, all right, so that's the picture that you have drawn there, um, what would the, what would the form of the solutions to this written algebraically be?
- J: Um... it would have form that looks a lot like that.
- I: OK.
- J: For, where  $x$  is less than zero. And the solution for... they're actually, technically, I guess there's constants that are up here...
- I: OK.
- J: ...but those really don't matter, we're just worried about that.
- I: Now, I, I noticed you now, you changed your sines and cosines to exponentials. Can you talk about why you did that?
- J: Um, mathematically when you solve the second-order differential, where the value of  $E$  minus  $V$  is a negative number, it turns into an exponential.
- I: OK.
- J: And... using boundary conditions on your (unint), well, when you use boundary conditions in solving the actual ener - for the full equation, this term would actually drop out, otherwise it would ultimately be...
- I: All right. You said something about - is that based on boundary conditions, or...?

J: Um, its either boundary conditions or initial conditions, I forget which.

I: OK. So, somewhere, you know,  $x$  less than zero, some combinations of sines and cosines,  $x$  greater than zero, some exponential function.

J: Yeah.

I: OK. Um, just one more point, sort of on both of these graphs, you, you earlier said that  $\psi^* \psi$  is a probability...

J: Yes.

I: ...and then you're graphing these sinusoidal functions.

J: Yes.

I: Does that mean, then, that its more likely to find whatever it is that this wave function is representing, uh, here and here than here and here.

J: Yes.

I: And is it also true then, I mean, do these touch the axis? Does that mean there's zero probability?

J: If they touch the axis. Well (unint) the zero point there, and (unint), thought you were at point five, like, if I was calling that point zero, where it actually touches the axis, then yes, there actually is no probability of finding the particle, or whatever you're searching for that has the wave function, at that point in space.

I: Maybe we should, uh, try to get some sort of physical reality to this scenario. Can you think of any sort of situation where you might see, uh, you know, potential energy at one region of some value, and then higher in a neighboring region? Or some sort of experiment that this would mimic, or...?

J: It's kinda hard to find a step-function potential.

I: And why is that?

J: In real life, usually most potentials drop off, um, have some kind of distance squared term, or, like, they're determined by like how far away you are from something, it's using the, quadratic rule or, maybe some kind of polynomial...

I: OK.

J: ...as opposed to just a, step function.

I: OK.

J: Um, my first guess would be if you have, had like a line of charges... and, if your two lines of charges, well, say like  $x$  goes that way...

I: Mmm-hmm.

J: ...um, in this area, the potential from the two lines of charges is gonna average out to zero, assuming this goes off to infinity, or you, or you can approximate it as going on to infinity.

I: All right.

J: But at regions here and here the potential - actually, no, potentials don't average out, or cancel, its just some constant value. Never mind. There's electric field, yeah, that's not... cause the electric field would

drop off, and then cancel off here, (unint), cancels out in there, and then spikes up and drops off... hmm...

I: All right, so maybe we, we don't have a good physical scenario that mimics this, um...

J: I think there is one, I just can't remember it.

I: OK. But if we think about this system we've said, OK, somehow we were able to build this, so that there is zero potential here and higher potential over here, and we shoot some sort of particle at this, um, do these pictures, then equations, represent what would happen to the particle?

J: Well, they don't necessarily represent what actually happens to the particle, they just represent the possibility, if we shot a whole stream of particles at say, said barrier, and we were measuring, particles as they went flying through, or if we measured where they are... stream of particles, right.

I: So you think this is a good model for a stream of particle?

J: Well, if you're...

I: Or a, a model for a stream of particles?

J: It's a model... well, it, it works for both a single particle and a stream of particles. If you measure a stream of particles, you can actually graph, perhaps, like the number of counts you find, on where particles are, and it should look something like that. Or if you, its kind of hard to graph a single particle, cause a soon as you make a measurement on that particle, you destroy its wave function, as for what it used to be, and it kind of spikes here at where you measured it at...

I: OK.

J: ...and it doesn't work anymore.

I: Wouldn't, then, making a measurement on a stream of particles destroy that wave function as well?

J: Well, you'd just, well, you'd destroy the wave function of the single particle that you measured, not the wave function of a stream of particles, cause each particle, assuming you'd make each particle in the stream have the same wave function, if you measured each particle, then it just ruins that one particle from the stream, but you still have the rest of the stream to measure from. And so you can measure at different points, and get, you know, something like that.

I: Then is that true, that there would be regions where there's no probability of finding particles? If you were to do these measurements.

J: It seems that way, its really kind of strange, if I were to think about it, yes.

I: How, then, does a particle, you know, get from one place to another, if there's no probability of being in between?

J: I've been wondering that for a couple of years now.

I: No good thoughts on that?

- J: Well, a lot of it has to do with... mathematically its kind of explained using Heisenberg's uncertainty principle, sort of, kind of.
- I: Which says what?
- J: Um, well, there's, there's various versions, uh, that's one of them... t... I think that's another one... Basically, you can only know so much between the position, or where a particle is, and where it's going. If you're aiming a stream of particles you'd know, more or less, quite a bit about where its going, and you can know less and less about where the particle actually is, and that kind of comes along into the zero points there.
- I: All right.
- J: This one explains quantum tunneling, and that, we, we only actually know so much about a particle's energy, so when you're, like talking about the case where energy's less than  $V_0$ , its actually kind of like a, plus or minus  $\Delta E$ . And, the error on, on that actually allows the particle to... go beyond where we think it usually could.
- I: All right. You uh, had some actual, probably foresight into where this is going to go. So what if we make this a little bit more complicated, and we talk about the situation where we have a barrier. I don't know if you want to refer to this at all, so we can set this up here, where you can still see it.
- J: Green now.
- I: So, yeah, lets stick with, um, the energy less than  $V_0$ .
- J: OK.
- I: What do the solutions look like for that system?
- J: (writes) All right, so... at the case where the energy of the particle is less than the potential, I guess we'll call it a potential barrier....
- I: OK.
- J: ...um... it's gonna look a lot more like that. It's gonna look very similar to, kind of, I guess the combination of the two cases before.
- I: OK.
- J: I was gonna, yeah. So this one, before it was just the, uh, potential (unint) off to infinity, it starts off, the probability starts off a sinusoidal function, and then it goes through an exponential decay...
- I: All right.
- J: ...well, then it looks like there's actually a point where the energy is greater than the particle, it goes back, the, yeah, the energy greater than the potential, it goes back to sinusoidal form, but it's gonna have a lower magnitude than it was out here.
- I: What would the algebraic solutions, then, look like in this case? What would the form of them be in, in each of those regions?
- J: Oh, the form of each of them. It, really'd just be kind of the same as those, except there'd be another term out here for... yeah... in this case actually the constants that technically come out in front here wouldn't

matter, cause they change the amplitude, so this constant's gonna be different from that constant, and that one is gonna be different from that one.

I: So, if I'm interpreting what you drew right, the amplitude is lower in  $x$  greater than  $a$ ... then it was in  $x$  less than  $a$ .

J: It, (unint) the probability of it...

I: So, in other words, those constants in, in the third equation, then, that you just wrote would be less than the constants in the first equation?

J: Yes.

I: All right. Um, previously when we did this, uh, potential step, I guess you could say, you tossed one of the solutions, uh,  $e$  to the plus  $y$   $x$ ...

J: Yes.

I: All right. Does that get tossed here as well, or not?

J: It may, it may not, it depends on your boundary conditions. The reason it was tossed before is because since the potential step went off to infinity...

I: OK.

J: ...if I left that term in there, then the probability of a function would have gone to infinity as  $x$  approaches infinity, but it's not allowed, it just doesn't work that way. Cause as you go  $\psi$  star  $\psi$ , (unint) infinity, it can only equal one. And if this is infinity at some point, the integral of it is not one.

I: OK.

J: Whereas now with the case, since this only goes from one point to another, its possible to leave that term in. You know, you can't just argue - I can't get rid of it, by, boundary conditions.

I: All right, um, I noticed, though, that what you drew would, looks to me like a decaying exponential.

J: Yes.

I: Which to me would only sort of match the first solution, there.

J: Well, it could if you had a small constant, too.

I: So, this could be a weighted combination...

J: Yes.

I: ...but the constant on the second term would always be less than the constant on the first? I mean, mathematically, it seems to be OK to have an increasing exponential in that middle region.

J: Well, its possible.

I: Is that at all a physical result, or is it tossed because its not physically real, or, or what?

J: I'm trying to work it out. Cause, in this case, in region where  $x$  is negative, that would actually become the decaying exponential, that would be a... (unint)... not really sure.

I: Is it possible that we could draw an increasing exponential here, or...?

J: No, you're not gonna draw an increasing exponential, cause physically that would mean that you're more likely to find it over here than you are over here, and since you're – the energy of the particle is less than the potential barrier, the probability of finding the particle farther and farther and farther along inside the potential barrier is going to be smaller and smaller and smaller. And so you're not going to have an increasing exponential there.

I: How would you find the particle inside the barrier?

J: That's a good question.

I: Or could we? I mean, is it possible?

J: Well, it depends on how you create a potential barrier. If you created a potential barrier by like putting a physical object there, you're going to have a hard time measuring it. But if you did it by having some, electric field potential, kind of, and measure where your particle is, still can't.

I: All right. Um, on the picture you drew, you have sinusoidal wave function in, in the region left of minus  $a$ , and to the right of plus  $a$ .

J: Yes.

I: And, uh, I think we previously mentioned the fact that the amplitude's decreased in, in the third region.

J: Yes.

I: And that corresponds to a lower probability of observing something there.

J: Yes.

I: How do, how do the other characteristics of the wave function – uh, the wavelength, for example – how do those compare in those two regions.

J: The wavelength should still be the same.

I: And why is that?

J: That's determined by the constants that are in front of  $x$ , and it's the same constant. Still, still the same sort of equation, and

I: That's determined by  $y$ ?

J: Yes.

I: OK. And everything in the  $y$  is, is the same...

J: Yes.

I: ...in those two regions.

J: True.

I: OK. Um, is that  $y$  the same in, in other words, are all these  $y$ 's the same?

J: Yes.

I: So,  $y$  here is identical to this  $y$ .

J: Yes.

I: It's just changes the sign there. How about the energy of the particles, so we sent it in with an energy  $E$ . How is its energy over there, to the right of plus  $a$ ?

- J: The energy of the particle no matter where it is to the graph is still going to be the same. It just – the only difference is the probability of finding the particle.
- I: All right. One more wrinkle, then, perhaps increasing the, I don't know, degree of difficulty, what would happen if you had a potential that looked like that?
- J: Ooh. Potential...
- I: I don't know if you want a new sheet, or just move to a different spot on there. But, so, same thing over here in, in the, in the first area. Less than negative  $a$  is zero, now its  $V_0$  up here, but then it goes to negative  $V_0$ , uh, over in  $x$  greater than  $a$ . How would that change anything about the solutions you just wrote down, uh, for the case three there?
- J: My first inclination is that it would look very similar to this, but posture the higher probability there. And then I rethink it, and, if I completely ignore this spot, when the particle's coming in, it has no idea what the potential is over here. All it sees is that wall. Which means that here and here are still gonna look the same. It makes me think that the probability function's still gonna look the same over here as it did over here. I guess it'd be kind of like, putting water under the dam. Like no matter how much, like no matter how low you put like the hill over here on the other side of the dam, only – the same amount of water's gonna come flying over. (unint) the same probability function.
- I: So everything would look exactly the same in this case, as the previous case? Yes? Not?
- J: Maybe and maybe not, cause I'm trying to think, because... OK, yeah. I was wrong before, then. The  $y$ 's actually aren't the same, because I wasn't remembering that changed. Cause the constant that's in front of the  $x$  is going to be the square root of  $2mE$  minus  $V$  of  $x$  over  $\hbar$  squared.
- I: All right.
- J: And  $V$  of  $x$  is different for that  $y$  and that  $y$ , but the same for that one.
- I: Different – these two  $y$ 's are different from each other?
- J: Yeah.
- I: OK. OK. It's OK, all right.
- J: Those  $y$ 's are gonna be the same.
- I: In this or this case?
- J: This case. Those  $y$ 's are the same, those ones are the same as each other, but different from those ones.
- I: All right.
- J: And there's some (unint) that  $E$  is less than  $V_0$  but is greater than zero, I'm assuming, for this case.
- I: Uh, yes, yes, yes.

- J: I don't know how... Mathematically it's gonna look very similar to this, it's just the value of this constant's gonna be different because that's changed. Huh... If that changes, that means that constant changes, and if that constant changes, that means its gonna change the wavelength of the function, so it's not gonna look the same... that number's gonna be greater... which means, this wave would have a smaller amplitude...
- I: Is that OK? Is that, are you troubled by that, or...?
- J: No, I'm just trying to make sure that I'm interpreting that correctly. Because if  $V \times$  is a negative number, then you're essentially adding a positive number. Which makes the magnitude of this larger than it was before, and if you have the cosine of say, a hundred  $x$  versus the cosine of ten  $x$ , that's gonna look like that... so assume you're like that...
- I: So now its wavelength in that region is...
- J: Shorter, than in this region. Yes.
- I: OK. How does the amplitude now in this, you know, post-barrier, um, compare with the amplitude that we had over here? The same amplitude, or might be different, or...?
- J: Should be the same amplitude, roughly. Actually, it could be a little different, but... I'm guessing it would be the same amplitude.
- I: So, equally likely to find the particle here as here, in these two scenarios.
- J: Like, in the whole region...
- I: Yeah, right, right, right. Not at a specific point.
- J: Yeah.
- I: OK. Um, how about the energy of the particle, then, after they make it through this barrier.
- J: Energy's still gonna be the same.
- I: And is that consistent with what we've written down here?
- J: I don't see why not. Cause, that's stayed the same the entire time. It's not changing.
- I: OK. Um, you went back partway here and said, wait a minute, I realize that the  $y$ 's are different for different situations.
- J: Yes.
- I: Right? Um, would that then change your answer to the first scenario here? In other words, the first scenario you had the square, or the psi star psi for the wave function looking the same in both of these regions.
- J: Yeah.
- I: Still believe that, or do you want to change your mind back here?
- J: I will go back and change my mind, cause that definitely did change. So if it was zero before, I'm actually subtracting a number, which makes this smaller, which makes a longer wavelength, so it actually is more like that.
- I: OK. And that's for the solution  $E$  greater than  $V_0$ .

J: Yeah.  
I: Does that shift at all the solution when  $E$  is less than  $V_0$ ?  
J: No.  
I: No change in that?  
J: No, no change there.  
I: OK. All right, that's all I got.

## Appendix G

### TRANSCRIPT OF FIRST SELENA INTERVIEW

- I: Um, OK, to start off with – and by the way, feel free to draw on any of these pictures if you want to, or to sketch on the butcher paper that's below you – but, here is a diagram of something that's often in, uh, modern physics or quantum physics called a potential barrier...
- S: Mmm-hmm
- I: ...and you may have seen something like this before. That is, there's some region in space where the potential energy is higher than in the surrounding regions, and so we've diagrammed that here with just a square barrier. An electron with an energy less than the energy of the barrier is going to be incident on the barrier in Region A.
- S: OK.
- I: Describe the behavior of the electron as it encounters the potential barrier.
- S: Well, you have your nice wave-particle duality, so it moves along in a wave form, and... or the probability of finding it at any point in this area is a wave function. When it hits the barrier, because it is a finite barrier, there is a small probability – you gotta do the math – of the electron tunneling through the p... the, um, potential area, er... It's one of those funny little things that. There's a possibility it will go through even though it doesn't have enough kinetic energy to overcome the potential energy barrier, but it still may make it through to the other side.
- I: So you think there's a chance that the electron could be found in Region C – could be detected over there...
- S: Yes.
- I: ...even though it's energy is less than the energy of the barrier.
- S: Yes.
- I: What determines the probability that it will be found in Region C? What kind of factors come into play here?
- S: That is the kinetic energy of the particle, the mass of the particle, and the size of the barrier... thrown in to the square root and some other stuff, do a little magic, and poof, there's your probability.
- I: So you're saying, throw them into the square root; are you...
- S: Well there's a square root function in the, there's a square root in, it was the potential minus the kinetic, and something with the mass, and... that's the part that I remember.
- I: What, what is this, is this a part of a larger equation that you're thinking of, or...?

