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Statistical Inference for the Risk Ratio in 2x2 Binomial Trials with Stuctural Zero

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STATISTICAL INFERENCE FOR THE RISK RATIO IN 2 X 2 BINOMIAL TRIALS WITH STRUCTURAL ZERO

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In some statistical analyses, researchers may encounter the problem of analyzing correlated 2x2 table with a structural zero in one of the off diagonal cells. Structural zeros arise in situation where it is theoretically impossible for a particular cell to be observed. For instance, Agresti (1990) provided an example involving a sample of 156 calves born in Okeechobee County, Florida. Calves are first classified according to whether they get a pneumonia infection within certain time. They are then classified again according to whether they get a secondary infection within a period after the first infection clears up. Because subjects cannot, by definition, have a secondary infection without first having a primary infection, a structural void in the cell of the summary table that corresponds with no primary infection and has secondary infection is introduced. For discussion of this phenomenon, see Tang and Tang (2002), and Liu (1998).

The risk ratio (RR) between the secondary infection, given the primary infection, and the primary infection may be a useful measure of change in the pneumonia infection

rates of the primary infection and the secondary infection. In this thesis, we will first develop and evaluate the large sample confidence intervals of RR. Then we will investigate the tests for RR and the power of these tests. An example from the literature will be provided to illustrate these procedures. Simulation studies will be carried out to examine the performance of these procedures.

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Chapter 1

INTRODUCTION

In order to compare two groups, statistical inference of the risk ratio, under independent binomial sampling, has been extensively discussed in the literature (Gart and Nam, 1988). However, there are situations in which the assumption of independent binomial sampling is not valid. Agresti (2002) has given an example in which calves were first classified according to whether they got primary infection and then reclassified according to whether they developed a secondary infection within a certain time period after the first infection cleared up. In this case, when assessing the risk ratio between a secondary infection, given a primary infection and the primary infection, the responses are taken from the same group of subjects and are not independent. Therefore, the statistical procedures, under independent binomial sampling are not appropriate. So the data can be summarized as

Table 1.1. Probability of Each Cell.

		Secondary Infection		Total
		Yes	No	
Primary Infection	Yes	p_{11}	p_{12}	$p_{1\bullet}$
	No	0	p_{22}	p_{22}

Notice that calves having no primary infection cannot have secondary infection and hence the frequency of such event is zero in the above table. This is known as structural zero as opposite to sampling zero. See Agresti (2002, page 392) for discussion and for the explanation.

In order to analyze such bivariate tables, Lui (2000) discussed the interval estimation of the simple difference between the proportion of the primary infection and the secondary infection, given the primary infection. He developed three asymptotic interval estimators using Wald's test statistic, the likelihood ratio test and the basic principle of Fieller's theorem. The simulation studies concluded that the asymptotic confidence interval using likelihood ratio test consistently perform well in all the situations.

On the other hand, Lui (1998) discussed the estimation of the risk ratio (RR) between a secondary infection, given a primary infection, and the primary infection. He developed three asymptotic interval estimators using Wald's test statistic (Agresti, 2002; Casella and berger, 2001) the logarithmic transformation, and Fieller's theorem (Casella and berger, 2001). On the basis of his simulation studies, he concluded that when the underlying probability of primary infection is large, all three estimators perform reasonably well. When the probability of primary infection is small or moderate, the interval estimator using the logarithmic transformation outperforms the other two estimators when the sample size does not exceed 100. In addition, the coverage probability of this estimator consistently exceeds the nominal value in all situations.

In addition to the references cited above, Tang and Tang (2002) studied small sample statistical inference for RR in a correlated 2 X 2 table with a structural zero in one of the off diagonal cells.

The purpose of the present investigation is to further study the statistical inference in the case of 2 X 2 correlated table with a structural zero. In section 2, we review the three confidence intervals of RR studied by Liu (1998) and derive a fourth confidence interval based on Rao's score test. An example is provided to compare the results. Simulation studies are carried out in section 3 to compare the performance of these four confidence intervals in terms of the coverage probability and the length of the confidence interval. The length of confidence interval estimated by Rao's score method is always shortest. However, the coverage probability of confidence interval by Rao's score method is low.

In section 4, we derive the Rao's score test for testing H_0: RR = 1 and compare the four tests with respect to the power by means of extensive simulation studies. The simulation studies suggest the Rao's score method is more consistent than the other three methods although it is not the most powerful test. Actually, there is no consistent most powerful test in this study.

Finally, in Chapter 5, we present some conclusion and comments.

Chapter 2

DERIVATION OF CONFIDENCE INTERVALS

In this chapter, we will first give some notations used throughout this research and briefly introduce the delta method that are needed to develop asymptotic confidence interval estimators of RR. Then we will illustrate the detailed steps to derive four asymptotic estimators of RR. Three of these were proposed by Liu (1998), and are based on Wald's test statistic, the logarithmic transformation, and Fieller's theorem. In addition to these, we propose a Rao's score test statistic to construct confidence interval of RR.

2.1. Notation

Consider a sample of n subjects, who are first classified according to whether they get a primary infection. After the primary infection clears up, subjects are reclassified according to whether they get a secondary infection within a certain time. Then possible results are as following:

Table 2.1. Observed Frequencies.

		Secondary Infection		Total
		Yes	No	
Primary Infection	Yes	n_{11}	n_{12}	$n_{1.} = n_{12} + n_{22}$
	No	0	n_{22}	n_{22}
	Total	n_{11}	$n_{12} + n_{22}$	n

The corresponding probabilities are:

Table 2.2. Corresponding Probabilities.

		Secondary Infection		Total
		Yes	No	
Primary Infection	Yes	p_{11}	p_{12}	$p_{1\bullet} = p_{11} + p_{12}$
	No	0	p_{22}	p_{22}
	Total	p_{11}	$p_{12} + p_{22}$	1

Note that the estimators of the probabilities are $\hat{p}_{11} = n_{11}/n$, $\hat{p}_{12} = n_{12}/n$,

and $\hat{p}_{22} = n_{22}/n$. Also $p_{1\bullet} = p_{11} + p_{12}$, $p_{11} + p_{12} + p_{22} = 1$, $n_{1\bullet} = n_{11} + n_{12}$ and $n_{11} + n_{12} + n_{22} = n$. The risk ratio (RR) between a secondary infection, given a primary infection and the primary infection is defined as $RR = (p_{11}/p_{1\bullet})/p_{1\bullet} = p_{11}/p_{1\bullet}^{2}$.

Suppose that we take a random sample of n subjects. Then the random vector (n_{11}, n_{12}, n_{22}) follows the trinomial distribution

$$f(n_{11}, n_{12}, n_{22}) = \frac{n!}{n_{11}!n_{12}!n_{22}!} p_{11}^{n_{11}} p_{12}^{n_{12}} p_{22}^{n_{22}}$$

Given n, if we know n_{11} and n_{12} then $n_{22} = n - n_{11} - n_{12}$. Given RR and p_{11}, then $p_{12} = p_{1\bullet} - p_{11} = \sqrt{\frac{p_{11}}{RR}} - p_{11}$, $p_{22} = 1 - p_{1\bullet} = 1 - \sqrt{\frac{p_{11}}{RR}}$. Therefore, in the terms of parameters RR and p_{11}, the above trinomial distribution can be expressed as follows:

$$f(n_{11}, n_{12}; RR, p_{11}) = \frac{n!}{n_{11}!n_{12}!(n - n_{11} - n_{12})!} p_{11}^{n_{11}} \left(\sqrt{\frac{p_{11}}{RR}} - p_{11}\right)^{n_{12}} \left(1 - \sqrt{\frac{p_{11}}{RR}}\right)^{n - n_{11} - n_{12}}$$

2.2. Delta Method

If a function $g(x)$ has derivatives of order r, then for any constant a, the *Taylor polynomial of order r* about a is $T_r(x) = \sum_{i=0}^{r} \frac{g^{(i)}(a)}{i!}(x-a)^i$.

The remainder from the approximation by the Taylor polynomial, $g(x) - T_r(x)$ always tends to zero faster than the higher-order explicit term, that is,

$$\lim_{x \to a} \frac{g(x) - T_r(x)}{(x-a)^r} = 0$$

For the statistical application of Taylor's Theorem, we are most concerned with the first-order Taylor polynomial, that is, an approximation using just the first derivative.

Let $X_1, \ldots, X_k$ be random variables with means $\theta_1, \ldots, \theta_k$, and define $\mathbf{X} = (X_1, \ldots, X_k)$ and $\theta = (\theta_1, \ldots, \theta_k)$. Suppose there is a differentiable function $g(X)$ (an estimator of some parameter) for which we want an approximate estimate of variance.

Define

$$g_i^{'}(\theta) = \frac{\partial}{\partial x_i} g(x) |_{x_1=\theta_1, \ldots, x_k=\theta_k}$$

The first-order Taylor polynomial expansion of g about θ is

$$g(x) = g(\theta) + \sum_{i=1}^{k} g_i^{'}(\theta)(x_i - \theta_i) + \mathrm{Re}\,mainder.$$

For our statistical approximation we approximate $g(x)$ as

$$g(x) \approx g(\theta) + \sum_{i=1}^{k} g_i^{'}(\theta)(x_i - \theta_i).$$

Now, we take expectations on both sides of above approximation to get

$$E_\theta(g(X)) \approx g(\theta) + \sum_{i=1}^{k} g_i^{'}(\theta) E_\theta(X_i - \theta_i) = g(\theta)$$

We can now approximation the variance of $g(X)$ by

$$Var_\theta(g(X)) \approx E_\theta[g(X) - g(\theta_i)]^2$$

$$\approx E_\theta\left(\sum_{i=1}^{k} g_i^{'}(\theta)(X_i - \theta_i)\right)^2$$

$$\approx \sum_{i=1}^{k} [g_i'(\theta)]^2 Var X_i + 2\sum_{i>j} g_i'(\theta) g_j'(\theta) Cov_\theta(X_i, X_j)$$

This approximation is very useful because it gives a variance formula using only simple variance and covariance.

2.3. Variance-Covariance Matrix of the Estimators

The risk ratio (*RR*) in our problem is given by *RR* = *P*(Secondary Infection | Primary Infection) / *P*(Primary Infection) = $(p_{11}/p_{1\bullet})/ p_{1\bullet} = p_{11}/p_{1\bullet}^2 = p_{11}/(p_{11} + p_{12})^2$. Let $RR = \varphi$ for the purpose of typing convenient.

Since φ is a function of p_{11} and $p_{1\bullet}$, we shall derive the variance-covariance matrix of p_{11} and $p_{1\bullet}$.

$$Var(\hat{p}_{11}) = Var(n_{11} / n) = Var(n_{11}) / n^2 = np_{11}(1 - p_{11}) / n^2 = p_{11}(1 - p_{11}) / n$$

Similarly, $Var(\hat{p}_{1\bullet}) = p_{1\bullet}(1 - p_{1\bullet}) / n$.

$$Cov(\hat{p}_{11}, \hat{p}_{1\bullet}) = Cov(\hat{p}_{11}, \hat{p}_{11} + \hat{p}_{12}) = Var(\hat{p}_{11}) + Cov(\hat{p}_{11}, \hat{p}_{12})$$

$$= p_{11}(1 - p_{11})/n - p_{11}\, p_{12}/n$$

$$= p_{11}\, p_{22}/n.$$

Hence the variance-covariance matrix of $\hat{p}_{11}, \hat{p}_{1\bullet}$ is given by

$$\Sigma = \begin{bmatrix} p_{11}(1 - p_{11}) & p_{11} p_{22} \\ p_{11} p_{22} & p_{1\bullet}(1 - p_{1\bullet}) \end{bmatrix}$$

2.4. Confidence Interval by Wald's Test Statistic

Let $\hat{\varphi} = \frac{\hat{p}_{11}}{\hat{p}_{1\bullet}^2} = \frac{X_1}{X_2^2}$, then $E(X_1) = p_{11} = \theta_1$ and $E(X_2) = p_{1\bullet} = \theta_2$.

