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Exact Results in Model Statistical Systems

Peter H. Kleban
Principal Investigator; University of Maine, Orono, kleban@maine.edu

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Principal Investigator: Kleban, Peter H. **Award ID:** 0536927

Organization: University of Maine

Submitted By:

Kleban, Peter - Principal Investigator

Title:

Exact Results in Model Statistical Systems

Project Participants

Senior Personnel

Name: Kleban, Peter

Worked for more than 160 Hours: Yes

Contribution to Project:

Peter Kleban is PI of this project, which is the continuation of a previously funded NSF grant.

Post-doc

Graduate Student

Name: Simmons, Jacob

Worked for more than 160 Hours: Yes

Contribution to Project:

Jacob J. H. Simmons completed his PhD in Physics in May 2007. His research involved applying conformal field theory to percolation in two dimensions. His thesis was very successful, resulting in three published articles (with one in Physical Review Letters), one preprint (just submitted), and two more articles almost completed. The most important result of his thesis solves an outstanding problem in percolation using a novel method that promises to shed light on some underlying, not yet understood symmetry. Jake also attended several meetings and gave two talks. He has taken a postdoctoral position with J. Cardy at Oxford.

This grant provided him salary support (he was also supported under the previous grant during this period) and travel support.

Name: Stone, Thomas

Worked for more than 160 Hours: Yes

Contribution to Project:

Tom worked on the project over the summer of 2007. He found factorizations in the critical q-state Potts models that generalize the percolation results reported in our article in Physical Review Letters. His results are verified by simulations (performed by Bob Ziff), so we expect a publication to come from this work.

This grant provided partial support for Tom for the summer.

Name: Flores, Steven

Worked for more than 160 Hours: Yes

Contribution to Project:

Steve is a third-year graduate student in Mathematics at the University of Michigan. He is in Maine for the summer of 2008 working on the project. He is partly supported by this grant, and partly by Prof. R. Ziff at Michigan. He is working on a connection between conformal field theory and stochastic Loewner evolution (SLE).

Undergraduate Student

Name: Commeau, Benjamin

Worked for more than 160 Hours: Yes

Contribution to Project:

A very bright freshman, Benjamin Commeau, showed up at my door during the Spring 2009 semester with some questions about special relativity, a course he has not yet taken. It was clear that he had gained considerable understanding of the subject on his own, and that he is very gifted for research in physics. We've had considerable contact since then, and Ben is working with my research group this summer (2009), supported by this grant. Part of his task will be to learn more physics and mathematics. I will have him go through parts of R. Feynman's introductory course in physics, which is at a level considerably above the courses given and UM. This will be coupled with his work on the research project.

Technician, Programmer

Other Participant

Research Experience for Undergraduates

Organizational Partners

University of Michigan Ann Arbor

We have been collaborating, under the previous NSF grant, and continue to collaborate very actively with Prof. R. M. Ziff, Department of Chemical Engineering (and Michigan Center for Theoretical Physics). Bob is a recognized expert in computer simulations of percolation.

Bob is co-author on several published articles (including one in Physical Review Letters) and preprints. He is also involved in several ongoing calculations. We are in very frequent contact by telephone and email. My current graduate student, Steve Flores, came to me via Bob.

In addition, Bob directs several undergraduate and graduate students who participate in the project (see the Contributions to Human Resource Development section).

Queen Mary, University of London

The PI is collaborating with Prof. Thomas Prellberg, who is in the dynamical systems group at Queen Mary, University of London. We are applying ideas and techniques from dynamical systems to the Farey spin chain model. Thomas is co-author on a previous publication in this area. Our paper (in collaboration with Prof. O. Bandtlow, also at Queen Mary and Jan Fiala, former graduate student), constitutes a rigorous derivation of the free energy of this model in the vicinity of its second-order phase transition, as a function of both temperature and magnetic field. We are only aware of one other such result in the literature. In addition, we find the interesting result that the dependence of the free energy on the temperature deviation from the critical value scales with the Lyapunov exponent of the associated dynamical system.

Thomas has provided some travel support for the PI. Thoms spent a sabbatical semester during Fall 2008 in Maine, supported by the University of Maine.