- S: Uh, I think it's derived from a larger equation, but this is part of a equation that predicts the tunneling probability or whatever, it's this and, you know, some other stuff.
- I: Now I think you, you mentioned, uh, the things that would affect it are the mass, the size of the barrier, and you said a third – was that the energy that you...?
- S: The energy of the particle.
- I: So how does the energy of the particle affect its chance of getting through to Region C?
- S: Well, you know, if you just look it... because of this barrier has a finite, um, limit. There's a certain number, a amount of energy that describes if the electron is closer to having the same (coughs) same amount of energy as this potential, there's a higher probability that it will get through, cause its path is going to be farther up here than down here.
- I: How does the mass come into play of, of deciding whether or not this particle will make it through the barrier?
- S: Um, I think it has something to do with the momentum, um... wait, just, I don't remember how it goes exactly, but intuitively, which doesn't really work with the quantum (laughs), um, but if its got more momentum, it's going to be able to get through easier, cause there's more to it.
- I: And the third thing you said was the barrier width? And how does that play into the probability of it making it through to Region C?
- S: No, I, I said the size of the barrier in height,...
- I: Oh, OK, excuse me, I misunderstood.
- S: ...it's more like...
- I: How does the height matter, then?
- S: Well, if its, if it requires a greater amount of energy to, you know, this is the potential barrier, say like the, uh, Coulomb force of keeping a particle in a nucleus. There's an amount of energy that's required to get past that. The particle by itself generally doesn't have enough kinetic energy to leave the nucleus, because of the amount of energy holding it in, so, the uh... if the height of the... if the amount of energy of the potential is greater, it's going to take more energy for the particle to get through, and if it's infinite, it's not going to get through.
- I: So if we extended this, and this energy barrier were in principle infinitely high...
- S: Mmm-hmm.
- I: ...you would say there would be no chance of it making it through to Region C?
- S: Um, yeah, you know, the idealized particle in a box. It's infinitely high, it will never get out.

- I: Ok. I noticed when you sketched, and when you were talking about the electron, that you drew some wavy lines here. Could you explain why you did that?
- S: Um, the, is it, the position of an electron can be described as a wave form. Um, that it's position as a function of time is wave-like. It's the wave particle duality.
- I: Ok. You mentioned wave form, um, often, I think, it's referred to as a wave function. Uh, could you sketch – why don't we do it on the butcher paper, so we've got more room – what the wave function as a function of position looks like in Region A?
- S: (sketches)
- I: Ok. Could you, could you label the axes on your graph, so I know what you're...
- S: Ok, um this will be energy and position. (unint). Yeah.
- I: Ok. Is it...
- S: Mmm-hmm.
- I: ... and E is for energy and x is for position?
- S: Yeah. You want me to write it in words?
- I: No, it's fine. Um, could you extend the diagram then, and sketch what the wave function would look like in Regions B and C, if, if indeed it would exist there?
- S: You've got your exponential curve, and the, is it the phase and angle have to match up on the other side, uh, wavelength doesn't change, neither does... but it's the slope and, uh, position have to be the same on each side or it doesn't actually go.
- I: Why do they have to be the same? Any idea?
- S: Um, well this is an indication of the amount of energy as a function of position of the particle. And it loses energy by going through the barrier. And whatever's left, it's a, uh, the wave function is dependent on the nature of the particle, not external conditions, so it's, it has to be the same wave on either side, but it's lost energy.
- I: Would you describe these as being the same wave?
- S: Yeah, I draw badly, but sure, you know...
- I: Is everything the same about them, or what?
- S: Well, there's less energy here, and depending on the size of this area the, um, wait a minute, that doesn't make any sense – this has less energy, the wavelength should be the same,... I think.
- I: Why do you think the wavelengths should be the same?
- S: Because it's the same particle, and the wave function is describing the particle. And the particle, the only thing that's changed about the particle by going through the barrier is the amount of energy that it has, which is indicated by the height of the waves.

- I: Ok. A minute ago when you first mentioned the wave that you sketched over here in Region C, you mentioned that the, the phase would be the same, and that the angle would be the same.
- S: Mmm-hmm.
- I: What, what did you mean by those? What did you mean by phase?
- S: Um, where in the wave it's at. Um, yeah. I, I don't know how to describe it better than that, um, it's on the downward slope as it's going into the barrier, and it's on the other end of the downward slope as it's coming out, it just loses energy, but we don't actually know what happens here... um, exponential decay, and the wave function itself... don't know, you know, what it looks like exactly, but when the wave, when the particle comes out on the other side, the wave has to be a continuance.
- I: Ok.
- S: Um, with the continuous functions, be able to do the derivative... it has to be that way for the math to come out right.
- I: Ok, um, if we look just back in Region A...
- S: Mmm-hmm.
- I: What would you say the electron's energy is in Region A? Is it constant? Is it changing?  
(17 second pause)
- S: Um, uh, I see what I did. This is actually the probability description, the probability of where it will be found. The energy is constant, cause it's just a particle moving along doing its thing, nothing much is happening until it tries to go through the barrier. This axis is actually the probability of finding the particle at these points. Um, so yes, the energy is constant.
- I: So you mentioned the word probability.
- S: Mmm-hmm.
- I: and that brings in to, in to my mind at least that it's, uh, perhaps more likely to be some place than some other place.
- S: Mmm-hmm.
- I: So where in Region A are you most likely to find an electron?
- S: Um, well, from the goofy drawing I did, it would be here and here.
- I: And why would it be there - could you say what you're thinking about that?
- S: Um, lets see, for the size of the boundary the, is it the... the  $n$ , I don't remember exactly what the, what the  $n$  was... the energy level of the particle is, determines the number of probability maxima in an area, so, say it stops here the way I've drawn it, this is the 2, uh, second energy level, and the top of the peak is indicating where it's most likely to be, and, you know, if I'd drawn this right, this should be the bottom of the axis, it should be zero right there.

I: Ok, you say it should be zero right there – you mean there's no probability of finding it in that region?

S: Mmm-hmm.

I: So the electron will never be in that region...

S: Uh, whether or not it actually is there we don't know, but we will never find it there.

I: Could you describe that a little bit further?

S: Uh, the math we have for describing these things is crappy, um, we don't actually know what's going on, we're assuming a whole lot of things, and, uh, according to the equations we have that work with observed stuff, we will not find it here.

I: So we could in principle take a measurement, you know, every second for the rest of our professional lives on this system, and never find the electron in this position.

S: Not if we're using these equations and the apparatus we've got, no.

I: Ok. Um, how does the electron's energy in Region C compare to it's region, energy in Region A, rather?

S: It's less.

I: It's less in Region C?

S: Mmm-hmm.

I: Ok, why is it less?

S: Uh, because it requires energy to go through this barrier.

I: How much energy is, is required to go through the barrier? What determines the amount of energy that's needed?

S: Um, the size of the barrier.

I: Ok, uh, let's go back to this picture then and assume, in principle, that the barrier height is twice the energy of the electron.

S: Mmm-hmm.

I: Um, any, any intuitive feel for how much of the electron's energy will then be lost in the barrier?

S: Probably about half.

I: And why would you say half?

S: Um, it's not going to lose everything, uh, tunneling doesn't uh... tunneling doesn't require the energy used by the particle to be the same as the potential it's overcoming. It's going to be some fraction of what it's got, just thinking about half.

I: Ok. If we were to then double this barrier height, would the amount of energy that the electron lost in doing this tunneling increase, decrease, stay the same?

S: Should stay the same, but it's much less likely to actually make it through.

I: So, you said earlier that the amount of energy that was lost in this barrier is dependent on the height of it, or the energy of the barrier. Am I correct...

S: Mmm-hmm.

I: ...in understanding that? And then you said that if I double the height, the energy loss will be the same.

S: Yeah, I did.

I: Which, uh, which of those two views do you favor? Or is it that I didn't increase the energy of the barrier dramatically enough to really affect the energy lost, uh, lets say that we made the barrier ten times as high...

S: Mmm-hmm.

I: ...would that change the amount of energy that you say is being lost in the barrier?

S: I'm actually not sure, um...

I: Does one of the views make more sense to you than the other?

S: Well, you know, making sense of the quantum isn't necessarily going to work, uh... For some reason, half of what it starts with is sticking in my head, um, regardless of the size of the barrier, because the size of the barrier impacts the probability of tunneling more than... anything else, as I remember, so...

I: So, again let me try to reiterate what I hear you saying.

S: Mmm-hmm.

I: Now you're saying that you don't believe really that it's the height of the barrier that affects the amount of energy that's lost; it's the height of the barrier that affects the probability or the chance that I'll actually get the electron through into Region C.

S: Mmm-hmm.

I: How about the width of the barrier? Does that affect anything about, about this tunneling... phenomena?

S: You know, I'm not sure.

I: If I made the barrier half as wide...

S: Mmm-hmm.

I: do you think that would change anything about the energy, lost...

S: Yeah, it does, it does. That was one of the, uh, one of the variables that had to go into the probability equation. So the wider barrier - well that sort of makes sense, if it's a wider barrier, there's going to be less chance it'll get all the way through.

I: Ok, so if I make a barrier wider, the probability is then less that I'll find it in Region C?

S: Yes.

I: Is the reverse of that true, as well, in other words...

S: Yes.

I: ... if I make the barrier narrower, there's a larger chance of finding it?

S: Yeah.

I: Does the barrier width at all affect the, uh, the energy between Region A and Region C? Um...

- S: Well, I suppose it's probably... well, I'm thinking in terms, in terms of classical physics where, you know, work is energy, er is, uh, force over distance, um, so if it's a wider barrier and more distance to go it'll require more energy. But I don't remember if that's how that was addressed in quantum. So...
- I: Gut feeling?
- S: Yeah. Wider barrier, gonna take more energy.
- I: Ok. And so, um, could I make the barrier, then, in principle wide enough that it would take more energy than the electron has?
- S: Sure.
- I: And so at some width, there would be, uh, zero probability of finding the electron in Region C?
- S: At some width.
- I: Any idea how wide that would have to be?
- S: Oh, um, I don't know. Uh... um, let's see. (9 second pause) I don't know. Several nanometers. It's probably going to have to be fairly large, since, since we discussed more the height of the barrier, and not the width. Pretty darn big, compared to an electron.
- I: I just, um, want to talk a little bit more about the wave that you've sketched here.
- S: Mmm-hmm.
- I: Previously, uh, if I'm remembering what you said correctly, you talked about the height of this wave being related to the amount of energy, and then later changed your mind to it being related to the probability of finding it in some place.
- S: Mmm-hmm.
- I: In your mind, what is the wavelength of this wave related to, in terms of the electron's behavior?
- S: Let's see, the wavelength... was... it's the total amount of energy it has, and the energy state it's in, I think. Yeah, cause electrons in the atom, it's an integral number of wavelengths around its orbital, so farther out, higher energy states require more wavelength. Um, maybe it's constant? Oh crap, I don't remember. Can you rephrase that somehow?
- I: I'm just wondering, um, you sketched this wave...
- S: Mmm-hmm.
- I: ...and you obviously made some choices, uh, this wave relative to this wave - how you chose to draw the amplitude, how you chose to draw the wavelength, and you, you talked a bit about what the amplitude meant to you in terms of the probability...
- S: Mmm-hmm.
- I: ...and so I was just wondering why you chose this wavelength for the wave that you drew.

- S: Um, let's see, when doing these sorts of sketches, it was the size of the region, and then within the region, the energy state dictated how many maxima you would have, and that dictated the wavelength...
- I: Ok.
- S: ...so it's, uh... like this would be wherever it is, um, most likely going to find it here, not going to find it here, and it's got a box this size to be in, so this is the wavelength, you've got, um, Oh God,... an energy state gives you the  $n$ , which gives you the number of maxima, and you have to get that into the size of the region you have, to sketch it in. It's that many maxima, evenly distributed through the region, and so that's the wavelength.
- I: So, what if I take this wall away over here that you've constructed, and so that I just say I, I send some electron in, but in principle, in all of Region A, which extends back as long as you want, the electron's got a certain energy.
- S: Mmm-hmm.
- I: How would you then choose, uh, what wavelength to draw in, so you drew the wave function in this region.
- S: Um...(41 second pause)... I'm not sure, I keep thinking of  $E = h \nu$ , but that only works for, uh, photons...
- I: Excuse me, what equals  $h \nu$ ?  $E$ ?
- S: Planck's constant, and, uh, frequency.
- I: OK. And this only works for photons?
- S: Yeah, its, the energy is, something like momentum... and something... um, but you know, pretend it was a photon, then it'd be, you know, here's your wavelength at constant energy, that's constant, that's constant, and there's your wavelength, so...
- I: So the wavelength...
- S: ...it'd just be.
- I: OK. So were this to be a photon, uh, you would determine its wavelength, then, based on its energy?
- S: Yeah.
- I: OK. Does that work for an electron, or no?
- S: No.
- I: Why not? What's, what's different?
- S: Um, cause they have mass, so they have rest energy, which throws off the whole thing.
- I: OK. Admittedly, and I think you've made some reference to this, quantum is some weird stuff to think about.
- S: Mmm-hmm.
- I: When you think about tunneling, do you think about any specific scenarios where tunneling occurs, or is it just something that's not even a part of real life, it's just...
- S: Oh, no, scanning tunneling microscopes use it all the time.

- I: Oh they do?
- S: Mmm-hmm.
- I: How convenient, because, this is actually a uh, a very simplistic diagram of a scanning, scanning-tunneling microscope, excuse me, and I've oversimplified things. There's some surface I want to study, there's of course some tip, I, I connect them across some potential, I can read the amount of current that's going through here, and then obviously, there's, uh, I'll offend the engineers here, but there's just some tip control device that works to move this...
- S: (unint) piezoelectrics, and, like a nice computer?
- I: Right. But all I'm concerned about in this conversation is the interplay between the tip and the surface.
- S: Mmm-hmm.
- I: So in your mind, how does a scanning-tunneling microscope work? What's going on?
- S: You have the, uh, potential applied to the tip, which gives the electrons in the tip from the current some energy. You get close to the surface, there's your barrier. You know, you get (unint) the tip and the surface, and there's, you know, oh a nanometer or something silly in there. You have some space, that's your barrier, that's your potential barrier, which classical physics, it ain't gonna happen. Well, it does. And they control it by keeping the current flow constant, so as you move over the surface, you'll have an atom here and an atom here, or whatever. You've got your current still flowing, you get to a dip, it stops flowing, so you bring your tip down closer. But it actually, the electrons actually tunnel through the barrier.
- I: OK. Um, and so, when we, when we have this name here - scanning-tunneling microscope, what does scanning refer to in the name?
- S: Oh, it's going back and forth across the surface.
- I: So, it's simply the motion across the surface? And tunneling refers to...?
- S: The electron tunneling through the potential barrier.
- I: What about microscope? Why is it called a microscope?
- S: Cause you're looking at little things.
- I: OK. Are you able to see those little things?
- S: To see the, uh, surface pattern. You get the, it's uh, sort of a topographical map. Course it doesn't do corners very well. It'll just kinda drop.
- I: And so in principle you can look into this device and see what the surface of the material looks like?
- S: Um, you, well it's hooked up to a computer that translates the information into a topographical map, and it'll give you, you know, mountains and some, you know, distance differences and uh, height differences, from here to here.

I: So could you ever visually see the surface?

S: No.

I: You have to have the computer interface?

S: Yes.

I: OK, uh, good. So let's, uh, simplify this picture, in fact you did so, um, you know, something like this.

S: Mmm-hmm.

I: You've got a tip, and a surface, but now I'm concerned only with the interplay between the, the atom on the tip, the yellow atom here in the picture...

S: Mmm-hmm.

I: ...and some surface atom, so let's say the green one there. OK?

S: Mmm-hmm.

I: Um, does this energy diagram, that we previously had, is it adequate for describing the tunneling behavior between the, uh, tip of the STM and the surface?

S: Well, it works along with simplified pictures like that.

I: So we could use this, this barrier, then, to describe the tunneling between this.

S: Sure.

I: And would anything need to change about this, in order for us to model what's going on.

S: Um... um, well let's see... I think the relative energy of the atoms on either side of the barrier is different then there. It would have to depend on the...

I: Could you sketch what you mean, over, perhaps, over on the side or even, uh, I think you've got enough room still on that sheet.

S: Well, say it's... this is the, uh, STM energy here, here's the barrier... and, I, I don't know what it would actually be, but the energy of the electr..., of the atom it's looking at may be higher than the energy of the STM.

I: You said it may be higher; could it be the same?

S: Uh, sure.

I: Could it be lower?

S: Yeah.

I: OK, why, why then did you choose to draw it higher?

S: I don't know - just cause.

I: Any idea why you want those two to be different, or why you chose them to be different, or is this just something you're remembering that you didn't understand?

S: Um, I don't know, I kind of, uh, kind of just think that, um... you know, I'm not sure. I guess I'm still kind of thinking about that as sort of a downhill process, uh...

I: What do you mean by that?

S: ...even though it's just not... Well, you know, classical physics you've got your, uh, nice hill, and you're at the top of the hill, and your sled's going to go down the hill, and then it's going to go up the hill, cause it still has some energy, and...

I: OK, and so how did that translate into this sketch?

S: Uh, the uh, applied voltage to the tip gives the electrons more energy than they have just hanging out. Um...

