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Tangled Up: Women’s Experiences in Mathematics

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TANGLED UP: WOMEN'S EXPERIENCES IN MATHEMATICS

by

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Abstract

This thesis is a bridge between two disciplines: Women's, Gender, and Sexuality Studies, and Mathematics. The first portion of the work synthesizes both theory and previously done studies to describe the state of women in mathematics as a whole, as well as historicizing the role of women in mathematics. Obstacles to the full and equal participation of women in mathematics are examined through a feminist lens. The second part of the thesis is a feminist biography crafted from an interview with a professor of mathematics, Dr. Erica Flapan. This provides information about her personal experiences as a woman in mathematics education in the late 1970s and early 1980s, and her perception of the field today and the status of women within it. Finally, the topological work of this professor is examined. This section introduces and explains some basics of knot theory and then begins to explore the work of Dr. Flapan in applications of knot theory to DNA in the process of site-specific recombination.

Dedicated to Sam: your constant support and friendship made this thesis happen, and I cannot thank you enough.

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Introduction

"Every time we get a chance to get ahead they move the finish line. Every time." This line, uttered by an African American female mathematician in the 2016 movie *Hidden Figures*, cuts to the core of our recent history surrounding race and gender in the professional world.[1] Women have to work in hostile environments to achieve less status and money than the average American professional male. This can be seen specifically in the field of mathematics. Currently, women outnumber men in higher education in general, both in enrollment and in degrees awarded. Despite this, in 2009, only 25% of mathematics and computer science bachelor's degrees awarded were to women.[2] This, along with multitudes of other factors to be explored, leaves a field of mathematics that is mostly male, and a culture of mathematics that is overwhelmingly masculine. The construction of this masculine culture of mathematics means that female mathematicians face obstacles like stereotype threat, sexist microaggressions, and other incidents that make it harder for them to thrive in the field. While there has undoubtedly been tangible progress over the last few decades, in policy and in public perception, it is clear that there is still observable inequality in the educational and professional experiences of men and women in mathematics. What is it that causes this phenomenon of women being held back or underrepresented in the field, when on paper, it appears that all the doors have been opened for women in higher education? This question will be explored not only

[1] *Hidden Figures*. Dir. Theodore Melfi. Perf. Taraji P Henson, Octavia Spencer, and Janelle Monae. N.p., 25 Dec. 2016. Web.

[2] Science and Engineering Indicators 2012. (2012). *S&E Indicators 2012 - Chapter 2. Higher Education in Science and Engineering - Undergraduate Education, Enrollment, and Degrees in the United States*. National Science Foundation. https://www.nsf.gov/statistics/seind12/c2/c2s2.htm#s2. Accessed 6 March 2017.

through the examination of research on the subject of women in STEM (Science, Technology, Engineering, and Mathematics), but also through an interview with mathematics professor Dr. Erica Flapan from Pomona College. Dr. Flapan's work in topology, specifically her applications of knot theory to DNA recombination, will be explored as well.

Women in Mathematics Through History

There is very little precedent for talking about both mathematics and feminist theory in one space—in 2008, a feminist theorist noted that "the boundary separating mathematics from women's studies and feminist theory, while not as forbidding as a prison wall, is nonetheless substantial and rarely crossed. Thinking mathematics and feminism together requires dealing with this boundary".[3] However, we know that women have been doing math for centuries. The first known, recorded female mathematician was Hypatia of Alexandria. Hypatia was active mathematically in the late 300s and early 400s Common Era (CE). Not much is known about the way she lived, and even less is known about her mathematics. What *is* known is that her father, Theon, supported her life learning and teaching geometry and early algebra. Theon is more commonly credited for his work, than Hypatia is for hers.[4] Whether that be a product of time or another woman being written out of history, we may never know, though it is likely both. Despite this, it is said that "Where Hypatia does quite clearly outshine Theon is in her reputation as a teacher. She was revered as such and no similar endorsement of Theon has come down to

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[3] Damarin, Suzanne. 2008. "Toward Thinking Feminism and Mathematics Together." *Signs: Journal Of Women In Culture & Society* 34, no. 1: 101-123. *Women's Studies International*, EBSCO*host* (accessed September 25, 2017).

us. We are left with a well-attested account of a popular, charismatic and versatile teacher."[4] Erica Flapan in her interview noted that in schools where mathematics education is offered, more women will take math classes in order to fulfill math education requirements.[5] This is perhaps the historical beginning of this sort of trend of female mathematicians becoming revered more as teachers and less as mathematicians. Another theme that Hypatia's story and life uncovers is the trend of our earliest known female mathematicians being the daughters of mathematicians. This is quite telling—in order to have exposure to the texts, the education, and the mathematical community that is necessary to be a mathematician, women like Hypatia and the many others who got into mathematics through their families had to use others' connections to even survive in the field, let alone thrive.

The first woman to obtain a PhD in mathematics was Sofia Kovalevskaya, a Russian mathematician born in 1850. Her PhD was awarded by the University of Göttingen in Germany in 1874, after she proved a theorem that now bears her name—the Cauchy-Kovalevskaya theorem.[6] Her advisor was the mathematician Weierstrass, a well known mathematician who is sometimes called "the Father of Modern Analysis."[7] Despite being a favorite student of one of the most important people in mathematics of

[4] Deakin, Michael A. "Hypatia and Her Mathematics." *Sherlock Holmes in Babylon: and Other Tales of Mathematical History*, edited by Marlow Anderson et al., 1st ed., Mathematical Association of America, 2004, pp. 52–59, www.jstor.org/stable/10.4169/j.ctt13x0n0r.10.

[5] "Thesis Interview with Erica Flapan ." Telephone interview by author. May 10, 2017.

[6] Keen, Linda. "Sonya Kowaleskaya- Her Life and Work." *Association for Women in Mathematics* 7.2 (1977): n. pag. Web. 6 Apr. 2017.

[7] Holt, Michael. "ABC of Mathematicians: Karl Wilhelm Weirstrass." *Mathematics in School* 5.3 (May 1976), pp. 18-19. Web. 26 Apr. 2017.

the era (and arguably of all time), Kovalevskaya was unable to find employment in the field of mathematics for quite a while. She spent years working in the field of literature and writing plays before finally becoming a professor of mathematics.[6] Though she was certainly brilliant enough to obtain her PhD and prove revolutionary results, she was still not awarded the respect that her male peers enjoyed at the time.

Following Kovalevskaya, the first PhD in mathematics awarded in the United States was to Winifred Edgerton at Columbia University in 1886. In addition to being a mathematician, Edgerton also was active in encouraging equal educational opportunities for women. At the time, there was no college in New York for women, and Edgerton was a part of the committee that petitioned Columbia to open Barnard College, a liberal arts school for women. Barnard is one of the Seven Sister Schools (along with Bryn Mawr, Wellesley, Mount Holyoke, Radcliffe, Vassar, and Smith Colleges) that historically were women's colleges in a time when most mainstream universities did not allow women to matriculate.[8] Since the nineteenth century, the number of women and the percentage of women in mathematics education in all levels of secondary education has increased, albeit slowly.[9] Despite this, it still seems somehow unnatural to discuss both mathematics and women together. There are many reasons that this could be true. Mathematics has a reputation of being a purely objective and unfeeling discipline—"There is a story about mathematics in which the discipline exists apart from the

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[8] Riddle , Larry . "Winifred Edgerton Merrill." *Biographies of Women Mathematicians*. Agnes Scott College . Web. 6 Apr. 2017.

[9] Science and Engineering Indicators 2012. (2012). *S&E Indicators 2012 - Chapter 2. Higher Education in Science and Engineering - Undergraduate Education, Enrollment, and Degrees in the United States*. National Science Foundation. https://www.nsf.gov/statistics/seind12/c2/c2s2.htm#s2. Accessed 6 March 2017.

messiness of human society. Answers are right or wrong, debates are not necessary, and judgments are impartial. This story was a comfort to me as a teenager; it suggested a calm, if clinical, fairness. Mathematics was a refuge from conflict and disagreement, and positive feedback from my teachers and standardized tests afforded a sense of safety. The pure objectivity story is, of course, incomplete, and fairness in the practice of mathematics is far from a necessary outcome. Mathematics happens in the context of human societies."[10] We know that there is great inequality in who has the privilege of doing mathematics professionally, and thus that math does not exist in the clinical environment that many seem to think it does. Mathematics education, workplaces, hiring processes, etcetera are processes that involve human beings who live in a gendered world that shapes their thinking. In order to become mathematicians, people have to get through the process of obtaining their degree or degrees and then be hired to a job. They then spend their lives in workplaces—gendered environments. There is recent research that specifically details the plight of millennial women, and the way that despite the still growing involvement of women in the workplace, "young women's experience of the workplace can be one of precarity and insecurity."[11] The myth that math and those who do it is somehow exempt from these human processes and biases is just that—a myth.

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[10] Bremser , Priscilla. " An Infusion of Social Justice into Teaching and Learning ." In *Mathematics Education: A Spectrum of Work in Mathematical Sciences Departments*, 328-46. Association for Women in Mathematics. Switzerland: Springer.

[11] Worth, Nancy. 2016. "Who we are at work: millennial women, everyday inequalities and insecure work." *Gender, Place & Culture: A Journal Of Feminist Geography* 23, no. 9: 1302-1314. *Women's Studies International*, EBSCO*host* (accessed October 15, 2017).

Literature Review

One factor that can affect women's performance in mathematics is something called stereotype threat. Stereotype threat is explained as "negative stereotypes raise inhibiting doubts and high-pressure anxieties in a test-taker's mind."[12] In a study by Catherine Good, *Problems in the pipeline: Stereotype threat and women's achievement in high-level math courses*, researchers examined people in higher level math classes. They were specifically looking to find out information about the performance of women who were enrolled in the most rigorous calculus class offered at a US public university. In conducting her study, Good found that women simply given a mathematics test without any mention of gender performance ended up performing worse than men, but women who were explicitly told that women and men performed equally as well on the diagnostic outperformed men. Good's study determined that stereotype threat still affects women in higher level math. This means that even though they have chosen to be in the field already, they are still susceptible to the phenomenon of stereotype threat. Another takeaway from this study is the idea that women generally outperform men when reassured of the equality of ability of men and women. Good discusses some reasons for this:

> It is likely that because women are more apt to self-select out of math and science fields early in their educational careers (American Association of University Women, 1998 and Eccles, 1994), the women in advanced mathematics classes, such as those in our sample, comprise the most motivated and prepared female

[12] "Stereotype Threat Widens Achievement Gap ." American Psychological Association . Accessed September 25, 2017. http://www.apa.org/research/action/stereotype.aspx

> mathematics students. Men, on the other hand, may be more likely to pursue math and science careers even if they are less prepared academically to do so.[13]

So, not only did stereotype threat influence women's performance, negatively changed it to make it seem as if they were unequal in ability to the men in the course. In reality they were superior in ability, just much more susceptible to stereotype threat. The methods of this study were distinct from other, previously done studies with similar premises. Typically, the way that stereotype threat is examined is by having a control group of students who were not told anything about gender or made to encounter sexist thought or language, and a group of students who were primed with some sort of sexist behavior, whether it be blatant, by telling them outright that women performed worse on the exam, or subtle, by wording the questions or directions in a way that implied a difference in ability between men and women.[14] [15] However, this study by Good instead examined the effects of how positive reinforcement and the reassurement of equality of ability can impact performance. Studies like these reveal that even the environment in which math classes exist inherently breeds conditions that cause stereotype threat. Good discusses some of the potential reasons behind this, though there are many more factors to be examined. The gender composition of the class, which in her study was a 2:1 ratio of male to female, impacts whether or not women felt they belonged. Additionally, Good

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[13] Good, Catherine, et al. "*Problems in the Pipeline: Stereotype Threat and Women's Achievement in High-Level Math Courses*." Journal of Applied Developmental Psychology, vol. 29, no. 1, Jan-Feb, 2008, pp. 17-28. EBSCOhost, doi:10.1016/j.appdev.2007.10.004.

[14] Adams, G., Garcia, D. M., Purdie-Vaughns, V., & Steele, C. M. (2006). The detrimental effects of a suggestion of sexism in an instruction situation. Journal of Experimental Social Psychology, 42, 602-615.

[15] Ambady, N., Shih, M., Kim, A., & Pittinsky, T. L. (2001). Stereotype susceptibility in children: Effects of identity activation on quantitative performance. Psychological Science, 12, 385-390.

identified another important factor—how people perceive others perception of their ability. Both men and women believed that in general, others thought that men were better at calculus than women.[16]

There are a few significant conclusions that can be drawn from this study. First of all, it reflects that although the concept of women feeling unwelcome in math begins in youth and early education, it continues to affect them observably through their higher education. Additionally, it seems to hint that simply ignoring gender in the classroom and not being blatantly sexist or unwelcoming towards female students is insufficient. In order to counteract the feeling of not-belonging and its impact on performance, there needs to be positive, affirming action taken to ensure female students that they have the same potential for success as male students in mathematics. As Good's study demonstrates, the culture of mathematics departments and mathematics classrooms does not necessarily facilitate this sort of action.[17] Additionally, this stereotype threat does not affect only women. People from racial, ethnic, and sexual minorities that are also greatly impacted by stereotype threat. To consider how these identities can interact with other identities like gender and socioeconomic class, we can consider the concept of intersectionality. Intersectionality, coined by Kimberlé Crenshaw and Patricia Hill Collins, is the academic framework that examines how different identities interact and

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[16] Good, Catherine, et al. "*Problems in the Pipeline: Stereotype Threat and Women's Achievement in High-Level Math Courses*." Journal of Applied Developmental Psychology, vol. 29, no. 1, Jan-Feb, 2008, pp. 17-28. EBSCOhost, doi:10.1016/j.appdev.2007.10.004.

[17] Good, Catherine, et al. "*Problems in the Pipeline: Stereotype Threat and Women's Achievement in High-Level Math Courses*." Journal of Applied Developmental Psychology, vol. 29, no. 1, Jan-Feb, 2008, pp. 17-28. EBSCOhost, doi:10.1016/j.appdev.2007.10.004.

how institutions treat these intersectional identities and those who experience them.[18] [19] For example, black women experience the world differently than black men do, and differently than white women. In the case of mathematics, this means that those with these intersectional identities can be more vulnerable to stereotype threat based on their race, gender, and/or sexuality. The prevailing image of a scientist or a mathematician today is not just a male, but a white straight one, and thus any deviations from this are considered deviant from the norm.