College de France

Collaboration with Prof. Don Zagier, who is there each fall. Don and the PI previously published a paper on percolation and number theory, supported by the predecessor to this

grant. This paper inspired a new area in number theory; higher-order modular forms. The PI continues to work in this area (see the paper with N. Diamantis listed below). Additionally, Don has been very helpful in our work on the Farey model. During fall 2007 the PI also worked with Michel-Stephane Dupertuis, Don's postdoc, on a Farey problem.

Cardiff University

The PI icollaborated with Prof. A. Zhigliavsky, School of Mathematics, on proving a new result involving Farey fractions. Anatoly provided some travel support for this purpose.

University of Nottingham

The PI is collaborating with Prof. N. Diamantis, a number theorist at the University of Nottingham. Nikos is an expert on higher-order modular forms, modular objects that arise in a previous paper on percolation by the PI and Don Zagier. The object of our collaboration is to further understand the connection between modular forms and percolation (or Stochastic Loewner Evolution processes). Nikos has provided some travel support.

The PI's paper with Nikos (see publications section) contributes to the area of higher-order modular forms. This recent work extends that fruitful interaction between physics and number theory.

University of Colorado at Boulder

The PI visited Prof. R. Maier, Mathematics Department, in June 2007 for discussions on special functions related to our studies of percolation.

Princeton University

The PI worked with Prof. Dmitry Beliaev, Mathematics Department, on calculations in stochastic Loewner evolution (SLE) that have interesting implications for conformal field theory. Steve Flores has also visited (May 2009) and developed a connection with Dmitry. The PI visited Dmitry several times, and spent the month of March 2008 in the Department as a guest of Prof. M. Aizenman, who provided him with an office, as he has for several periods after that. The PI also had extensive discussions re percolation with Michael Damron, a postdoctoral fellow.

Other Collaborators or Contacts

1.) Ali Ozluk, Mathematics Department, University of Maine. Ali is a long-time collaborator on the Farey fraction part of this project. He co-authored a paper on the Farey spin chain (together with the PI and former grad student Jan Fiala).

2.) Discussions with M. Aizenman, T. Spencer, J. Cardy, and Y. Sinai (Princeton).

Activities and Findings

Research and Education Activities:

One major goal of our research is a deeper understanding of critical percolation, and related critical models in two dimensions. These are important and very extensively studied model systems, to which we are bringing new and unexpected approaches. We treat these systems by means of conformal field theory, number theory and computer simulations (the simulations are mainly done by Bob Ziff). We have obtained new results for crossing and connection probabilities as well as cluster densities via these methods.

These indicate a symmetry in percolation that is not understood.

One interesting area that we have discovered is exact factorizations of certain higher-order correlation functions in terms of less complicated correlation functions. We have found several such relations for percolation in two dimensions. Some of them are also valid for q-state Potts models, or other critical systems. In some cases the factorization is not quite exact but holds to within a few percent. Here the deviation from exactness only depends on one coordinate, which hints that some new symmetry is at play. Further, one such factorization has been shown numerically to hold within a few percent for critical percolation in three dimensions.

To our knowledge there are no other such exact factorization results available for any fluid. Our results seem to be the only ones available that express higher-order correlation functions in terms of lower-order ones.

Our work on percolation (especially the paper with Don Zagier) has inspired a new area in number theory, higher-order modular forms. This work is extended to our recent results in percolation in a paper by the PI and N. Diamantis.

More recent work calculates the probability density of critical clusters in a wide variety of models in the upper half plane or any conformally equivalent geometry. Even though this calculation requires determining a six-point correlation function, we have succeeded in a complete solution of the PDEs arising from conformal field theory. The result exhibits an interesting independence of one coordinate in a rectangle. In addition, various formulas arise which transform as Jacobi modular forms of weight and index zero.

The other main goal of this project is an understanding of the Farey fraction spin chain, a set of one dimensional models that exhibits interesting phase transition behavior. These spin chains are connected to multifractals and dynamical systems. We investigate them with various analytical methods, including operator theory and number theory, and to some extent computer simulation. A recent paper presents a rigorous calculation of the free energy of the model as a function of both temperature and field near the phase transition. To our knowledge, there is only one other such result.