I: OK.

S: So when they do go through the barrier, they've got quite a bit of energy, um... and then there are on the atom which is, however it's energized it, I don't...

I: Was this sketch of electron motion, or a wave function, or what?

S: Uh, just random movement.

I: OK.

S: Don't read anything into that (laughing).

I: Would, would the wave function look like this? Would it look like what you drew back here?

S: It would look like this.

I: OK, um...

S: Cause this is just a real-life application of this concept.

I: OK.

S: Good for as we get a useful thing out of that equation.

I: Um, if we can use this model, to use this, though you think that the energy level may be different on the side of the atom...

S: Mmm-hmm.

I: ...um, you said previously that the energy is lost in the barrier, some amount of energy.

S: Mmm-hmm.

I: Where does that energy go? Like in a system like this - what would happen to that energy?

S: Hadn't actually thought of that before. Um, well energy loss is, you know, usually given up as heat, um, and you get electrons running around, they're going to smack into the electrons on your, uh, sample you're looking at, and get them excited, and maybe, uh, change the energy state that they're in.

I: OK. Um, somewhere back here you'd said the tip has to be on the order of magnitude of a nanometer away from the surface. Why? What, what changes if I take the tip farther away from the surface, or closer to the surface?

S: Uh, that's the height of the barrier, then. The distance between the tip and the surface is the height of the barrier.

I: OK. If I take the tip farther away from the surface, what happens to the barrier?

S: It increases.

I: What if I, and... is the reverse true as well, then, if I bring it closer to the surface?

S: Mmm-hmm.

I: Um...

S: Then the barrier gets smaller and it's easier.

I: OK. What happens if we increase the potential difference between the tip and the surface? Does that change anything about the tunneling phenomenon?

S: Yeah, the potential difference, or the potential applied to the tip is what gives the electrons doing the tunneling their energy, and that's um, the higher their energy, the more likely they are to tunnel.

I: So if I increase the potential, it's more likely that the electrons will tunnel.

S: Mmm-hmm.

I: OK. If I shut the battery off, would the electrons ever tunnel?

S: Yeah.

I: Why do you think so?

S: Um, there's always a probability given whatever energy they have, that there's, a, assuming the electrons have some energy, and there's a diff, uh, barrier, um, there is, as long as the barrier's not too large, a finite probability that they will tunnel. Whether or not we're making use of it, they don't care.

I: So, let's play engineer here a little while...

S: Mmm-hmm.

I: ...and, and say if this thing works, regardless of whether I have a battery, why would I ever want to hook a battery up to it?

S: Because then you'd get em going faster, you'd get more tunneling.

I: So if I didn't have a battery, in principle tunneling could occur - is it at a rate to be useful to me, or no?

S: No.

I: The probability is small that it would happen?

S: Mmm-hmm.

I: OK. That is all.

S: That's it?

I: That's it. Pretty painless.

S: Here I thought I'd get to gripe about Schrödinger.

I: Oh! Please! Tell us what you think about Schrödinger.

S: OK, that whole probability of the cat being half dead and half alive, and that being dependent on who's looking - it's just a big lie. That equation is for the cat, not the observer.

I: So...

S: Somebody finds out the cat's alive or dead, the cat's alive or dead, it doesn't matter whether Joe in the next room knows or not.

- I: Is the cat, then, in the box, either definitely alive or definitely dead, we just can't know?
- S: Yeah. But once somebody finds out, that cat has a definite state, period end. It's not dependent on who finds out who else knows, cause it's an equation for the cat, not for me looking at the cat.
- I: What about the electron here in this situation?
- S: Yeah.
- I: Is the electron definitely in Region A, or in Region B, or in Region C, we just can't maybe know which region for sure it's in?
- S: Uh, we know that we will find it here and we'll find it here, and we'll never find it here; we don't know what the hell is going on in here, and we don't know how it gets from here to there.
- I: Does at any instant in time, it, does the electron have a definite location? That is, it's either in Region A, or in Region B, or in Region C?
- S: Maybe.
- I: Maybe?
- S: Maybe.
- I: Not sure?
- S: Don't know. You have to go find out to know for sure, and in order to go find out, you have to mess with it, and that changes where it's going to go.
- I: In a hundred years, will physics be able to tell you where an electron is for sure?
- S: Maybe, I don't know. Well, and, you know, other than that, is it really necessary?
- I: Is any of this?
- S: No, not really. But it's fun.

## Appendix H

### TRANSCRIPT OF SECOND SELENA INTERVIEW

- I: At the top of that sheet is the time-independent Schrödinger equation, written in two different forms, though algebraically of course they're equivalent...
- S: Right.
- I: ...I just don't know which form you're more used to seeing and working with.
- S: We did both.
- I: OK. Um, are you OK with all the symbols as written there, or are you used to seeing things differently?
- S: Nope, this is what we, what we did...
- I: And I...
- S: Uh, of course it should be  $\hbar$  and not  $h$ .
- I: Yes.
- S: Do I get extra points for that?
- I: I can't figure out how to write  $\hbar$  in Word's equation editor.
- S: Oh... it won't let you enter symbols? Well, it let you do that one.
- I: Uh, you could probably enter a symbol, but these are written in the equation editor, and I don't know whether the equation editor lets you enter symbols or not.
- S: Oh. I don't know, I haven't tried it – I just skipped over to MathCad. Of course, it doesn't have  $\hbar$  either.
- I: OK.
- S: Yeah, anyway.
- I: So, um, I want to think about a system that has a potential as listed there, so it's zero if  $x$  is less than zero and it's some  $V_0$  if  $x$  is greater than zero.
- S: Mmm-hmm.
- I: What do the solutions to the Schrödinger equation in that region look like if we assume that the energy  $E$  is greater than  $V_0$ .
- S: If  $E$  is greater than you have, um, I think it was an exponential. It's, it's still a, a wave, you know, sines, cosines, it's exponential, however you want to write it. But, uh, generally the, the um, what do you call it, the intensity, or uh amplitude of the wave would decrease a little bit going over the barrier, and there's that finite very small probability of something being reflected, even though it's got (unint) energy.
- I: OK. So, could you write down what the form of those solutions are?
- S: Yeah, you want me to use markers, or...?

I: Sure, sure, markers is better... You can write on here, you can write on butcher paper, wherever you want.

S: Uh, let's see...  $\sin x$  is equal to the, uh there's usually a square root of  $2mE$ , um  $V$  minus  $E$  difference over  $\hbar$ , in all that... Yeah, something like that.

I: OK, so this applies then in both of those regions?

S: Yeah, then it depends on... oh, in both of these regions, with the energy higher?

I: Yeah, yeah...

S: Uh...

I: ...I mean is this the same answer in  $x$  less than zero and  $x$  greater than zero?

S: I think so, well, cause in this area,  $V$  is zero, so you just put in zero  $V$  there...

I: OK.

S: ...and that's your equation.

I: OK. Um, what would it look like graphically then, in both of those regions?

S: Let's see... amplitude would just shrink a little bit, and then there's a little tiny bit that might come back.

I: So... in general sinusoidal over here?

S: Mmm-hmm.

I: Still sinusoidal over here?

S: Mmm-hmm. Wavelength should stay the same.

I: Wavelength's the same, but it loses amplitude.

S: Right.

I: And that's.... is there an amplitude representation in what you wrote here, or, um?

S: Sort of. (Laughs.) Uh, you know the amplitude's a little bit higher on this side than on this side.

I: Sure, how, how does that show up mathematically in what you wrote for the  $\psi$ .

S: Oh, oh, well, here it doesn't, let's see, oh, the resolve? Constant for your, uh,  $\psi$  at zero, put in the boundary conditions, blah, blah, blah...

I: OK.

S: Of course, yeah. There, there is a number here that tells you the amplitude...

I: OK.

S: ...in this form.

I: OK. What would change, if anything, if, uh we had the condition where  $E$  was less than  $V_0$ ?

S: Uh, then you would get tunneling, uh, exponential decay inside the barrier.

I: What would that look like?

S: (sketches)...same kind of thing, and when it hits here, well, I didn't do that very well, cause the slopes and everything have to match up, but it goes like so... OK, since they're inside the barrier.

I: So, slopes have to match up where?

S: Oh, at the, at the interface, the, um, incident wave... slope has to match the transmitted wave slope...

I: OK.

S: ...to keep the math...

I: It, it looks like from what you've drawn that it's...

S: It's not - connected badly, yeah...

I: ...it's an increasing slope here, is that OK, or should it be decreasing?

S: ...no, it should be...

I: You can use this to, if you want to do...

S: ...OK, so, so say you have this kind of decay, it would be like this (sketches)... incoming wave...

I: Would this formula still work in all of those regions?

S: Yes. Then you get, you get the decaying because  $V$  is greater than  $E$ , and you get a negative, and it, you know, decays.

I: How do you get a negative out here if  $V$  is greater than  $E$ ?

S: Oh, OK, maybe it was  $E$  minus  $V$  - somehow it worked out with the - I don't remember that thing, oh yeah, it has to be  $E$  minus  $V$ , cause that's how it's written here. The energy minus the potential.

I: So, you... so this is negative if  $E$  is less than  $V$ .

S: Right.

I: What does that do to the function that  $\psi$  - it looks sinusoidal here and it looks exponential here.

S: Right.

I: How does, how does that negative change the character of the function?

S: Well, that negative gives you the exponential decay. And, in this, in this region  $V$  is zero, so you just have the sinusoidal...

I: OK.

S: ...part.

I: OK. How do you get a sinusoidal part from the exponential part?

S: Well you can write  $e$  to the  $i$  whoever is cosine and sine.

I: But I guess I, I didn't notice that you wrote an  $i$  anywhere in here. Is this whole term up here imaginary?

S: Oh, um... no, no. Where did that  $i$  go? Oh, I, I don't know. Is it minus  $2m$ ? There was something from this whole part that gets the square... nope.... I forget where that comes from. Something like that seemed to work.

I: Does this solution fit the equation as you've written it?

S: Let's see, second derivative... I think... putting a minus there, and, yeah...

I: If it would help to write out what you're thinking you can, you don't have to...

S: Well, I, I need a smaller writing utensil.

I: OK. (unint) pen here?

S: Oh, yeah, that'll work. OK. OK, you don't need the negative, cause its... I didn't even put an x in there... Yeah, you need an i... otherwise it doesn't work out, so the... e to the...

I: So you have an i in there? OK. i and an x?

S: Yeah.

I: OK.

S: Yeah.

I: Does this, does this then match with what you said over here - will this be sinusoidal here and exponential here? And if so, how?

S: So over here where V is zero, then you'd have e to the... that way,  $2m E - \hbar^2 k^2$ ... This (unint) sine and cosine. Then you'd get this stuff. Then over here you have...  $2m E - V_0 - \hbar^2 k^2$ ... and this will be an exponential decay because then you have another... this is negative, so you can pull out a square root of minus one, multiplies by the x, and then the whole thing is negative, and it's exponential decay for here.

I: All right. So you're happy that this matches your solutions to...?

S: Yes, it's fantastic...

I: OK.

S: ...don't tell Unertl though.

I: OK. Um, all right, uh, let's see - how does, so case one we said when the energy was, um, um, greater than  $V_0$ , excuse me, that you drew this sinusoidal in both regions...

S: Mmm-hmm.

I: ...higher amplitude here than here. Can you talk about why the amplitude here is greater than it is over here?

S: Um, same form with the, uh,  $E - V_0$ ...

I: OK.

S: ...so on the left it's  $E - 0$ , so your amplitude is A, and then once you get to the barrier, this term is, is smaller than E, so the amplitude is smaller than the original amplitude.

I: So is it this term that then drives the amplitude of the wave function? Drive is probably a bad word...

S: Yeah, well, determines?

I: Determines? There you go.

S: Uh, yeah, I believe so. Yeah, cause this is the... this is sort of doing like y is a function of x, you know, you're plotting the waveform...

I: OK.

S: ...versus x, so, uh, yeah, inputting your x value and then this term is zero, y-axis value...

I: OK.

S: ...so yeah.

I: Um, if, if this wave function is describing, you know, some particle – electron, photon, whatever we want to talk about in this system, how does the energy of that compare in here to over here?

S: The energy of the particle, the, average energy of the particles from one side to another, here will be higher because only higher energy particles are able to tunnel...

I: OK.

S: ...you know, on average higher a, higher average energy. This side the average energy is the same because the vast majority get through – it's only a very small probability that anything will be reflected...

I: OK.

S: ... and that will be even lower average energy. So on average its all fine.

I: You sort of – I said particle and you switched to particles.

S: Yeah, cause you don't generally do this with one particle.

I: Why not?

S: Because it's all probabilities. Psi is a non-measurable entity. Psi-squared you can measure.

I: OK.

S: And you need to do a whole lot of measurements on a whole lot of different things in the same system before you can get any kind of information, because one particle has the probability of doing something, but, you can't really measure just one, and get any useful information.

I: Why is that? Why couldn't you measure a single particle?

S: Because it's quantum and its silly (laughs), cause if, if you measure one particle, and say it's, the energy is higher than the barrier, you could get the one that reflects.

I: OK.

S: ...which only happens like, a hundredth of a percent of all the little particles that might go.

I: So you're saying a, a single measurement wouldn't really give you the characteristics of the system...

S: Right.

I: ...because there's a chance that you would get one that in an ensemble would do the odd thing, or the....

S: Right, right...

I: ...the minority thing.

S: Yeah.

I: OK. Um, is it OK to talk about probability if you're talking about single things? Single particles, single things?

S: You can, it's not necessarily effective, because probability assumes a large sample. Otherwise, you know, you have to take a large sample

and, of a lot of particles in the same setup before you can get an idea of what's going to happen. And individual particles can go do whatever the heck they want.

I: All right. Um, if – you said something in, in this situation – greater energy over here than here?

S: Yeah, tunneling only happens for those particles that have larger energy.

I: All right.

S: Um, the average energy is the same, you know, there – energy is conserved, particles are conserved, nothing disappears.

I: All right.

S: So the energy stays the same, uh, barriers don't change it, there's no friction, there's no way to take away energy from particles in these situations.

I: So highly unphysical, but if we were able to get a system where all the particles had exactly the same energy, then they would have the same energy everywhere?

S: Right.

I: OK, all right. Um, all right, let's uh... you've already sort of talked about over here, so...

S: OK.

I: ...might as well give you that actual, you know...

S: Oh, over there.

I: ...so now we have an actual barrier where we go back to some zero here at a less than  $x$ . So first off, um, are the solutions, let's just talk about the  $E$  less than  $V_0$  case here...

S: Mmm-hmm.

I: ...are the solutions here, um, similar to what you've written down here?

S: Yeah. Yeah.

I: OK, graphically, let's.. maybe we'll start there, what is the wave function for this system look like?

S: Uh, do you want  $E$  less than  $V_0$ ...?

I:  $E$  less than  $V_0$ , yeah.

S: All right, so then you have, this and...  $k$ ... and, like that. Ooh, it doesn't actually peter out, it just continues.

I: OK. So, sinusoidal...

S: Yep.

I: ...exponential, sinusoidal?

S: Right.

I: OK. Um, compare, uh, the characteristics of the wave function in the, in the two, um, regions here, the left and the right – amplitude the same in both?

S: No. Amplitude is less because you have fewer particles in the (unint) region.

I: OK.

S: OK.

I: Um, how does the wavelength for what you've sketched compare?

S: Um, I did it badly - it's supposed to be the same.

I: Why should it be the same?

S: Because the, uh, average energy, assuming all equal, you know, the energy doesn't get taken away.

I: OK. And how is energy related to wavelength?

S: Let's see...  $h\text{-bar over } \lambda$ ... yeah, right?... that's per meter...

I: What's, what's... that's OK.

S: ...de Broglie's wave relation...

I: OK.

S: ...the  $h\text{-bar}$ , and it's gotta be  $\lambda$ , that way, for the units. Right? Maybe. ...a second... is this right?... Oh, anyway, what was the question again?

I: Um, you, you said something about how the wavelengths had to be the same, because the energy was the same...

S: Oh, yeah...

I: ...so I was asking what's the connection between energy and wavelength.

S: ...yeah, the, the energy is related to the... momentum of the particle...

I: OK.

S: ...and the momentum isn't changing because it's not losing mass, we're neglecting the relativity stuff, um, it's not losing mass, its not losing energy, so it has to be traveling the same rate, so the wavelength would be the same.

I: What did you mean when you alluded to 'we're not worried about the relativity stuff'?

S: It was undergrad quantum.

I: Would, would the answer change if we were....

S: Well, if it starts going closer to the speed of light, the mass increases, and then the momentum changes, and the velocity changes, and everything just goes crazy.

I: OK.

S: (Laughs.) Which is why we don't know that in undergrad quantum.

I: OK. Um, what would the solution look like if  $E$  was greater than  $V_0$ , then...?