The central conflict of women in mathematics is well-explained in the piece *Images of mathematics, values and gender: A philosophical perspective* by Paul Ernest:

> Since mathematics is stereotypically male, conforming to mathematical standards conflicts with standards of femininity. This, at its simplest, means that women must choose to be feminine or choose to be successful at mathematics.[20]

This suggests some sort of incompatibility between math and femininity. Masculinity has been associated with mathematics not only because the majority of mathematicians have been men, but also because of the traditional domains of knowledge and the way they are associated with different genders. Women are more associated with types of knowledge and thus professions related to helping others and providing others with services, whereas

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[18] Crenshaw, Kimberlé Williams. 1995. "Mapping the margins: intersectionality, identity politics, and violence against women of color." *Critical Race Theory: The Key Writings That Formed The Movement.* 357-383. *Women's Studies International*, EBSCO*host* (accessed September 25, 2017).

[19] Collins, Patricia Hill, Women Memphis State University. Center for Research on, and Women Research Clearinghouse and Curriculum Integration Project on Women of Color and Southern. 1989. *'Toward a New Vision: Race, Class and Gender as Categories of Analysis and Connection.'*. n.p.: 1989. *Women's Studies International*, EBSCO*host*(accessed November 3, 2017).

[20] Ernest, P. 2004. *Images of mathematics, values and gender: A philosophical perspective*. In Mathematics education: Exploring the culture of learning, ed. B. Allen and S. Johnston-Wilder. New York, NY: Routledge-Falmer.

men are more associated with "careers that grant them agency, most commonly in the form of status and financial gain."[21] Cheryan's research also suggests that these associations impact us more than we might expect. "Math-related fields such as computer science and engineering are perceived as less likely to fulfill communal goals (i.e., helping humanity and having interpersonal interactions) than other math related fields that have attracted more women in recent decades, such as medicine and architecture."[20] Femininity is more easily reconciled with these "helping" fields that involve applications of math, rather than the more abstract fields.

In the book *Delusions of Gender: How Our Minds, Society, and Neurosexism Create Difference*, Cordelia Fine discusses this same issue of mathematics and femininity being incompatible in the context of education and the workplace. She cites countless studies, many similar to Catherine Good's study, that demonstrate that women perform worse in mathematics when they are exposed to sexism, and discusses that this leads to women leaving the field or undervaluing their performance. Fine discusses the concept of women who "make it" in mathematics choosing mathematics over femininity—but what does this really look like? Fine offers a few examples of women "shedding the feminine attributes they perceived as a liability."[22] One of these is altering their dress and physical presentation by not wearing things that are stereotypically feminine, like makeup, jewelry, skirts, and dresses. Additionally, women began to become hostile towards other

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[21] Cheryan, Sapna. 2012. "Understanding the Paradox in Math-Related Fields: Why Do Some Gender Gaps Remain While Others Do Not?." *Sex Roles* 66, no. 3-4: 184-190. *Women's Studies International*, EBSCO*host* (accessed October 15, 2017).

[22] Fine, Cordelia. *Delusions of gender: how our minds, society, and neurosexism create difference*. New York: W.W. Norton, 2010. Print.

women, invalidating or shunning their emotions, and taking up "anti-female attitudes." [22] These seemed to echo the more general, non-career related popular trend of women ensuring people that they 'aren't like other women,' as if being like other women is something to be ashamed of. This is a reflection of internalized misogyny, which is defined by scholars as "a fear of femininity and a hatred and devaluing of women and female related characteristics."[22] In order to be taken seriously, women feel as if they must change themselves, or at least set themselves apart from the category of 'woman'. This is how they are taken more seriously—by establishing themselves as non-feminine, or not like other women, they feel like they will be respected more. This also ties in with the undervaluing of what is considered female work in general. Because our society tells us that women's work is not valuable, in order to be of value, women cannot be feminine.

Beyond the specific case study of women in mathematics, Fine's work overall challenges the belief that men and women are fundamentally, neurologically different. This is one of a few schools of thought about what creates difference between the sexes––observed in Fine's text are those who believe in social construction of gender, and those who believe in biological difference.[23] Fine's text is an attempt to explore the presumed neurological differences between men's brains and women's brains, and to debunk the idea that there are differences neurologically that impact the way men and women learn, grow, and experience the world in a gendered way. She tends towards the theory that gender is socially constructed—outlined by sociologist Judith Lorber in *The Social Construction of Gender*, where she argued that it was human behavior and social norms and expectations that shaped us into the gendered beings of "men" and "women."[25]These

[23] Lorber, Judith. *The Social Construction of Gender*. Newbury Park, Calif.: Sage, 1991

theories push up against the type of biological essentialism displayed in the common arguments that somehow men are "naturally" or "biologically" better at mathematics, and argue that it is instead social factors and social norms that make it seem as if men excel in math naturally. This explains why we see so many examples of exceptional women in mathematics history. Rather than fighting their biological impediments, women who exceed in mathematics are simply breaking a gender norm—something Lorber describes as "breachable."[25]

Initially, most of the computing work of computer science was done by women because computing (as it was then called) was seen as tedious, menial work.[21] It was later when the importance of computer science was fully realized that it was masculinized. An interesting tidbit of this transition from feminine to masculine field is noted in a study by Sapna Cheryan, in which she hypothesized that "Movies such as *Revenge of the Nerds* and *Real Genius*… crystallized the image of the "computer geek" in the cultural consciousness."[24] Clearly the media and the way it presents people in certain career fields has shaped the way the rest of the world views that field. Computer science today is known as a masculine field, much like mathematics, as it developed out of math. But how does this affect women, specifically? An NBC article that summaries Cheryan's findings, *Geeks drive girls out of computer science*, says "women can be turned off by just the physical environment, say, of a computer-science classroom or office that's strewn with

[24] Cheryan, S., Plaut, V. C., Davies, P. G., & Steele, C. M. (2009). Ambient belonging: How stereotypical cues impact gender participation in computer science. Journal of Personality & Social Psychology, 97(6), 1045–1060.

objects considered "masculine geeky," such as video games and science-fiction stuff."[25] This can either discourage them from entering the field, or make them feel as if they do not fit in if they are in the field already. An added difficulty independent of media representations of fictional STEM folk is the availability of actual real role models for young girls. Cheryan also points out in her study that most of the big names we associate with computer science are male. While these men are brilliant, they are also a replication of what we already see in fictional media—white men who fit into the concept of what a computer scientist should be. There are no real household names in computer science of women, though computer science was built on the backs of important women like Ada Lovelace and Grace Hopper. There are few films about their accomplishments (like Alan Turing in *The Imitation Game*), nor are their faces in our news accompanying announcements about advancements in technology (like Bill Gates). Therefore, people are not exposed to females in this field in the same way they are to male computer scientists. Though not covered by Cheryan in her study, the same can be said about mathematics. Women do not have high visibility as mathematicians in our media. In more recent years, there have been a few exceptions. One of the most notable ones is the 2016 movie *Hidden Figures*. The premise of *Hidden Figures* is to show a dramatized, fictionalized, account of the lives of three black female mathematicians who actually worked at NASA during the 1960s Space Race. Katherine Johnson, the main focus of the film, was a mathematician at NASA who was promoted to work on the Space Task Group, the most important group of people responsible for the calculations and

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[25] Bryner, J. (2009, December 15). *Geeks drive girls out of computer science*. Retrieved March 26, 2017, from http://www.nbcnews.com/id/34437233/ns/technology_and_science-science/#.WNhzM_nyvIV

mathematics behind the first missions sending American astronauts to space. Dorothy Vaughan had asked multiple times to be promoted to supervisor of the "colored" computing group, but as NASA was segregated during that time period, her requests were denied. She was, however, expected to do the work of a supervisor without being paid like one. The third person the movie focuses on is Mary Jackson, who was NASA's first black female engineer.[26] Hidden Figures stars Taraji P. Henson, Octavia Spencer, and Janelle Monae brought Katherine Johnson on stage during the Oscars while they were presenting the award for outstanding documentary. They did this in order to share the success of the film with her and acknowledge her contributions to computing and mathematics, saying— "Movies about the lives of men and women in the history books have long been a staple of storytellers. Sometimes, the names and deeds are the heroes and their names are known to all, and then there are those films that shine the spotlight on those whose names are known to only a few, but whose stories are deserved to be told."[27] This sort of acknowledgment on a national platform with so many viewers is invaluable. Even those who did not see the film had the opportunity to learn about a female mathematician through this media exposure.

Another important factor that can impede women in their pursuit of higher education regardless of their academic discipline is the constant threat of gender based harassment and sexual assault. The Rape, Abuse & Incest National Network found that "among undergraduate students, 23.1% of females and 5.4% of males experience rape or

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[26] Shetterly, Margot Lee. "Mary Jackson Biography." *NASA*. NASA, 22 Nov. 2016. Web. 15 Mar. 2017.

[27] Pearlman, Robert Z. "Oscars Honor Real-Life NASA Hero Katherine Johnson, But Pass On 'Hidden Figures'." Space.com. February 27, 2017. Accessed October 15, 2017. https://www.space.com/35841-oscars-honor-nasa-katherine-johnson-hidden-figures.html.

sexual assault through physical force, violence, or incapacitation." For graduate students, this goes down to about 8.8% of females and 2.2% of males.[28] Many college campuses have excellent programs that acknowledge this serious problem and attempt to address it. However, these numbers remain high, and the threat of violence is something that all college-aged women have to live with. All universities that receive federal funding in the United States must comply with Title IX, a part of the Education Amendments of 1972. Title IX is a short statement that allows schools to interpret it as loosely or strictly as they wish—

> "No person in the United States shall, on the basis of sex, be excluded from participation in, be denied the benefits of, or be subjected to discrimination under any education program or activity receiving federal financial assistance"[29]

Gender-based harassment and sexual assault can both be reported to a Title IX compliant school's Title IX point person. When these complaints are made, the Title IX coordinator is "required to be prompt when receiving a complaint of sex discrimination, sexual harassment, or sexual violence in order to remedy any hostile educational environment created by such behavior"[29] This can take a number of forms—switching class schedules around to isolate perpetrator from victim, making housing accommodations, or even pursuing a formal investigation of the allegation. Obviously, this issue of gender based discrimination impacts the lives of every college woman, regardless of their discipline.

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[28] "Campus Sexual Violence: Statistics ." RAINN (Rape, Abuse & Incest National Network). 2017. Accessed September 13, 2017. https://www.rainn.org/statistics/campus-sexual-violence.

[29] "Title IX ." Know Your IX . 2017. Accessed September 13, 2017. https://www.knowyourix.org/college-resources/title-ix/

However, in their 2012 evaluation of Title IX's 40th year, the National Women's Law Center examined the impact of Title IX specifically on Women in STEM. They propose a strengthening of Title IX to help women and girls in STEM and specifically target the discrimination they face in the classroom, and to ensure they have equal access to their programs. [30]When women are under constant threat of harassment and discrimination, they cannot perform to their best ability—especially women in STEM fields, who are already vulnerable to the aforementioned stereotype threats. If there was a way to encourage more cooperation between STEM departments and the Title IX and harassment prevention teams at universities, perhaps these women would be more seamlessly integrated into their programs with less worry of discrimination.

Some studies have suggested that there are ways to combat the issues of stereotype threat, hostile environments, and self-selecting out of the field. One of these proposed solutions is the introduction of strong female role models into the lives of female math students. In a study on the effects of these role models, it was found that "In the end, increasing the number of female role models in math and engineering classes may allow female students to view the negative gender stereotypes that confront them as surmountable barriers rather than ones that are insurmountable and therefore potentially inspire more women, who may not be initially identified with math, to pursue careers in these academic areas."[31] So these role models cannot eliminate the fact that it is still

[30] "The Next Generation of Title IX: STEM – Science, Technology, Engineering, and Math." National Women's Law Center . June 2012. Accessed September 13, 2017. https://nwlc.org/wp-content/uploads/2015/08/nwlcstem_titleixfactsheet.pdf.

[31] Marx, David M., and Jasmin S. Roman. "Female Role Models: Protecting Women's Math Test Performance." *Personality And Social Psychology Bulletin* 28, no. 9 (September 2002): 1183. *Women's Studies International*, EBSCO*host*(accessed September 10, 2017).

incredibly difficult to be a woman in the field, but it can help to provide proof that other women have made it through those obstacles and achieved success. However, this ray of hope also brings to light another problem, which is the low numbers of women in mathematics professorial roles in American universities. This problem is not unique to math, and is the focus of the article *Why Aren't More Women Tenured Science Professors?* by Katherine Harmon. She cites a study, commissioned by congress, that gives the latest numbers of the gender breakdown of math and science professors—with 26.5 percent of assistant math professors being women, and only 9.7 percent of full professors.[32] The article guesses that these low numbers are a reflection of a few causes––women not applying to these jobs and instead working in other careers within their respective fields, and also women being "held back at the assistant professor level."[32]

Another study, just published in September 2017, examines another possible way that female professors are being held back from tenure and full professor status. It does not examine by subject, but rather gives an overview of all professors across disciplines and their teaching evaluations. This study examines how a professor's gender can impact their teaching evaluations. They found, in line with many previous studies, that female professors get lower evaluations than their male colleagues, even when their performance is identical.[33] What is unique about this study is that it looks at the students that are rating

[32] Harmon, Katherine . "Why Aren't More Women Tenured Science Professors?" Scientific American . June 19 , 2009. Accessed August 15 , 2017. https://www.scientificamerican.com/article/women-tenured-science-professors/.

[33] Mengel , Friederike , Jan Sauerman, and Ulf Zolitz. "Gender Bias in Teaching Evaluations ." *Journal of the European Economic Association* , September 2017. Accessed September 9, 2017 . https://www.briq-institute.org/wc/files/people/ulf-zoelitz/working-papers/gender-bias-in-teaching-evaluations.pdf.

these female professors low, and also which female professors are particularly susceptible to these discrepancies. Their findings are disheartening—they found that the occurrence of lower evaluations "is stronger for male students who seem to question the teaching abilities of, in particular, junior female teachers and in math related courses."[33] These findings are not insignificant, because teaching evaluations are used in considerations for promotions, teaching prizes, and other important things that can impact the professional futures of younger professors. It can also impact how these women view themselves and their success in their careers, or even how their supervisors and colleagues view them. The conclusion of this study goes so far as to say that "The fact that female PhD students are in particular subject to this bias may contribute to explaining why so many women drop out of academia after graduate school."[33] The trials of women in the field of math do not halt after they finish schooling, but continue through the remainder of their professional life. Regardless of the reasons behind their exclusion from academia, low numbers of women teaching math at the collegiate level mean that depending on their college, and area of study, students can encounter very few female professors, and thus female role models, along the way.