Let $g(x) = \dfrac{X_1}{X_2^2}$, then

$$g_1^{'}(x) = \left.\frac{1}{x_2^2}\right|_{x_1=\theta_1, x_2=\theta_2} = \frac{1}{p_{1\bullet}^2}$$

$$g_2^{'}(x) = \left.\frac{-2x_1}{x_2^3}\right|_{x_1=\theta_1, x_2=\theta_2} = \frac{-2p_{11}}{p_{1\bullet}^3}$$

The variance of $\hat{RR}$ is as follows:

$$Var(g(X)) = (\frac{1}{p_{1\bullet}^2})^2 Var(X_1) + (\frac{-2p_{11}}{p_{1\bullet}^3})^2 Var(X_2) + 2(\frac{1}{p_{1\bullet}^2})(\frac{-2p_{11}}{p_{1\bullet}^3})Cov(X_1, X_2)$$

$$= \frac{1}{p_{1\bullet}^4}\frac{p_{11}(1-p_{11})}{n} + \frac{4p_{11}}{p_{1\bullet}^6}\frac{p_{1\bullet}(1-p_{1\bullet})}{n} - \frac{4p_{11}}{p_{1\bullet}^5}\frac{p_{11}p_{22}}{n}$$

$$= \frac{1}{n(p_{1\bullet}^4)}\left[p_{11}(1-p_{11}) + \frac{4p_{11}^2}{p_{1\bullet}}(1-p_{1\bullet}) - \frac{4p_{11}^2 p_{22}}{p_{1\bullet}}\right]$$

$$= \frac{1}{n(p_{1\bullet}^4)}\left[p_{11}(1-p_{11}) + \frac{4p_{11}^2}{p_{1\bullet}}(1-p_{11}-p_{12}) - \frac{4p_{11}^2(1-p_{11}-p_{12})}{p_{1\bullet}}\right]$$

$$= \frac{p_{11}(1-p_{11})}{n(p_{1\bullet}^4)}.$$

Thus, the variance of $(n)^{1/2}(\hat{RR} - RR)$ is $\dfrac{p_{11}(1-p_{11})}{p_{1\bullet}^4}$, denoted as var_1. We can estimate this variance by using $\hat{\text{var}}_1 = \hat{p}_{11}(1-\hat{p}_{11})/\hat{p}_{1\bullet}^4$.

The asymptotic (1-α)100% confidence interval for RR is then

$$\hat{\varphi} \mp \sqrt{\frac{\hat{p}_{11}(1-\hat{p}_{11})}{n(\hat{p}_{1\bullet}^4)}} Z_{\alpha/2}.$$

(1-α)100% confidence interval is (d_l, d_u), where

$$d_l = Max(\hat{\varphi} - \sqrt{\frac{\hat{p}_{11}(1-\hat{p}_{11})}{n(\hat{p}_{1\bullet}^4)}} Z_{\alpha/2}, 0)$$

$$d_u = \hat{\varphi} + \sqrt{\frac{\hat{p}_{11}(1-\hat{p}_{11})}{n(\hat{p}_{1\bullet}^4)}} Z_{\alpha/2}$$

2.5. Confidence Interval by Logarithmic Transformation

When n is small, the normal approximation of the sampling distribution of $\hat{RR}$ may not be accurate enough to allow the interval estimator by the above method to perform well. To alleviate this problem, Liu (1998) applied logarithmic transformation on *RR*, which had been successfully applied in interval estimation of the risk ratio for cohort studies (Katz et al., 1978; Liu, 1995).

Define $f_2(X_1, X_2) = \log(X_1 / X_2^2)$ Note that $f_2(\hat{p}_{11}, \hat{p}_{1.}) = \log(\hat{RR})$

$$\ln RR = \ln \varphi = \ln(\frac{p_{11}}{p_{1\bullet}^2})$$

$$\ln \hat{\varphi} = \ln(\frac{\hat{p}_{11}}{\hat{p}_{1\bullet}^2}) = \ln \hat{p}_{11} - 2\ln \hat{p}_{1\bullet}$$

$$\therefore g(X) = \ln \hat{\varphi} = \ln \hat{p}_{11} - 2\ln \hat{p}_{1\bullet} = \ln X_1 - 2\ln X_2$$

$$E(X_1) = p_{11} = \theta_1 \qquad E(X_2) = p_{1\bullet} = \theta_2$$

$$g_1'(x) = \left.\frac{1}{x_1}\right|_{x_1=\theta_1, x_2=\theta_2} = \frac{1}{p_{11}}$$

$$g_2'(x) = \left.\frac{-2}{x_2}\right|_{x_1=\theta_1, x_2=\theta_2} = \frac{-2}{p_{1\bullet}}$$

The variance of $\hat{RR}$ is as follows:

$$Var(g(X)) = (\frac{1}{p_{11}^2})^2 Var(X_1) + \frac{4}{p_{1\bullet}^2} Var(X_2) + 2(\frac{1}{p_{11}})(\frac{-2}{p_{11}}) Cov(X_1, X_2)$$

$$= \frac{1}{p_{11}^2} \frac{p_{11}(1-p_{11})}{n} + \frac{4}{p_{1\bullet}^2} \frac{p_{1\bullet}(1-p_{1\bullet})}{n} - \frac{4}{p_{11}p_{1\bullet}} \frac{p_{11}p_{22}}{n}$$

$$= \frac{1}{p_{11}} \frac{(1-p_{11})}{n} + \frac{4}{p_{1\bullet}} \frac{(1-p_{1\bullet})}{n} - \frac{4}{p_{1\bullet}} \frac{(1-p_{1\bullet})}{n}$$

$$= \frac{1-p_{11}}{n(p_{11})}$$

$$\mathrm{var}_2 = (1-p_{11})/n(p_{11})$$

$$\therefore \hat{\mathrm{var}}_2 = (1-\hat{p}_{11})/n(\hat{p}_{11})$$

$$\sqrt{n}(\ln(\hat{\varphi}) - \ln(\varphi)) \sim N(0, \mathrm{var}_2)$$

$$\ln(\hat{\varphi}) - \ln(\varphi) \sim N(0, \mathrm{var}_2/n)$$

Therefore, the C.I. for *ln(RR)* is

$$\ln(\hat{\varphi}) \mp \sqrt{\frac{1-\hat{p}_{11}}{n(\hat{p}_{11})}} Z_{\alpha/2}$$

Thus, $$r_l = \ln(\hat{\varphi}) - \sqrt{\frac{1-\hat{p}_{11}}{n(\hat{p}_{11})}} Z_{\alpha/2}$$

$$r_u = \ln(\hat{\varphi}) + \sqrt{\frac{1-\hat{p}_{11}}{n(\hat{p}_{11})}} Z_{\alpha/2}$$

C.I. for *ln(RR)* is (r_l, r_u), and C.I. for *RR* is (e^{r_l}, e^{r_u}).

2.6. Confidence Interval by Fieller's Theorem

Following Fieller's theorem (Casella and Berger, 2001), Liu (1998) defined that

$$Z = \hat{p}_{11} - RR(n\hat{p}_{1\bullet}^2 - \hat{p}_{1\bullet})/(n-1)$$

Then $\sqrt{n}Z$ has asymptotic normal distribution with mean = 0 and variance = var_3 by use of the delta method and the central limit theorem again.

$$E\left(\frac{n\hat{p}_{1\bullet}^2 - \hat{p}_{1\bullet}}{n-1}\right) = \frac{1}{n-1}[nE(\hat{p}_{1\bullet}^2) - E(\hat{p}_{1\bullet})]$$

$$= \frac{1}{n-1}[n\{Var(\hat{p}_{1\bullet}) + (E(\hat{p}_{1\bullet}))^2\} - p_{1\bullet}]$$

$$= \frac{1}{n-1}[n\{\frac{p_{1\bullet}(1-p_{1\bullet})}{n} + p_{1\bullet}^2\} - p_{1\bullet}]$$

$$= \frac{1}{n-1}[p_{1\bullet}(1-p_{1\bullet}) + np_{1\bullet}^2 - p_{1\bullet}] = p_{1\bullet}^2$$

$$Z = \hat{p}_{11} - \varphi(n\hat{p}_{1\bullet}^2 - \hat{p}_{1\bullet})/(n-1)$$

$$E(Z) = p_{11} - \varphi p_{1\bullet}^2 = 0$$

Let $\hat{p}_{11} = X_1$, $\hat{p}_{1\bullet} = X_2$ and $g(x_1, x_2) = Z = x_1 - \varphi(nx_2^2 - x_2)/(n-1)$, then

$$g_1'(x) = 1$$

$$g_2'(x) = \left.\frac{-\varphi(2nx_2 - 1)}{n-1}\right|_{x_1=\theta_1, x_2=\theta_2} = \frac{-\varphi(2np_{1\bullet} - 1)}{n-1}$$

$$Var(g(X)) = (1)^2 Var(X_1) + \frac{\varphi^2(2np_{1\bullet} - 1)^2}{(n-1)^2} Var(X_2)$$

$$+ 2(1)(\frac{-\varphi(2np_{1\bullet} - 1)}{n-1})Cov(X_1, X_2)$$

$$= (1)^2 Var(\hat{p}_{11}) + \frac{\varphi^2(2np_{1\bullet} - 1)^2}{(n-1)^2} Var(\hat{p}_{1\bullet})$$

$$+ 2(1)(\frac{-\varphi(2np_{1\bullet} - 1)}{n-1})Cov(\hat{p}_{11}, \hat{p}_{1\bullet})$$

$$= \frac{p_{11}(1-p_{11})}{n} + \frac{\varphi^2(2np_{1\bullet}-1)^2}{(n-1)^2}\frac{p_{1\bullet}(1-p_{1\bullet})}{n}$$

$$- \frac{2\varphi(2np_{1\bullet}-1)}{n-1}\frac{p_{11}p_{22}}{n}$$

$$= \frac{1}{n}[p_{11}(1-p_{11}) + \frac{\varphi^2(2np_{1\bullet}-1)^2 p_{1\bullet}(1-p_{1\bullet})}{(n-1)^2}$$

$$- \frac{2\varphi(2np_{1\bullet}-1)p_{11}p_{22}}{n-1}]$$

Thus, the variance of $\sqrt{n}Z$, var_3, is as follows:

$$p_{11}(1-p_{11}) + \frac{\varphi^2(2np_{1\bullet}-1)^2 p_{1\bullet}(1-p_{1\bullet})}{(n-1)^2} - \frac{2\varphi(2np_{1\bullet}-1)p_{11}p_{22}}{n-1}.$$