Our major educational goal is the training of graduate students in these areas of study. Undergraduates have also been involved in the research, both in Maine (with the PI) and in Michigan (working with Bob Ziff). In addition, the talks given at various institutions contribute to the diffusion of knowledge and to training in physics and mathematics.

Presentations:

Peter Kleban, invited seminar on 'The Farey fraction spin chain' at the School of Mathematics, Cardiff University, Wales, October 2006.

Peter Kleban, invited seminar on 'The Farey fraction spin chain' at the School of Mathematics, University of Nottingham, England, October, 2006.

Peter Kleban, invited talk on 'Crossing Probabilities, Modular Forms and Anchored Percolation Clusters' at the dynamical systems seminar in the School of Mathematics, Queen Mary, University of London, England, November, 2006.

The three presentations above were supported in part by a travel grant from the London Mathematical Society (written by the PI but submitted and administered by Prof. Prellberg).

Peter Kleban, invited talk on 'CFT and Modular Forms Applied to Crossing and

Connection Probabilities in Critical 2-D Percolation' at the workshop entitled Stochastic Geometry and Field Theory: From Growth Phenomena to Disordered Systems, held at the Kavli Institute for Theoretical Physics, UCSB, December 2006.

Graduate student Jake Simmons gave a contributed talk on 'Critical 2D Percolation Crossing and Connection Probabilities' at the 96th Statistical Mechanics Conference, Mathematics Department, Rutgers University. R. M Ziff was listed as a third author.

Peter Kleban, invited talk on 'Critical 2-D Percolation: Crossing Probabilities, Modular Forms and Factorization' at the March American Physical Society meeting, Denver (co-authors: J. Simmons and R. Ziff) March 2007.

Peter Kleban, invited informal talk on 'Crossing Probabilities in 2D Critical Percolation' at the Mathematical Physics bag lunch seminar, Physics Department, Princeton University, March 2007.

Peter Kleban, invited talk on 'Crossing Probabilities in Percolation and Stochastic Loewner Evolution and Modular Forms' at the American Mathematical Society 2007 Spring Western Section Meeting, University of Arizona, Tucson (Don Zagier listed as co-author), April 2007.

Peter Kleban, invited talk 'On Cardy's Crossing Formula and Related Formulas in Percolation' at the 97th Statistical Mechanics Conference, Mathematics Department, Rutgers University (J. Simmons and R. Ziff were listed as co-authors), May 2007.

Jake Simmons, contributed talk on 'Exact Factorization of Correlations in 2D Critical Percolation' at the 97th Statistical Mechanics Conference, Mathematics Department, Rutgers University (R. Ziff and Peter Kleban were listed as co-authors), May 2007.

Peter Kleban, oral presentation (refereed) on 'Exact Factorization of Correlations in 2D Critical Percolation' at STATPHYS 23, Genoa, Italy (J. Simmons and R. Ziff were listed as co-authors), July 2007.

Peter Kleban, contributed talk on 'Exact Factorization of Correlation Functions in Critical Sysems' at the 99th Statistical Mechanics Conference, Mathematics Department, Rutgers University (J. J. H. Simmons and R. Ziff were listed as co-authors), May 2008.

Peter Kleban, invited talk on 'Exact Factorization of Correlations in 2-D Critical Systems' at the program: The Theory and Practice of Fluctuation-Induced Interactions, Kavli Institute for Theoretical Physics, UCSB.
http://online.kitp.ucsb.edu/online/fluctuate08/kleban/

Peter Kleban, contributed talk on ?Factorization of Cluster Density Correlations in Critical 2-D Percolation in Rectangles? (J. J. H. Simmons and R. M. Ziff listed as co-authors), givenat the 100th Rutgers Statistical Mechanics Meeting.

Peter Kleban, contributed talk on ?A Hamburger Theorem for Percolation? (N. Diamantis listed as coauthor), given at the 101st Rutgers Statistical Mechanics Meeting.

Peter Kleban, invited talk on ?Exact Factorization of Correlations in 2-D Critical Systems?, Department of Theoretical Physics, Australian National University, Canberra, September 2009.