S: (sketches)... x, how does that go, OK, there's the probability that something reflected, at each edge, and it, you know... goes back to what it was, you know...

I: So, the amplitude, uh, you said back to what it was here?

S: Mmm-hmm.

I: So it decreases here, goes back to what it was here?

S: Right.

I: Wavelength the same all the way across, or what?

S: Should be, yeah, cause again you're not losing any energy.

I: All right. Um, one more scenario to think about then, one more wrinkle for this – what would happen if we dropped the potential, uh, to the right of the barrier, so that it was some negative  $V_0$ . Does that change anything about the solutions to the wave function? Uh, let's take the  $E$  less than  $V_0$  but greater than zero case.

S: Right. So if it's here, you get the decay. And then I know we discussed the negative potential, but I don't remember how that went. It didn't seem like it made a significant difference, um, but I, I don't remember, we, we didn't do it very much.

I: Do you think with, uh... first of all, do you think the characteristics remain the same, then – sinusoidal, exponential, sinusoidal? That appears...

S: Yeah. Yeah.

I: OK. Um, what are you then wondering whether you're drawing right or, or you're thinking about?

S: Well, the uh, the amplitude... of the wave when the energy is a lot higher than the potential.

I: OK. Do you, do you think, so if we say that these are the same amplitudes...

S: Mmm-hmm.

I: ...are these the same amplitudes? I mean, should they be, or should they be different?

S: They should be different because the potential is different. And that's pretty much what determines the amplitude there, the relative energy difference between the particles and the barrier.

I: All right. In this case with the lower potential over here, is the amplitude going to be greater than or less than what it was over here?

S: I think it would be greater than.

I: And why so? Greater in this case?

S: Mmm-hmm.

I: OK, and why would it be greater?

S: Because the difference between the energy of the particles and the energy of the barrier is, is so much larger in this case.

I: So, is amplitude somehow then tied to that difference?

S: Yeah, cause the, the number, the actual number of particles that are going to get through, which is the amplitude, is determined by the barrier size, in relation to the energy of the particles.

I: OK.

S: Um, yeah there was something else you had to do when setting up the wave equations to solve for each of these regions, for this wave...

I: All right.

S: ...um...

I: Any idea - remember what that was, or...

S: Well, you have to, you, you solve this chunk for this interface, and this chunk for this interface, and you have to then do all the algebra and the crap to smash it all together and get all of your constants correct. Um, so it sort of assumes that a wave of particles incident at this side has some, like pre-knowledge of what's going to happen over here.

I: OK.

S: That, in that, this is the part that here, because the numbers that you need in your wave equation are determined by these... end values.

I: OK. So then you were saying that you think the amplitude here is greater than here.

S: Yeah.

I: Does that match then, if you go back and think about the, I can't remember which is the final form we agreed on, here, I think it's something like this...

S: Yeah, I think it was that one.

I: Does that, does that agree then?

S: It's still going to be something of that form, but all the, this stuff is going to be different.

I: All right, different how?

S: It's gonna be...

I: What's going to happen to it?

S: Well, there's, there's going to have several chunks of stuff, because  $2m$   $\hbar$  and  $E$  are going to be constant, but  $V$  of  $x$  is changing, so you have to put in your different  $V$  of  $x$  for each of these things, and that's going to change, your wave equation.

I: So previously, uh, in the first region,  $V$  is zero...

S: Mmm-hmm.

I: ...you said this is essentially just this, I think.

S: Right.

I: Yeah, OK.

S: Yeah.

I: And that this can be written as sines and cosines, therefore we get the sinusoidal function, right?

S: Right. Mmm-hmm.

I: Um, what, do now we're talking about the region where  $V$  of  $x$  is minus  $V_0$ , how does this factor change, and what effect, if any does it have on these sine and cosine functions.

S: Ah, then you're going to get...  $2m$ ... This is going to be some, well it's  $E$  minus minus  $V$ , so you get some  $e$  to greater than  $E$  in there...  $\hbar$ ... so this is still a positive thing, an  $x$  and the  $i$ , so you can still write this as sines and cosines.

I: All right.

S: Instead of an exponential decay.

I: What changes about those sines and cosines?

S: Uh, now it's going to be the amplitude and the, the argument is changing, so that the rate of, uh, oscillation is going to be different, it's going to be higher in this side than in this side.

I: Does that mean that the wavelength over here is, is now different, or what? You said wave oscillation, are you talking frequency...

S: Right.

I: ...or are you...

S: Well, let's see, you get... let's see... I think this is the frequency now... yeah, I think the wavelength is changing here. Has - it's gotten higher. Or shorter, whatever the heck it is, it's, you know, closer together, waves. Cause the argument is larger. Ooh, does that make sense? So OK, longer wavelength, maybe (laughs). OK, just trying to think, you know...

I: Sure.

S: ...say  $E_2$  is twice  $E$  of the initial energy, then...

I: Sure.

S: ....uh, does that mean that it comes up faster, or... slower... Well, it's definitely changing, I'll give - I'll state that for a fact - I would have to actually, like, pull out my calculator and calculate these things, cause I never bothered to memorize cosine of what is, you know, what numbers.

I: OK. Sure, so but somehow it's changing.

S: Yeah.

I: All right, it's different than it was in the first region.

S: Right.

I: Um, when we were back here you, you talked about how... I think it was this scenario you first mentioned it in, that, um, the energy is greater than here if we're talking about an ensemble of particles with a range of energy...

S: Right.

I: But, then I said, highly non-physical but if we assume they all have the same energy, then you would say the same energy here.

S: Right.

I: Is that then true over here - do, do these particles have the same energy as they had here if we have the whole ensemble has the same energy to start with?

S: Yeah, it has to, cause there's no way to get rid of any energy in this, in this sys, system. There's no friction of any kind of loss, thing.

I: Is that true in this situation, then?

S: Well, yeah, I mean, a potential barrier doesn't take away any energy, you can't - the particles don't have any way to lose energy in this stuff.

I: So they're going to have the same energy over here that they had over here...

S: Yeah.

I: ...as long as we could get our ensemble...

S: Yeah.

I: ...to have the same energy.

S: Assuming, you know, we could force it.

I: OK. Um, but, a few minutes ago you said something about the fact that energy is related to wavelength.

S: Right.

I: So, are you troubled at all that now you're (unint) wavelength?

S: Well, the energy and momentum, uh, velocity can change, too. Cause the, the energy and momentum being related, momentum is both, the, wavelength and the frequency. So you can change wavelength as long as you're changing frequency, and still maintain the energy. So it'd be longer wavelength and, what is it, longer wavelength, higher frequency? Shorter frequency? If the individual waves are longer, they'll come faster, to maintain the energy.

I: All right. So you're saying, um, that, even though you previously had sad energy's related to wavelength...

S: Mmm-hmm.

I: ...you still think, you're still OK with that, but that in perhaps this situation the frequency's also changing?

S: Yeah.

I: To keep the energy constant.

S: Right.

I: OK. Um, you, you mentioned probability quite a few times, and how the fact that we could measure probability, but not a wave function, I think...

S: Right.

I: ...I'm trying to remember back 10 or 15 minutes here. What do you mean by that, when we say we, we, you know, we can measure probability, we can't measure a wave function?

S: Um, you can't measure  $\psi$  of  $x$ . You can't say at this time this particle is in this spot.

I: OK.

S: Um, cause it's not allowed. (Laughs.)

I: OK.

S: The uncertainty principle says we're not allowed to do that.

I: All right.

S: You can measure the square of a wave function, which gives you an average of where a whole lot of stuff would be over time.

I: OK.

S: And the, you can measure the probability of something being somewhere at some point, but you can't actually nail down one particle at some time and say it's in this spot.

I: You said the square of the wave function is probability?

S: Right.

I: So, could we, uh, you say we can't, we drew these wave functions, we could never measure these wave functions, but we could measure probabilities that correspond to them?

S: Mmm-hmm.

I: So what would the square of these wave functions look like? Let's, let's start back in our first scenario, if you want to...

S: OK, um, let's see it's the... peak, and... you just, literally square it.

I: So, this one corresponds to, this corresponds to this shape?

S: Right, right. And, you know, drawn badly, but multiplied by itself, and everything's positive, then.

I: All right, so all numbers are positive.

S: Yeah.

I: Are these values in the troughs positive, or zero, or what?

S: Um, it depends on what this is. These, I didn't draw this to be exactly zero, so it would be some positive number.

I: Does this, OK, as you've drawn it, does this mean the wave function at all values is always positive?

S: The way I drew it here, um... Wait a minute... Yeah. The way I drew it here all values are already positive.

I: OK. And then, over here you said sines and cosines. Are sines and cosines always positive?

S: No.

I: So, is this, does this match, or is there some discrepancy, or do you want to change...?

S: This is the arbitrary opposite. (Laughs.)

I: OK. All right.

S: For my own convenience, I drew it like that.

I: OK.

S: Uh, you could, if you wanted to, you could do like this where it's exactly like the thing, and then, you, well, that makes it really crappy, but - it's just prettier when you're going into a barrier and you can draw it like this; it's easier to look at, but, no, sines and cosines do go negative.

I: All right, so is the actual wave function arbitrary, or does it oscillate between positive and negative values like here, or is it all positive here, or does it matter? In other words, where I'm going with this is, is like you said, if you take this function that is all positive and you square it...

S: Mmm-hmm.

I: ...all it's squared values are all positive.

S: Mmm-hmm.

I: If you take a function that oscillates between positive and negative and square it, there obviously will be zeros in that...

S: Yes.

I: ...in that function, so I'm saying, is this an arbitrary choice in terms of the wave function, is one more correct than the other?

S: Um... it depends on how you label your axes and what you're calling the energy of the wave, and, you know, how you want to actually do it.

I: OK.

S: But yes, you will get zeroes.

I: Does that mean you would get zeroes in your square as well?

S: Yeah.

I: So you'll - is that OK to have zero probability of being a certain place?

S: Absolutely.

I: How, how so, can you talk about that?

S: Well, the, it, not really here, but if you did the particle in a box where it's in a potential well that it can't get out of...

I: OK, all right.

S: ... um, you can have a wave that is in here, and this is just the wave, and then you square it, like this, there is zero probability of being at these two points.

I: So, you'd never find it at the two ends.

S: Right.

I: I guess where I'm going with that, sort of, is that, OK...

S: How can it get from one place to the other without being in one spot?

I: Yeah, right, it's, there's a probability of being here and a probability of being here, but not here, what gives there?

S: Such is the mystery of quantum.

I: So it's just one of those, uh, one of those...

S: It's one of those things. I don't know, I, uh, it kind of makes sense in a, in a strange way if it's really just a wave, but if you're talking about electrons, and it's a particle, and it can just go there if it wants to, this is one of the fundamental weirdnesses of, of quantum.

I: OK.

S: It can, it can be here, it's most likely to be here...

I: All right.

S: ...it will never, ever be found here. Which is not to say if you measured this same setup, one million times, you know over the rest of your life that you would never, ever, ever find one here, the math says you won't, but, you know, if you did it enough times, you might.

I: In other words, the math might not be accurate.

S: I don't buy that Schrödinger's equation is a hundred percent right. Yeah, it works, but, you know, derivative twice with respect to

position, derivative once with respect to time, it works, I think we, we're missing something.

I: So, in other words, in, is it something along, akin to the fact that, you know, here comes Newton in whenever writing down  $F_{\text{net}} = ma$ ...

S: Mmm-hmm.

I: ...and this works great for a couple hundred year, until we find very small applications where it doesn't apply anymore.

S: Yeah.

I: Along comes quantum, we have all these equations, so they seem to work for a, a different set of problems, but they're not entirely accurate either?

S: Right.

I: OK.

S: Yeah.

I: Um, just a couple of quick ones here at the end, then. You said something about how this, well, depending on what you labeled your axes, was it? I, I, maybe I was misunderstanding...

S: Yeah, you know, that, you're - uh, what does Wittmann say, the, the, uh, references are arbitrary but necessary. You pick.

I: OK. So depending on where you pick your zeroes to be and everything you might get...

S: Right.

I: ...different...

S: And you can call  $V_0$ , you could call that zero...

I: All right.

S: ...and, you know, this site negative, if you wanted to, it's just, whatever you want.

I: When you are plotting  $a$ , a, uh, a graph of wave function as you've done....

S: Mmm-hmm.

I: ...in all these different situations. Um, and the horizontal axis is position...

S: Mmm-hmm.

I: ...what is on the vertical axis when you're plotting a wave function?

S: Potential energy.

I: So this, these, the vertical axis of this graph right here is energy?

S: Yeah, which makes half of what I've said wrong, because I didn't keep the amplitude the same. But this is the part that I always get messed up with when I'm thinking about it, cause I remember pictures that look like this, but not necessarily where the axes were, precisely, so this is one of those things that you can't quote me on.

I: OK. Sure. Does that say then, I mean, this, this is the energy is really oscillating about some average value here?

S: The way that's been, the way it's drawn, yeah, that's what it says, that's not what I think it's supposed to mean.

I: What do you think it's supposed to mean?

S: Well, you're not losing any energy, there's nowhere for the energy to go, the particles have to have the same amount of energy all the time.

I: Right.

S: Um, this is more of the, I, you know, I think these are... if you just said these waves are  $\psi$ ...

I: OK.

S: ...of  $x$ . We have no problems.

I: So, vertical axis being  $\psi$  of  $x$ , not the energy?

S: Yeah.

I: Do you think it's one or the other, or do you think it re, it is energy, or what?

S: Uh, I think it is energy, but because you can't lose any energy it doesn't make any sense.

I: Would it be OK to write  $\psi$  of  $x$  equals  $V$  of  $x$ ? I mean, if it's energy, could we...

S: Yeah, it would, you could say...

I: ...(unint) with these?

S: ...yeah, you could easily say the  $\psi$  of  $x$  for the potential is a constant starting at  $x$  equals zero.

I: OK.

S: And, then everything would be OK.

I: So then you're essentially writing a wave function for the potential energy of the region?

S: Yeah.

I: Can you write a wave function for anything?

S: Sure.

I: You, I mean, so we've talked about writing wave functions for photons, electrons, etcetera, things that are particles or particle-waves, or whatever they are...

S: Yeah.

I: ... but in my mind, things. Is energy a thing, I guess, it's OK to write wave functions for energy? I-, in other words, can you write a wave function for anything, I guess?

S: Well you can, as long as there's... let's see, a, yeah, as, as long as there's momentum, it's got energy, it's got a wavelength, you can write a wave function for it.

I: So, this wave function, I don't, I, I don't know how in the world we would, I... is this a wave function, then? You could write a step function, and that's a wave function?

S: Oh yeah.

I: OK.

S: Yeah. You do that whole fancy Fourier transform slapped together with a bunch of sines and cosines until you get enough to, make it approximate a step function...

I: OK.

S: ...if you want to go (unint), yeah.

I: All right. So it doesn't only have to be exponential and sines and cosines.

S: No.

I: OK.

S: No, you can sum up as many exponentials and sines and cosines as you want until you get something approximating a step.

I: All right. And then, last question over here when you were doing this, um, we had this function oscillating about some lev-, some median value here.

S: Mmm-hmm.

I: And this function oscillating about some median value that looks to be lower than this one.

S: Yeah.

I: Is that just an artifact of the way you drew it, or is that the actual way that it is, that this wave function shifts to a lower, uh, average point of oscillation?

S: It actually has to do that, because you get exponential decay of the wave function inside a potential barrier, so the average oscillation point after the barrier has to be lower than before the barrier.

I: OK. Does that mean the energy's less after the barrier than before the barrier, or no, that's just...

S: No.

I: ...that's just where. So is there anything, I guess is there any physical significance of where we choose to draw the wave function oscillating about?

S: Um, uh, let's see. There is, and I can't remember what the proper way to do it is. Cause it does matter, it does make a difference. I just don't remember what the difference is. Cause it, it all depends on what your, your axes, actually represent.

I: OK. But it's not telling you, for example, that the energy is less over here than it was over here?

S: No.

I: OK. It is telling you something but we're not sure what it is.

S: Yes, it's telling you something, and, uh, whatever it is, it's not energy decreasing.

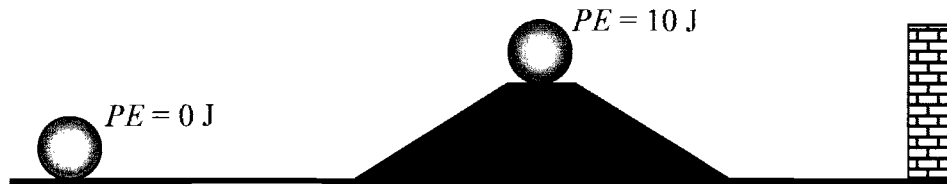
I: OK.

S: That's the one thing I'm certain about.