Thus, for large n

$$P[Z_{\alpha/2} \le \frac{Z-0}{\sqrt{\text{var}(g(X))}} \le -Z_{\alpha/2}] = 1-\alpha$$

$$P[Z^2 \le \text{var}(g(X))Z^2_{\alpha/2}] = 1-\alpha$$

$$P\{[(\hat{p}_{11} - \varphi(n\hat{p}_{1\bullet}^2 - \hat{p}_{1\bullet})/(n-1)]^2 \le \text{var}(g(X))Z^2_{\alpha/2}\} = 1-\alpha$$

$$P\{[(\hat{p}_{11} - \varphi(n\hat{p}_{1\bullet}^2 - \hat{p}_{1\bullet})/(n-1)]^2 \le \frac{1}{n}[\hat{p}_{11}(1-\hat{p}_{11}) + \frac{\varphi^2(2n\hat{p}_{1\bullet}-1)^2 \hat{p}_{1\bullet}(1-\hat{p}_{1\bullet})}{(n-1)^2}$$

$$- \frac{2\varphi(2n\hat{p}_{1\bullet}-1)\hat{p}_{11}\hat{p}_{22}}{n-1}]Z^2_{\alpha/2}\} = 1-\alpha$$

$$P\{\varphi^2[\frac{(n\hat{p}_{1\bullet}^2 - \hat{p}_{1\bullet})^2}{(n-1)^2} - \frac{(2n\hat{p}_{1\bullet}-1)^2 \hat{p}_{1\bullet}(1-\hat{p}_{1\bullet})}{n(n-1)^2}Z^2_{\alpha/2}]$$

$$- \varphi[2\hat{p}_{11}\frac{n\hat{p}_{1\bullet}^2 - \hat{p}_{1\bullet}}{n-1} - \frac{2(2n\hat{p}_{1\bullet}-1)\hat{p}_{11}\hat{p}_{22}}{n(n-1)}Z^2_{\alpha/2}]$$

$$+ [\hat{p}_{11}^2 - \frac{1}{n}\hat{p}_{11}(1-\hat{p}_{11})Z^2_{\alpha/2}]\} = 1-\alpha$$

The above can be written as a quadratic inequality: $A(RR^2) + B(RR) + C \le 0$, where

$$A = \frac{(n\hat{p}_{1\bullet}^2 - \hat{p}_{1\bullet})^2}{(n-1)^2} - \frac{(2n\hat{p}_{1\bullet} - 1)^2 \hat{p}_{1\bullet}(1-\hat{p}_{1\bullet})}{n(n-1)^2} Z_{\alpha/2}^2$$

$$B = 2\hat{p}_{11}\frac{n\hat{p}_{1\bullet}^2 - \hat{p}_{1\bullet}}{n-1} - \frac{2(2n\hat{p}_{1\bullet} - 1)\hat{p}_{11}\hat{p}_{22}}{n-1} Z_{\alpha/2}^2$$

$$C = \hat{p}_{11}^2 - \frac{1}{n}\hat{p}_{11}(1-\hat{p}_{11})Z_{\alpha/2}^2$$

If both $A > 0$ and $B^2 - 4AC > 0$, then the asymptotic $1-\alpha$ confidence interval of RR for large n is given by $[f_l, f_u]$, where $f_l = \max[(-B - (B^2 - 4AC)^{1/2})/(2A), 0]$ and $f_u = [(-B + (B^2 - 4AC)^{1/2})/(2A)$.

2.7. Confidence Interval by Rao's Score Test Statistic

Suppose that $X_1, \ldots, X_n$ are a random sample from a distribution with p.d.f. $f(x; \boldsymbol{\theta})$, where $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_k)'$ is a vector of unknown parameters taking on value in a set **S**. Let $L(\boldsymbol{\theta})$ be the likelihood function for $\boldsymbol{\theta}$, then $L(\theta) = \prod_{i=1}^{n} f(x_i;\theta)$. Let $\hat{\theta}$ be a point in set **S** at which $L(\boldsymbol{\theta})$ is maximized; then $\hat{\theta}$ is the maximum likelihood estimate of $\boldsymbol{\theta}$. It is usually obtained by solving the following maximum likelihood equations.

$$U_i(\theta) = \frac{\partial \log L(\theta)}{\partial \theta_i} = 0, \text{ where } i = 1, \ldots, k.$$

The $U_i(\boldsymbol{\theta})$'s are called scores, and the k x 1 vector $U(\boldsymbol{\theta}) = [U_1(\boldsymbol{\theta}), \ldots, U_k(\boldsymbol{\theta})]^T$ is called the score vector (Lawless, 1982). $U(\boldsymbol{\theta})$ has mean **0** and covariance matrix $\boldsymbol{I}(\boldsymbol{\theta})$, with entries

$$I_{ij}(\theta) = E\left(\frac{-\partial^2 \log L(\theta)}{\partial \theta_i \partial \theta_j}\right) \text{ where } i, j = 1, \ldots, k.$$

The matrix $\boldsymbol{I}(\boldsymbol{\theta})$ is called the Fisher information matrix. The matrix $\boldsymbol{I}_0$, with entries

$$I_{0,ij}(\theta) = \left. \frac{-\partial^2 \log L(\theta)}{\partial \theta_i \partial_j} \right|_{\theta=\hat{\theta}}$$

is a consistent estimator of $\boldsymbol{I}(\boldsymbol{\theta})$ under mild conditions (Lawless, 1982).

In addition, $\boldsymbol{U}(\boldsymbol{\theta})$ is asymptotically distributed as $N_k[\mathbf{0}, \boldsymbol{I}(\boldsymbol{\theta})]$. Therefore, under the hypothesis H_0: $\boldsymbol{\theta} = \boldsymbol{\theta}_0$, $U^T(\theta_0) I(\theta_0)^{-1} U(\theta_0)$ is asymptotically distributed as $\chi^2_{(k)}$. We can use it to test H_0: $\boldsymbol{\theta} = \boldsymbol{\theta}_0$ and to obtain confidence interval of $\boldsymbol{\theta}$.

In our case, the p.d.f. $f(x;\, \boldsymbol{\theta})$ is as follows:

$$f(n_{11}, n_{12}; \varphi, p_{11}) = \frac{n!}{n_{11}! n_{12}! (n - n_{11} - n_{12})!} p_{11}^{n_{11}} \left(\sqrt{\frac{p_{11}}{\varphi}} - p_{11}\right)^{n_{12}} \left(1 - \sqrt{\frac{p_{11}}{\varphi}}\right)^{n - n_{11} - n_{12}}$$

The likelihood function for *RR* is

$$\ln L(\varphi, p_{11}) = C + n_{11} \ln p_{11} + n_{12} \ln\left(\sqrt{\frac{p_{11}}{\varphi}} - p_{11}\right) + (n - n_{11} - n_{12}) \ln\left(1 - \sqrt{\frac{p_{11}}{\varphi}}\right)$$

$$U_1(\varphi, p_{11}) = \frac{\partial \ln L(\theta)}{\partial \varphi} = \frac{n_{12}}{\sqrt{\frac{p_{11}}{\varphi}} - p_{11}} \left(\sqrt{\frac{p_{11}}{\varphi}} - p_{11}\right)' + \frac{n - n_{11} - n_{12}}{1 - \sqrt{\frac{p_{11}}{\varphi}}} \left(1 - \sqrt{\frac{p_{11}}{\varphi}}\right)'$$

$$= \frac{-n_{12}}{2\varphi(1 - \sqrt{\varphi p_{11}})} + \frac{(n - n_{11} - n_{12})\sqrt{p_{11}}}{2\varphi(\sqrt{\varphi} - \sqrt{p_{11}})}$$

$$U_2(\varphi, p_{11}) = \frac{\partial \ln L(\theta)}{\partial p_{11}} = \frac{n_{11}}{p_{11}} + \frac{n_{12}}{\sqrt{\frac{p_{11}}{\varphi}} - p_{11}} \left(\sqrt{\frac{p_{11}}{\varphi}} - p_{11}\right)' + \frac{n - n_{11} - n_{12}}{1 - \sqrt{\frac{p_{11}}{\varphi}}} \left(1 - \sqrt{\frac{p_{11}}{\varphi}}\right)'$$

$$= \frac{n_{11}}{p_{11}} + \frac{n_{12}(1 - 2\sqrt{\varphi p_{11}})}{2p_{11}(1 - \sqrt{\varphi p_{11}})} - \frac{n - n_{11} - n_{12}}{2\sqrt{p_{11}}(\sqrt{\varphi} - \sqrt{p_{11}})}$$

The entries of the Fisher information matrix are as follows:

$$I_{ij}(\theta) = E(\frac{-\partial^2 \log L(\theta)}{\partial\theta_i \partial_j \theta_j}), \text{ where } i, j = 1, \ldots, 2.$$

$$I_{11}(\theta) = E(\frac{-\partial^2 \log L(\varphi, p_{11})}{\partial^2 \varphi})$$

$$= \frac{n_{12}(2 - 3\sqrt{\varphi p_{11}})}{4\varphi^2(1 - \sqrt{\varphi p_{11}})^2} - \frac{(n - n_{11} - n_{12})(3\sqrt{\varphi p_{11}} - 2p_{11})}{4\varphi^2(\sqrt{\varphi} - \sqrt{p_{11}})^2}$$

$$I_{12}(\theta) = E(\frac{-\partial^2 \log L(\varphi, p_{11})}{\partial \varphi \partial p_{11}})$$

$$= \frac{-n_{12}}{4\sqrt{\varphi p_{11}}(1 - \sqrt{\varphi p_{11}})^2} + \frac{n - n_{11} - n_{12}}{4\sqrt{\varphi p_{11}}(\sqrt{\varphi} - \sqrt{p_{11}})^2}$$

$$I_{21}(\theta) = E(\frac{-\partial^2 \log L(\varphi, p_{11})}{\partial p_{11} \partial \varphi}) = I_{12}(\theta)$$

$$I_{22}(\theta) = E(\frac{-\partial^2 \log L(\varphi, p_{11})}{\partial^2 p_{11}})$$

$$= \frac{n_{12}(2 - 3\sqrt{\varphi p_{11}})}{4\varphi^2(1 - \sqrt{\varphi p_{11}})^2} - \frac{(n - n_{11} - n_{12})(3\sqrt{\varphi p_{11}} - 2p_{11})}{4\varphi^2(\sqrt{\varphi} - \sqrt{p_{11}})^2}$$

Thus, the information matrix is given by

$$I(\theta) = \begin{bmatrix} I_{11}(\theta) & I_{12}(\theta) \\ I_{21}(\theta) & I_{22}(\theta) \end{bmatrix}$$

and its inverse matrix is given by

$$I^{-1}(\theta) = \begin{bmatrix} I^{11}(\theta) & I^{12}(\theta) \\ I^{21}(\theta) & I^{22}(\theta) \end{bmatrix}$$

$$= \frac{1}{I_{11}(\theta) I_{22}(\theta) - I_{12}(\theta) I_{21}(\theta)} \begin{bmatrix} I_{22}(\theta) & -I_{21}(\theta) \\ -I_{12}(\theta) & I_{11}(\theta) \end{bmatrix}$$

Therefore, for testing the hypothesis H_0: $RR = RR_0$,

$$U_1(\varphi, p_{11})^T I(\varphi, p_{11})^{-1} U_1(\varphi, p_{11})$$

is asymptotically distributed as $\chi^2_{\alpha,1}$.