Feminist Biography of Dr. Erica Flapan

Part of this look into women in mathematics was an interview with a woman who has spent her life studying math, Dr. Erica Flapan. Flapan is a professor of Mathematics at Pomona College in Claremont, California. She studies knot theory and the applications of topology to chemistry and molecular biology. She was chosen as the interviewee because of her willingness to share stories about her experiences in education and share her research on site specific recombination and DNA modeling using tangles.

The story of Flapan's life as presented here is used as a glimpse into the world of mathematics education and how women experienced it in the late 1970s and early 1980s. The text *The Challenge of Feminist Biography: Writing the Lives of Modern American Women* discusses the ways in which feminist biographies can contribute to knowledge—"We would argue that feminist biography not only expands our knowledge about women's lives, but alters the frameworks within which we interpret historical experience."[34] Not only can Flapan's experiences tell us something about her, but it can tell us something about the field overall and how it shapes women who engage with it. This is not to say that all women in the mathematics world have experiences that parallel Flapan's experience. Each school has unique policies that can influence how women encounter the college environment. Every math department is different, the attitudes of professors and fellow classmates are unique from class to class and from school to school. Additionally, individuals' experiences with education varies based on other aspects of identity like race, social class, and ability. A black woman in education has a markedly different experience than a white woman, for example. Higher education, especially in STEM, was designed for men, but it is important to acknowledge that it was designed for white, able-bodied, and higher-class men. These intersections of identities are important to consider. Despite these differences, Flapan's story still provides valuable insight into some of the obstacles that women encounter along their paths in mathematics as a field.

[34] Alpern, Sara, Joyce Antler, Elisabeth Israels Perry, and Ingrid Winther Scobie. *The Challenge of Feminist Biography: Writing the Lives of Modern American Women* . Urbana and Chicago, IL : University of Illinois Press, 1992.

Flapan’s early childhood education pushed her into math in a unique way. She attended an experimental elementary school with few formal guidelines for learning, and from this early age she gravitated towards mathematics. This is not the typical experience of young children, and thus her childhood experiences do not necessarily represent the “average.” As she entered the formal school system in junior high school, she felt inadequately prepared in other subjects, and as a result felt compelled to advance into a math-related career—

> I felt inadequate in those other areas. For that reason, I felt like I didn’t have any choice—I had to go into math which at that age, meant be a math teacher. But later on, it morphed into being a college math professor. So that was important, in terms of that decision. And I think that it was important later on because I felt like I didn’t have any other options. So when things got harder in math, I still felt like I had to do them. I didn’t feel like I could switch and do something else.

Her background is clearly unique, and it propelled her into a field that was at the time very predominantly populated with men. If she had attended a traditional elementary school, she may have not been pushed in the direction of math so strongly. As she started advancing through higher education, this became evident. She attended the now non-existent Kirkland College, an exclusively women’s college in New York. She graduated from Kirkland College in 1977. It was the sister school of Hamilton College, and Hamilton eventually absorbed Kirkland in a school merger in the year after Flapan’s graduation. When she attended, Kirkland students could attend classes at Hamilton, and the mathematics major was only available as a Hamilton major, so she spent a great deal of time in Hamilton College’s math department. Flapan was the only woman in her graduating class with a major in math, and the only woman in many of her math classes.

While at Kirkland, Flapan wrote an undergraduate thesis in set theory, a subset of the larger field of logic. However, when she was in graduate school, she decided to pursue topology instead, and redirected her focuses to that field. She cited a few different factors when asked about this shift. First of all, Flapan considers herself a visual learner, and topology is much better suited to visual learning than logic is. She cited the difference in environment in the two fields as something that pushed her in the direction of topology:

> I was of course the only woman, and I found the atmosphere [in logic] among the students very unpleasant. In topology, for whatever reason, I find that topologists are more kind of informal and warm than other fields. It's a small thing but if you go to a topology conference, people are going to be less dressed up than if you go to another math conference. That represents some kind more, informal atmosphere. I got really attracted to topology because of the visual intuition and also the people in topology. By people I mean the other grad students. They were much nicer to me. People worked collaboratively together as compared to logic… where the people seemed competitive and unpleasant.

This preference for a more comfortable and casual environment seems to echo the discussion that Sapna Cheryan began in her discussion of the "geeky" classroom and the importance of environment and interpersonal relations to the success of female students.[35] Competition and increased competitiveness can mean increased hostility, which can heighten the feeling of not belonging. Perhaps women feel more comfortable in fields that are not hyper-professional and don't place as much of an emphasis on dress and formality.

Flapan had some strong examples of how women in math are often treated as "others" from both her undergraduate career and her experience as a graduate student.

[35] Cheryan, S., Plaut, V. C., Davies, P. G., & Steele, C. M. (2009). Ambient belonging: How stereotypical cues impact gender participation in computer science. Journal of Personality & Social Psychology, 97(6), 1045–1060.

Most notably, an interaction with an undergraduate professor in the late 1970s who made her feel unwelcome in the classroom. The professor, who was an older male gentleman, would refer to the class as a whole as “boys,” saying things like “now I know you boys would rather be outside playing football…” (Flapan 2017), despite Flapan’s presence in the class. Additionally, when Flapan got a bad grade on a test that brought her class average down to around a B, she went to speak with this professor one-on-one and he suggested she drop out of the course, saying that “she didn’t belong” (Flapan 2017). Of course, this professor did *not* say “you don’t belong because you are a woman,” but in combination with his unwillingness to change his vernacular in the classroom to include her, it is clear that he was uninterested in creating a welcoming environment for women in his classes. Those who study linguistics and discourses recognize how impactful it can be when we eliminate these uses of sexist language— “Language does, indeed, have the power to influence other parts of society; it can reinforce the status quo, or it can work to facilitate change. An awareness of sexist language is essential if we are to understand the traditional rules of interaction between women and men.”[36] Unfortunately, sexist language is entrenched in our culture. Even if professors are not explicitly referring to their students as “boys,” as in this case, there are still plenty of ways that gendered language slips into the classroom environment. For example, students and professors alike referring to gender-mixed groups as “you guys” is a part of our vernacular of English that subtly imposes maleness on the group. This is not unique to the field of mathematics, but can be a factor that alienates women in any discipline.

[36] Adams, Karen , and Norma Ware. "Sexism and the English Language: The Linguistic Implications of Being a Woman." Women: a feminist perspective (1995).

For graduate school, Flapan attended the University of Wisconsin, but spent a year studying and teaching during her graduate school years at the University of Texas at Austin. While at the University of Texas, she received a teaching award for teaching a section of Calculus. The undergraduate chair of Mathematics at the time called her into his office after she received the award. She assumed, logically, that he was going to congratulate her for her hard work. However, when she got there, she remembers—

> He said that he wanted to give me advice. He said "you're a great teacher, but I want to give you some advice… I think you should drop out of graduate school. You know, my wife never went to college, and she plays harpsichord, and she's very happy." Which seemed strange to me. And then he said "If you continue on in math, and get a PhD, no one's ever going to hire you. Because if you are married, then you'll have to spend something like a third of your time taking care of your husband, a third of your time taking care of your children, and then you'll only have a third of your time for your job. So people wouldn't want to hire someone like that. Or, if you don't get married, then you'll become neurotic, and no one will want to hire someone that's neurotic."

The chair is not only projecting the supposed aspirations of his wife onto Flapan, purely because she is another woman, but is directly suggesting that she halt her education. It brings back the archaic, but somehow persistent, argument that women are not qualified workers because their responsibilities at home somehow impede their ability to throw themselves into their work with the same dedication as men. To someone who has already experienced "I don't belong" feelings through their education, this sort of "advice" could be devastating. Furthermore, the use of the term "neurotic" suggests that women who deviate from the norm of heterosexual reproduction are somehow mentally unfit. As if it weren't enough to be questioning Flapan's role in the math department, the undergraduate chair continued—

> And then he said "and here in my department," which was a department that didn't have any women, "I have always voted against any woman hire, because of these reasons, and I will continue to do so."

Outraged at this, Flapan sought out the help of the graduate chair in mathematics. Instead of taking any actions against the comments made by the undergraduate chair, he asked Flapan if she was convinced by his comment about dropping out. When she replied that she of course was not, the graduate chair was satisfied, and said that there was no harm in him being the undergraduate chair because everyone knew he was crazy, and that his opinions were not swaying students' decisions to stay in the program. This of course ignored that he was in a position of power not only over female students who wanted to major in math and required his approval to do so, but also in the hiring process for new professors. It shows a much larger problem than the actions and direct sexism of the undergraduate chair. The graduate chair, when presented with this information, was complicit in institutionalizing sexism within the math department. This is an excellent example of how one individual's experience can reveal something larger about the state of women at an institution, and a chilling reminder to take reports of gender discrimination seriously and not trivialize someone's personal experience as an anomaly. These incidents occurred in the early 1980s, and it is somewhat difficult to imagine something as blatantly awful occurring without consequence today. However, this is not to say that women in math do not encounter professors or classmates with sexist attitudes, or that these people do not say anything harmful anymore. These attitudes manifest instead through microaggressions, which are defined by psychologist Derald Sue as being "the everyday verbal, nonverbal, and environmental slights, snubs, or insults, whether intentional or unintentional, which communicate hostile, derogatory, or negative

messages to target persons based solely upon their marginalized group membership”[37] An example of a microaggression in this context would be a professor suggesting to a female student that she would be a better fit in the math education program—it’s not inherently negative, or explicitly about gender, but indicates that they do not belong in the math department, and would be better off in what happens to be a more stereotypically feminine field.

When asked how she proceeded with both of these incidents at both the undergraduate and graduate levels, Flapan said that she did not further report either incident, and noted that this was one of the major differences between experiencing harassment and gender discrimination versus when she was in school—”There wasn’t an infrastructure. These concepts of discrimination didn’t even exist then.” Today, as we know, someone who experiences gender discrimination in the workplace or in an educational environment has the Title IX policy behind them should they wish to report any incidents. While Title IX was passed in 1972 and was in place in this time, public knowledge of all it could do for those who were harassed or discriminated against in education was not like it is today. As Flapan reflected, the infrastructure for reporting discrimination was either nonexistent or not accessible enough to be a viable option for victims. When they are well-executed, these sorts of policies can not only help to bring justice to the individual who was targeted, but also help to make sure that others are not targeted by the same individual in a similar way. In Flapan’s grad school experience, this could have helped prevent the undergraduate mathematics chair from discriminating

[37] Sue, Derald Wing. *"Microaggressions in everyday life: race, gender, and sexual orientation"*. Hoboken, NJ: John Wiley and Sons , 2010.

against other students outright, though it cannot stop all forms of discrimination, especially those that are more subtle and coded such as the aforementioned microaggressions.

Flapan attended the University of Wisconsin for graduate school in the late 1970s and early 1980s. She estimated that there were around a hundred math faculty members at this time, and among these hundred, only three were women. Of these three women, Flapan only had one as a professor—

> Of the three women, only one of them was teaching the introductory class, which I took with her. I really liked her, she seemed very encouraging and warm to me, and I sort of wanted to do my dissertation with her, but I wasn't that interested in her area. So I sort of debated whether to do my dissertation with her, even though I wasn't interested in it. But I didn't end up doing that. The other two I never took a class with because they were in different fields than me, so they never taught an upper level class in my specialty.

Female role models can greatly improve female students' relationship with the field of mathematics and help them through the often hostile and unwelcoming environment of higher education. With only 3% of a department being staffed by women, then there's a very low chance that female students will get the amount of time with female professors that they need to have any meaningful impact. Women deserve and need positive role models in their real lives just like they do in media—the "we can't be what we can't see" phenomenon. If all the professors they encounter in their field are men who are not relatable to them, regardless of how encouraging they may be, women will have a harder time visualizing themselves in the field. For some insight on the state of this today, Flapan reflected on the mathematics department at the university she works at, Pomona College, saying that of eleven tenured or tenure track professors, five are women. This is

much more promising than the situation at Wisconsin when Flapan attended, and perhaps shows some sort of progress, but not for all schools—Flapan says:

> And I think that's true of most small colleges. It's not true at the more research oriented schools. I think that's related to— there's this pipeline problem, but in the graduate level, in the very elite schools. So the number of women in graduate programs is about 30% I believe, but if you focus on MIT, Harvard, Princeton, places like that, the percent of women is low. And then if you're thinking about getting jobs at a research facility like that, they will typically only consider people who went to one of the top universities as a graduate student. So there's a pipeline issue there.

This information is important because it reminds us that progress in some places does not mean progress overall. One university with good representation and balance of genders in their tenured faculty does not mean that it is reflective of all universities. In this case, the universities that produce the nation's best thinkers and top mathematicians are not the ones who have high numbers of female faculty. This means that the few women in these elite programs do not have visible role models to encourage them along the way in the same way that those in smaller colleges do, and then the cycle continues to perpetuate itself—fewer female students, fewer female professors, less representation, repeat.

Reflecting overall on her early life, higher education, and beyond, Flapan acknowledges that today, women have a different experience with higher education. Largely today, women do not experience the overt and shocking sexist incidents that Flapan did in her undergraduate and graduate education, and are more plagued by microaggressions. However, she believes that women her age have had largely similar experiences to her own. When asked about the universality of her overall experience—

> I think it might be typical of people my age. I think the experience of my students is quite different than my experience. There aren't many women in math my age, but the ones that I know had relatively similar experiences. They had, like me, some negative experiences, but also some people who were encouraging, and they were relatively isolated in terms of their gender in their education.

Today, we can see progress in the inclusion of women in math education, and especially with the institutional responsibility to provide a harassment and discrimination-free learning environment. This does not mean that women today do not experience barriers and obstacles in their higher education, or that institutions have foolproof systems that eradicate discrimination completely, but rather that we have progressed significantly. This progress does not negate or overshadow the fact that there are still issues that impede women from thriving in the field of mathematics, like the previously discussed stereotype threat, self-selection phenomenon, microaggressions, and ingrained ideas of an incompatibility of math with femininity. This progress should not stop us from fighting for even better possibilities in the future for the coming generations of women in math and women in STEM.