## Appendix I

### PRETEST FOR THE FIRST TUNNELING TUTORIAL

A ball is placed in the system shown below. When the ball is on the lowest level, which we designate as height = 0, it has no potential energy. When the ball is on top of the hill, it has 10 J of potential energy. (Recall that gravitational potential energy is defined as  $PE = mgh$ , where  $m$  is the mass of the object,  $g$  is the gravitational acceleration, and  $h$  is the height of the object.)

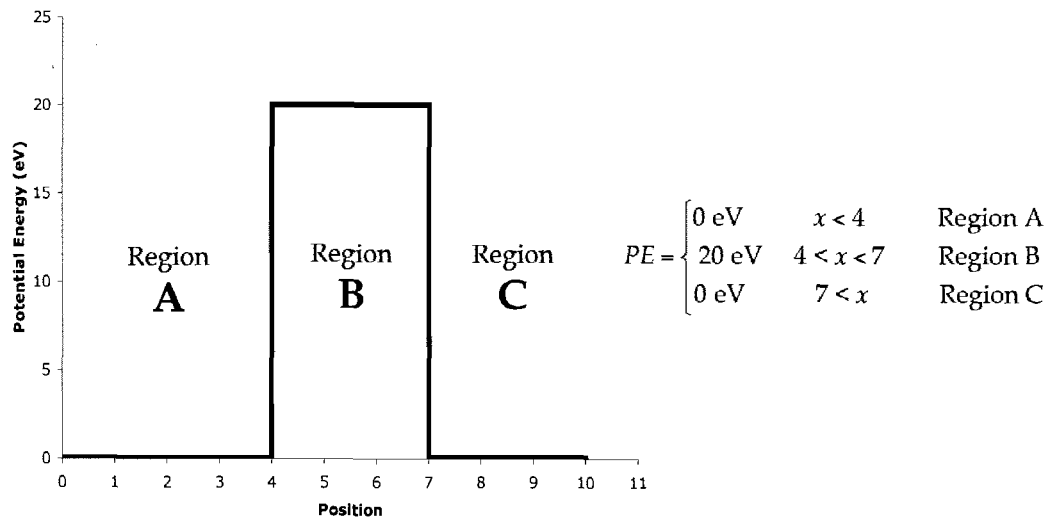


The ball is rolled into the system from the left with 5 J of kinetic energy, as shown below. (Recall that kinetic energy is defined as  $KE = 1/2 mv^2$ , where  $m$  is the mass of the object, and  $v$  is its speed.)

Is there any chance the ball will hit the brick wall? Explain why or why not.



The potential energy of three regions, A, B, and C, is sketched below as a function of position:



A beam of electrons, each with a total energy of 10 eV, is sent into this system from the left. Describe what happens to the electrons as they encounter the region of increased potential energy (Region B).

Is there any chance of electrons being detected in Region C? Explain why or why not.

## Appendix J

### FIRST TUNNELING TUTORIAL

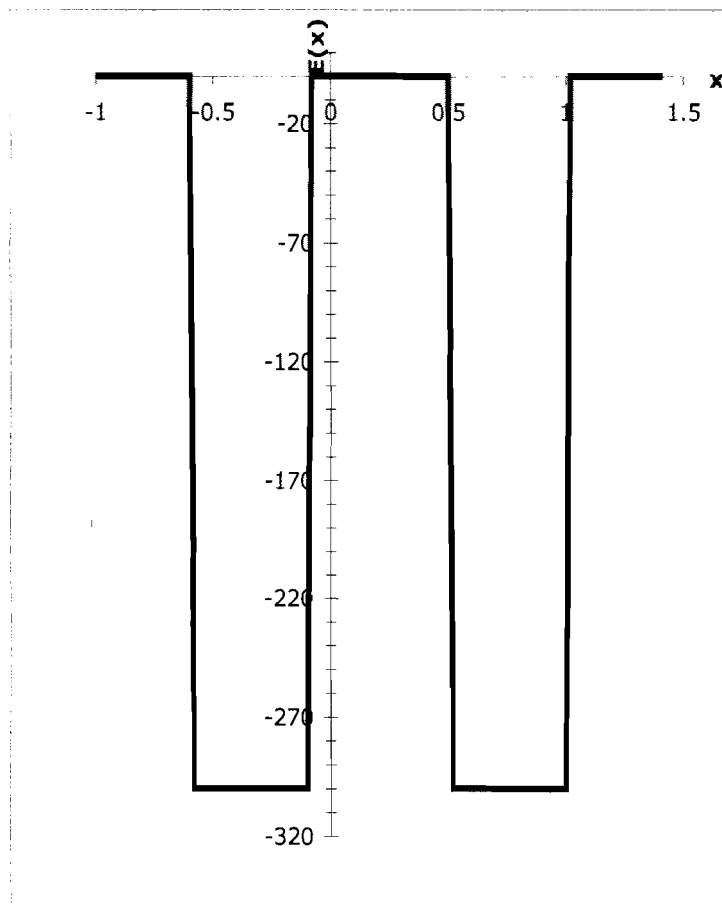
#### I. Board Meeting I – Molecules Revisited

All groups should answer all questions.

Last week, we spent some time with the double potential well scenario, and ended with finding the six bound states for the potential energy scenario diagrammed at the right.

Think back to last week's work, and answer the questions below:

1. What, exactly, is this model describing?
2. In what ways is the model useful? In what ways is the model limited?
3. Tell the story of an electron trapped in this system.



## II. The Bowling Ball Model of Large Objects

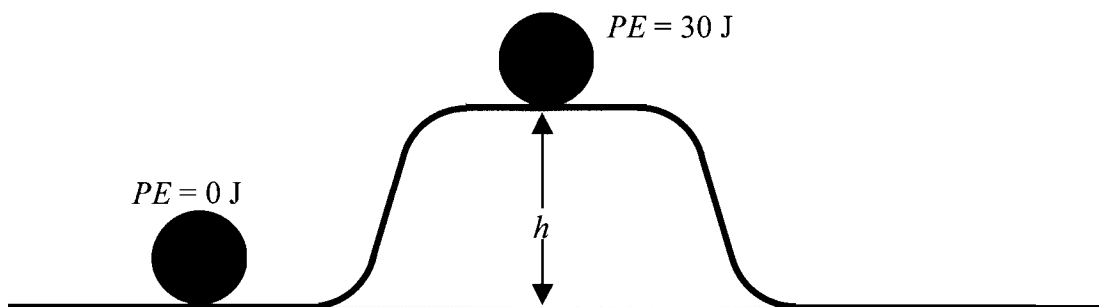
About 100 hundred years ago, the model for light was that it was a wave/particle, but the model for electrons was that they were only particles. In thinking of electrons as having only particle properties, one arrives at different predictions for the probability density than if one considers the electron to have both particle and wave properties.

Recall from previous tutorials that kinetic energy and gravitational potential energies have been defined as

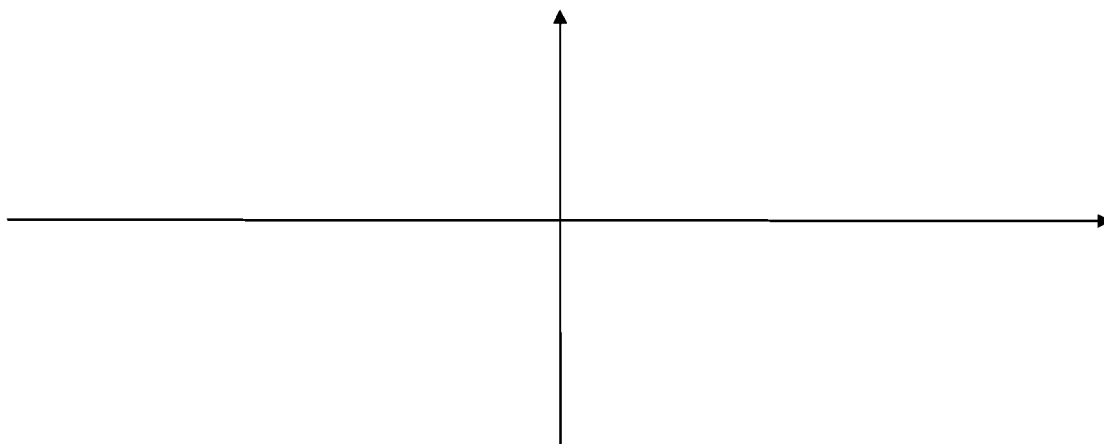
$$KE = \frac{1}{2}mv^2 \quad PE = mgh$$

We can use the idea that the total energy (the sum of the kinetic and gravitational potential energies) is conserved in many situations to predict the behavior of systems, that is,  $TE = KE + PE = \text{constant}$ .

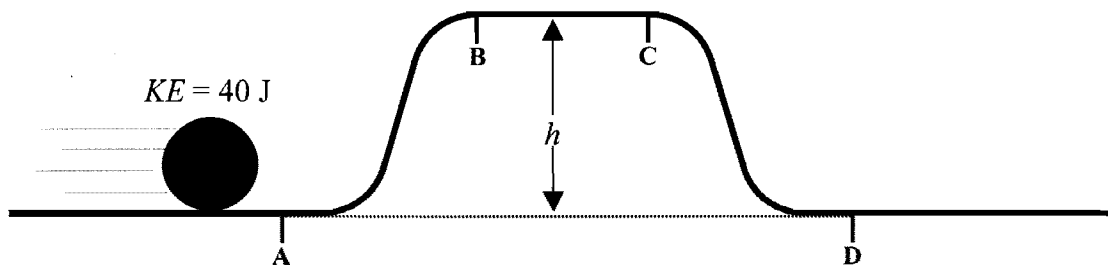
A bowling ball is in a two-tiered system, shown below. If we define the lower level to be at height zero, the bowling ball has a gravitational potential energy of 0 J. The height of the upper level is such that the gravitational potential energy of the ball on that level is 30 J.



- A. Sketch a picture graph of the gravitational potential energy of the bowling ball as a function of position. Label your axes.



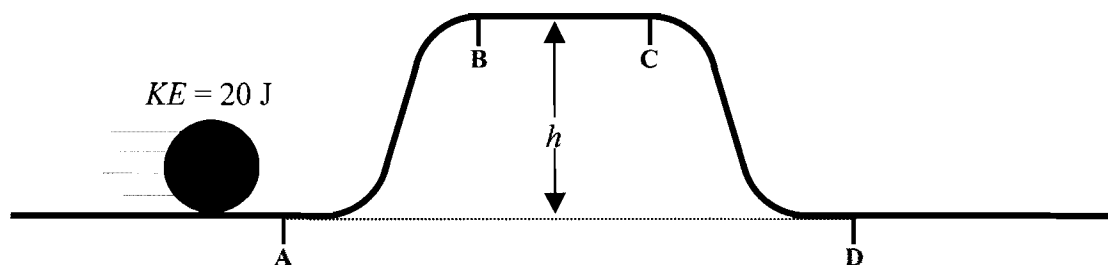
- B. Consider the situation where the bowling ball is incident on the same system from the left with a kinetic energy of 40 J, as shown below.



Use the first column on the sheet titled 'Reference Sheet – Bowling Ball' to:

- Record the potential, kinetic, and total energy of the ball at points A, B, C, and D.
- Note how, if at all, the ball's speed changes as it moves through the system.
- Describe where in the system you are most likely to observe the ball (you might use the '100 frames of video' idea here.)
- Sketch a picture graph of the probability density of the bowling ball between points A and D. (Make sure you label your axes!)

- C. The experiment is repeated, but this time the incident bowling ball has a kinetic energy of 20 J, as shown below. Fill in the second column on the reference sheet for this scenario.

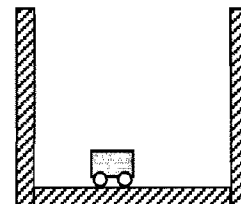


Look back at the graphed you sketched in question A. This potential energy arrangement, where there is a higher potential energy in some area than in its surroundings, is called a **potential energy barrier**. (Contrast this with the potential energy *well*, where the potential energy in some area is lower than its surroundings.)

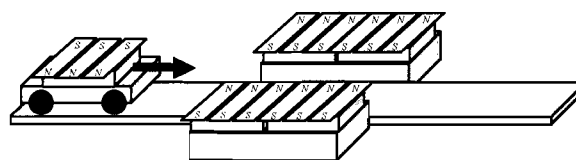
### III. Extending the Cart Model for Electrons

For several weeks now, we've been building a model of electrons that includes wave-like properties. This is not by accident – scientists believe the wave-particle model to be the best in describing the actual behavior of electrons.

However, one hallmark of a good model is that it is *simple*, and it would certainly be easier to think about electrons as just particles (really tiny carts, perhaps). In fact, it wasn't too long ago that the particle model was the prevalent idea regarding electrons. However, as we saw in a previous tutorial, using the cart between two walls to model an electron in a potential well has limitations – it worked for some ideas, but not for others.

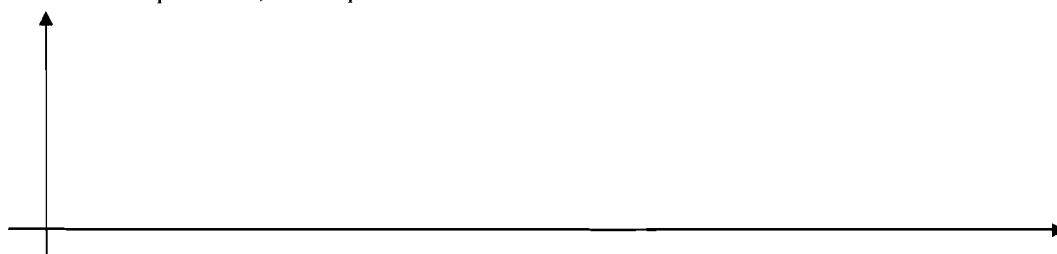


In this section, we're going to return to using a cart, and see if it's a good model to use in thinking about electrons interacting with potential energy barriers. To do so, we'll use the magnet cart setup that we first saw in the 'Energy and Probability' tutorial.



A. Go to the magnet cart setup in the front of the room.

1. Practice rolling the cart through the magnets until you can start it at a speed that allows it to just make it through. Use the computer to produce a kinetic energy picture graph of your experiment.
2. Reproduce the kinetic energy picture graph on the axes below. Label your axes. Label the values of your axes – we haven't made you do this much previously, but for this experiment, it's important.



Return to your table so another group can use the apparatus.

3. Sketch a potential energy picture graph on the axes below. Label your axes. Label the values of your axes as well.



4. Sketch a picture graph of probability density for the magnet cart. Label your axes.



B. Lets think about an incident cart with less energy.

1. What was the kinetic energy of the cart at the beginning of its motion (that is, before it went past the magnets)?
2. What would happen to the cart if you started it with half as much kinetic energy? (If no other group is using the apparatus, return and check your predictions.)

3. Sketch a picture graph of probability density for the magnet cart in this scenario. Label your axes.



How is this graph different from the one you sketched in A.4?

4. Is there any chance, given the initial energy you gave the cart in the second experiment, that the cart will make it past the magnets? Explain.

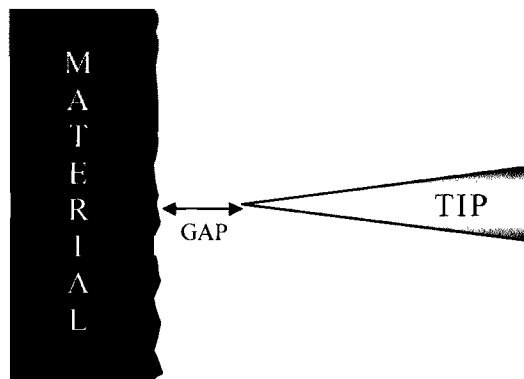
All right, you didn't think we'd let you go a whole tutorial without wave functions, did you? We're going to explore the predictions of quantum physics for potential energy barriers, then return and contrast them with the predictions of the 'cart' model.

#### IV. A Scanning-Tunneling Microscope

The scanning tunneling microscope (STM), invented in 1981, is a device widely used in both industrial and fundamental research to obtain atomic-scale images of metal surfaces. It provides a three-dimensional profile of the surface that is very useful for characterizing surface roughness, observing surface defects, and determining the size and conformation of molecules on the surface.

The Laboratory for Surface Science and Technology (LASST) at the University of Maine uses a scanning-tunneling microscope and other similar technologies to study the surfaces of materials they develop. You can find out more about them at <http://www.umaine.edu/lasst>, or visit them in the new wing of Barrows Hall.

In this section, we're going to develop a model of how a scanning-tunneling microscope works. Very crudely, a STM consists of a very pointed tip that passes over the surface of a material, but does not touch it. The closer the tip is to the material, the more electrons are measured in the tip. This is a very surprising effect, since previous physics models predicted that they shouldn't be there!