The Chi-square test statistic is

$$T_s = U_1(\varphi_0, \hat{p}_{11})^T I(\varphi_0, \hat{p}_{11})^{-1} U_1(\varphi_0, \hat{p}_{11})$$

where,

$$U_1(\varphi_0, \hat{p}_{11}) = \frac{-n_{12}}{2\varphi_0(1-\sqrt{\varphi\hat{p}_{11}})} + \frac{(n-n_{11}-n_{12})\sqrt{\hat{p}_{11}}}{2\varphi_0(\sqrt{\varphi_0}-\sqrt{\hat{p}_{11}})}$$

$$I_{11}(\varphi_0, \hat{p}_{11})^{-1} = \frac{I_{22}(\theta)}{I_{11}(\theta)I_{22}(\theta) - I_{12}(\theta)I_{21}(\theta)} = \frac{(1-\hat{p}_{11})\varphi^2{}_0}{n\hat{p}_{11}}$$

Therefore, the test statistic is given by

$$T_s = \left(\frac{-n_{12}}{2\varphi_0(1-\sqrt{\varphi_0\hat{p}_{11}})} + \frac{(n-n_{11}-n_{12})\sqrt{\hat{p}_{11}}}{2\varphi_0(\sqrt{\varphi_0}-\sqrt{\hat{p}_{11}})}\right)^2 \frac{(1-\hat{p}_{11})\varphi_0^2}{n\hat{p}_{11}}$$

To obtain the CI of RR, we will solve the following equation for RR.

$$\left(\frac{-n_{12}}{2\varphi(1-\sqrt{\varphi\hat{p}_{11}})} + \frac{(n-n_{11}-n_{12})\sqrt{\hat{p}_{11}}}{2\varphi(\sqrt{\varphi}-\sqrt{\hat{p}_{11}})}\right)^2 \frac{(1-\hat{p}_{11})\varphi^2}{n\hat{p}_{11}} \le \chi^2_{\alpha,1}$$

or

$$\left(\frac{-n_{12}(\sqrt{\varphi}-\sqrt{\hat{p}_{11}}) + (n-n_{11}-n_{12})(1-\sqrt{\varphi\hat{p}_{11}})\sqrt{\hat{p}_{11}}}{2(1-\sqrt{\varphi\hat{p}_{11}})(\sqrt{\varphi}-\sqrt{\hat{p}_{11}})}\right)^2 \frac{(1-\hat{p}_{11})}{n\hat{p}_{11}} \le \chi^2_{\alpha,1}$$

or

$$\left\{\frac{-\sqrt{\varphi}[n_{12} + (n-n_{11}-n_{12})p_{11}] + (n-n_{11})\sqrt{\hat{p}_{11}}}{2(1-\sqrt{\varphi\hat{p}_{11}})(\sqrt{\varphi}-\sqrt{\hat{p}_{11}})}\right\}^2 \frac{(1-\hat{p}_{11})}{n\hat{p}_{11}} \le \chi^2_{\alpha,1}$$

or

$$\{-\sqrt{\varphi}[n_{12}+(n-n_{11}-n_{12})p_{11}]+(n-n_{11})\sqrt{\hat{p}_{11}}\}^2$$

$$-4(1-\sqrt{\varphi\hat{p}_{11}})^2(\sqrt{\varphi}-\sqrt{\hat{p}_{11}})^2\frac{n\hat{p}_{11}\chi^2_{\alpha,1}}{(1-\hat{p}_{11})}\leq 0$$

or

$$4(1-\sqrt{\varphi\hat{p}_{11}})^2(\sqrt{\varphi}-\sqrt{\hat{p}_{11}})^2\frac{n\hat{p}_{11}\chi^2_{\alpha,1}}{(1-\hat{p}_{11})}$$

$$-\{-\sqrt{\varphi}[n_{12}+(n-n_{11}-n_{12})p_{11}]+(n-n_{11})\sqrt{\hat{p}_{11}}\}^2\geq 0$$

This leads us to solve the following two quadratic equations for $\sqrt{RR}$ as forms

$$A_1\varphi+B_1\sqrt{\varphi}+C_1=0 \quad and \quad A_2\varphi+B_2\sqrt{\varphi}+C_2=0$$

$$2\varphi\hat{p}_{11}\sqrt{\frac{n\chi^2_{\alpha,1}}{1-\hat{p}_{11}}}-\sqrt{\varphi}\{\sqrt{\frac{4n\hat{p}_{11}\chi^2_{\alpha,1}}{(1-\hat{p}_{11})}}(1+\hat{p}_{11})+n_{12}+(n-n_{11}-n_{12})p_{11}\}$$

..................(2.1)

$$+2\hat{p}_{11}\sqrt{\frac{n\chi^2_{\alpha,1}}{1-\hat{p}_{11}}}+(n-n_{11})\sqrt{\hat{p}_{11}}=0$$

$$2\varphi\hat{p}_{11}\sqrt{\frac{n\chi^2_{\alpha,1\alpha,1}}{1-\hat{p}_{11}}}-\sqrt{\varphi}\{\sqrt{\frac{4n\hat{p}_{11}\chi^2_{\alpha,1}}{(1-\hat{p}_{11})}}(1+\hat{p}_{11})-n_{12}-(n-n_{11}-n_{12})p_{11}\}$$

..................(2.2)

$$+2\hat{p}_{11}\sqrt{\frac{n\chi^2_{\alpha,1}}{1-\hat{p}_{11}}}-(n-n_{11})\sqrt{\hat{p}_{11}}=0$$

Suppose there are four distinctive roots as $RR_1 > RR_2 > RR_3 > RR_4$, then CI of RR will be $RR_2 - max(RR_3, 0)$. If there are only three distinctive roots, then CI of RR is 0. It is impossible that there are only two distinctive roots or one roots since $A_1 = A_2 > 0$, $B_1 > B_2$, and $C_1 < C_2$ unless $n_{11} = n$ or $n_{11} = 0$. In the case $n_{11} = n$, Rao's score method cannot apply to obtain CI of RR because the denominators of above expressions will be zero. In the case $n_{11} = 0$, all the methods are not applicable. Therefore, we apply the commonly used adjustment for sparse data in the contingency table analysis by adding 0.5 to each n_{ij} to avoid this limitation.

2.8. Example

To illustrate above four methods, we consider the calves' example again (Agresti 2002). 156 calves were born in Okeechobee County, Florida. Calves are first classified according to whether they get a pneumonia infection within 60 days after birth. They are then classified again according to whether they get a secondary infection within 2 weeks after the first infection clears up. We have $n_{11} = 30$, $n_{12} = 63$, and $n_{22} = 63$. With these given data, the estimate of risk ratio $\hat{RR}$ is 0.541. Applying interval estimators developed previously, we obtain the 95% confidence intervals of RR as in Table 2.1.

Table 2.3. 95% Confidence Intervals of *RR* for Four Methods in Calves Example

Method	C.I.	Length of C.I.
Wald	[0.367, 0.715]	0.342
Log	[0.392, 0.746]	0.354
Fieller	[0.381, 0.746]	0.365
Score	[0.464, 0.660]	0.196

From this table we can see that score method gets the shortest length. Since all upper limits of resulting confidence intervals are less than 1, the primary infection does generate a natural immunity to reduce the likelihood of a secondary infection.

Chapter 3

SIMULATION STUDIES TO EVALUATE THE PERFORMANCE OF THE FOUR CONFIDENCE INTERVALS

3.1. Generation of the Data

In order to evaluate and compare the performance of the four methods, described earlier, in constructing confidence intervals of RR, we have written SAS programs to generate data sets with different parameter combinations. Then, for each method, we calculate the average length of the confidence intervals and the coverage probability.

We selected three sample size n = 50, 100, 200, four primary infection rate $p_{1.}$ = 0.2, 0.3, 0.5, 0.8, and four values of the risk ratio RR = 0.25, 0.5, 1.0, 1.5 for generating data.

Notice that having $p_{1.}$ and RR (φ) one can obtain p_{11}. Thus the parameters of the model become p_{11} and φ. We generate data set according to the following trinomial distribution:

$$f(n_{11}, n_{12}; RR, p_{11}) = \frac{n!}{n_{11}! n_{12}! (n - n_{11} - n_{12})!} p_{11}^{n_{11}} \left(\sqrt{\frac{p_{11}}{\varphi}} - p_{11}\right)^{n_{12}} \left(1 - \sqrt{\frac{p_{11}}{\varphi}}\right)^{n - n_{11} - n_{12}}$$

............(3.1)

For each parameter combination (n, $p_{1.}$, and RR), 10, 000 data sets were generated. Then we can estimate p_{11} for each data set. Next, we calculate the lower bound and upper bound of 95% CI for each data set. The length of 95% CI is the upper bound minus the lower bound.

The coverage probability of confidence interval is determined by the following way. First, calculate confidence interval. Then check whether the parameter *RR* was covered by the confidence interval. If the parameter *RR* equal or greater than the lower bound and equal or less than upper bound, then we say that *RR* was covered by the confidence interval. Otherwise, *RR* was not covered by the confidence interval. Count the number that *RR* was covered by the confidence interval for all generated data sets for each parameter combination. Then the coverage probability is number of *RR* was covered by the confidence interval divided by simulation times for each parameter combination. In this study the simulation times are 10,000.

3.2. Results of Simulation

The primary results of the simulation study are displayed through Figure 3.2, Figure 3.3, and Table 3.1. From Figure 3.2 we can see that the lengths of confidence interval of Rao's score method is lowest among the four methods. However, the coverage probability (Figure 3.3) of Rao's score method is too low when comparing with other three methods.

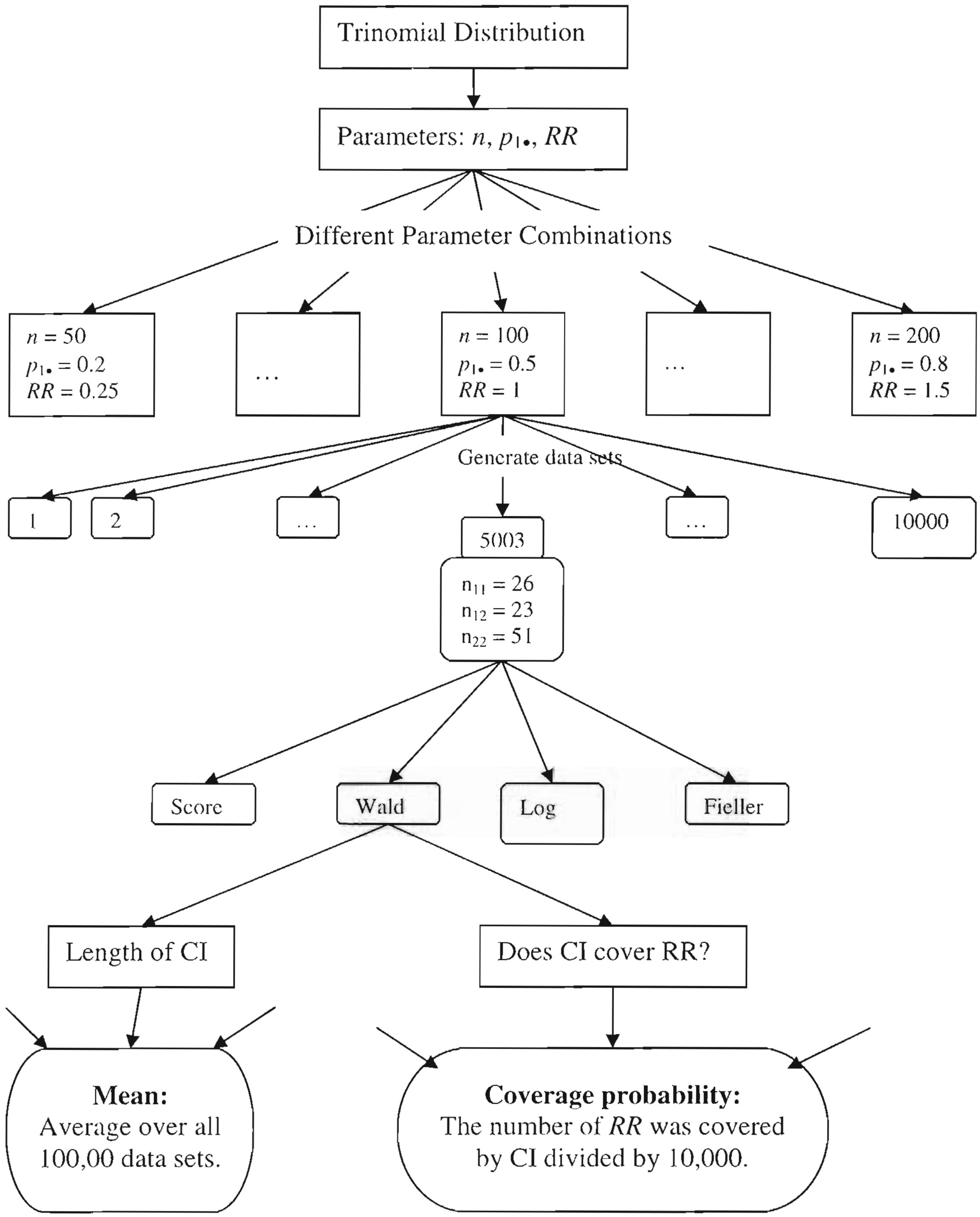

Figure 3.1. The Flow Chart of Simulation Study.