Peter Kleban, refereed talk on ?Factorization of density correlation functions for clusters touching the sides of a rectangle?, at the Conference 'Conformal Maps from Mathematics to Physics', Monte Verit?, Ascona, Ticino, Switzerland May 2010. (This talk was contributed but only accepted after refereeing. Out of about 70 participants, there

were 16 contributed talks presented.)

Peter Kleban, invited talk on ?New percolation crossing formulas and second-order modular forms?, Maine-Qu?bec Number Theory Conference, October 2010 co-author N. Diamantis http://www.math.umaine.edu/numbertheory/qm10.html

104th Statistical Mechanics Conference, Rutgers University, December 2010
Peter Kleban, contibuted talk on, 'Crossing cluster densities in rectangular geometries at 2-D critical points' co-authors J. Simmons, R. M. Ziff, and S. Flores http://www.math.rutgers.edu/events/smm/smm104shorttalks.html, 104th Statistical Mechanics Conference, Rutgers University, December 2010

Peter Kleban, Statistical Mechanics seminar, Physics Department, Princeton University 12/01/10 'Hidden symmetries at the?percolation point in two dimensions' http://www.princeton.edu/physics/events_archive/viewevent.xml?id=129

?Crossing probabilities, their densities and modular forms?, Statistical Mechanics seminar, Physics Department, Princeton University 12/14/11 http://www.princeton.edu/physics/events_archive/viewevent.xml?id=310 .

Presentations about the project by Bob Ziff:

R. M. Ziff. 'Density Profiles in Critical Percolation, ' Invited talk at IPAM (Institute for Pure and Applied Mathematics) workshop on Random Curves, Surfaces and Transport, U.C.L.A., April 16-20, 2007. https://www.ipam.ucla.edu/schedule.aspx?pc=rsws2

R. M. Ziff, Crossing problems and thresholds in percolation and the Potts model (oral presentation, session 2C) Statphys23 meeting. Genova, Italy, July 9-13, 2007

R. M. Ziff - ?Simulations of crossing and new exact results in percolation? (Invited talk) V Brazillian Meeting on Simulational Physics, Ouro Preto - MG ? Brasil, July 31st - August 3rd 2007

R. M. Ziff (invited talk), ?Percolation and the Quasi-static state of dynamical processes,? 98th Statistical Mechanics Conference, Rutgers University (December 16-18, 2007).

M. Ziff, Peter Kleban, Jacob Simmons and Christian Scullard, ?Percolation density profiles and lattice structures (Poster session C1) American Physical Society March Meeting New Orleans, March 10-14, 2008.

Presentations about the project by Jake Simmons:

J. J. H. Simmons, ?Watts? Formula and Logarithmic CFT?, at the workshop on Stochastic Loewner Evolution and Scaling Limits, CRM, University of Montreal, August 2008.

Findings:

Farey fraction spin chain:

By use of operator identities, we have extended our previous approximate calculation of the free energy of this model into a completely rigorous derivation of the free energy as a function of both the temperature and the magnetic field, in the vicinity of the second order phase transition point. To our knowledge, there is only one other rigorous result

of this type. The operator cluster method that we have developed here seems to be of interest to dynamical systems theorists. One interesting feature that emerges is that the free energy depends on the temperature deviation from its critical value scaled by the Lyapunov exponent of the associated chaotic map (the Farey map).

We had also found an interesting result on the sum of lengths of half of the intervals between 'new' Farey fractions. We have proved, rigorously, that the lim inf of this sum vanishes in the limit of infinite level. The numerical evidence makes it clear that in fact the limit of this sum vanishes. This very simple geometric property of the Farey fractions is not very apparent. The intervals chosen are alternating, and there seems no obvious reason why the sum of their lengths should vanish in this limit. This work has inspired a recent paper (M. Kesseboehmer and B. O. Stratmann, to appear [arXiv: math/0509603]) which proves the property (and also determines the limiting behavior) using infinite ergodic theory.