Imagine that 100 electrons are measured in some material. The tip of the scanning-tunneling microscope is brought close to the surface, and 25 electrons are measured in the tip.

A. We're going to start thinking about this system in terms of probability.

1. In the first box on the 'Reference Sheet – Scanning Tunneling Microscope', sketch a probability density picture graph for this system.
2. The tip is moved farther away from the surface, and now only 10 electrons are measured in the tip. How does this change your graph of probability density? Sketch the new graph on the reference sheet.
3. The tip is now moved closer to the surface than it originally was, and now 40 electrons are measured in the tip. How does this change your graph of probability density? Sketch the new graph on the reference sheet.

- B. We can refine our model a bit by working with some energy values. A very rough model of this scanning-tunneling microscope system is given by the values below:

$$PE_{system} = \begin{cases} 0 \text{ eV} & \text{Material} \\ 30 \text{ eV} & \text{Gap} \\ 0 \text{ eV} & \text{Tip} \end{cases}$$

1. 100 electrons, each with  $TE = 20 \text{ eV}$ , are measured at the surface of some material. 25 of them are subsequently measured in the tip.
  - a. How much kinetic energy does each electron have while it is in the material?  
How do you know?
  - b. How much kinetic energy does each electron that is found in the tip have?  
Explain your reasoning.
2. Two students (pretend they're members of your group – you may even want to have two of your group members read the statements out loud) are discussing this situation, and make the following statements:

Student 1: "The barrier's energy is higher than the electron's energy, so the electrons that make it through the barrier lose energy in the process. They'll probably lose about half, meaning the electrons that are in the tip will have about 10 eV of energy."

Student 2: "No, energy is conserved, so the electrons in the tip will have the same energy, the same  $\Psi$  – everything about them is the same. That means the same probability density in the tip – wait, then shouldn't all the electrons make it to the tip? I don't get it."

Discuss the student statements with the members of your group. With which parts of the statements do you agree? With which parts do you disagree? Record your comments in the space below.

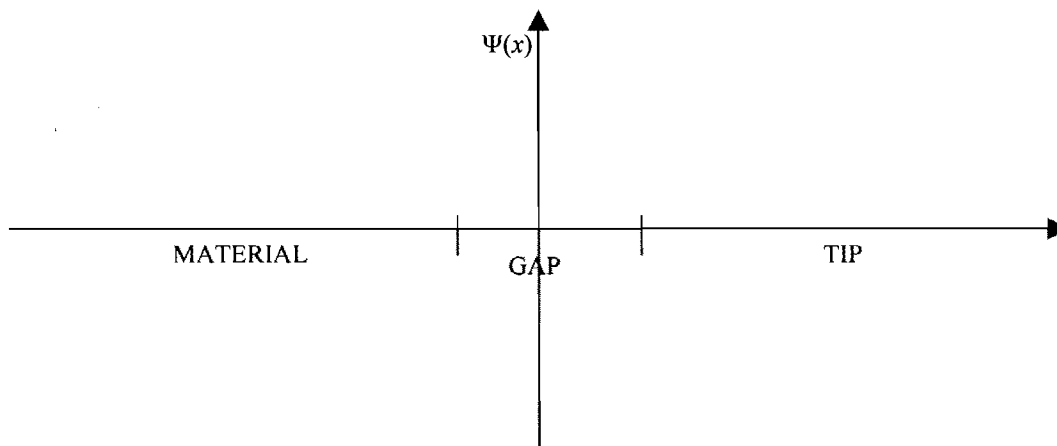
3. What type of wave function would the electrons have in the material region? How do you know?
4. What type of wave function would electrons have in the tip region? How do you know?
  - a. How would *wavelength* in the tip region compare to the wavelength in the material region?
  - b. How would *amplitude* in the tip region compare to the amplitude in the material region?
5. What type of wave functions must exist in the gap region? Support your answer by considering and analyzing...
  - a. ... the energy of the electron and energy of the system.
  - b. ... the amplitudes in the material and tip regions.

*Keep in mind that your answers must be consistent when answering the same question from two different perspectives.*

6. Sketch the wave functions for each scenario on the reference sheet, making sure to change your sketches appropriately for each situation.
- C. Imagine that the gap was somehow changed – filled with a different gas, perhaps, so that the potential energy of the system in the gap is 50 eV. Everything else stays the same.
1. How, if at all, would this change  $\Psi$  in...
    - a. ...the material?
    - b. ...the gap?
    - c. ...the tip?

Be sure to think about the type of function, the amplitude, the wavelength, and the curviness at a given value in each region!

2. Sketch a picture graph of the wave function for this new situation.



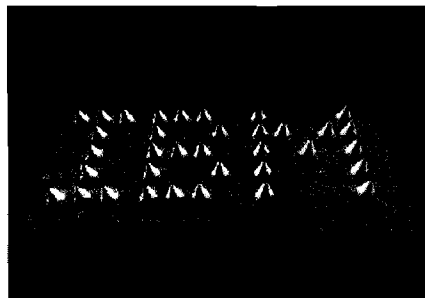
The phenomenon that we're describing with this wave function graph is called **tunneling**. As we've previously said, the energy arrangement where the potential energy is higher in some region than its surroundings is called a **potential energy barrier**. These words, with obvious links to our everyday usage, often lead to confusion about what's going on.

- D. We think of barriers as obstacles that are difficult or impossible to get through.
1. Would classical ("cart") electrons make it to the tip? How do you know?
  2. This barrier does involve loss, but it's not the loss that many students link it with.
    - a. What, specifically, is lost?
    - b. What, specifically, is not lost?

## V. Board Meeting 2 - Mapping surfaces

We've spent some time on the tunneling part – now we're going to explore the scanning part of the scanning tunneling microscope name.

Researchers at IBM were able to manipulate atoms on a surface and produce the image seen at right, where each of the points you see represents the location of a single atom. The image of this arrangement was then produced by information gathered by a scanning-tunneling microscope.



The following questions should be addressed by all groups:

Imagine you were using a scanning-tunneling microscope to probe this surface. How could you tell that the surface spelled out I-B-M? What specifically would you have to do? Be sure to address such issues as...

1. Moving the tip
2. Moving the surface.
3. The type(s) of information you'd have to collect.
4. How you would interpret that information.

## VI. Cart-like Electrons vs. Quantum Electrons

Compare the predictions about the behavior of the 'cart' electron to those of the quantum electron.

- A. In what ways, if any, is the behavior of the 'cart' electron similar to the behavior of the quantum electron?
- B. In what ways, if any, is the behavior of the 'cart' electron different than the behavior of the quantum electron?
- C. What evidence do we have that we have to choose one description over the other?

## VII. When Quantum Applies – deBroglie Wavelengths

- A. Have you ever observed a bowling ball to be where the laws of classical physics predict it cannot be? That is, if you roll it at a wall, would you ever expect to observe it on the other side of the wall?

Quantum physics seems to apply in some situations, like electrons at relatively small energies (10's of eVs), but we never see bowling balls travel where they're not supposed to go. To explore the reason, we introduce something physicists call the deBroglie wavelength, which all objects have:

$$\lambda_{deBroglie} = A \cdot \frac{1}{\sqrt{KE}}$$

where  $A$  is a number that depends on the amount of mass something has, and  $KE$  is the object's kinetic energy in Joules. The table below lists the value of  $A$  for some objects:

Object	$A \text{ (J} \cdot \text{s} \cdot \text{kg}^{-0.5})$
Electron	$5.0 \times 10^{-19}$
Bowling Ball	$2.0 \times 10^{-34}$
Earth	$2.0 \times 10^{-46}$

(Don't worry too much about the units; if you calculate a deBroglie wavelength with this formula, the units of the wavelength turn out to be meters.)

- B. Calculate the deBroglie wavelength for an electron with a kinetic energy of 20 eV, a bowling ball with a kinetic energy of 20 J, and the Earth, with a kinetic energy of  $2.7 \times 10^{33}$  J. To do so, follow these steps:
1. If necessary, convert the energy to Joules (recall that  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ ).
  2. Take the square root of the kinetic energy.
  3. Find the inverse of the square root of the kinetic energy.
  4. Multiply the inverse of the square root of the kinetic energy by the appropriate constant  $A$ .

Object	Energy (eV)	Energy (J)	$\sqrt{KE}$	$\frac{1}{\sqrt{KE}}$	$A \times \left( \frac{1}{\sqrt{KE}} \right)$
Electron	20 eV				
Bowling ball		20 J			
Earth		$2.7 \times 10^{33} \text{ J}$			

One idea that we can use to discern whether or not we'll see an object behave in manners predicted by quantum physics is to examine the size of its deBroglie wavelength. If the deBroglie wavelength is on the order of a length that can be discerned by some measuring device, we might be able to observe the quantum properties of the object. If not, we have no way of knowing whether or not the object is behaving like a quantum object.

For reference, an ordinary microscope cannot resolve objects much smaller than  $10^{-6}$  meters. An electron microscope can resolve objects down to about  $10^{-10}$  meters.

- C. Is the deBroglie wavelength for a bowling ball on the order of a length that can be experimentally discerned? Do we expect to be able to observe the quantum behavior of a bowling ball?
  
- D. What about the deBroglie wavelength for an electron? Is it on the order of a length that can be experimentally discerned? Do we expect to be able to observe the quantum behavior of a bowling ball?
  
- E. Given enough time, do you expect that technology will advance to the point where one would be able to observe the quantum properties of everyday objects (*i.e.* bowling balls)? Why or why not?

If you're interested in this topic, there are some researchers who are actually finding quantum phenomena in "large" objects, and don't believe this deBroglie wavelength argument. You can read more at <http://www.quantum.univie.ac.at/>.

## VIII. When Quantum Applies – Curviness Arguments

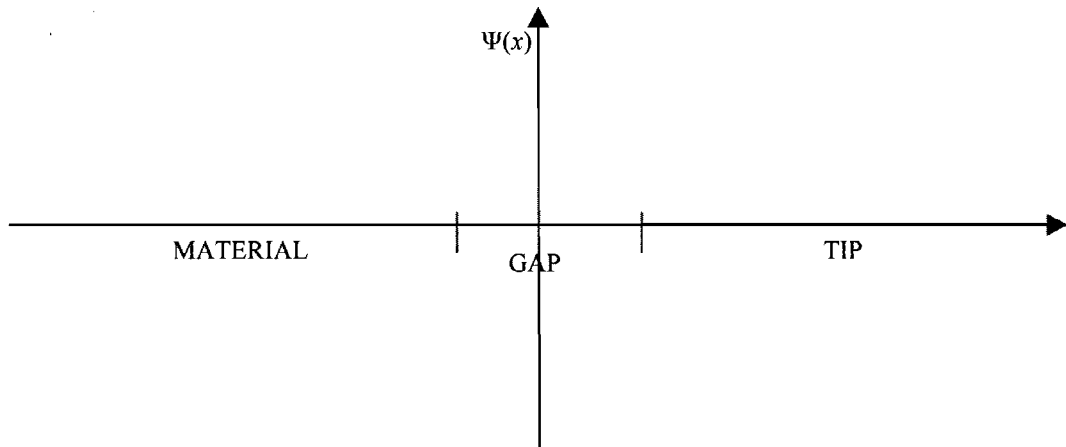
There's another way we can reason about when to apply quantum ideas. Although we haven't said so explicitly, it turns out that an object's wave function depends on mass. In the Schrödinger Equation

$$-k \cdot (TE_{particle} - PE_{system}) \cdot \Psi(x) = Curv \Psi(x)$$

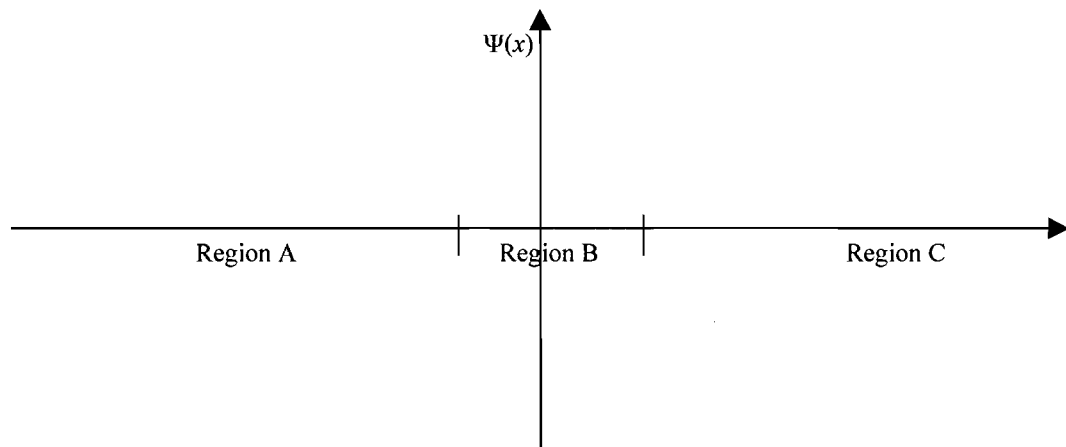
$k$  depends on mass. If  $m$  doubles, so does  $k$ .

- A. What happens to the curviness of the wave function as mass increases? What about the wavelength?

- B. Back on page 10, you sketched the wave function for 20 eV electrons that tunnel from the material to the tip, where 25 are detected for each 100 at the surface.
1. For comparison purposes (to what? Hang on...), reproduce that sketch here.

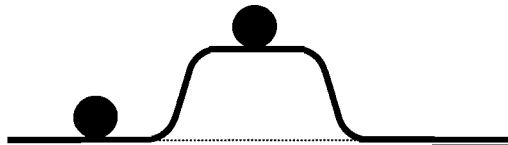


2. Imagine (OK, its not very physical!) that you had an identical system and a 20 eV bowling ball. The bowling ball's mass, however, is about  $10^{31}$  times greater than the electron's. Sketch what the bowling ball's wave function would look like on the axes below:



3. What's the probability of finding the bowling ball in Region C?

**Since we've had two board meetings, check your reasoning with your instructor before leaving.**



### Reference Sheet – Bowling Ball Model

		Scenario 1: Initial Kinetic Energy = 40 J			Scenario 2: Initial Kinetic Energy = 20 J		
Energy of the bowling ball	Point A	PE:	KE:	TE:	PE:	KE:	TE:
	Point B	PE:	KE:	TE:	PE:	KE:	TE:
	Point C	PE:	KE:	TE:	PE:	KE:	TE:
	Point D	PE:	KE:	TE:	PE:	KE:	TE:
Speed of the bowling ball							
Where is the bowling ball most likely to be?							
Picture graph of the probability density ( $P(x)$ ) as a function of $x$ of the bowling ball between points A and D.							

### Reference Sheet – Scanning Tunneling Microscope

Scenario	Probability Density Picture Graph	Wave Function Picture Graph
25 electrons measured in the tip.		
10 electrons measured in the tip.		
40 electrons measured in the tip.		

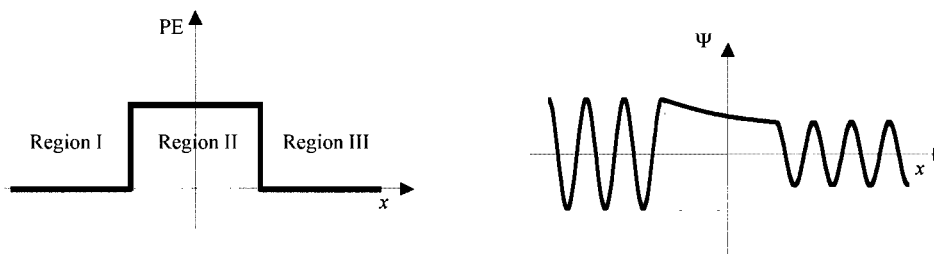
## Appendix K

### PRETEST FOR THE SECOND TUNNELING TUTORIAL

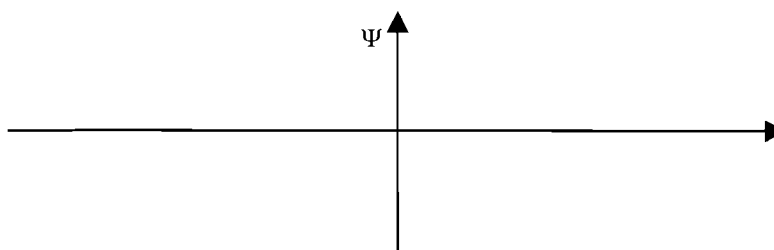
A system has the potential energy shown in the  $PE$  picture graph below at left, where the potential energy is higher in Region II than it is in Regions I and III. An electron is incident on this system from the left (Region I), and the electron has a total energy greater than the potential energy of Region I and less than the potential energy of Region II, that is

$$PE_{\text{Region I}} < TE_{\text{Particle}} < PE_{\text{Region II}}$$

A sketch of the wavefunction for the electron is shown below at right.