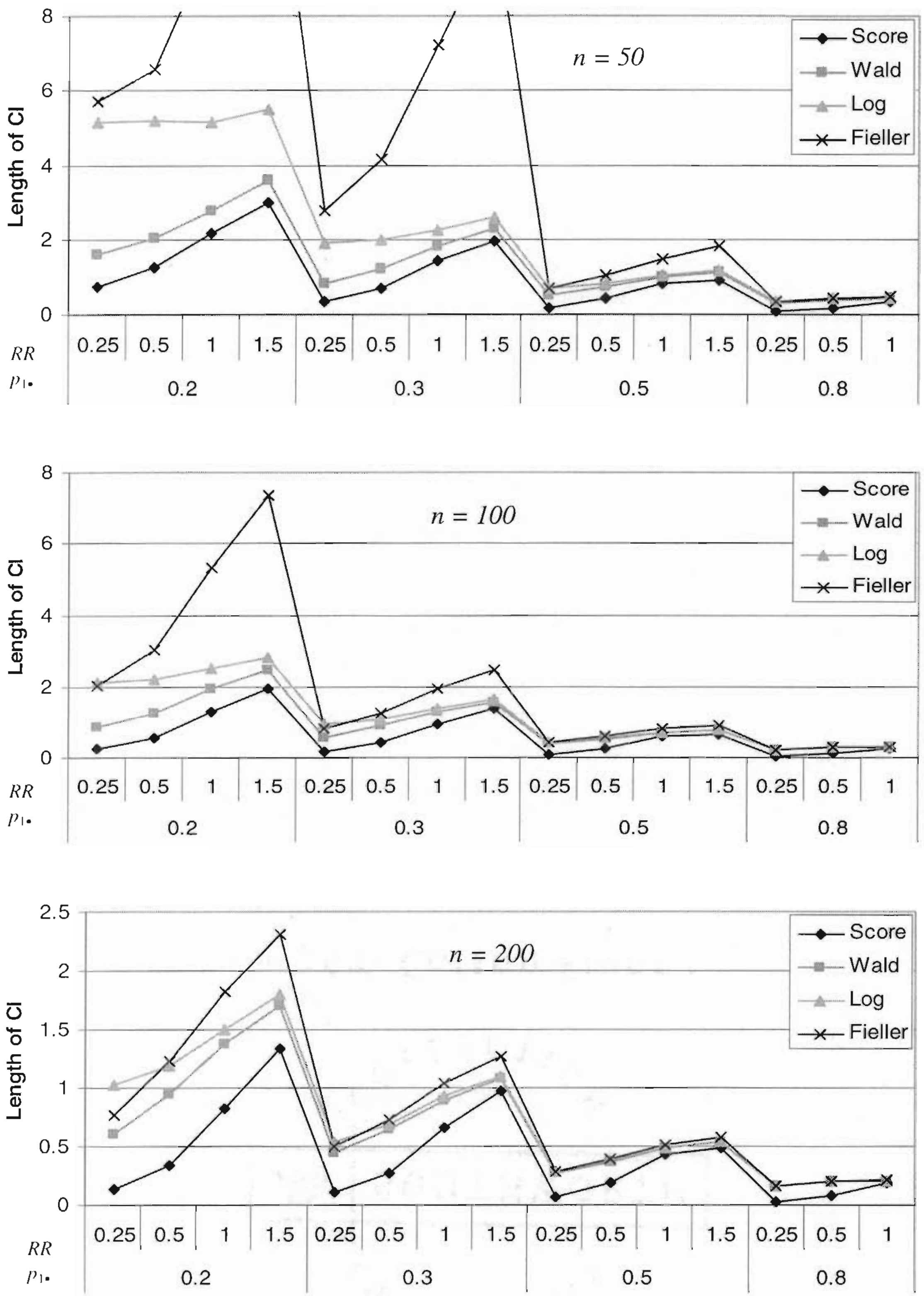

Figure 3.2. The Length of Confidence Interval of the Simulation Study.

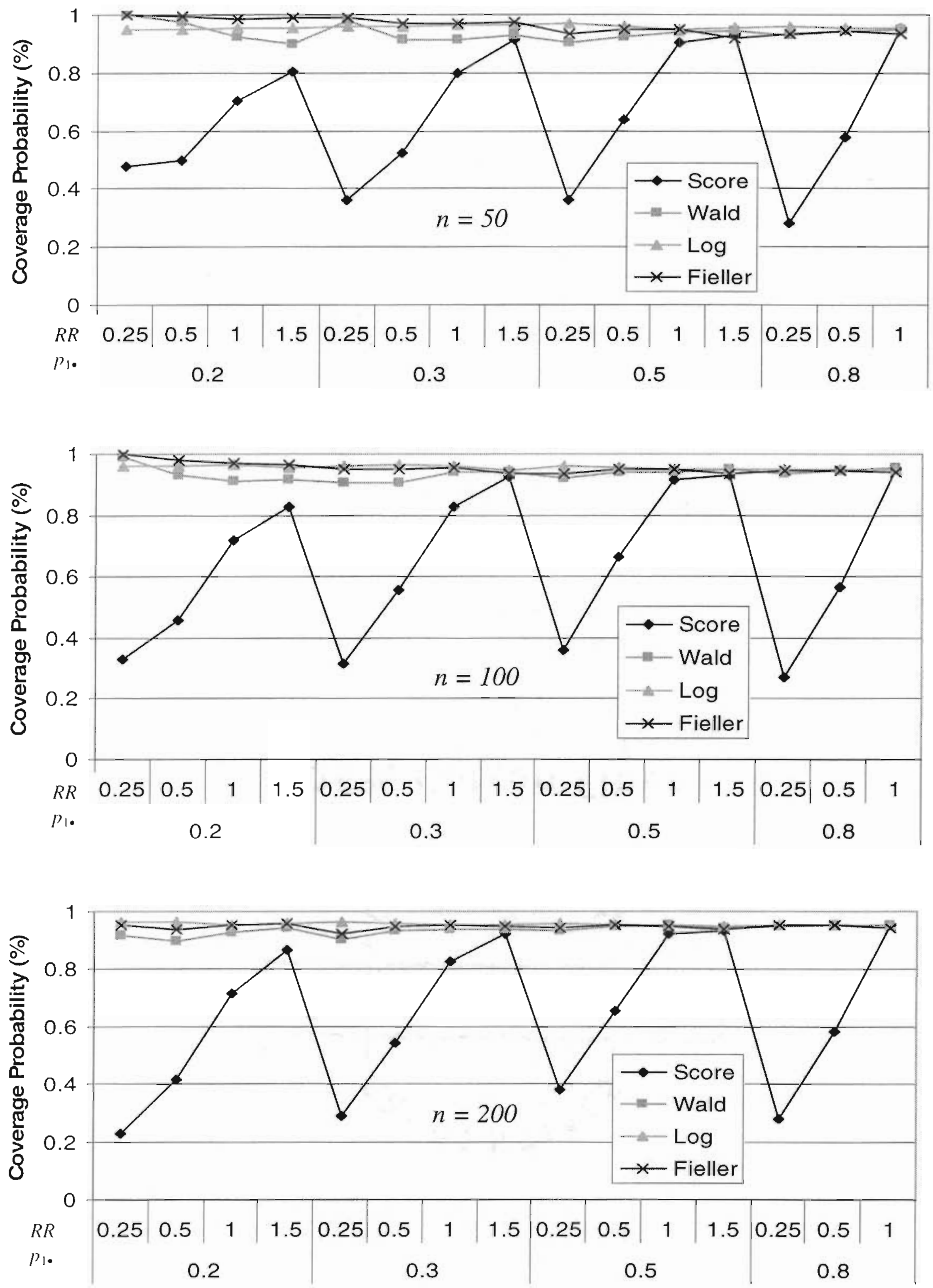

Figure 3.3. The Coverage Probability of the Simulation Study.

Table 3.1. The Coverage Probability and Length of the 95% Confidence Intervals for the Risk Ratio between a Secondary Infection, Given a Primary Infection, and the Primary Infection.