Percolation:

In collaboration with Prof. R. M. Ziff, the PI and his graduate student Jacob Simmons previously examined the behavior of clusters attached to boundaries in 2-D critical percolation. In particular, we considered the densities of such clusters. These quantities, given by correlation functions, were calculated by use of conformal field theory and computer simulations. We found that they are simple functions of the potentials of 2-D electrostatic dipoles, and that an exact superposition cum factorization in terms of lower-order correlation functions applies.

Here, an exact and universal factorization result (reminiscent of the Kirkwood superposition approximation) emerged. More recently, we have found several other exact (and universal) factorizations of higher-order correlation functions in 2-D critical percolation. We are not aware of any other exact result of this type in the theory of fluids.

Further, it appears that this kind of factorization also applies to correlations on Fortuin-Kastelyn clusters at the critical points of the two?dimensional q?state Potts models. This research involved graduate student Tom Stone. In addition, we have found some cases (involving fixed boundary conditions) where the factorization is not quite exact but holds to within a few percent. Bob Ziff has shown numerically that one such factorization holds to within a few percent for critical percolation in three dimensions. In another situation, (almost) exact factorization holds in a rectangle, and the deviation from exactness only depends on one co-ordinate. This was first shown numerically, then later calculated from conformal field theory. This curious behavior points to some unknown symmetry of percolation and other two-dimensional critical systems.

Exactly why and when such factorizations apply is not at all clear, and is a major focus for our research. Progress in this direction has been made: one of our papers identifies all possible factorizations of a certain type allowed by conformal field theory.

Perhaps the most significant result of Jake Simmons' thesis posits a novel operator identity in conformal field theory (for central charge c = 0, which applies to percolation). This gives rise to several new crossing formulas, which are used to unify the previously known crossing results, thereby solving an outstanding problem in the area. Further, this new method appears to shed light on hidden symmetries in percolation. These new crossing formulas have also led to results showing their relation to number theoretic objects (see the paper with N. Diamantis).

More recent work calculates the probability density of critical clusters in a wide variety of models in the upper half plane or any conformally equivalent geometry. Even though this calculation requires determining a six-point correlation function, we have

succeeded in a complete solution of the PDEs arising from conformal field theory. The result exhibits an interesting independence of one coordinate in a rectangle. In addition, various formulas for physical quantities arise which transform as Jacobi modular forms of weight and index zero.

Training and Development:

A graduate student, Jacob J. H. Simmons, has been involved on the project. Jake has completed his PhD. in Physics. His thesis involves applying conformal field theory to percolation in two dimensions. The research assistantship provided by this grant greatly aided his progress. Jake is co-author of six published papers in this area (including one in
Physical Review Letters), five of which were supported under this grant, and of one preprint. Two other articles are nearing completion. Jake was a postdoctoral fellow with J.
Cardy in Oxford and is currently a postdoc at the University of Chicago. He continues to collaborate on various aspects of this project.

A second graduate student, Thomas Stone, worked on the project during summer 2007. Tom made very good progress on extending our factorization results to the critical q-state
Potts models. This work is being done in close collaboration with Bob Ziff, who is performing computer simulations. Tom has not continued in this research area.

A third graduate student, Steven M. Flores, is a PhD student in Applied Mathematics at the
University of Michigan (Ann Arbor), but working under the PI?s direction. Steve came to Maine via Bob Ziff, who is one of the PI's collaborators. Steve is working on some calculations in stochastic Loewener evolution (SLE) that promise to give interesting information about conformal field theory. This project grew out of conversations with Prof.
D. Beliaev at Princeton.

An undergraduate at Maine, Ben Commeau, are participate in the project during the summers of 2009 and 2010. He is described in the Participants section.

Several undergraduate and graduate students have or are participating in the project at the
University of Michigan (see the Contributions to Human Resource Development section).