There are a few other ways in which women in math can become more seamlessly integrated into the traditionally unwelcoming field. Flapan provided some insight into some of these other programs that went beyond traditional academic support. Originally, these services were constructed to help undergraduate women in mathematics thrive in the field. For fifteen years, she taught in a summer program for female freshman and sophomore math students from across the country funded by the NSF. There were classes taught by women and colloquia given by women. The classes provided the women an opportunity to bond and find a community within their academic world. Unfortunately, the program’s funding was cut in 2015. She identified a few more similar programs run out of other universities that lost funding, just like the program she once taught at. Flapan’s daughter personally attended one of these programs when she was an

undergraduate, and still enjoys the benefits as a PhD candidate in mathematics today. When she attends conferences in algebraic geometry, she still meets up with women whom she met in that undergraduate experience. It helped her to make what Flapan refers to as “a cohort of women” (Flapan). This kind of experience is invaluable. Unfortunately, there are few that remain active, and with more and more women entering the field of mathematics as years go on,[38] these programs become more competitive and less able to serve a significant proportion of female math students. This is perhaps another instance of institutions that make up our society “moving the finish line” just as those who have been disenfranchised are starting to become more active participants in traditionally unfriendly fields.

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[38] Science and Engineering Indicators 2012. (2012). *S&E Indicators 2012 - Chapter 2. Higher Education in Science and Engineering - Undergraduate Education, Enrollment, and Degrees in the United States*. National Science Foundation. https://www.nsf.gov/statistics/seind12/c2/c2s2.htm#s2. Accessed 6 March 2017.

Dr. Flapan's Research: DNA applications of knot theory

Flapan's work is an application of topology to molecular biology. It uses knot theory to understand parts of enzymology, the action of enzymes on DNA. Knot theory is a part of the mathematical field of topology, which is "the study of the properties of geometric objects that are preserved under deformations."[39] A knot is loosely defined in *The Knot Book* as "a closed curve in space that does not intersect itself anywhere."[39] A mathematical knot resembles, and can be intuitively thought of, just like any other knot—it's like a knotted string without any thickness. The image in Figure 1 is what we call a projection of a knot, which is simply how knots are represented visually.

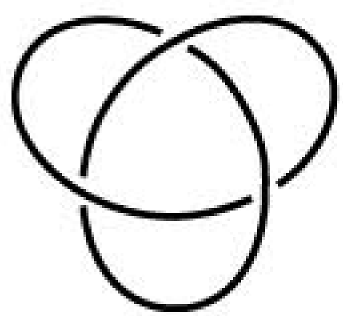

Figure 1: Trefoil Knot

Sometimes two knots are the same, although they may be presented differently—their projections may not appear to be identical. However, if you can manipulate one knot into another, then it is the same knot. For example, the two knots in Figure 2 are the same because you can untwist the second to make it into the first.

[39] Adams, Colin. *The Knot Book: An Elementary Introduction to the Mathematical Theory of Knots*. Providence, RI: American Mathematical Society, 2010.

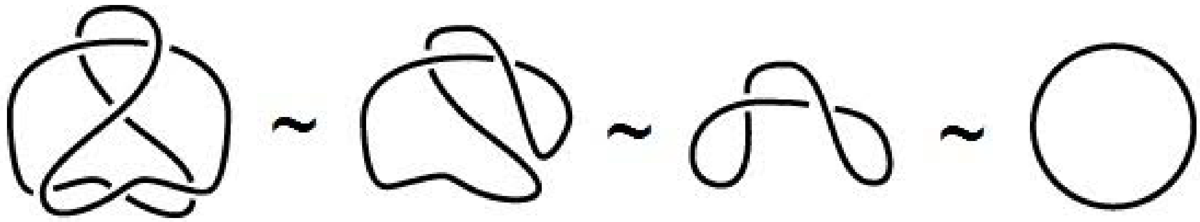

Figure 2: Representations of the Unknot

All the knots in Figure 2 are examples of the unknot, or the trivial knot, which is usually just presented as a circle. If two knots are the same, they are called equivalent. This intuitive idea will suffice and knot equivalence will not be formally defined.

The point of depicted overlap in these projections is called a crossing. Knots can be categorized by the number of crossings they have. For example, the knot in Figure 1 is a trefoil knot, and it has three crossings in that representation of the knot. A knot is called an *n*-crossing knot if *n* is the minimal number of crossings in any projection of it. In Figure 3 we show various knots with 0-7 crossings.

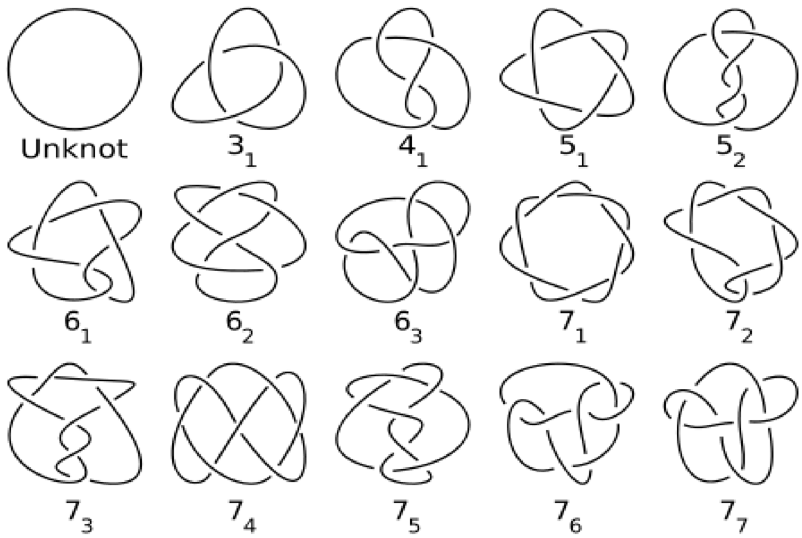

Figure 3: Knot Table [40]

A tangle, as explained in *The Knot Book* by Colin Adams, is a region in a knot or link projection in the plane that is surrounded by a circle in such a way that the link or the knot will cross the circle four times exactly.[41] A tangle diagram is the image created of a tangle when the tangle is surrounded by a circle with its four endpoints generally lining up with the compass points NW, NE, SE, and SW. Figure 4 shows an example of a tangle diagram.

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[40] "Knot Table ." Wikipedia . Accessed November 22, 2017. https://en.wikipedia.org/wiki/File:Knot_table.svg.

[41] Adams, Colin. *The Knot Book: An Elementary Introduction to the Mathematical Theory of Knots*. Providence, RI: American Mathematical Society, 2010.

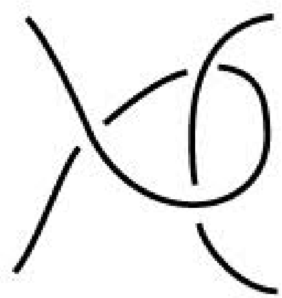


Figure 4: Tangle Diagram

Tangles are of interest because they provide a way we can understand and talk about knots. Tangles can be combined and manipulated to create and manipulate knots, and to code their structure. Knots can be built with local pieces that are tangles, that are combined to form the global structure of the knot. This connects to the applications to DNA because operations on DNA, like recombination, are local, and tangle theory helps us to understand global impacts of these processes.

An integer tangle (denoted $[n]$) is one of the possible components of a tangle. It is a horizontally oriented tangle of two stands that contains $|n|$ crossings, as illustrated in figure 5. The value of n can be positive or negative, depending on the orientation of the crossing in the diagram. Examples of integer tangles are shown in Figure 5.

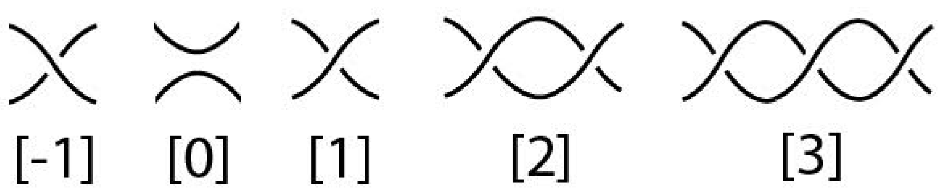


Figure 5: Integer Tangles

A reciprocal tangle (denoted $\frac{1}{[n]}$) is another type of tangle. It is a vertically oriented tangle of two strings that contains $|n|$ crossings, as illustrated in Figure 6.

Reciprocal tangles can also be positive or negative, again depending on the orientation of the crossing in the diagram.

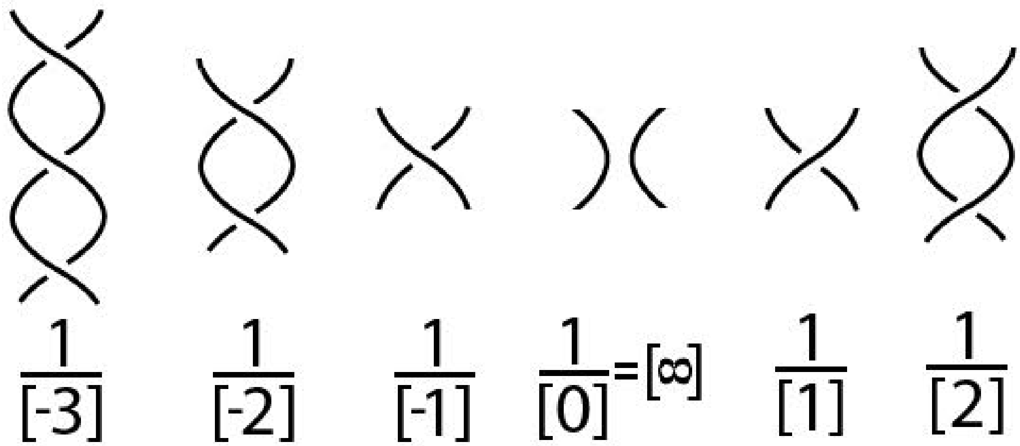


Figure 6: Reciprocal Tangles

Two tangles are equivalent if one can be deformed to the other while holding the four ends fixed. This intuitive idea will suffice and tangle equivalence will not be formally defined. The two tangles in Figure 7 are equivalent. Note that in the right hand tangle, the second and third crossings have the same strand on top, and therefore in that part of the tangle the top strand can be slid off the bottom strand, eliminating those two crossings and starting a deformation that results in the tangle on the left.

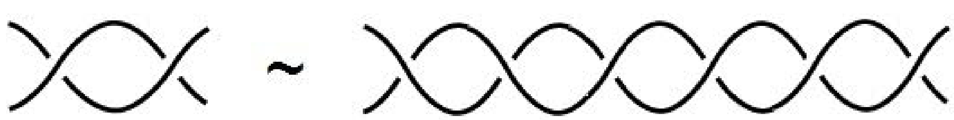


Figure 7: Tangle Equivalence

Two tangles can be added, resulting in a tangle sum. When tangles are added, they are aligned horizontally and their endpoints are connected. Figure 8 shows tangle addition, $T_1 + T_2$.

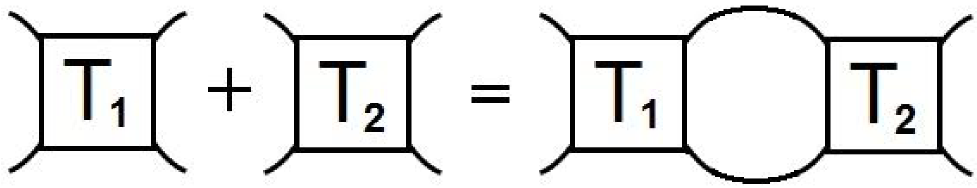


Figure 8: Tangle Addition

Two tangles can also be multiplied, resulting in a tangle product. When tangles are multiplied, they are aligned vertically and their endpoints are connected. Figure 9 shows tangle multiplication, $T_1 * T_2$.

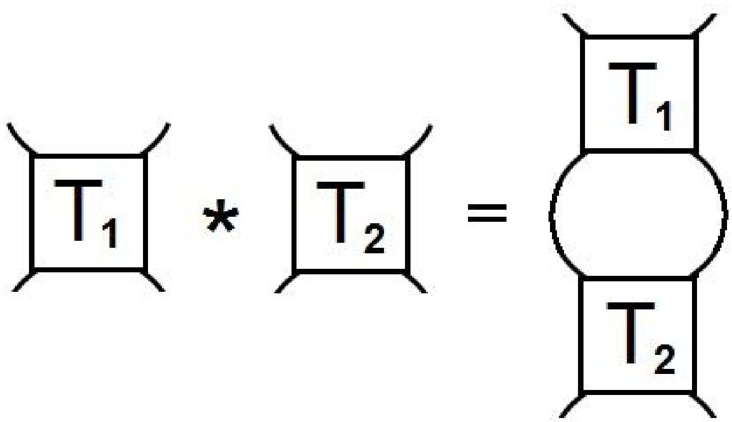


Figure 9: Tangle Product

Given a tangle Q, the mirror of Q, denoted $-Q$, a tangle is taken by switching all the crossings of Q. The inverse of Q, denoted Q^i, is the mirror of the clockwise rotation of Q by 90 degrees. Figure 10 shows the inverse of a tangle, and Figure 11 shows the mirror of a tangle.

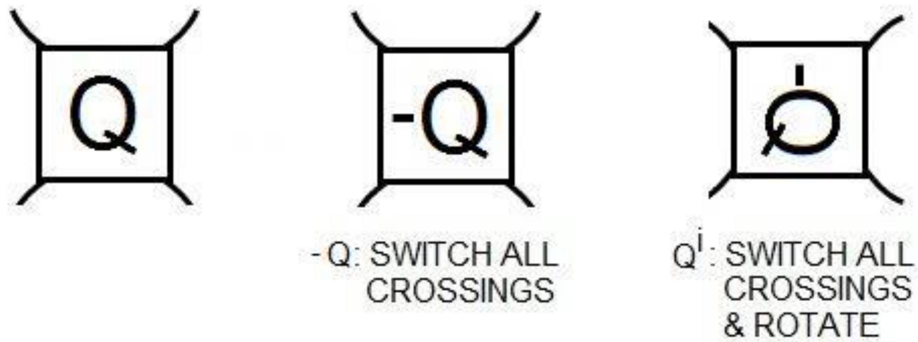


Figure 10: Inverse

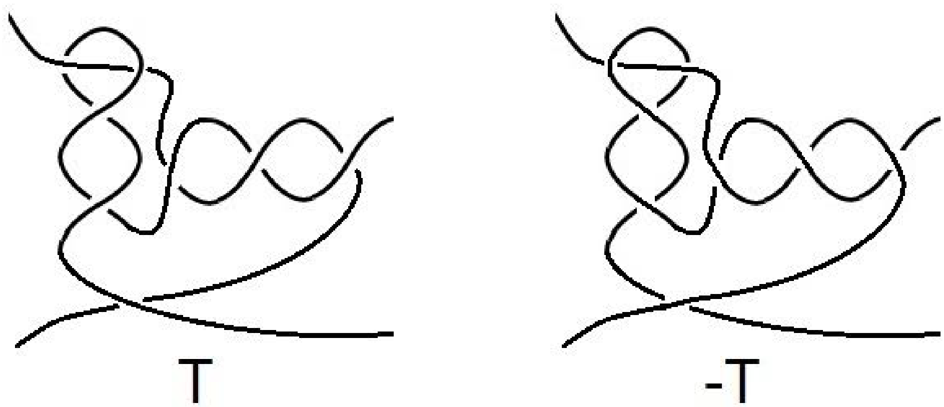


Figure 11: Mirror

Numerator closure and denominator closure are two operations that transform a tangle into a knot. The numerator closure of a tangle T is denoted $N(T)$ and is formed by connecting the end-strands of a tangle T as shown in Figure 12.