n	p1•	RR	Coverage Probability				Length of CI			
			Score	Wald	Log	Fieller	Score	Wald	Log	Fieller
50	0.2	0.25	0.475	0.999	0.951	0.999	0.751	1.602	5.159	5.716
		0.5	0.495	0.976	0.951	0.995	1.243	2.011	5.179	6.575
		1	0.702	0.926	0.956	0.987	2.150	2.771	5.147	10.091
		1.5	0.806	0.900	0.957	0.989	2.967	3.582	5.501	14.254
	0.3	0.25	0.360	0.982	0.959	0.991	0.330	0.821	1.912	2.767
		0.5	0.524	0.915	0.961	0.972	0.713	1.210	1.995	4.150
		1	0.801	0.916	0.965	0.970	1.434	1.828	2.241	7.218
		1.5	0.915	0.928	0.966	0.976	1.943	2.298	2.576	10.315
	0.5	0.25	0.364	0.904	0.969	0.933	0.173	0.505	0.691	0.684
		0.5	0.640	0.926	0.961	0.950	0.426	0.743	0.831	1.039
		1	0.904	0.938	0.952	0.949	0.838	0.989	1.032	1.465
		1.5	0.930	0.947	0.953	0.919	0.894	1.121	1.148	1.816
	0.8	0.25	0.282	0.928	0.962	0.937	0.067	0.313	0.337	0.330
		0.5	0.576	0.946	0.956	0.947	0.194	0.404	0.416	0.429
		1	0.956	0.949	0.955	0.933	0.344	0.419	0.422	0.457
100	0.2	0.25	0.331	0.991	0.963	0.998	0.267	0.869	2.116	2.042
		0.5	0.456	0.930	0.962	0.982	0.579	1.259	2.210	3.036
		1	0.719	0.910	0.967	0.971	1.296	1.939	2.496	5.320
		1.5	0.830	0.916	0.956	0.964	1.960	2.449	2.809	7.355
	0.3	0.25	0.316	0.908	0.962	0.953	0.171	0.580	0.935	0.812
		0.5	0.555	0.908	0.964	0.951	0.418	0.889	1.081	1.270
		1	0.829	0.942	0.961	0.958	0.965	1.287	1.386	1.931
		1.5	0.924	0.943	0.947	0.938	1.380	1.554	1.626	2.470
	0.5	0.25	0.358	0.920	0.963	0.936	0.107	0.376	0.423	0.421
		0.5	0.665	0.943	0.955	0.953	0.278	0.523	0.549	0.590
		1	0.916	0.942	0.950	0.952	0.605	0.692	0.707	0.801
		1.5	0.933	0.950	0.953	0.936	0.670	0.775	0.784	0.923
	0.8	0.25	0.271	0.937	0.953	0.944	0.044	0.224	0.232	0.229
		0.5	0.565	0.947	0.953	0.946	0.128	0.285	0.289	0.293
		1	0.950	0.954	0.952	0.942	0.262	0.295	0.296	0.307
200	0.2	0.25	0.230	0.918	0.965	0.954	0.137	0.604	1.030	0.775
		0.5	0.414	0.901	0.967	0.940	0.341	0.942	1.190	1.226
		1	0.717	0.927	0.955	0.954	0.830	1.378	1.505	1.830
		1.5	0.868	0.946	0.958	0.959	1.333	1.701	1.797	2.317
	0.3	0.25	0.289	0.904	0.967	0.925	0.105	0.442	0.535	0.502
		0.5	0.542	0.933	0.959	0.951	0.272	0.642	0.692	0.735
		1	0.825	0.940	0.953	0.952	0.660	0.898	0.929	1.045
		1.5	0.924	0.941	0.952	0.947	0.978	1.075	1.099	1.273
	0.5	0.25	0.382	0.934	0.957	0.944	0.071	0.268	0.281	0.281
		0.5	0.655	0.950	0.956	0.954	0.188	0.368	0.377	0.389
		1	0.923	0.955	0.955	0.950	0.428	0.484	0.489	0.517
		1.5	0.932	0.946	0.948	0.937	0.491	0.542	0.545	0.586
	0.8	0.25	0.280	0.950	0.956	0.955	0.030	0.158	0.161	0.160
		0.5	0.586	0.952	0.954	0.953	0.087	0.202	0.203	0.204
		1	0.952	0.953	0.953	0.946	0.194	0.208	0.208	0.212

Chapter 4

HYPOTHESIS TEST

4.1. Wald Test Statistic

We have proved in Chapter 2 that $\sqrt{n}(\hat{\varphi}-\varphi)$ is asymptotic normal distribution with mean = 0 and variance = var_1 for large n. For the null hypothesis test H_0: $\varphi = \varphi_0$, the test statistic is:

$$T_1 = \sqrt{n}(\hat{\varphi}-\varphi)/\sqrt{\mathrm{var}_1} = \sqrt{n}\left(\frac{\hat{p}_{11}}{\hat{p}_{1.}^{\,2}} - \varphi_0\right)/\sqrt{\frac{\hat{p}_{11}(1-\hat{p}_{11})}{\hat{p}_{1.}^{\,4}}}$$

$$= \sqrt{n}\left(\frac{\hat{p}_{11}}{\hat{p}_{1.}^{\,2}} - \varphi_0\right)/\frac{\hat{p}_{1.}^{\,2}}{\sqrt{\hat{p}_{11}(1-\hat{p}_{11})}}$$

$$= \sqrt{n}\left(\frac{\frac{n_{11}}{n}}{\left(\frac{n_{11}+n_{12}}{n}\right)^2} - \varphi_0\right)/\frac{\left(\frac{n_{11}+n_{12}}{n}\right)^2}{\sqrt{\frac{n_{11}}{n}\left(1-\frac{n_{11}}{n}\right)}}$$

$$= \frac{nn_{11} - \varphi_0(n_{11}+n_{12})^2}{\sqrt{nn_{11}(n-n_{11})}}$$

If $T_1 > Z_{\alpha/2}$, or $T_1 < -Z_{\alpha/2}$, we will reject H_0 and accept H_1. Otherwise, we will accept H_0.

4.2. Logarithmic Transformation Test Statistic

For large n, $\sqrt{n}(\ln(\hat{\varphi}) - \ln(\varphi))$ is asymptotic normal distribution with mean = 0 and variance = var_2 (see Chapter 2). For the null hypothesis test H_0: $RR = RR_0$, the test statistic is:

$$T_2 = \sqrt{n}(\ln(\hat{\varphi}) - \ln(\varphi)) / \sqrt{\text{var}_2}$$

$$= \sqrt{n}(\ln(\frac{\hat{p}_{11}}{\hat{p}_{1.}^{\ 2}}) - \ln(\varphi_0)) / \sqrt{\frac{(1-\hat{p}_{11})}{\hat{p}_{11}}}$$

$$= \frac{\sqrt{n_{11}n}(\ln(nn_{11}) - 2\ln(n_{11}+n_{12}) - \ln(\varphi_0))}{\sqrt{n-n_{11}}}$$

If $T_2 > Z_{\alpha/2}$, or $T_2 < -Z_{\alpha/2}$, we will reject H_0 and accept H_1: $RR \neq RR_0$. Otherwise, we will accept H_0.

4.3. Fieller's Test Statistic

Once again, we have proved that $\sqrt{n}[\hat{p}_{11} - \varphi(n\hat{p}_{1.}^{\ 2} - \hat{p}_{11})/(n-1)]/\sqrt{\text{var}_3}$ is asymptotically normal distributed with mean = 0 and variance = var_3 for large n in Chapter 2. For the null hypothesis test H_0: $RR = RR_0$, the test statistic is

$$T_3 = \sqrt{n}[\hat{p}_{11} - \varphi_0(n\hat{p}_{1.}^{\ 2} - \hat{p}_{11})/(n-1)]/\sqrt{\text{var}_3}$$

$$= \sqrt{n}[\frac{n_{11}}{n} - \varphi_0(\frac{(n_{11}+n_{12})^2}{n} - \frac{n_{11}+n_{12}}{n})/(n-1)]/\sqrt{\text{var}_3}, \quad \textit{where}$$

$$\text{var}_3 = \hat{p}_{11}(1-\hat{p}_{11}) + \varphi_0^{\ 2}(2n\hat{p}_{11}-1)^2\hat{p}_{1.}(1-\hat{p}_{1.})/(n-1)^2 - 2\varphi_0(2n\hat{p}_{11}-1)\hat{p}_{11}\hat{p}_{22}/(n-1)$$

$$= \frac{n_{11}(n-n_{11})}{n^2} + \varphi_0^{\ 2}(2n_{11}+2n_{12}-1)^2\frac{(n_{11}+n_{12})(n-n_{11}-n_{12})}{n^2(n-1)^2} - 2\varphi_0\frac{n_{11}(n-n_{11}-n_{12})(2n_{11}+2n_{12}-1)}{n^2(n-1)}$$

If $T_3 > Z_{\alpha/2}$, or $T_3 < -Z_{\alpha/2}$, we will reject H_0 and accept H_1: $RR \neq RR_0$. Otherwise, we will accept H_0.

4.4. Score Test Statistic

It is showed that $U^T(\varphi, p_{11})I(\varphi, p_{11})^{-1}U(\varphi, p_{11})$ is asymptotically distributed as $\chi^2_{\alpha,1}$. For testing the hypothesis H_0: $RR = RR_0$, the test statistic is

$$T_4 = U^T(\varphi_0, \hat{p}_{11})I(\varphi_0, \hat{p}_{11})^{-1}U(\varphi_0, \hat{p}_{11})$$

where,

$$U(\varphi_0, \hat{p}_{11}) = \frac{-n_{12}}{2\varphi_0(1-\sqrt{\varphi\hat{p}_{11}})} + \frac{(n-n_{11}-n_{12})\sqrt{\hat{p}_{11}}}{2\varphi_0(\sqrt{\varphi_0}-\sqrt{\hat{p}_{11}})}$$

$$I(\varphi_0, \hat{p}_{11})^{-1} = \frac{I_{22}(\theta)}{I_{11}(\theta)I_{22}(\theta)-I_{12}(\theta)I_{21}(\theta)} = \frac{(1-\hat{p}_{11})\varphi^2{}_0}{n\hat{p}_{11}}$$

Therefore,

$$T_4 = \left(\frac{-n_{12}}{2\varphi_0(1-\sqrt{\varphi_0\hat{p}_{11}})} + \frac{(n-n_{11}-n_{12})\sqrt{\hat{p}_{11}}}{2\varphi_0(\sqrt{\varphi_0}-\sqrt{\hat{p}_{11}})}\right)^2 \frac{(1-\hat{p}_{11})\varphi^2{}_0}{n\hat{p}_{11}}$$

$$= \left(\frac{-n_{12}}{2(1-\sqrt{\varphi_0\hat{p}_{11}})} + \frac{(n-n_{11}-n_{12})\sqrt{\hat{p}_{11}}}{2(\sqrt{\varphi_0}-\sqrt{\hat{p}_{11}})}\right)^2 \frac{(1-\hat{p}_{11})}{n\hat{p}_{11}}.$$

If $T_4 > \chi^2_{\alpha,1}$, we will reject H_0 and accept H_1: $RR \neq RR_0$. Otherwise, we will accept H_0.

4.5. Example

For the calves example, we have $n = 156$, $n_{11} = 30$, $n_{12} = 63$, and $n_{22} = 63$. For the hypothesis test: H_0: $RR = 1$ vs. H_1: $RR \neq 1$, the four test statistics are as in Table 4.1.

Table 4.1. Test Statistic of Four Methods in Calves Example.

Method	p_{11}-D*		p_{11}-M	
	Test Statistic	p-value	Test Statistic	p-value
Wald	-5.169	1.18x10-7	N/A**	
Log	-3.743	9.10x10-5	N/A	
Fieller	-3.607	1.55x10-4	N/A	
Score	19.706	9.03x10-6	26.714	2.36x10-7

** We have two ways to estimate nuisance parameter* p_{11}. *One is direct estimate* p_{11} *by* $\hat{p}_{11} = n_{11}/n$, *symbolized by* p_{11}-D. *Another way is Maximum Likelihood Estimation (MLE) of* p_{11} *(Tang and Tang 2002), symbolized by* p_{11}-M.

***It is not applicable for Wald, Log, and Fieller methods in calculating test statistic when* p_{11}*-M is used. Since* $\hat{p}_{11}$ *is obtained by solving maximum likelihood equation. Then we use the formula*

$RR = \dfrac{p_{11}}{p_{1\bullet}^2}$ *to estimate* $p_{1\bullet}$. $\hat{p}_{1\bullet} = \sqrt{\hat{p}_{11}/RR_0}$. *Therefore,*

$$T_1 = \sqrt{n}(R\hat{R} - RR)/\sqrt{\text{var}_1} = \sqrt{n}(\frac{\hat{p}_{11}}{\hat{p}_{1.}^{\ 2}} - RR_0)/\sqrt{\text{var}_1} = 0$$

$$T_2 = \sqrt{n}(\ln(R\hat{R}) - \ln(RR))/\sqrt{\text{var}_2} = \sqrt{n}(\ln(\frac{\hat{p}_{11}}{\hat{p}_{1.}^{\ 2}}) - \ln(RR_0))/\sqrt{\text{var}_2} = 0$$

$$T_3 = \sqrt{n}[\hat{p}_{11} - RR_0(n\hat{p}_{1.}^{\ 2} - \hat{p}_{11})/(n-1)]/\sqrt{\text{var}_3}$$

$$= \sqrt{n}[\hat{p}_{11} - RR_0(n\frac{\hat{p}_{11}}{RR_0} - \hat{p}_{11})/(n-1)]/\sqrt{\text{var}_3} = 0 \quad when \quad RR_0 = 1.$$

Thus, no matter what kind of data, T_1, T_2, *and* T_3 *are always be zero if* $RR_0 = 1$.