Outreach Activities:

Journal Publications

Jacob J H Simmons, Peter Kleban, Kevin Dahlberg, and Robert M. Ziff, "The density of critical percolation clusters touching the boundaries of strips and squares", J. Stat. Mech., p. , vol. P06012, (2007). Published, 10.1088/1742-5468/2007/06/P06012

Jan Fiala and Peter Kleban, "Intervals Between Farey Fractions in the Limit of Infinite Level", Annales des sciences mathematiques du Quebec, p. 63, vol. 34, (2009). Published,

Jacob J. H. Simmons, Peter Kleban, and Robert M. Ziff, "Percolation crossing formulas and conformal field theory", J. Phys. A: Math. Theoret., p. F771, vol. 40, (2007). Published, 10.1088/1751-8113/40/31/F03

Jacob J. H. Simmons, Peter Kleban, and Robert M. Ziff, "Exact factorization of correlation functions in 2-D critical percolation", Phys. Rev. E, p. 0, vol. 76, (2007). Published, 10.1103/PhysRevE.76.041106

Jacob J. H. Simmons and Peter Kleban, "First Column Boundary Operator Product Expansion Coefficients", arXiv, p. , vol. 0712.35, (2007). arXiv preprint,

Simmons, Jacob J. H., Ziff, Robert M. and Kleban Peter, "Factorization of percolation density correlation functions for clusters touching the sides of a rectangle", J. Stat. Mech., p. P02067, vol. , (2009). Published, 10.1088/1742-5468/2009/02/P02067

Jacob J. H. Simmons and Peter Kleban, "General solution of an exact correlation function factorization in conformal field theory", Journal of Statistical Mechanics, p. P10002, vol. , (2009). Published, 10.1088/1742-5468/2009/10/P10002

N. Diamantis and P. Kleban, "New percolation crossing formulas and second-order modular forms", Communications in Number Theory and Physics, p. 1, vol. 3, (2009). Published, http://www.intlpress.com/CNTP/p/2009/CNTP-3-4-A4- diamantis.pdf

Oscar F. Bandtlow, Jan Fiala, Peter Kleban and Thomas Prellberg, "Asymptotics of the Farey Fraction Spin Chain Free Energy at the Critical Point", Journal of Statistical Physics, p. 447, vol. 138, (2010). Published, http://www.springerlink.com/content/102588/?k=bandtlow

Jacob J H Simmons and Peter Kleban, "Complete conformal field theory solution of a chiral six-point correlation function", J. Phys. A: Math. Theor., p. 315403, vol. 44, (2011). Published, doi:10.1088/1751-8113/44/31/315403

Jacob J H Simmons, Peter Kleban, Steven M Flores and Robert M Ziff, "Cluster densities at 2D critical points in rectangular geometries", J. Phys. A: Math. Theor., p. 385002, vol. 44, (2011). Published, doi:10.1088/1751-8113/44/38/385002

Ziff, Robert M.; Simmons, Jacob J. H.; Kleban, Peter,, "Factorization of correlations in two-dimensional percolation on the plane and torus", J. Phys. A: Math. Theor., p. 065002, vol. 44, (2011). Published, doi:10.1088/1751-8113/44/6/065002

Books or Other One-time Publications

Web/Internet Site

URL(s):

http://online.itp.ucsb.edu/online/sle06/
http://online.kitp.ucsb.edu/online/fluctuate08/kleban/

Description:

Website with the PI's talk on "CFT and Modular Forms Applied to Crossing and Connection Probabilities in Critical 2-D Percolation" at the workshop entitled Stochastic Geometry and Field Theory: From Growth Phenomena to Disordered Systems, held at the Kavli Institute for Theoretical Physics, UCSB, fall 2007.

Website with the PI's talk on "Exact Factorization of Correlations in 2-D Critical Systems" at the workshop entitled The Theory and Practice of Fluctuation-Induced Interactions, held at the Kavli Institute for Theoretical Physics, UCSB, fall 2008.

Other Specific Products

Contributions

Contributions within Discipline:

The paper 'Intervals Between Farey Fractions in the Limit of Infinite Level', by (former graduate student) Jan Fiala and Peter Kleban, Annales des sciences math?matiques du Quebec 34 (2010), no 1, 25-36 [arXiv: math-ph/0505053] contains an interesting result on the sum of the lengths of half of the intervals between 'new' Farey fractions. It proves, rigorously, that the lim inf of this sum vanishes in the limit of infinite level. The

numerical evidence indicates that in fact the limit of this sum vanishes. This very simple geometric property of the Farey fractions is not very apparent. The intervals chosen are alternating, and there seems no obvious reason why the sum of their lengths should vanish (as has now been proved) in this limit. This work was also supported by the original grant, and the NSF grant for the Kavli Institute, where part of it was done, while the PI attended a workshop during the fall of 2007. This work has inspired a paper by two mathematicians, (M. Kesseboehmer and B. O. Stratmann, to appear [arXiv: math/0509603]) which proves the property (and also determines the limiting behavior) using recent developments in infinite ergodic theory.