1. Region II is now made half as wide. For example, in the scanning-tunneling microscope model, this would correspond to the tip moving closer to the surface. How, if at all, does the wave function change in...
  - a. Region I?
  - b. Region II?
  - c. Region III?
2. On the axes below, sketch the wave function for this scenario, and note any differences from the picture graph above.



3. Relative to the original scenario, has the probability of finding the electron in Region III *increased*, *decreased*, or *remained the same*? Explain.

## Appendix L

### SECOND TUNNELING TUTORIAL

Many of us have heard of radioactivity, because of nuclear power, medical technologies, or maybe radon in homes. Radioactivity is a form of radiation in which an unstable nucleus becomes more stable by emitting a high-energy particle and/or a high-energy photon. Exposure to too much radiation can have negative health effects. Increased exposure to radon (a radioactive gas often found in places with a large amount of granite, like most of Maine) has been linked to an increased probability of developing lung cancer.

Ideas from quantum physics are needed to accurately describe the properties of the particles and photons emitted as radiation. The purpose of today's tutorial is to build a model for a specific type of radiation – alpha decay.

#### I. Board Meeting I – Agreeing on Tunneling through a Potential Barrier

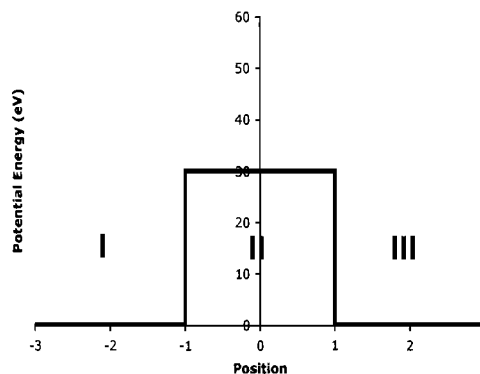
The purpose of this board meeting is to summarize last week's work, and to make sure everyone starts out with the same solution to the potential energy barrier scenario. Recall that for the past several weeks we've used the Schrödinger equation to describe the behavior of particles with wave properties:

$$\text{Curv } \Psi(x) = -k \cdot (TE_{\text{particle}} - PE_{\text{system}}) \cdot \Psi(x)$$

Last week we considered systems similar to that described below, where the potential energy is:

$$PE_{\text{system}} = \begin{cases} 0 \text{ eV} & x < -1 & \text{Region I} \\ 30 \text{ eV} & -1 < x < 1 & \text{Region II} \\ 0 \text{ eV} & x > 1 & \text{Region III} \end{cases}$$

A sketch of the potential energy of the system is shown on the graph at right.

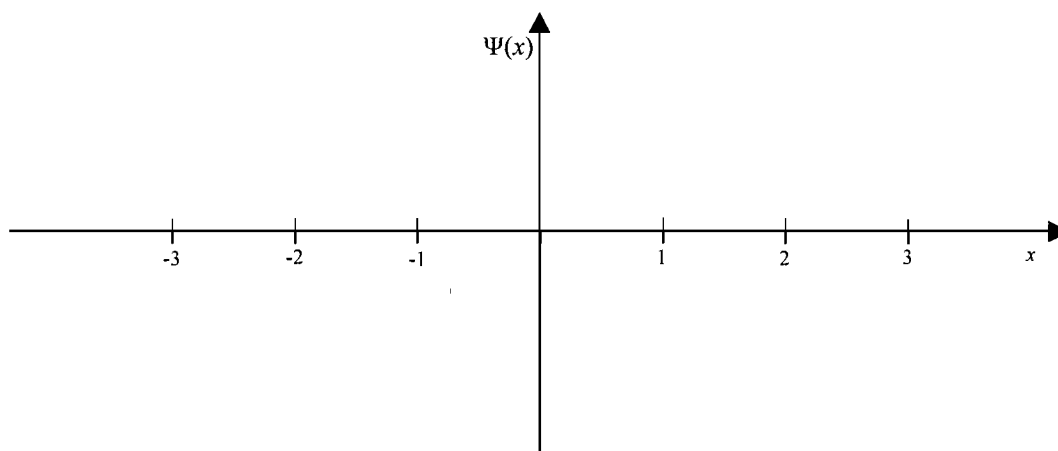


Think about an electron with total energy 10 eV is incident on the system from the left.

1. Sketch a picture graph of the wave function for this scenario on your whiteboard. Be sure to label and number your axes.

2. Examine your sketch of the wave function, and answer the following questions.
- How does the amplitude of the wave function in Region III compare to the amplitude of the wave function in Region I? Explain.
  - How does the wavelength of the wave function in Region III compare to the wavelength of the wave function in Region I? Use your knowledge of the relationship between energy and wavelength to help you answer.

Following the board meeting, you may find it useful to sketch a correct wave function for this scenario on the axes below:



**Hey! Read This! (It might help you do better on the rest of the tutorial):**

We're going to use the results from the board meeting as a reference experiment, and compare the results of changing (i) the electron's energy, (ii) the potential energy ("height") of the barrier, (iii) the width of the potential energy barrier, and (iv) the type of incident particle to the results we found here. Be sure you have a good understanding of the board meeting scenario before moving on to the next section.

## II. Summarizing the Effects of Change in Pictures

The table that was distributed with the tutorial accompanies this portion of the tutorial. Your task in this section is to fill in the blanks on the table, but do so carefully – this activity is not trivial! You should discuss your ideas about each situation with your group members, using your whiteboard to share ideas, and only sketch things on the table after you've agreed on the answer.

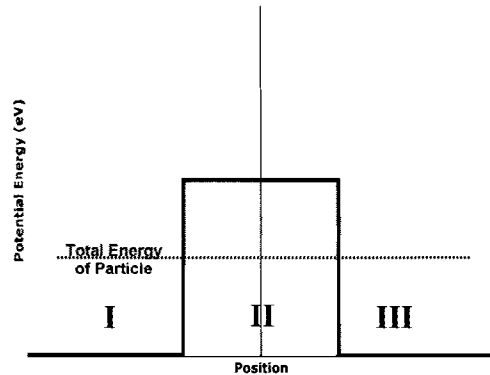
The table contains five columns. An explanation of each column is below:

- **Particle:** whether the particle we're talking about is an electron or proton. (A proton is a positively charged particle often found in the nucleus of an atom, and its mass is about 2000 times that of an electron.
- **System Potential Energy:** This column contains picture graphs of the potential energy of the system. Note that there are no numbers, but relative "heights" and "widths" of this barrier are important!
- **Particle Total Energy:** This column contains picture graphs of the total energy of the particle that is incident on the system. It, too, is numberless, but just because numbers are absent doesn't mean that the relative height of the energy line isn't important.
- **Wave Function:** This column contains picture graphs of the wave function corresponding to the particle type, system potential energy, and particle total energy.
- **Probability Comparison:** In all these scenarios, we're assuming that the particles are incident on the system from the left. In this column, we want you to compare the probability of finding the particles to the right of the barrier in the given scenario to the probability of finding the particles to the right of the barrier in the reference scenario – the first line in the table (which you'll notice is completely filled in, since you can't compare it to itself).

### III. Summarizing the Effects of Change in Words

One of the emphases in this course has been on the many ways we can model something – sketches, written explanations, story graphs, picture graphs, etc. To build a more complete understanding of tunneling, we need to not only understand graphical representations, but written representations as well.

You'll notice as you review the table you just completed that regardless of the changes we make for each scenario, there are common elements – three regions of potential energy (we'll call these Regions I, II, and III), some value of total energy for the particle, a wave function describing the particle's behavior.



This activity involves describing system changes in words. The other side of your reference table – Written Representations – will be used for this activity.

The table contains five columns. An explanation of each column is below:

- **Scenario Change:** This column details the change that has taken place to the particle and/or the system. All the changes are relative to the reference scenario – row 1 of our Graphical Representations chart.
- **Change of  $\Psi$  in Region II:** In this column, describe how (if at all) the wave function will be different in Region II. Be sure to think about curviness, wavelength, and/or amplitude as appropriate.
- **Change of  $\Psi$  in Region III:** Similarly, in this column describe how (if at all) the wave function will be different in Region III. Be sure to think not only about curviness, wavelength, and/or amplitude, but about the fact that the wave function in this region must stitch smoothly to the wave function in Region II.
- **Effect on Probability:** Here, you need to describe whether the probability of finding a particle in Region III has *increased*, *decreased*, or *remained the same*, relative to the reference scenario.
- **Effect on Particle Energy:** Here, you need to describe how the energy of a particle detected in Region III has *increased*, *decreased*, or *remained the same*, (i) relative to its energy in Region I, and (ii) relative to the reference scenario.

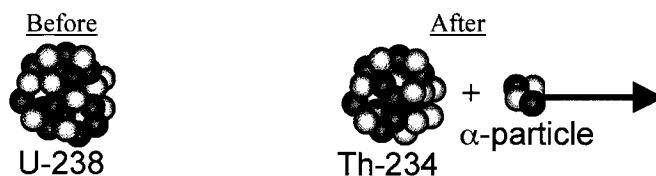
If possible, have your instructor check your table before proceeding, as we'll carry these ideas forward to address a more challenging problem.

#### IV. Introduction to Alpha Decay

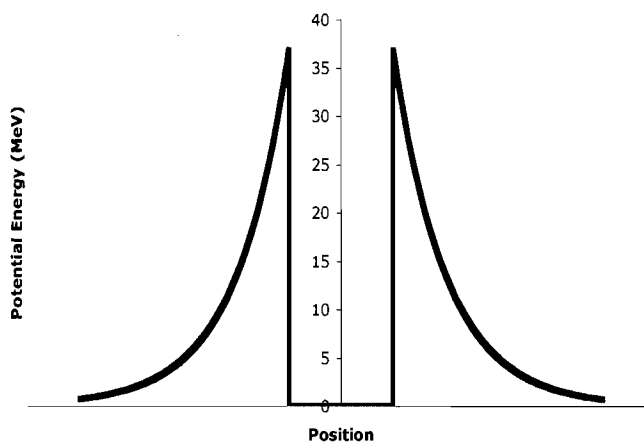
All right – so why all the focus on changes to the barrier and the particle energy? Well, we’re going to use the knowledge we’ve built to model a more complicated situation – one type of radiation.

One particle emitted by unstable nuclei is an alpha particle, made up of two protons and two neutrons. When an alpha particle is emitted, the atomic number of the atom (the number of protons in the nucleus) is reduced by two, and the element changes to a different element. This event is often called *alpha decay*.

For example, the nucleus of uranium-238 (U-238) contains 92 protons and 146 neutrons. When an alpha particle is emitted, the “new” nucleus is left with 90 protons and 144 neutrons, and the atom is now thorium-234 (Th-234). (If you’ve had some chemistry, this might make sense. If not, don’t worry too much about the numbers involved – we’re going to focus on the tunneling side of this problem.)



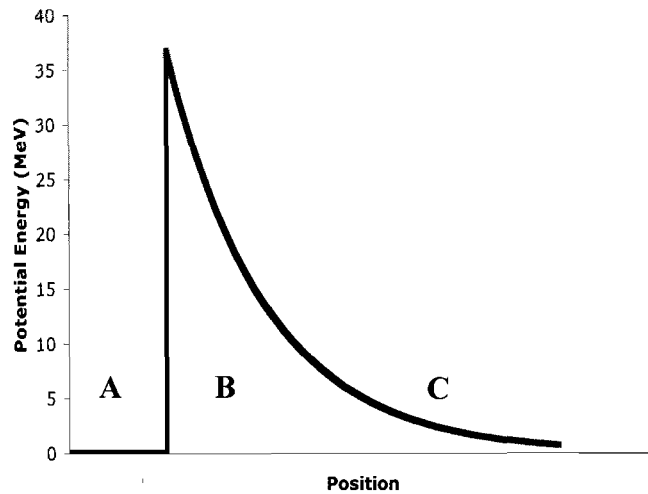
The potential energy of the system that the alpha particle is in can be modeled by the system shown below: Note that this potential energy graph is a lot more complicated than anything we’ve thought about so far. The energy is represented in MeV (mega electron-volts, where 1 MeV = 1,000,000 eV).



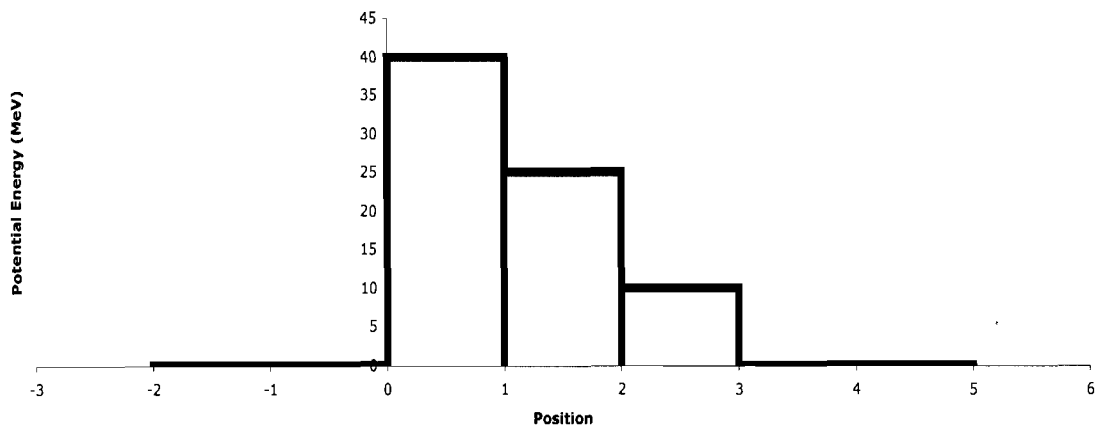
- A. If we imagine a classical (‘cart-like’)  $\alpha$ -particle (no wave properties, no ability to tunnel), how much energy would you expect an alpha particle that had escaped from the nucleus to have? Explain your reasoning.

Your answer to A should have been  $\approx 37$  MeV; if it's not, go back and re-check your reasoning. It has been found, however, that alpha particles that have escaped from the nucleus have a total energy between 4-9 MeV, a result that could not be explained by using only a particle model.

Since the potential energy of the system is symmetric (meaning for a particle that starts in the middle of the well, the potential energy “looks” the same regardless of whether it moves to the left or the right), we'll concentrate on the right-hand barrier:

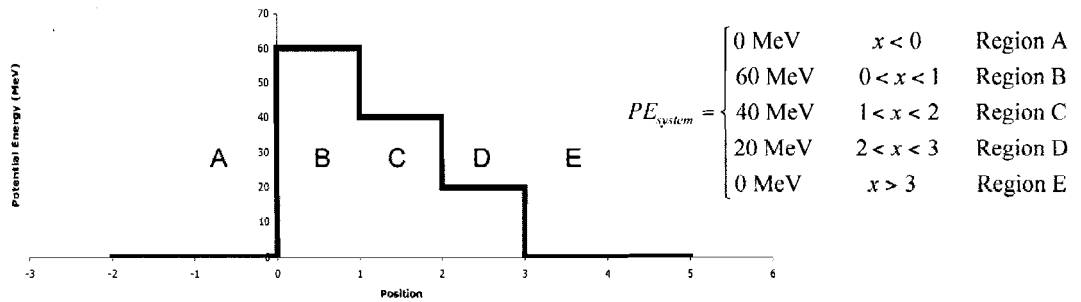


This is still a very complicated situation, so we're going to model it with a simpler situation – three rectangular barriers put side-to-side, as shown below:



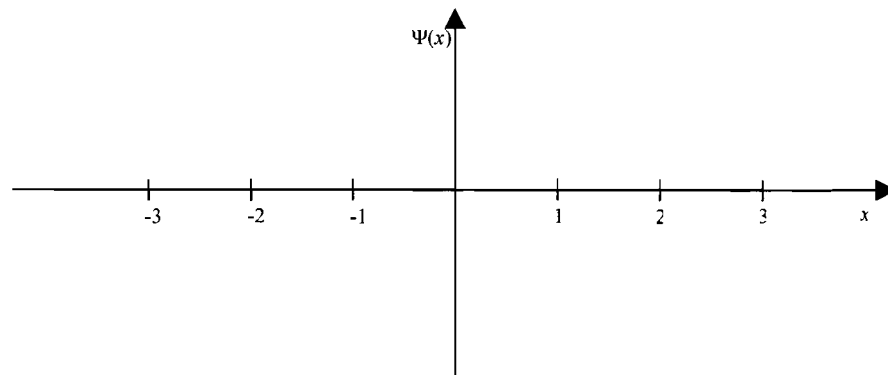
## V. Tunneling in Complex Systems

Consider a more complicated system with potential energy as shown on the graph below:



A particle with total energy 30 MeV is incident on the system from the left.