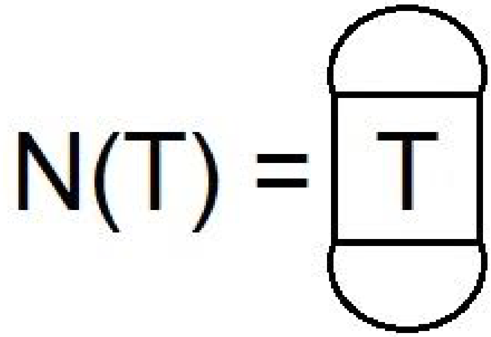


Figure 12: Numerator Closure

The denominator closure of a tangle T is denoted $D(T)$ and is formed by connecting the end-strands of a tangle T as shown in Figure 13.

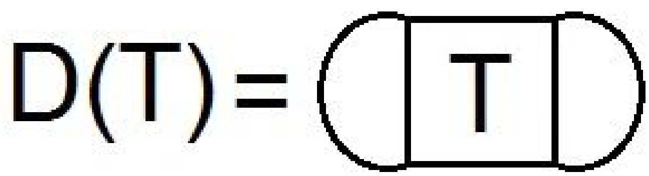


Figure 13: Denominator Closure

For example, the numerator closure of [3] results in the trefoil knot, as shown in Figure 14.

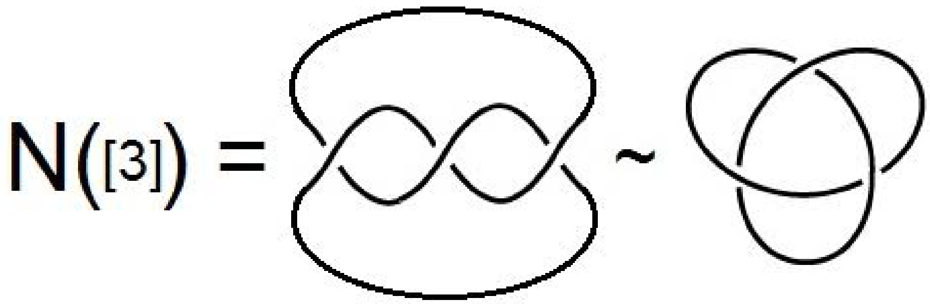
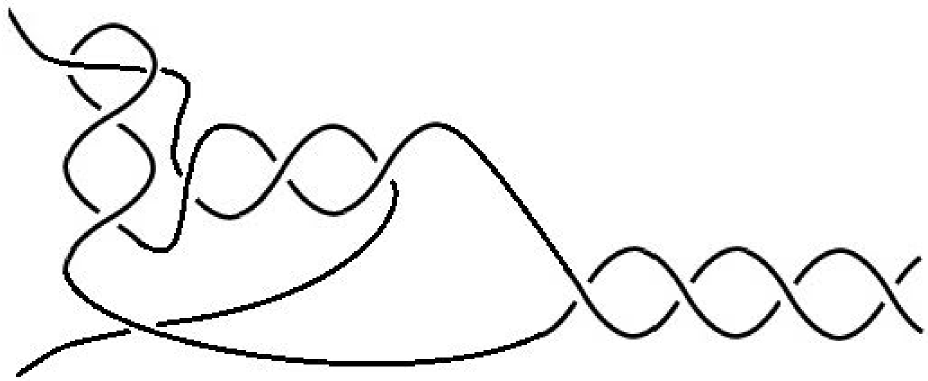


Figure 14: Closure of [3]

Rational tangles are tangles that are constructed recursively. Integer and reciprocal tangles are rational tangles that can be thought of as start pieces T_1 for a rational tangle T_n. If T_k is a rational tangle then rational tangle T_{k+1} is obtained by adding an integer tangle to T_k or multiplying T_k by a reciprocal tangle.

A twist diagram is a diagram of a rational tangle that shows the sequential parts of the tangle, that is the integer and reciprocal tangles that are combined to form it. By looking at a twist diagram you can break a tangle down into its integer and reciprocal parts and express the tangle as a product and sum of these parts. For example, the twist diagram in Figure 15 can be broken down and expressed in products and sums. We call this the products and sums notation.



$$\left(\left(\left([-2] * \frac{1}{[2]}\right) + [3]\right) * \frac{1}{[1]}\right) + [-4]$$

Figure 15: Products and Sums Notation

Tangles can be flipped both vertically and horizontally. The vertical flip of a tangle Q is taken by rotating the tangle 180° over a vertical axis, and is denoted Q^{Vflip}. The horizontal flip of a tangle Q is taken by rotating the tangle 180° over a horizontal axis, and is denoted Q^{Hflip}. Vertical and Horizontal Flips are shown in Figure 16.

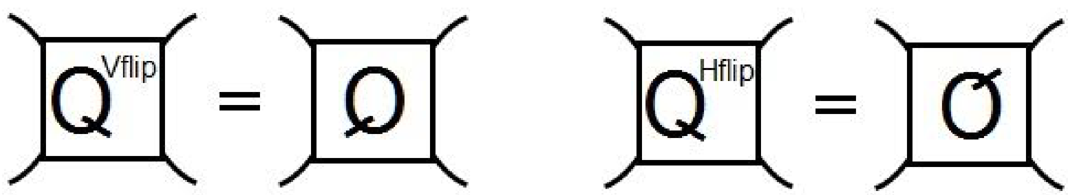


Figure 16: Vertical and Horizontal Flips

Note that every integer tangle is identical to its vertical flip and its horizontal flip, as illustrated in the Figure 17.

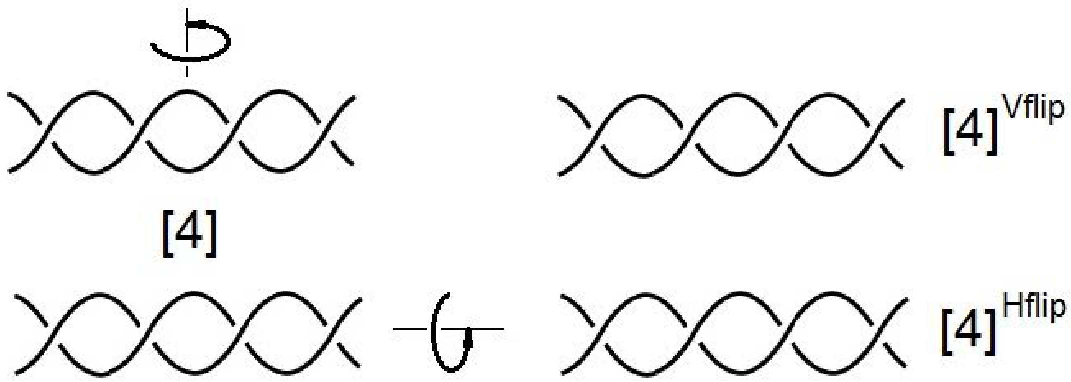


Figure 17: Horizontal and Vertical Flips of [4]

Clearly, the same holds true for reciprocal tangles. Remarkably, it turns out that all rational tangles are equivalent to their vertical and horizontal flips. This is illustrated in Figure 18.

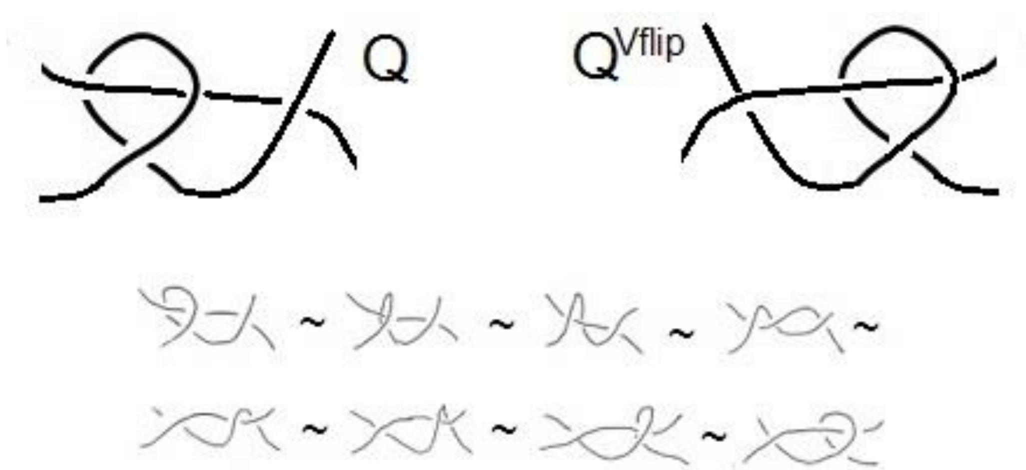


Figure 18: Flip Equivalence Example

The Flip Theorem: For a rational tangle Q, $Q \sim Q^{Vflip}$ and $Q \sim Q^{Hflip}$.

<u>Proof of the Flip Theorem:</u>

The Flip Theorem states $Q \sim Q^{Vflip}$ and $Q \sim Q^{Hflip}$, that for a rational tangle Q. We will prove this by induction on n, the number of crossings Q has.

Base case: $n = 1$

In this case we are considering vertical and horizontal flips of [1] or [-1]. We observed above that these tangles are equivalent to their flips.

Inductive step: We assume that $Q \sim Q^{Vflip}$ and $Q \sim Q^{Hflip}$ is true if Q has n crossings and prove it is true for tangles with $n + 1$ crossings.

A rational tangle P with n $+ 1$ crossings can be expressed as one of the following, where Q is a rational tangle with n crossings:

$Q + [1], Q + [-1], \; [1] + Q, \; [-1] + Q,$

$$Q * \frac{1}{[1]}, \; Q * \frac{1}{[-1]}, \; \frac{1}{[1]} * Q, \; \frac{1}{[-1]} * Q$$

We consider the case where $P = [1] + Q$. Each of the other cases follows similarly or via commutativity of tangle addition or tangle multiplication.

Note the vertical flip of $[1] + Q$ appears in Figure 19.

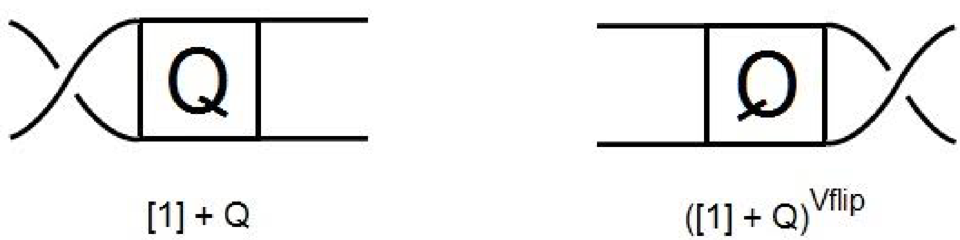


Figure 19: Flip Theorem Proof 1

We want to show these tangles are equivalent. First start with $[1] + Q$ and rotate in the horizontal axis, holding the four ends fixed. The left pair of strands untwists while the left

pair twists, as shown in Figure 20.

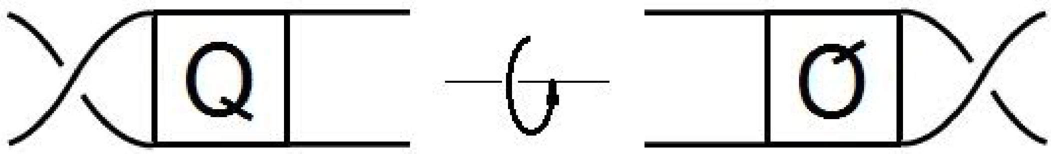


Figure 20: Flip Theorem Proof 2

By induction $Q{\sim}Q^{Vflip}$ and $Q^{Vflip}{\sim}(Q^{Vflip})^{Hflip}$ and therefore we have the sequence in Figure 21.

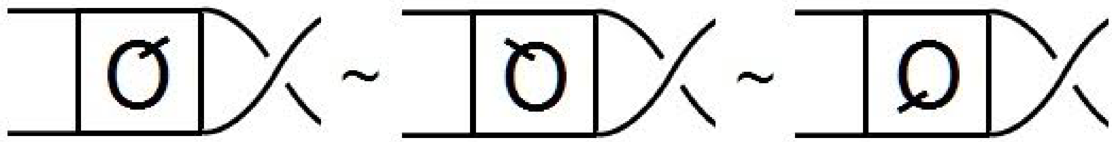


Figure 21: Flip Theorem Proof 3

The latter is the vertical flip of $[1] + Q$ demonstrating that with the inductive hypothesis, $[1] + Q{\sim}([1] + Q)^{Vflip}$. So $Q{\sim}Q^{Vflip}$.

Next we show that $[1] + Q{\sim}([1] + Q)^{Hflip}$.

Note the horizontal flip of $[1] + Q$ appears in Figure 22.

Figure 22: Flip Theorem Proof 4

We want to show these tangles are equivalent. First start with $[I] + Q$. By induction, $Q \sim Q^{Hflip}$ and therefore we have the image in Figure 23.

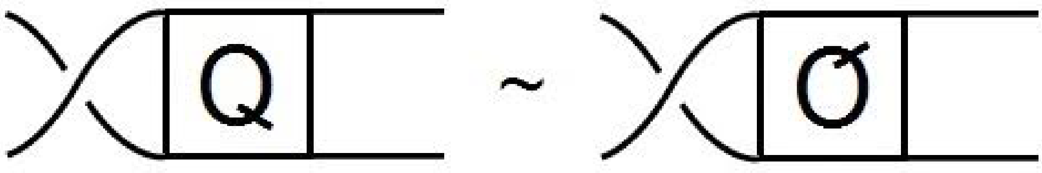


Figure 23: Flip Theorem Proof 5

Notice that the latter is the horizontal flip of $[I] + Q$demonstrating that with the inductive hypothesis, $[I] + Q \sim ([I] + Q)^{Hflip}$. Thus it follows that $Q \sim Q^{Hflip}$ and $Q \sim Q^{Vflip}$, then $[I] + Q \sim ([I] + Q)^{Hflip}$ and $[I] + Q \sim ([I] + Q)^{Vflip}$. A similar approach can be taken for the cases of $[1] + Q, \frac{1}{[1]} * Q$, and $\frac{1}{[-1]} * Q$.