Chapter 5

POWER OF THE TESTS AND SIMULATION RESULTS

5.1. Introduction

The power of a statistical test is the probability of rejecting the null hypothesis H_0 when the alternative hypothesis is true (Montgomery, Runger and Hubele. 2004).

We know that for any estimator of $\hat{\theta}$ *of* θ which satisfies

$$n^{1/2}(\hat{\theta}-\theta) \rightarrow N(0, g(\theta)^2)$$

An approximate α level Neyman-Pearson test can be constructed based on the critical region $\{z : n^{1/2} \mid \hat{\theta}-\theta_0 \mid / g(\theta_0) \geq c\}$, where α satisfies

$$\alpha = P\{n^{1/2} \frac{\mid \hat{\theta}-\theta_0 \mid}{g(\theta_0)^2} \geq c; H_0\} \rightarrow 2\{1-\Phi(c)\}$$

and $\Phi(\cdot)$ is the standard normal distribution function.

Welsh (1996) showed that under the local alternative hypotheses of the form H_1: $\theta = \theta_n = \theta_0 + \xi/n^{1/2}$, the power of test with critical region $\{z : n^{1/2} \mid \hat{\theta}-\theta_0 \mid / g(\theta_0) \geq c\}$ is

$$P\{n^{1/2} \frac{\mid \hat{\theta}-\theta_0 \mid}{g(\theta_0)} \geq c; H_1\} = P\{n^{1/2} \frac{\hat{\theta}-\theta_n}{g(\theta_n)} \geq -\frac{\xi}{g(\theta_n)} + \frac{g(\theta_0)c}{g(\theta_n)}; H_1\}$$

$$+ P\{n^{1/2} \frac{\hat{\theta}-\theta_n}{g(\theta_n)} \leq -\frac{\xi}{g(\theta_n)} - \frac{g(\theta_0)c}{g(\theta_n)}; H_1\}$$

provided $n^{1/2} \mid \hat{\theta}-\theta_n \mid / g(\theta_n)$ is asymptotically normal under H_1 and g is continuous and positive at g_0.

From this, we can develop the formula for calculating the power of first two tests in Chapter 4.

5.2. Power of Wald Test

We know that $\sqrt{n}(\hat{\varphi}-\varphi)$ is asymptotic normal distribution with mean = 0 and variance = var_1. For the hypothesis test H_0: $RR = RR_0$ *vs.* H_1: $RR = RR_n = RR_0 + \xi_1/n^{1/2}$, where $\xi_1 = n^{1/2}(RR_n - RR_0)$, according to Welsh (1996), the power of this test is:

$$P\{n^{1/2}\frac{|\hat{\varphi}-\varphi_0|}{\sqrt{\mathrm{var}_{10}}} \geq c; H_1\} = P\{n^{1/2}\frac{\hat{\varphi}-\varphi_0}{\sqrt{\mathrm{var}_{10}}} \geq c; H_1\} + P\{n^{1/2}\frac{\hat{\varphi}-\varphi_0}{\sqrt{\mathrm{var}_{10}}} \leq -c; H_1\}$$

$$= P\{n^{1/2}\frac{\hat{\varphi}-\varphi_n+\varphi_n-\varphi_0}{\sqrt{\mathrm{var}_{10}}} \geq c; H_1) + P\{n^{1/2}\frac{\hat{\varphi}-\varphi_n+\varphi_n-\varphi_0}{\sqrt{\mathrm{var}_{1n}}} \leq -c; H_1)$$

$$= P\{n^{1/2}\frac{\hat{\varphi}-\varphi_n}{\sqrt{\mathrm{var}_{10}}} + \frac{n^{1/2}(\varphi_n-\varphi_0)}{\sqrt{\mathrm{var}_{10}}} \geq c; H_1\} + P\{n^{1/2}\frac{\hat{\varphi}-\varphi_n}{\sqrt{\mathrm{var}_{10}}} + \frac{n^{1/2}(\varphi_n-\varphi_0)}{\sqrt{\mathrm{var}_{10}}} \leq -c; H_1\}$$

$$= P\{n^{1/2}\frac{\hat{\varphi}-\varphi_n}{\sqrt{\mathrm{var}_{1n}}}\frac{\sqrt{\mathrm{var}_{1n}}}{\sqrt{\mathrm{var}_{10}}} + \frac{\xi_1}{\sqrt{\mathrm{var}_{10}}} \geq c; H_1\} + P\{n^{1/2}\frac{\hat{\varphi}-\varphi_n}{\sqrt{\mathrm{var}_{10}}}\frac{\sqrt{\mathrm{var}_{1n}}}{\sqrt{\mathrm{var}_{10}}} + \frac{\xi_1}{\sqrt{\mathrm{var}_{10}}} \leq -c; H_1\}$$

$$= P\{n^{1/2}\frac{\hat{\varphi}-\varphi_n}{\sqrt{\mathrm{var}_{1n}}} \geq (c - \frac{\xi_1}{\sqrt{\mathrm{var}_{10}}})\frac{\sqrt{\mathrm{var}_{10}}}{\sqrt{\mathrm{var}_{1n}}}; H_1\} + P\{n^{1/2}\frac{\hat{\varphi}-\varphi_n}{\sqrt{\mathrm{var}_{10}}} \leq (-c - \frac{\xi_1}{\sqrt{\mathrm{var}_{10}}})\frac{\sqrt{\mathrm{var}_{10}}}{\sqrt{\mathrm{var}_{1n}}}; H_1\}$$

$$= P\{Z \geq \frac{c\sqrt{\text{var}_{10}} - \xi_1}{\sqrt{\text{var}_{1n}}}\} + P\{Z \leq -\frac{c\sqrt{\text{var}_{10}} + \xi_1}{\sqrt{\text{var}_{1n}}}\}$$

$$= 1 - \Phi(\frac{c\sqrt{\text{var}_{10}} - \xi_1}{\sqrt{\text{var}_{1n}}}) + \Phi(-\frac{c\sqrt{\text{var}_{10}} + \xi_1}{\sqrt{\text{var}_{1n}}})$$

5.3. Power of Logarithmic Transformation Test

$\sqrt{n}(\ln(\hat{\varphi}) - \ln(\varphi))$ is asymptotic normal distribution with mean = 0 and variance = var_2. The hypothesis test H_0: $RR = RR_0$ *vs.* H_1: $RR = RR_n$ is equivalent to the hypothesis test H_0: $\ln RR = \ln RR_0$ *vs.* H_1: $\ln RR = \ln RR_n = \ln RR_0 + \xi_2/n^{1/2}$, where $\xi_2 = n^{1/2}(\ln RR_n - \ln RR_0)$. The power of this test is:

$$P\{n^{1/2} \frac{|\ln\hat{\varphi} - \ln\varphi_0|}{\sqrt{\text{var}_{20}}} \geq c; H_1\}$$

$$= P\{n^{1/2} \frac{\ln\hat{\varphi} - \ln\varphi_n}{\sqrt{\text{var}_{2n}}} \geq (c - \frac{\xi_2}{\sqrt{\text{var}_{20}}})\frac{\sqrt{\text{var}_{20}}}{\sqrt{\text{var}_{2n}}}; H_1\}$$

$$+ P\{n^{1/2} \frac{\ln\hat{\varphi} - \ln\varphi_n}{\sqrt{\text{var}_{2n}}} \leq (-c - \frac{\xi_2}{\sqrt{\text{var}_{20}}})\frac{\sqrt{\text{var}_{20}}}{\sqrt{\text{var}_{2n}}}; H_1\}$$

$$= P\{Z \geq \frac{c\sqrt{\text{var}_{20}} - \xi_2}{\sqrt{\text{var}_{2n}}}\} + P\{Z \leq -\frac{c\sqrt{\text{var}_{20}} + \xi_2}{\sqrt{\text{var}_{2n}}}\}$$

$$= 1 - \Phi(\frac{c\sqrt{\text{var}_{20}} - \xi_2}{\sqrt{\text{var}_{2n}}}) + \Phi(-\frac{c\sqrt{\text{var}_{20}} + \xi_2}{\sqrt{\text{var}_{2n}}})$$

5.4. Power of Fieller Test and Rao's Score Test

Exact expressions for power of Fieller and Rao's score tests are difficult to obtain. We will use empirical power calculation instead of exact power calculation in these two cases in our simulation study for the power of the tests.

To compute empirical power with $\alpha = 0.05$, we first generate many data sets for each sample size and parameters combination. For each data set, we will calculate Fieller's and Score's test statistic under H_1. The proportion of test statistic great than $Z_{\alpha/2}(1.96)$ or less than - $Z_{\alpha/2}(-1.96)$ for Fieller test, and the proportion of test statistic great than $\chi^2_{(\alpha, 1)}$ (3.81) for Rao's score test represents the empirical power of these two test with type I error $\alpha = 0.05$, respectively.

5.5. Simulation Study

To compare the power of the four tests about RR in the Chapter 4, we will generate large number and variety of data sets according to trinomial distribution with different sample size and parameter combinations. We have selected three sample size n (50, 100, 200), four primary infection rate $p_{1.}$ (0.2, 0.3, 0.5, 0.8), and four risk ratio RR (0.25, 0.5, 1.0, 1.5) with four methods (Score, Wald, Log, and Fieler), two estimations of p_{11} (D and M) and two kind of power calculation methods (empirical power (E) and exact power(X)).

We first generated 10, 000 data sets for each combination of sample size and parameters. Then we will estimate p_{11} in two ways (p_{11}-D and p_{11}-M) for each data set. By using p_{11}-D we will calculate the empirical power for all four tests and exact power for Wald's test and logarithmic transformation test. Since we do not have the expression for power calculation for score test and Fieller's test, we can not calculate the exact power for these two tests. By using p_{11}-M we can only calculate the empirical power of score test because all other three tests are not applicable in p_{11}-M situation. We have shown that why all other three tests not applicable in M situation in the end of Chapter 4. See the process of simulation study (Figure 5.1).

5.5.1. Comparison Between Empirical Power and Exact Power

Figure 5.2. shows that empirical power and exact power are general match each other, especially for Wald test, those two power are very close to each other in most points. For the Log test, although the trend is same, the difference is also obvious.

5.5.2. Comparison Between Score-M-E and Score-D-E

Score-D-E does not perform well because it has unexpected high power at point $H_1 = 1$ (Figure 5.3.).

5.5.3. Comparison of Four Methods

The empirical power simulation study indicates that the power of score-M-E test is consistent than these of the other tests although it is not the most powerful test (Figure.5.4.-6.).