The paper 'Anchored Critical Percolation Clusters and 2-D Electrostatics', by Peter Kleban, Jacob J. H. Simmons, and Robert M. Ziff, Phys. Rev. Letters 97, 115702 (2006) [arXiv: cond-mat/0605120], was supported by the previous grant, and forms the basis of some of our ongoing research. It considers the densities of clusters, at the percolation point of a two-dimensional system, which are anchored in various ways to an edge. These quantities are calculated by use of conformal field theory and computer simulations. We find that they are given by simple functions of the potentials of 2-D electrostatic dipoles, and that a kind of superposition cum factorization applies. we find that an exact and universal result similar to the Kirkwood superposition approximation emerges, whereby higher-order correlation functions are simply expressed in terms of lower-order ones. To our knowledge, there is no previous such result in the theory of fluids. Further, our subsequent research demonstrates similar factorizations in various contexts as described below.

We have, under this grant, extended this work in the article 'Exact factorization of correlation functions in 2-D critical percolation', by Jacob J. H. Simmons, Peter Kleban, and Robert M. Ziff, Phys. Rev. E76, 041106 (2007). http://link.aps.org/abstract/PRE/v76/e041106 [arXiv: 0706.4105].

Perhaps the most significant result of Jake Simmons' thesis was published as 'Percolation crossing formulas and conformal field theory', by Jacob J. H. Simmons, Peter Kleban, and Robert M. Ziff, J. Phys. A: Math. Theor. (fast track communication) 40, pp F771-F784 (2007) [arXiv: 0705.1933]. I quote the (only) referee's report in full: 'This interesting paper shows how various percolation crossing formulas can be unified by positing an operator identity in the CFT, of a rather novel form. This not only leads to new formulae, it probably sheds light on possible hidden symmetries in percolation CFT. The paper is well-written and deserves publication.'

An article on 'The density of critical percolation clusters touching the boundaries of strips and squares', by Jacob J. H. Simmons, Peter Kleban, Kevin Dahlberg and Robert M. Ziff, J. Stat. Mech. P06012 (2007) [arXiv:0704.0901] considers the density of two-dimensional critical percolation clusters, constrained to touch one or both boundaries, in infinite strips, half-infinite strips and squares, as well as several related quantities for the infinite strip. Our theoretical results follow from conformal field theory and are compared with high-precision numerical simulation. Note that Kevin is an undergraduate student (at Michigan).

An article on 'Factorization of percolation density correlation functions for clusters touching the sides of a rectangle' with J. J. H. Simmons and R. M. Ziff, J. Stat. Mech. P02067 (2009). Here we show both analytically and numerically that our factorization result for percolation is also approximately valid for rectangles with 'wired' (fixed) boundary conditions on the vertical ends. The deviation from exact factorization is, interestingly, independent of the vertical coordinate, indicating some unknown symmetry. More recently (not yet published), we have shown that this property follows exactly from conformal field theory, and holds for many other two-dimensional critical models as well. Its physical origin remains mysterious, however.

An article on ?New percolation crossing formulas and second-order modular forms? with N. Diamantis, published in Communications in Number Theory and Physics, considers the three new crossing formulas found in Jake Simmons? thesis (and published as noted above). This paper proves that all three of them (i) may be simply expressed in terms of Cardy's and Watts' crossing probabilities, (ii) are (weakly holomorphic) second-order modular forms of weight 0 (and a single particular type) on the congruence group $\Gamma(2)$, and (iii) under some technical assumptions (similar to those used in the PI?s work with Don Zagier), are completely determined by their transformation laws. The only physical input in (iii) is Cardy's crossing formula, which suggests an unknown connection between all crossing-type formulas.