- Sketch a line representing the total energy of the electron on the potential energy graph.
- Consider possible wave functions that could be used to describe the particle in each region.
  - How does the energy of the particle in each region compare to the energy in the other regions?
  - Compare the curviness of the wave function at a given value of the wave function in Region B to Region C.
  - Compare the wavelength of the wave function in Region A and Region E.
  - Is  $\Psi$  the same in Regions C and D? How do you know?
  - How does the amplitude of the wave function in Region A compare to the amplitude in Region E?
- On the picture graph below, sketch the wave function in Regions A, B, C, D, and E. Make sure it is stitched smoothly at  $x = 0$ ,  $x = 1$ ,  $x = 2$ , and  $x = 3$ .



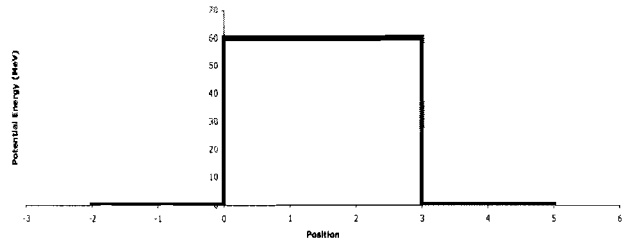
- D. The sketch you just completed is pretty hard to draw (depending, of course, on your artistic ability). If you think it would help you remember the changes, annotate your graph – that is, add written descriptions of the changes taking place.

## VI. Comparing Models

- A. Compare the behavior in the system you just reasoned about to another system described below:

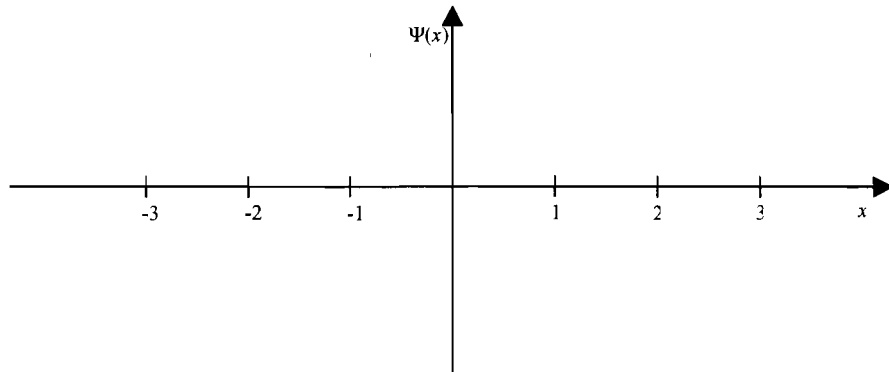
$$PE_{\text{system}} = \begin{cases} 0 \text{ MeV} & x < 0 & \text{Region I} \\ 60 \text{ MeV} & 0 < x < 3 & \text{Region II} \\ 0 \text{ MeV} & 3 < x & \text{Region III} \end{cases}$$

A sketch of the potential energy of the system is shown on the graph at the right.



A particle with total energy 30 MeV is incident on the system from the left.

- On the graph below, sketch the wave function picture graph for the new scenario.



- How does the probability of detecting an electron in Region III compare with the probability of detecting an electron in Region I in the previous scenario? Explain your reasoning.

- B. Think about the potential barrier a particle “sees”.

- How wide is the potential barrier described in A to a...
  - ...50 MeV particle?
  - ...30 MeV particle?
  - ...10 MeV particle?

2. How wide is the potential barrier described in part V to a...
  - a. ...50 MeV particle?
  - b. ...30 MeV particle?
  - c. ...10 MeV particle?
3. If the three-barrier model is more complicated, why use it?

## VII. Board Meeting II

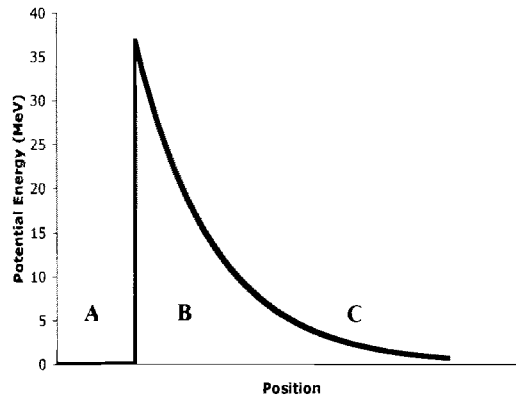
All groups should address the following question in the *FINAL!* board meeting.

Refer to the scenario from Part V to answer the following questions. (A careful sketch of the wave function may be helpful.)

1. How does the amplitude of the wave function in Region D compare to the amplitude of the wave function in Region E?
2. How does the wavelength of the wave function in Region D compare to the wavelength of the wave function in Region E?

### VIII. Back to Alpha Decay

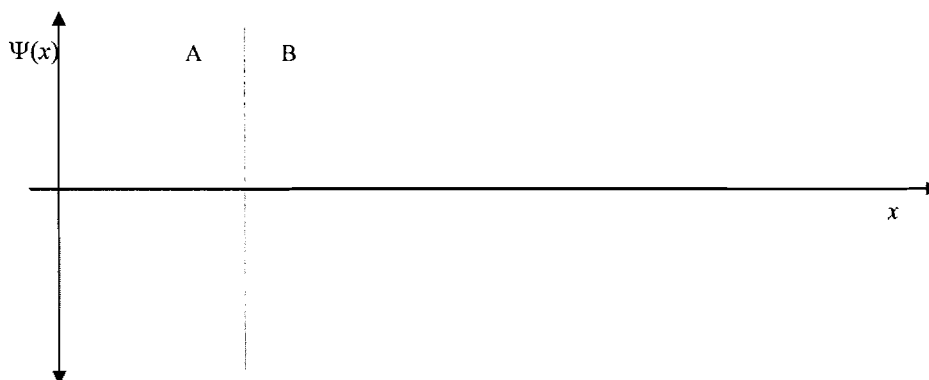
We'll now return to the more complicated (but more realistic) potential energy graph for alpha decay.



Imagine a quantum alpha particle is originally in Region A with an energy of 5 MeV.

- A. Sketch a line representing the total energy of the  $\alpha$ -particle on the graph.
- B. Consider possible wave functions that could be used to describe the  $\alpha$ -particle in each region. What type should be used in...
  1. Region A?
  2. Region B?
  3. Region C?
- C. Is it possible for the  $\alpha$ -particle to be found in Region C? Explain why or why not.

- D. On the picture graph below, sketch the B-C boundary that the 5 MeV particle “sees”. Sketch the wave function in Regions A, B, and C. Label the regions.



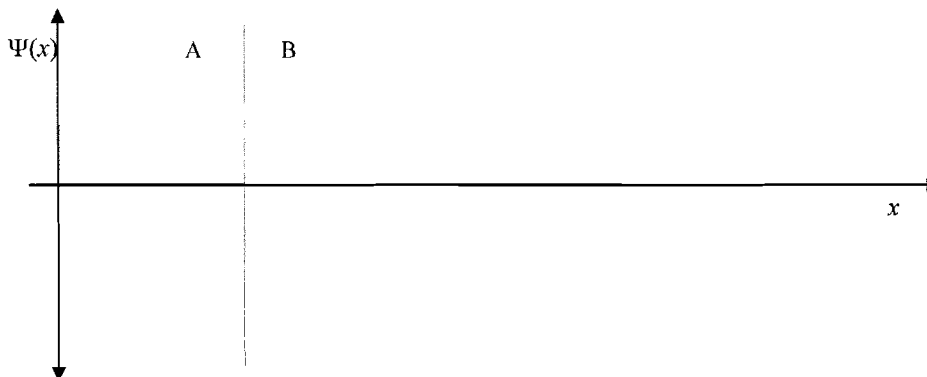
1. How does the amplitude of the wave function in Region C compare to the amplitude of the wave function in Region A?
2. How does the wavelength of the wave function in Region C compare to the wavelength of the wave function in Region A? (Is it the same everywhere in Region C?)

Now consider another alpha particle in an identical system with a total energy of 9 MeV. Use your reasoning from sections II and IV to help you answer the questions below.

- E. Consider, now, the characteristics of the regions that this 9 MeV “sees”. Which region(s) are different for a 9 MeV particle, compared to a 5 MeV particle? Explain your reasoning.
- F. Consider possible wave functions that could be used to describe the  $\alpha$ -particle in each region.
1. Does  $\Psi(x)$  change in Region A (compared to  $\Psi(x)$  used for a 5 MeV particle)? If so, how?
  2. Is  $\Psi(x)$  in Region B any different from  $\Psi(x)$  used to describe a 5 MeV  $\alpha$ -particle?

3. Only 1 region left, right? Any changes to  $\Psi(x)$  in Region C?

- G. On the picture graph below, sketch the B-C boundary that the 9 MeV  $\alpha$ -particle sees. (Be careful – it's different!) Then sketch the wave function in Regions A, B, and C that can be used to describe it.



1. How does the amplitude of the wave function in Region C for a 9 MeV  $\alpha$ -particle compare to the amplitude of the wave function in Region C for a 5 MeV  $\alpha$ -particle? Relate your answer to the changes in...
  - a. ...the width of Region B.
  - b. ...the total energy of the  $\alpha$ -particle.
2. Is the probability of finding a 9 MeV  $\alpha$ -particle in Region C *greater than*, *less than*, or *equal to* the probability of finding a 5 MeV  $\alpha$ -particle in Region C? Explain.
3. How does the wavelength of the wave function in Region C for a 9 MeV  $\alpha$ -particle compare to the wavelength of the wave function in Region C for a 5 MeV  $\alpha$ -particle? Explain.

## IX. Half-Life

Another issue in alpha decay not explained by classical (“cart-like”) physics is the fact that different elements take (on the average) very different amounts of times to decay. Some elements decay in fractions of a second, others take (on average) millions of years. Scientists use the term **half-life** to characterize this decay; the half-life of a given type of radioactive material is the amount of time it takes for half the nuclei in a given sample to decay.

The table below shows some average half-lives for various elements:

$\alpha$ -emitting nucleus	$\alpha$ -particle energy	Half Life
Polonium-212	8.8 MeV	$4.4 \times 10^{-7}$ s
Radon-220	6.3 MeV	79 seconds
Radium-224	5.7 MeV	5.3 days
Radium-226	4.8 MeV	2300 years
Uranium-238	4.3 MeV	$6.5 \times 10^9$ years

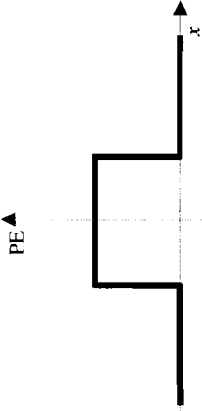
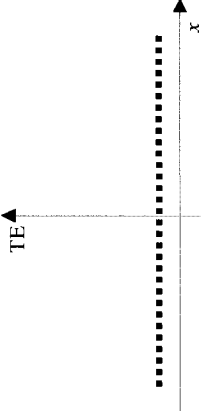
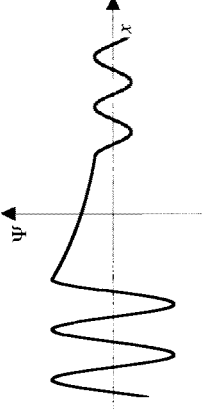
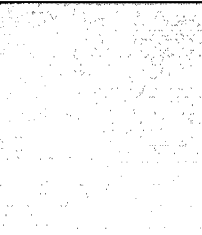
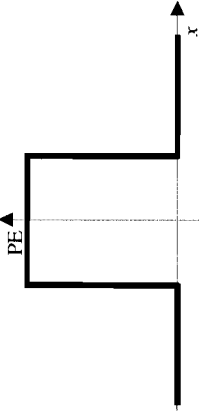
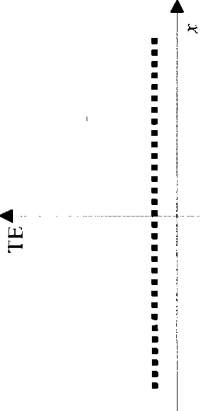
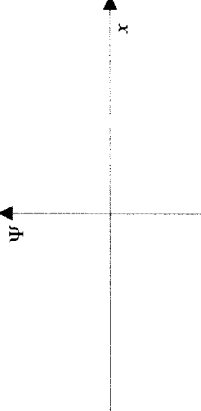
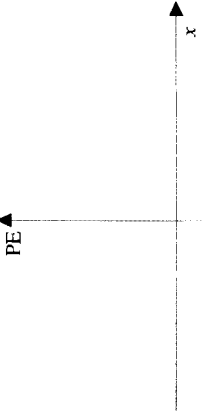
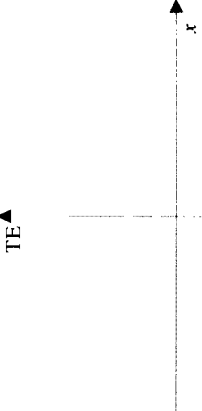
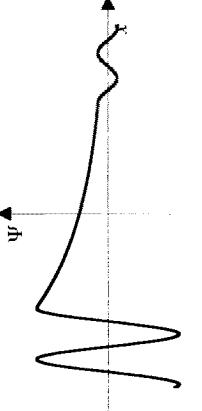
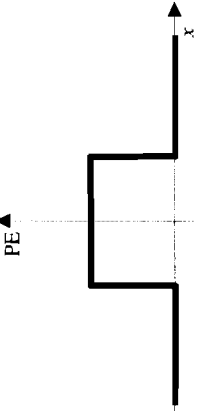
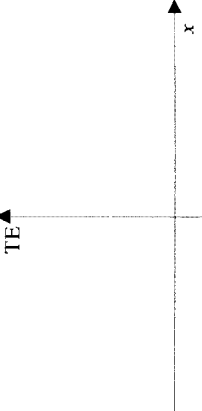
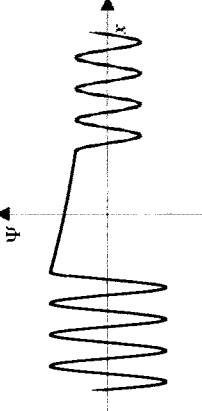
- A. Compare the data in the table to the scenarios you previously examined.
  1. Which  $\alpha$ -particle most closely corresponds to the wave function you sketched in VII.D?
  2. Which  $\alpha$ -particle most closely corresponds to the wave function you sketched in VII.G?
- B. What do the sketches of the wave functions say about the probability of an alpha particle tunneling out of the nucleus (being detected in Region C) of radium compared with polonium?
- C. How does the probability of a particle tunneling out of the nucleus (being detected in Region C) relate to the average half-life for various elements?

Some people think that the longer the half-life of a material, the longer the amount of time it takes a particle to tunnel through the barrier. By the laws of physics, it’s actually impossible to know the amount of time it takes a particle to get from the inside to the outside of a potential barrier. Tunneling time, however, is not the same as half-life. To help us see this, answer the following questions:

1. How much time (on the average) do you spend in Bennett Hall?
2. How much time (make an estimate) do you think your instructor spends in Bennett Hall?
3. How much time (again, a crude estimate is OK) does it take you to exit Bennett Hall? What about your instructor?

We all spend considerably different amounts of time here, but take about the same time to leave; the same is possible for particles tunneling out of the nucleus.

Tunneling Reference Table – Graphical Representations

Particle	System Potential Energy	Particle Total Energy	Wave Function	Probability Comparison
electron				
electron				
electron				
electron				

**Tunneling Reference Table – Written Representations**

Scenario Change	Change of $\Psi$ in Region II	Change of $\Psi$ in Region III	Effect on Probability of Detecting Particle in Region III	Effect on Particle Energy in Region III
<i>The total energy of the particle is decreased</i>				(i)  (ii)
<i>The width of the barrier is decreased</i>				(i)  (ii)
<i>The mass of the particle is decreased</i>				(i)  (ii)
<i>Decreasing the potential energy of Region II (still greater than <math>TE_{\text{particle}}</math>)</i>				(i)  (ii)
<i>Increasing the potential energy <u>and</u> width of Region II</i>				(i)  (ii)

## BIOGRAPHY OF THE AUTHOR

Jeffrey Morgan was born in Rutland, Vermont on December 22, 1973. He graduated from Union Springs Academy in Union Springs, New York in 1992. He attended Walla Walla College in College Place, Washington, and graduated in 1997 with a Bachelor of Science degree in Physics. Before enrolling in graduate school at The University of Maine in 2001, Jeffrey taught physics and physical science at Hixson High School in Hixson, Tennessee.

At The University of Maine, Jeffrey has been employed as a teaching assistant, research assistant, and instructor. Along with Michael Wittmann, he has published early results from the work described in this thesis in the *Proceedings of the Physics Education Research Conference* and the *European Journal of Physics*. Jeffrey is a member of the American Association of Physics Teachers. He is a candidate for the Doctor of Philosophy degree in Physics from The University of Maine in May, 2006.