Thus by induction on n, $Q \sim Q^{Vflip}$ and $Q \sim Q^{Hflip}$ as needed. □

A consequence of the flip theorem is the commutativity of tangle multiplication and tangle addition.

Theorem: $T_1 + T_2 = T_2 + T_1$ for a tangle T.

Proof: As modelled in Figure 24, Start with $T_1 + T_2$. Apply a vertical flip to $T_1 + T_2$. Then individually apply vertical flips to T_1 and T_2. This gives $T_2 + T_1$, as needed. □

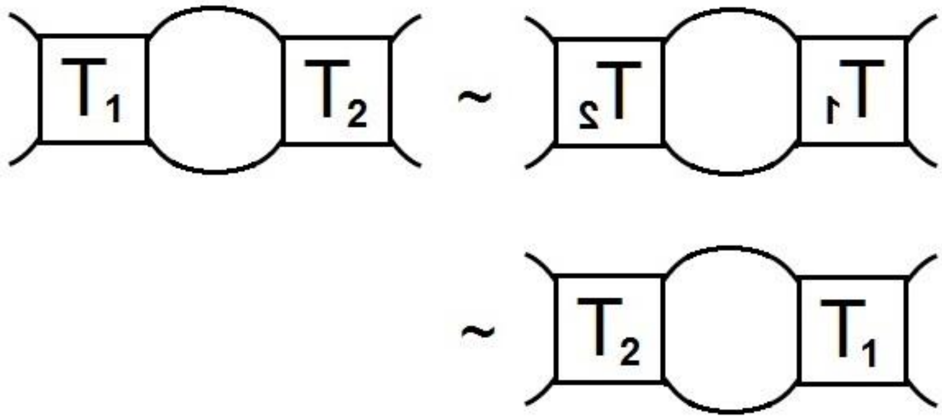


Figure 24: Commutativity of Addition

The Product-to-Inverse Equivalence is a theorem that allows us to take the product and sums notation and turn it into an alternate continued fraction notation.

The Product-to-Inverse Equivalence Theorem: For a rational tangle, P, and $a \in \boldsymbol{Z}$,

$P * \frac{1}{[a]} \sim \frac{1}{[a]+\frac{1}{P}}$ and $\frac{1}{[a]} * P \sim \frac{1}{\frac{1}{P}+[a]}$.

Proof:

Start with $P * \frac{1}{[a]} \sim \frac{1}{[a]+\frac{1}{P}}$. These are shown in Figure 25.

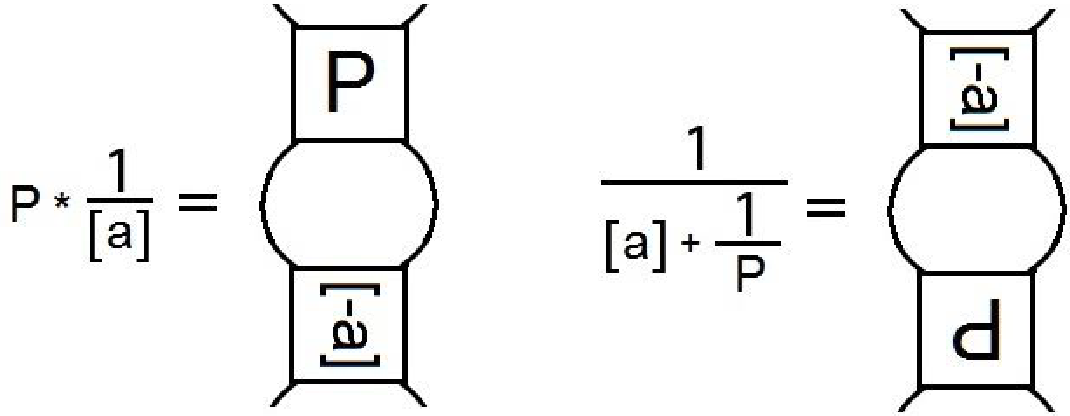


Figure 25: Product to Inverse Equivalence Proof 1

We must show that these are equal in order to prove the Product-to-Inverse Equivalence.

Start with $P * \frac{1}{[a]}$ and apply a horizontal flip.

Since $[a]$ is an integer tangle, it does not change when flipped, so we can horizontally flip *[a]*. Then apply a vertical flip, and we are given $\frac{1}{[a]+\frac{1}{P}}$. These actions can be seen in Figure 26.

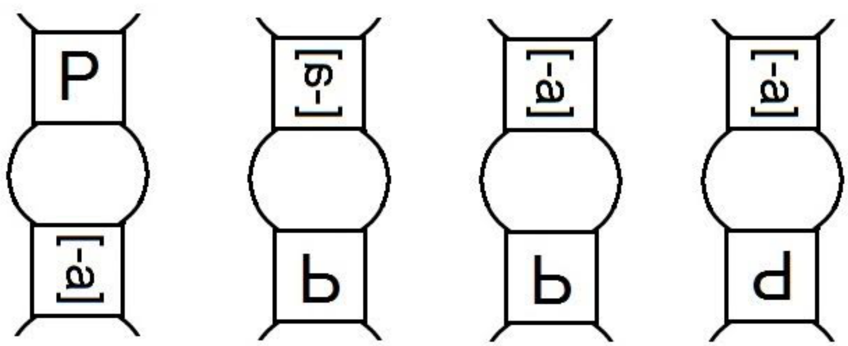


Figure 26: Product to Inverse Equivalence Proof 2

Thus $P * \frac{1}{[a]} \sim \frac{1}{[a]+\frac{1}{P}}$ as needed, and by commutativity of multiplication, $\frac{1}{[a]} * P \sim \frac{1}{\frac{1}{P}+[a]}$ as well. $\square$

In order to use this to turn products and sums into continued fractions, the products are taken and rewritten with the inverse expressions as in the theorem.

The continued fraction form of rational tangles is in the form

$[a_n] + \frac{1}{[a_{n-1}]+\ldots+\frac{1}{[a_{n-2}]+\frac{1}{[n_1]}}}$. All the $a_i \in \boldsymbol{Z} - \{\boldsymbol{0}\}$ except a_n which can be zero.

It is always possible to translate products and sums form to continued fraction form—this is demonstrated through the following example:

Take the tangle in product and sums notation:

$$— [5] + ([3] + (\frac{1}{[2]} * [6] * \frac{1}{[10]}) + [-8]))$$

$$[5] + ([3] + (\frac{1}{[2]} * [6] * \frac{1}{[10]}) + [-8]))\sim[5] + \frac{1}{[4]+\frac{1}{([3]+(\frac{1}{[2]}*[6]*\frac{1}{[10]})+[-8]}}$$

$$\sim[5] + \frac{1}{[4]+\frac{1}{([-5]+(\frac{1}{[2]}*[6]*\frac{1}{[10]})}}$$

$$\sim[5] + \frac{1}{[4]+\frac{1}{([-5]+([6]*\frac{1}{[12]})}}$$

$\sim[5] + \frac{1}{[4]+\frac{1}{([-5]+\frac{1}{[12]+\frac{1}{[6]}})}}$ which is in continued fraction form.

If we take the numerical version of continued fraction form, we obtain a simplified fraction. For example, the corresponding fraction for $[5] + \frac{1}{[4]+\frac{1}{\left([-5]+\frac{1}{[12]+\frac{1}{[6]}}\right)}}$ is $\frac{7174}{1363}$. This value that the continued fraction reduces to is called the fraction of the tangle. In 1970

John Conway proved the significant result that two rational tangles are equivalent if and only if their fractions are equivalent.[42]

Tangle equations are important in the application of topology to DNA. Tangle equations are equations involving and relating the numerator closure, denominator closure, and known and unknown tangles and knots. An example of a tangle equation is shown in Figure 27.

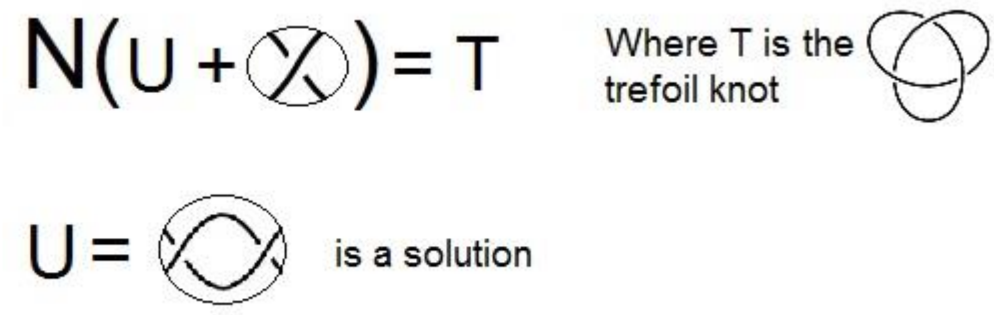


Figure 27: Tangle Equation

It is simple to observe that *U*=[2] is a solution to this tangle equation, but determining all of the solutions to a tangle equation or that this is the only solution to this specific equation is a nontrivial topological task.

An important part of the work of Erica Flapan and others modeling DNA involves solving tangle equations. Erica Flapan has written multiple papers on applications of knots and tangles to molecular biology. The one that most guided this project was a summary written with Dorothy Buck, *Summary of A Model of DNA Knotting and Linking*, which is an overview of results from other papers Flapan and Buck published on the same topics. This article discusses enzymes, catalysts that aid in the performance of

[42] Johnson, Inga, and Allison K. Henrich. An Interactive Introduction to Knot Theory. New York, NY: Dover Publications, 2017.

recombination. This process occurs on the molecular level in DNA, and is a transformation that help organisms sustain life. Buck and Flapan's research uses knot and tangle theory to understand recombination.

Site specific recombination is the action of the enzyme recombinase on duplex DNA. Duplex DNA is two strands of linear sugar and phosphorus. The four bases of DNA, adenine, thymine, cytosine, and guanine, attach to the sugar. Adenine bonds with thymine, and cytosine bonds with guanine. This bonding is what forms the familiar ladder-like structure of DNA. Recombinases act on short segments of DNA, which usually only have about 10-15 base pairs of adenine/thymine and cytosine/guanine. During recombination, two of these sites first become very close together, and then are bound to each other by the recombinase. This step is called synapsis, and the result of these sites bonding is called the local synaptic complex. After synapsis, the recombinase enzyme breaks both strands of the DNA at the sites, and then recombines them as illustrated in Figure 28. We can view this process as eliminating or creating a crossing in a diagram of a knot.

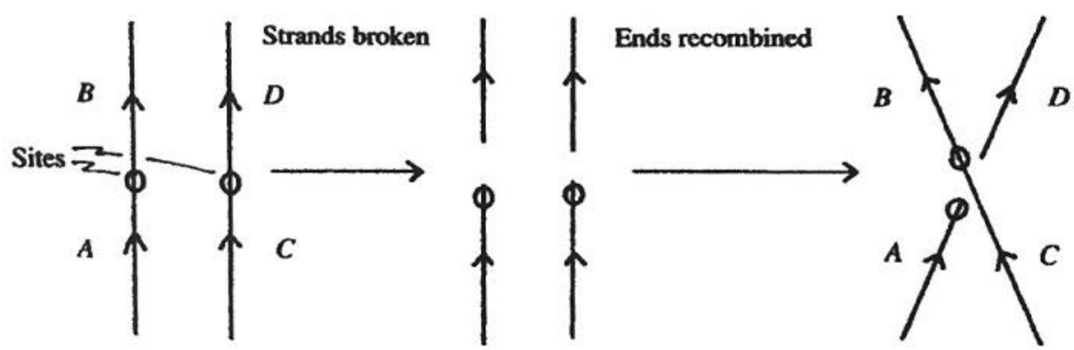


Figure 28: Site Specific Recombination[43]

[43] Ernst, C., and D. W. Sumners. "A Calculus for Rational Tangles: Applications to DNA Recombination." *Mathematical Proceedings of the Cambridge Philosophical Society* 108, no. 03 (1990): 489. doi:10.1017/s0305004100069383.

After this recombination, the enzyme lets go of the sites and the process is complete. The result of the process is called the product. Site specific recombination on a substrate involves a global move and a local move. The global move involves the bonding of the sites to the recombinase enzyme. Once these processes are observed, they can be modeled topologically; the components of the local synaptic complex are represented as an enzyme ball with a two-string DNA tangle in it. It is assumed that the entire process of recombination takes place within the enzyme ball, and that everything outside of the enzyme ball is unaffected by recombination.

We can model these smaller local operations on circular DNA by using tangles, and use these models and how they are manipulated to infer global results about DNA, just as we use tangles and tangle operations locally to understand knots globally. Because the recombination process involves two strands, the model that best demonstrates recombination is a two-string tangle. When the enzyme acts upon a two-string tangle, it essentially replaces the tangle with another tangle. Modeling this process involves tangle equations—the primary task in investigating the impact of recombination on DNA is to solve tangle equations modelling the process. Finding complete solutions to tangle equations is not a simple task and involves sophisticated topological tools.

In their article *Outline of A Calculus for Rational Tangles: Applications to DNA Recombination*s, Ernst and Sumners outlined their models for these tangles focusing on the specific resolvase (or enzyme) Tn3. They describe the recombination in the following way:

The enzyme divides the substrate into two tangles, which we call O and T, where O is the outside tangle, and T is the site tangle. The outside tangle is the part of the tangle unaffected by the recombination process, and the site tangle is the part that is impacted by recombination. Recombination then is effectively the cutting and pasting of tangles. It replaces $T$with a new tangle, R called the recombinant tangle that is the local synaptic complex, post-recombination. This process can also be represented in equations—

$N(O + T) = substrate,$ the substrate equation, and

$N(O + R) = product,$ the product equation.

This shows that the outside tangle, O, remains unchanged while T, the site tangle, is replaced with R, the recombinant tangle. This gives two equations with three unknown variables, so the furthest we can generally get to solving these equations is by solving for a variable in terms of other variables.

Sometimes recombination occurs multiple times—for example, in processive recombination, where "multiple rounds of recombination occur at a single binding encounter between DNA and enzyme, after which the DNA is released and undergoes no more recombination" (Ernst and Sumners). Processive recombination can be modeled through tangle addition, where each additional recombination adds a copy of R, the recombinant tangle.