The article ?General solution of an exact correlation function factorization in conformal field theory' with J. J. H. Simmons uses conformal field theory, to consider necessary conditions for factorization of the type discovered in percolation. We find that only one combination of Kac operators (with integral or half-integral indices) satisfies the condition.

'Asymptotics of the Farey Fraction Spin Chain Free Energy at the Critical Point', with O. Bandtlow, J. Fiala, and T. Prellberg. This work gives a rigorous derivation of the free energy of this model, in the vicinity of its second-order phase transition, as a function of both temperature and magnetic field. There is to our knowledge only one other such rigorous result in the literature. The cluster operator method developed in this paper may be of interest in other applications, e.g. to dynamical systems. In addition, we find that the dependence of the free energy on the temperature deviation from the critical value scales with the Lyapunov exponent of the associated dynamical system.

The papers ?Complete conformal field theory solution of a chiral six-point correlation function? and ?Cluster densities at 2D critical points in rectangular geometries? calculate the probability density of critical clusters in a wide variety of models in the upper half plane or any conformally equivalent geometry. Even though this calculation requires determining a six-point correlation function, we have succeeded in a complete solution of the PDEs arising from conformal field theory, and in interpreting the results physically for a wide variety of models. The result exhibits an interesting independence of one coordinate in a rectangle. In addition, various formulas for physical quantities arise which transform as Jacobi modular forms of weight and index zero, which points to an interesting extension of our work with Zagier and Diamantis.

Contributions to Other Disciplines:

The PI?s paper on percolation with Don Zagier has inspired number theorists to study new modular objects, ?higher-order modular forms?, which arise in that paper. This area in number theory has attracted the attention of several researchers in recent years, and a number of papers have been published. One of these experts, N. Diamantis, is collaborating with the PI on exploring further connections of number theory and percolation (see the paper listed in the Findings section).

The PI's experience in conformal field theory (and more pertinently, conformal mapping) led to some useful results in what is seemingly a very different area. The PI has been involved (with others) in a project to develop spectrometers with greatly improved throughput. This has been supported by NSF under the MRI grant DMR-9977800 'Development of a Fourier transform-based time-of-flight spectrometer with ultra-high resolution' and also by a grant from the Maine Technology Institute, which funded the incorporation of a company, Stillwater Scientific Instruments, of which the PI is a founding partner. The instrument involves a Bradbury-Nielsen gate (two interleaved conducting 'combs' with opposite voltages), which interrupts the beam of ions when voltage is applied to it, thus creating time-of-flight pulses. The PI has derived new analytical formulas for the ion scattering angles (especially those not large enough to cause the ions to miss the

detector, so they are erroneously counted) and also for the number of ions with a given excess voltage ('energy corruption') when the voltage is turned on or off. These quantities are of central importance in understanding the performance of the instrument.

Contributions to Human Resource Development:

An undergraduate was supported by the project at the University of Maine during the summers of 2009 and 2010:

1.) Benjamin Commeau, an undergraduate in Electrical and Computer Engineering. Ben?s participation is described in the Participants section.

Several undergraduate and graduate students at the University of Michigan worked on the project under Bob Ziff's direction:

1.) Kevin Dahlberg, an undergraduate student in Chemical Engineering, did computer simulation of crossing density problems; his results were published in our J. Stat. Mech. paper, of which he is a co-author.

2.) Steven M. Flores, second-year graduate student in Applied Math, worked on crossing probabilities on Fortuin-Kastelyn clusters at Michigan. Steve spent the summers of 2008 and 2009 at Maine. The PI is directing his thesis research.

3.) Paul Shearer, second-year graduate student in Applied Math, is studying Sochastic Loewner Evolution with applications to crossing problems.

4.) Navid Dianaty, a second-year physics graduate student, is working on percolation threshold and related q-state Potts model problems.

Contributions to Resources for Research and Education:

Contributions Beyond Science and Engineering:

Please see the entry under 'Contributions to Other Disciplines'.

Conference Proceedings

Categories for which nothing is reported:

Activities and Findings: Any Outreach Activities

Any Book

Any Product

Contributions: To Any Resources for Research and Education

Any Conference