Ernst and Sumners proved that in experimentation, through the first two rounds of processive recombination, the tangle equations have four solutions for O and R. They also proved that a third round of processive recombination can eliminate three of these solutions, and leave the solution that is assumed to be biologically valid.

Flapan and Buck's work goes beyond Ernst and Sumners', extending to other substrates. Their model is an attempt to predict which knots and links can be the products of site-specific recombination overall. They look at both instances of a singular recombination and processive recombination. They start with a substrate that is either an unknot, an unlink, or a T(2,m) torus, shown in Figure 29. The inclusion of the unlink and T(2,m) is what makes their work unique—it expands the work of Ernst and Sumners to allow for the inclusion of more substrates with more sites for recombination than the unknot—the unlink has the possibility to have a site on each component.[44]

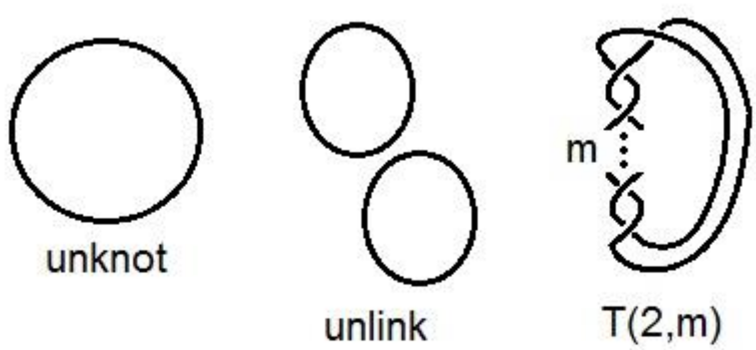


Figure 29: Unknot, Unlink, Torus

The sites where recombination takes place are called crossover sites by Flapan and Buck. Crossover sites are where where two molecules of the site specific recombinase bind to each of two specific DNA sites. There are two forms of site-specific recombinases, the serine recombinases, and the tyrosine recombinases.

[44] Flapan, E. and Dorothy Buck. A topological characterization of knots and links arising from site-specific recombination. In Journal of Physics A: Mathematical And Theoretical 40 (2007).

Serine recombinase performs recombination through what is known as “subunit exchange mechanism.” This process has the recombinase make two double-stranded breaks in the sites. It then rotates the opposite sites together inside of the productive synapse and reseals the ends to the opposite partners. When there is processive recombination, the individual instances of recombination are all the same. After recombination catalyzed by a tyrosine recombinase, the projection of the crossover sites has at most one crossing between the sites at the most, and there are no crossings within single sites.

Flapan and Buck found that all the knotted or linked products that arise as a result of using the unknot, unlink, or T(2,m) fall into one specific family of knots and links, shown in Figure 30.

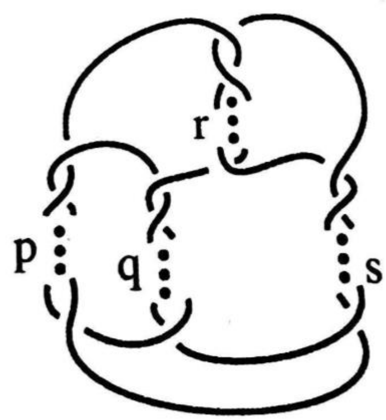


Figure 30: Family of Products[45]

Beyond this, there are subfamilies that the products of recombination with these substrates can fall into, as illustrated in Figure 31.

[45] Flapan, Erica, and Dorothy Buck. "A Model of DNA Knotting and Linking ." July 2008. Accessed April 22, 2017.

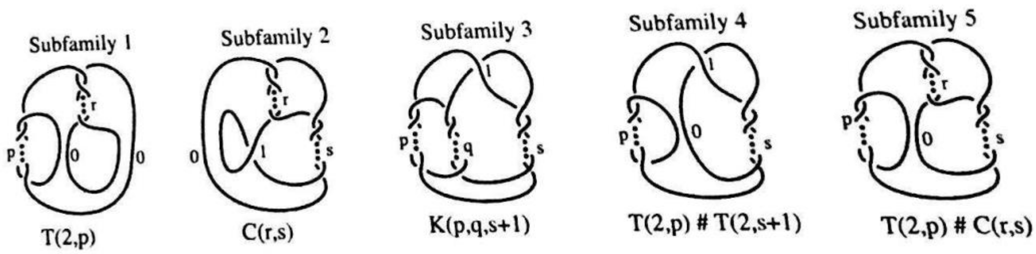


Figure 31: Subfamilies of Products[45]

All of these topological findings are important because they tell us more about recombination, and the enzymes involved in the process. Experimenters also found when doing their work that all the products of site-specific recombination fall into certain categories of knots. However, Flapan and Buck were able to narrow down the possibilities with the results of their topological manipulations. For example, experimenters such as Bath et al. found certain 7-crossings and 9-crossings as a result of their studies.[55] Flapan and Buck's work made it possible to narrow down the 56 possibilities for these knots to six possibilities.

Conclusion

An intriguing thought is brought up in an article about representation of mathematics in popular culture, *Nerds? Or Nuts? Pop culture portrayals of mathematicians* by Wilson and Latterell:

> When you meet a mathematician who fails to fit the stereotype as here presented, do you revise your image of mathematicians, or simply assume that the person is an exception to the rule?[46]

In this case, does our society allow women to be mathematicians by including them in our image of what a mathematician is, or do we count them as outliers? The contributions to mathematics by women like Erica Flapan are many, but the barriers to entry like stereotype threat, and sexist microaggressions still exist despite policy and law put in place to counteract them. Despite these, Flapan has excelled in mathematics. In addition to her paper on knot theory and DNA that was explored in this work, she has written textbooks, such as *When Topology Meets Chemistry*, received grants, and done research on many other topics. She persisted through the things that were in place to stop her—through her undergraduate professor's exclusion and the direct suggestion to drop out. Had she not been so persistent, all of these incredible contributions would be lost. It is sad, though also valuable, to think about the women who do not make it as far—what knowledge have we lost because of the aforementioned barriers? In order to fully integrate women into the fields that have traditionally been inhabited by men, our current

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[46] Wilson, Janelle L., and Carmen M. Latterell. "NERDS? OR NUTS? POP CULTURE PORTRAYALS OF MATHEMATICIANS." *ETC: A Review of General Semantics*, vol. 58, no. 2, 2001, pp. 172–178., www.jstor.org/stable/42578095.

policies are insufficient. This kind of large scale change requires not only structural and legal protections but also a shift in culture, as Wilson and Latterell suggest. As a society, we must stop considering women as the exception, and start reconciling femininity and women with mathematics.

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Appendix A: Interview Transcript

LL: Okay, so just getting started, can you tell me a little about your early education?

EF: So basically, our school, we didn't have any curriculum, we didn't have any grades, any exams. We could basically do what we wanted. There was a small amount of supervision like we were supposed to learn to read via a self study thing they had in first grade, and some basic math, and I think in maybe third grade they taught us cursive. Basically, most of the day, most every day was free, and then they would use us when somebody wanted to do some research. They would send home a note and make sure it was okay with our parents. And then we'd participate in whatever project they wanted. And these were never like, unpleasant, they weren't giving us electric shocks or anything like that. I can give you an example—when I was in first grade, I participated in something where they taught us the words "ascending" and "descending" instead of going up and going down, and then they showed us flash cards really fast, and we had to say whether it was ascending or descending. I'm not sure what they were testing exactly, how quickly you could learn a new fancy word or something, that's an example. Nothing unpleasant. But what ended up happening was that I just enjoyed doing math so I spent most of my time working through the math books they had, and didn't spend much time doing other things. Except, I did learn to read and stuff. But what happened after that, was that school ended after 6th grade, and I went to a regular junior high school and I really didn't know anything outside of math, so it was a very difficult transition. I didn't know anything about American history, geography, science, spelling, grammar. And I never sat through a lecture, or took notes, or took an exam or did homework. None of

these things. So it was quite traumatic, and I felt very unconfident in everything but math. By contrast in math, I basically skipped into the high school in terms of math. It was a feeling that I never recovered from of not knowing what was going on in my other subjects. Basically from 7th grade on, I felt inadequate in those other areas. For that reason, I felt like I didn't have any choice—I had to go into math which at that age, meant be a math teacher. But later on, it morphed into being a college math professor. So that was important, in terms of that decision. And I think that it was important later on because I felt like I didn't have any other options. So when things got harder in math, I still felt like I had to do them. I didn't feel like I could switch and do something else. Now, I realize later in my life that I really enjoy writing. Of course now, it's writing about math but certainly at the time, I felt like I couldn't write an essay because I didn't know how. So all of this is the background to how I ended up in math. Let's see. Your other question was about school.For various family reasons, like, I did not have a good family life, I wanted to leave home and go to college after three years in high school, and so I did that by sort of doubling up on some courses but the problem was that I was starting college when I was just turning 17. Which meant that I was still a minor, which meant that a lot of schools weren't open to me. And I also didn't have any advice about where to go, anything.There was a girl in my apartment building who went to where I ended up going. So I ended up at Kirkland College, which doesn't exist anymore. It was the Hamilton college sister school. It's located in upstate New York, now it's co-ed. And I was in the last graduating class of Kirkland. It was only around for a very short time. I believe the first class was in 1968, and I graduated in 1977. A few days before my graduation, they announced that Kirkland was out of money and would be merged with

Hamilton, so Hamilton would be co-ed. But in any case, I didn't really go there for any clear reason. There was that girl that went there, and they were willing to consider someone who was not over 18, and I got in, and I went there. I didn't really understand what I was getting into, in the sense that Kirkland was designed to complement Hamilton. The idea was that you could take classes at both schools and so the idea was that anything offered at Hamilton wouldn't be offered at Kirkland. So, at Kirkland they offered computer science because that was a new field and it wasn't offered at Hamilton, so there wasn't any math at Kirkland, it was just offered at Hamilton. What this meant, actually, which I really didn't think about because I had no guidance, was that there were almost no students at Kirkland that wanted to major in a Hamilton subject. There were almost like no other girls in math. And of course, there were already few girls in math because of the time that it was in. But I think that may have been more so at a school that didn't offer a math major. I mean it wasn't a problem, I was allowed to major in math at Hamilton, and I did their requirements, and I think that was why there weren't more girls than there were. I was the only girl in my graduating class that majored in math, and I was the only girl in most of my math classes. There was one girl a year older than me, and a girl a year younger than me. So that explains where I went to college. And for graduate school, that's also sort of a weird story. So I went to the University of Wisconsin, which is a good school. I had actually been admitted to Berkeley, which was a better school. However, basically because of a boy, in the opposite direction than you might think, I was breaking up with a boy, and he was admitted to Berkeley. He was in computer science. At that time Wisconsin didn't have a good computer science department. And so in particular he couldn't go to Wisconsin. I was having a hard time

breaking up with him. That is to say I kept breaking up with him and he kept pressuring me to go back, and so it seemed like a bad idea to go to grad school with him. Which in hindsight, maybe I shouldn't have done that, maybe I should have gone to Berkeley, but at the time it seemed like a reasonable thing to do to avoid him. So that explains how I went to that school. And that might be the end of your question, I don't remember.

LL: Yeah, yeah, definitely. Super interesting. So the next question I have kind of delves into your professional life. So I have preliminarily looked at a little bit of your research in DNA and knot theory, and I was wondering if you've perceived a difference in the way that your research has been received by male or female colleagues.

EF: I have not really noticed that, no. Definitely when I was a student I was not treated the same as my male counterparts, but in terms of my research, I haven't noticed that.

LL: Okay. Can you maybe talk a little bit about not feeling like you were treated equally in school?

EF: Sure. So for example, when I was in college, I took a geometry course where I was again the only girl in the class. The teacher was about to retire, he was a lot older, and he would always refer to us as boys, and he would say things like "now, I know you boys want to be out there playing football, but you need to..." whatever. Like he would talk to the class like that. And then I did not do well on the first exam. I actually have had my whole life, exam anxiety. I think that was the result of not taking any exams until I was in

7th grade and then doing badly on exams other than math. Math was different, because I was so accelerated that I didn't have to worry, but still I didn't really know how to study to take an exam. So then when I went to college, and it was more challenging math-wise, the fact that I didn't know how to study meant that I fell into the same exam anxiety that I had in my other subjects. In any case, there was the one exam that I didn't do well on, in geometry, and I went to talk to the professor, and he told me that I didn't belong in the class, and that I should drop. I mean, like I was getting a B in the class, it wasn't like I was getting an F. So that was an example of that. When I was in graduate school, there was a particular professor that I wanted to do my dissertation with. He would not take me as a student. He didn't say that it was because I was a woman, but he said that he didn't have time. Shortly after that, he took a man as a student that was in my class, who had not yet passed his qualifiers while I had already passed my qualifiers with honors, and I felt that it was because I was a woman. So those are examples of that. Keeping in mind, again, that all through graduate school, there were no other women in my class. In undergrad, I think if I went to another school, there would have been more than zero other women in my class, whereas graduate school at the time, very few women were in PhD programs at math. So it might have been that no matter where I went to grad school, I would've been the only woman in my class.

LL: Did you have many professors that were women, or any at all?

EF: At Wisconsin when I was there, there were a hundred faculty members, three of whom were women, and I took a class with one of those women. The thing was with grad

school, you had to take like standard classes, and then after that you take classes that are in your subspecialty. Of the three women, only one of them was teaching the introductory class, which I took with her. I really liked her, she seemed very encouraging and warm to me, and I sort of wanted to do my dissertation with her, but I wasn't that interested in her area. So I sort of debated whether to do my dissertation with her, even though I wasn't interested in it. But I didn't end up doing that. The other two I never took a class with because they were in different fields than me, so they never taught an upper level class in my specialty.

LL: Just keeping with the theme of your research and all of that, do you have a specific reason or do you know what caused you to go more into topology and knot theory?

EF: I would say that I'm a very visual thinker, which I don't really have any proof of this, but I think it's some kind of genetic thing. And this is somewhat based on my students, because some of them *hate* topology, because they feel like "how do you know it's true, you're just relying on this intuitive visualization." And this is students who go to grad school. Whereas other people just love topology because it has this very strong visual component. When I was in undergraduate, I did an undergraduate senior thesis in set theory, and when I went to grad school, I went with the idea that I would do my thesis in set theory. And we had a really great group at Wisconsin of people who researched set theory. However, the other graduate students in the field were quite competitive. I was of course the only woman, and I found the atmosphere among the students very unpleasant. In topology, for whatever reason, I find that topologists are more kind of informal and

warm than other fields. It’s a small thing but if you go to a topology conference, people are going to be less dressed up than if you go to another math conference. That represents some kind more, informal atmosphere. I got really attracted to topology because of the visual intuition and also the people in topology. By people I mean the other grad students. They were much nicer to me. People worked collaboratively together as compared to logic (set theory is a subfield of logic) where the people seemed competitive and unpleasant.

LL: So you spoke a little bit about your one female professor being warm and encouraging you, what about your other professors and teachers, or even your family and friends? Was there a lot of encouragement, and people that were supporting you?

EF: No, probably not. Certainly not in my family. Like I said, I had a dysfunctional, not good family situation. In elementary school, the teachers didn’t really pay attention to us, so no one particularly encouraged me in math. When I was in middle school, I was taking a class with someone who I thought was elderly, she probably wasn’t, but anyway, she did have grey hair, and she was very encouraging and very sweet. I believe I took two courses with her, and she was very encouraging. My other high school math teachers, who were young men, were not particularly encouraging, and seemed possibly to be intimidated by me. When I was in college, there was a female professor that was hired only in my junior year I think. But there were also two male professors who were very encouraging and helpful to me. Needless to say, not the guy that taught the geometry class. The guy that taught the geometry class retired after my first year in college so I

never really encountered him again. But none of my teachers in college were discouraging, at all, so the one who supervised my senior thesis happened to be a man, because he was the set theorist, but there was a woman in the department who I took possibly two classes with, and she was very nice, and now she's retired, and I've actually seen her over the years at meetings and conferences, and she's been very wonderful. You know, telling me she was so proud of me, when I was giving talks. The other male professors, there was one who I took multiple classes with, he had gone to Wisconsin himself and he thought I should go there. I didn't tell him about the boy thing. The other one who I did my thesis with was nice and encouraging. In graduate school, there wasn't anybody except that woman who I took the one class with. But when I ran into her in the hallway or whatever, she would always be very nice.

LL: I kind of wanted to ask- did you find yourself consciously thinking about gender in the classroom when you were the only woman in these math classes? Was it something that was frequently on your mind, or was it more like something that was just kind of an accepted fact of life?

EF: I think for the most part it was an accepted fact of life. I mean it definitely annoyed me when that geometry professor would refer to us as boys, but it didn't really seem that strange to me. Like in my high school I was maybe one of two girls in my calculus class, just something like that. Already, at that point, in advanced math in high school, there weren't as many girls. I don't think I really thought about it, and most of my teachers didn't do anything that made me feel like I was being singled out because I was a woman.

LL: When you experienced that remark from your geometry teacher, and he said you didn't belong in the class, did you report that incident or was there even an infrastructure in place for you to report that?

EF: No. There wasn't an infrastructure. These concepts of discrimination didn't even exist then. It just occurred to me that I have another story, which happened when I was in graduate school. Although it was not at Wisconsin. There was a year when I was at a different school. So I was a teaching assistant, and I taught my own calculus class, and I won a teaching award. And there was a guy who was the undergraduate chair. Most grad programs, there's the overall chair, the graduate chair, and the undergraduate chair. Anyway, he asked me to come to his office, and I thought he asked me to his office because I had just won this award and he was going to say congratulations or something, and I went to his office and he said that he wanted to give me advice. He said "you're a great teacher, but I want to give you some advice." He said "I think you should drop out of graduate school. You know, my wife never went to college, and she plays harpsichord, and she's very happy." Which seemed strange to me. And then he said "If you continue on in math, and get a PhD, no one's ever going to hire you. Because if you are married, then you'll have to spend something like a third of your time taking care of your husband, a third of your time taking care of your children, and then you'll only have a third of your time for your job. So people wouldn't want to hire someone like that. Or, if you don't get married, then you'll become neurotic, and no one will want to hire someone that's neurotic." And then he said "and here in my department," which was a department that

didn't have any women, "I have always voted against any woman hire, because of these reasons, and I will continue to do so." and I was very upset. Like I was mad, that he said this to me. And I immediately went to the office of the graduate chair. And I told him what happened, and I said to him that I was worried about it because this guy is the undergraduate chair, it means all the undergraduate math majors have to go to him to get them to sign off on their program—if he thinks women shouldn't be majoring in math, this is terrible! And the graduate chair said to me, "are you going to drop out of graduate school because he told you to?" And I said, "No, of course not, he's crazy." And he said, "See? There's no danger in him being the undergraduate chair because everyone knows he's crazy." And that was the end of that. And I didn't see anyone else I could go to. There was the chair of the department, but I wasn't going to go to him. There wasn't any office or whatever to make a complaint to, so that was the end of that incident. And like I said, I was only there for a year, so then I went back to Wisconsin, and I didn't see that guy again.

LL: Wow, okay. Good. Now I kind of want to ask you about your life as a professor. So I know you said that you were impacted by the female professor that you had in grad school. Do you strive to encourage the women that go through your mathematics program at your college to continue on in mathematics?

EF: I don't treat the women differently from the men—I try to encourage everybody, and whatever, however recently, I've started to stop encouraging people to go to grad school in math because the job market is really bad. I have a number of my former students who

have gone to grad school who keep getting one year jobs here, one year jobs there, and it's a horrible life. A lot of times, math faculty don't think about that when they're advising students. They think, "oh, I love my life as a professor, you should go to grad school so you can be like me." They're not thinking about what's good for the students. So recently, I try to tell students about other options. I'll say look, if you really want to go to math grad school and be a professor, so much so that you're willing to possibly end up bouncing around and then ending up in a very rural location, with not very motivated students, because that's where the jobs are, if you're devoted enough to math that you want to do that, then you should do it. But in you're not willing to accept that as your life, then you're much better off going on in statistics or computer science, and then there's different areas like finance, etc. that are better job wise than going on in grad school in math. So I'll make a little speech like that. Actually in all my upper level division classes, I talk about that. And then if a student comes to me and says that they really love math enough to do this, I don't discourage them, I'll help them. But I think a lot of times the faculty and parents try to get the people they're mentoring to be like them. Like, "you should be like me," and not only should you go to math grad school, but you should go into topology! But that's not really what's best for the student. But I do try to encourage everybody. I try to work with a student. I encourage them to come to my office, even if they don't have a math question, to talk about their future plans, even if they're freshmen or sophomores. And then I'll try to help them identify the direction that they want to go in by having them imagine themselves in different directions. I think also a lot of professors think it's like tainted if you start talking to students about jobs. But you need to start planning as soon as possible. I'm not a statistics professor, so even though

statistics is a great field as far as the job market, I'll encourage students to take statistics classes. Statistics isn't my field, but I know it's a good field to go into.

LL: No that's kind of refreshing to hear. Do you think you've seen progress over time with respect to women in math? Like both progress in numbers enrolled and progress in treatment?

EF: Oh, definitely, yes. Like now, I think it's roughly 30% of graduate students in math, are women. At the undergraduate level I think it's 30-50% depending on if the college offers teaching as an option or track, so I think that universities with education as an option increases the total pool of women in math classes. At schools like that, it's often 50% of students. In schools that don't have math education, like for example we don't at Pomona, I think it's closer to 30%. But nonetheless, it's not just one or two, you know? And of course, overall, the administrative awareness of issues like discrimination or creating a hostile environment is on people's' radar and there are resources to go to if something like that happens. Even in my own department. When I was hired at Pomona, I was the only woman in my department. And now we have 4… let me count them. Four in addition to me. So five total, out of eleven tenured or tenure track faculty. And I think that's true of most small colleges. It's not true at the more research oriented schools. I think that's related to— there's this pipeline problem, but in the graduate level, in the very elite schools. So the number of women in graduate programs is about 30% I believe, but if you focus on MIT, Harvard, Princeton, places like that, the percent of women is low. And then if you're thinking about getting jobs at a research facility like that, they

will typically only consider people who went to one of the top universities as a graduate student. So there’s a pipeline issue there. And I think there’s also a pipeline issue in the level of undergraduate students in those schools like Harvard and Yale, the percentage of women who are math majors is lower than what it is overall. I think that’s not so much due to discrimination as it is due to having an unsupportive environment in the math department towards students in general, the faculty tends to only care about their own research, and maybe their graduate students, but not about the undergraduate students. They often have non-long term postdocs teaching undergraduate courses. And so what happens, because women tend to be more insecure, they react more negatively to this kind of environment than male students. For example, my daughter, who is actually getting her PhD in math, she was an undergraduate at Yale, and a math major, in her first class, which was an honors Calc III and Linear Algebra class, had 50 students and I think 5 of them were women, so already, at Yale, which is you know, Yale, they were perhaps afraid to take the honors level class. And that class was taught by a postdoc, who didn’t speak english well, who never turned to face the class, who didn’t answer questions. So by the end of that class, I think there were only three of the women who were going on to be math majors. There were three women who graduated with her. Only she and one other person went to grad school in math overall, and the other person was a male. So I think it was an unsupportive environment in general, but it sort of disproportionately affected women. Because I think, you know, their personalities. That was a long winded answer.

LL: So some of the research that I've been doing in reading more of of Feminist theory that talks about women in stem in general talks about the idea that there's this ideal typical mathematician and that mathematician is a male, stereotypical, idealized version of what mathematicians aspire to be, and I wanted to ask if you think that's a still a relevant critique, and if you think you've been impacted by that societal stereotype of what a mathematician looks or acts like?

EF: So in my personal experience, certainly here at Pomona college, I don't feel like that stereotype exists for our students. We have a quite diverse department with five women. So I'm not aware of that as a stereotype. And even in terms of race and stuff like that we have some faculty who are originally not from this country, and even if you add in the temporary faculty which would put us up to thirteen, there's only one person who is a white male. I mean, it's a question of what you define as a white male. We have someone from Iran, but in any case, we have one person who is a white male from America. And there's no one from Europe. We have a female who's from Albania. I don't think it affected me in the sense of trying to visual myself as a mathematician. Like it just didn't happen. Yes, I was aware of not being treated fairly but I also— maybe it's like, I saw myself as different, so even when I had this really nice older woman as my math teacher, she was a woman, but I was aware of the fact that I was one of two girls in the class. It wasn't that I was comparing myself to my teachers and saying "oh, I'm not like this," as much as I had already accepted the fact that I was not your typical girl. You know, maybe something that may or may not be relevant, but I'll tell you this. When I was a child, my mother only dressed my sister and me in pants, which back then, I was born in the fifties,

was very unusual for girls. If you look at those Dick and Jane books, girls always wore dresses. And I remember girls asking me why I was wearing pants, and I didn't really have an answer. I don't recall feeling bad about it or even thinking about it, you know? It was just what I wear. Anyway, it's kind of an example that from a young age, I thought of myself as different. So maybe the fact that I didn't have specific role models just didn't come up. And the fact of the matter is, the women that I had maybe weren't role models either. Like they were elderly, or from the south, the one I had in college was not elderly or from the south, but in any case I don't really think I thought of myself as either becoming like them or not becoming like them, or whatever.

LL: Looking back on your whole experience, through childhood, your undergraduate and graduate education, and your career, do you feel like your experience has been the typical experience for a woman in math, or do you feel like it's been atypical?

EF: I think it might be typical of people my age. I think the experience of my students is quite different than my experience. There aren't many women in math my age, but the ones that I know had relatively similar experiences. They had, like me, some negative experiences, but also some people who were encouraging, and they were relatively isolated in terms of their gender in their education. That's what I would say pretty much.

LL: The last question I have, more generally, is there anything that you feel like I may not have asked, but that you feel is relevant to the questions and the places that we were going? Anything that you wanted to talk about related to anything we've discussed?

EF: I guess I'll tell you one other thing. For many years, from 2000 to 2015, every summer I would teach in a program for women in math, focused on freshmen and sophomore women, based out of a college, and was funded by the NSF. And I felt like it was a really great program, where we had 18-20 undergraduate women from all over the country, and they took classes with women, and there were colloquia by women, and it was just like a sort of really great bonding experience for them. Unfortunately, the project has now been unfunded by the NSF so that's kind of unfortunate. You know, the thing is, that it was a very successful program, and the thing is that for policy reasons, which probably comes from congress rather than within the NSF, although I don't know, decided that they only wanted to fund new programs, and not existing programs that were successful. And so 2014 was the last summer that the program was offered. So that's just an unfortunate thing that I thought I would share with you.

LL: Interesting, I hadn't really heard about any programs like that. It sounds like a really awesome opportunity.

EF: Yeah, there were also a few other programs that have been unfunded. So one was a program at George Washington University for junior and senior undergraduate women, that one is just called the GW program I think. There is one called EDGE, which is for women and minorities, especially minority women, for the transition between undergraduate and graduate school. That program still exists, but it no longer has NSF funding. And then there was a program at Smith college, for women who wanted to go to

math grad school but for whatever reason didn’t have a strong math background in undergraduate. Like maybe they didn’t major in math, or something. That was funded by the NSF, and that has also been cut at this point. So basically the NSF has cut all of the programs for women. The only one that is still funded, is this program at the Institute for Advanced Study, which is funded by the NSF. So the institute is funded by the NSF, and then they give the program, so the program isn’t directly funded by the NSF. It’s a program in the last two weeks of May where they bring together students and faculty and postdocs to Princeton, where they are lectured in a specific field. My daughter went to this, she’s in algebraic geometry. She went for the two weeks. She felt really great, because the program itself was great, and also almost all the women who are her age in algebraic geometry were at that program back when she was a senior undergraduate, so she feels like she has like a cohort of women in algebraic geometry. She’s getting her PhD at UCLA, and they’re not necessarily at UCLA with her, but they’re all over the country. And so I think that’s been good for her to have this network so that when she was first going to math conferences as a graduate student in algebraic geometry, she would run into these other women that she knew, so she would always have these other women that she could bond with. So that is a good program that continues but I think it continues because it isn’t directly funded by the NSF.

LL: I’m definitely going to have to look into that, especially the cutting of funds.

EF: Yeah, unfortunate.

LL: So again, that’s all I have unless there’s anything else you wanted to share.

EF: I think that’s all on my end.

Author's Biography

Lori Loftin was born in Tampa, Florida on March 1, 1996. She was raised in Tampa and graduated from Sickles High School in 2014. Lori is majoring Women's, Gender and Sexuality Studies and Mathematics. She is co-chair of the Feminist Collective, co-director of the Women's Resource Center on campus, and is a member of Pi Mu Epsilon. Upon graduation, Lori plans to attend graduate school with the goal of obtaining her PhD in Gender Studies.