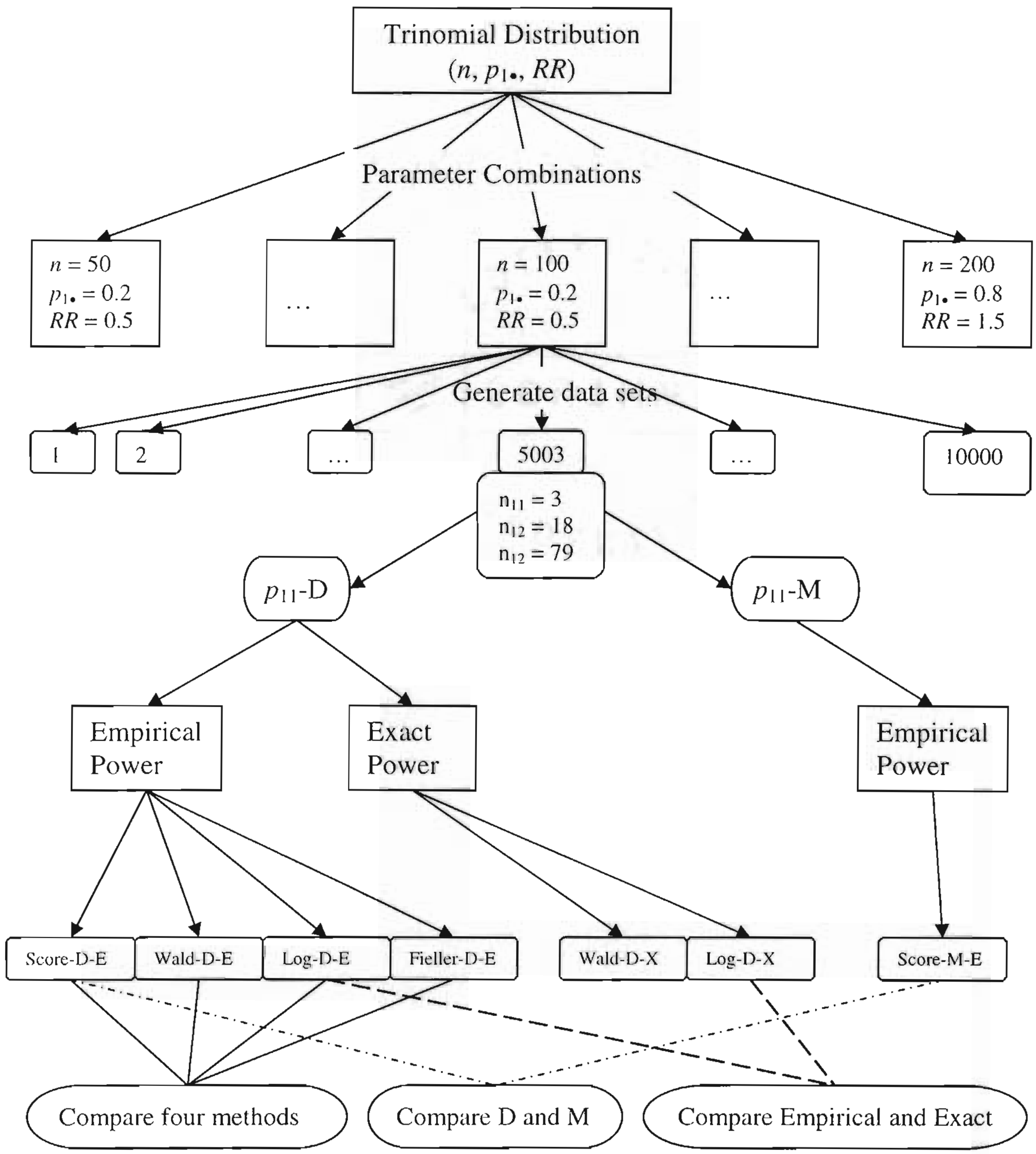

Figure 5.1. The Flow Chart of Simulation Study.

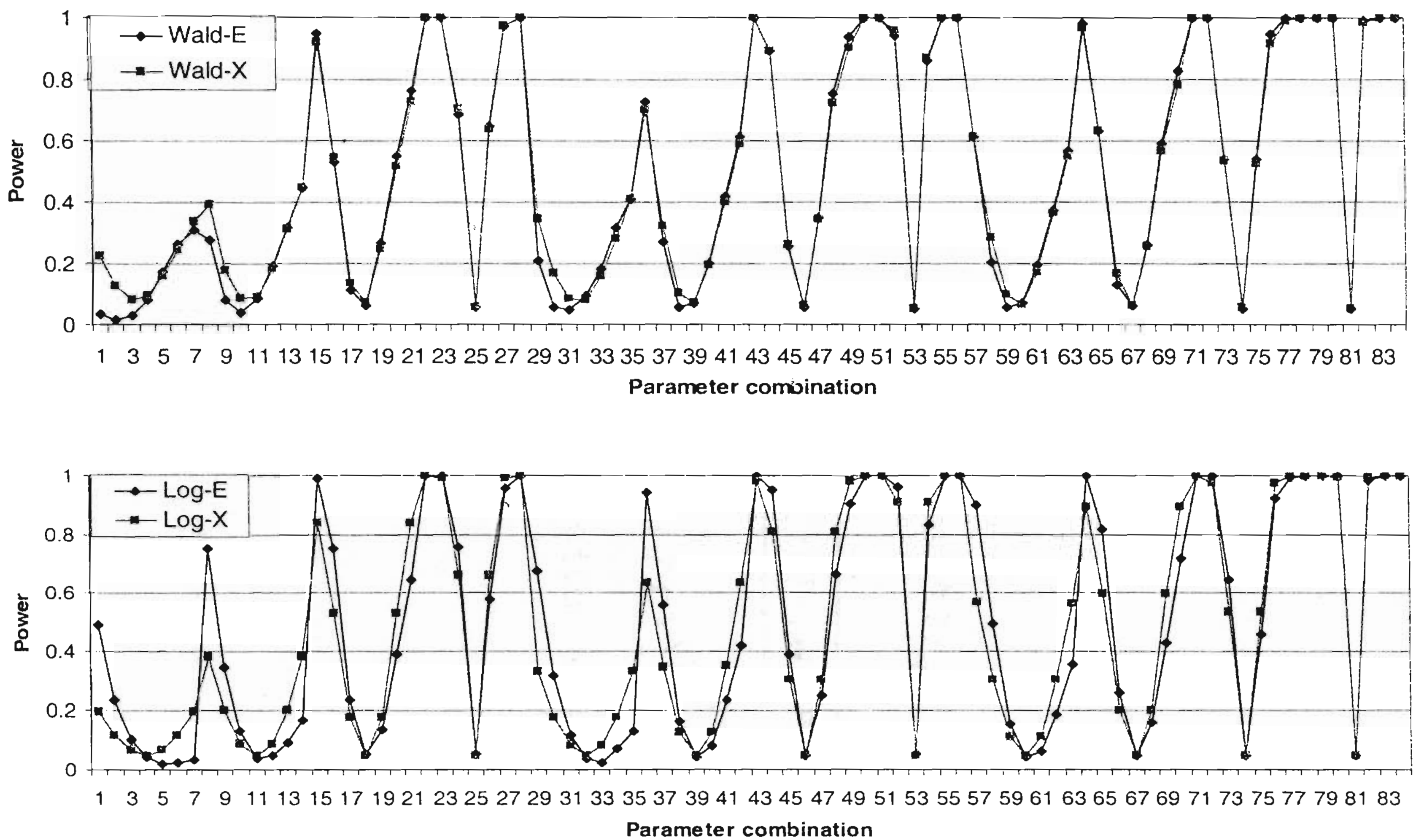

Figure 5.2. Comparison between Empirical Power and Exact Power.

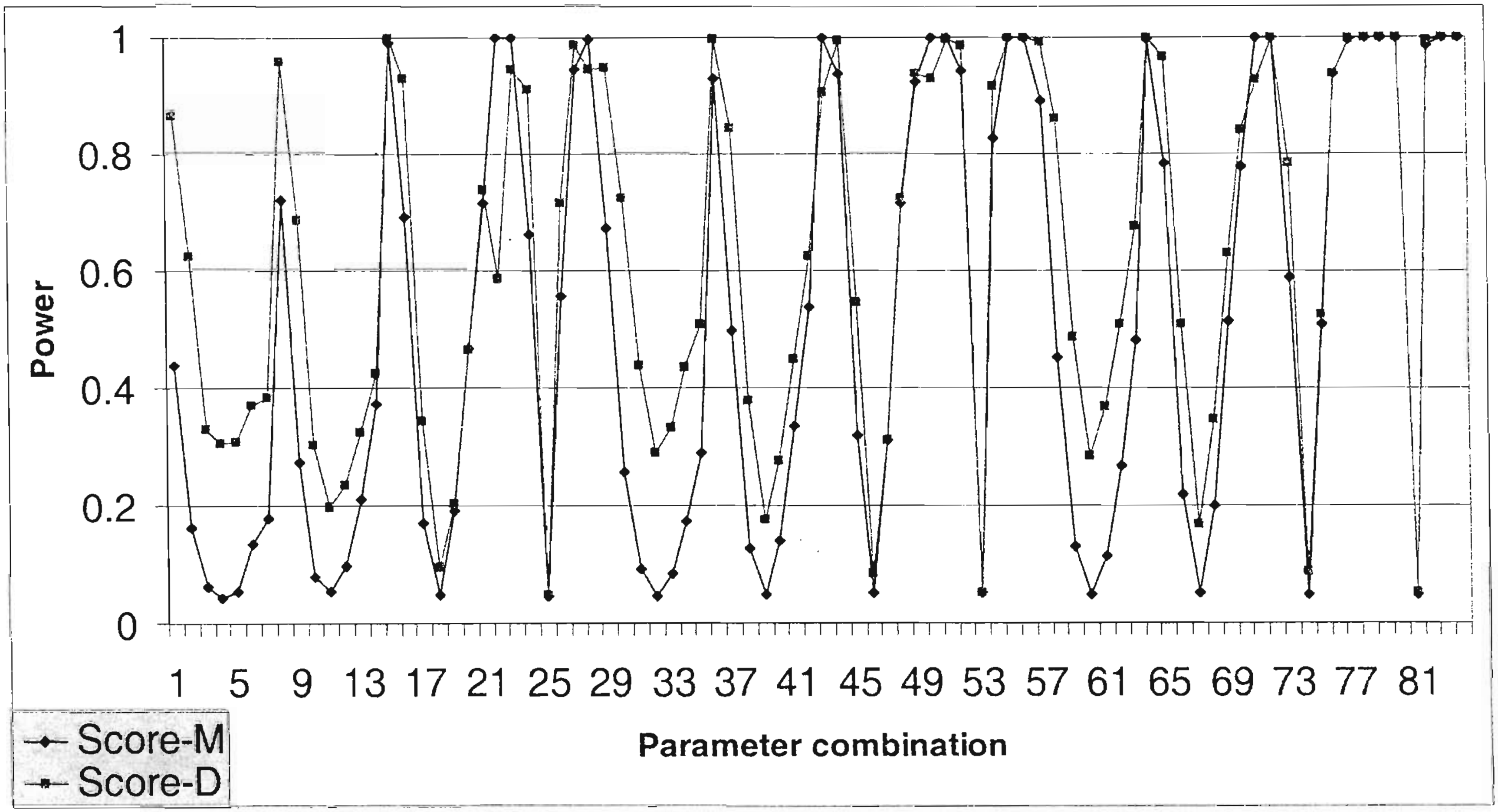

Figure 5.3. Comparison Power between Score-D and Score-M.

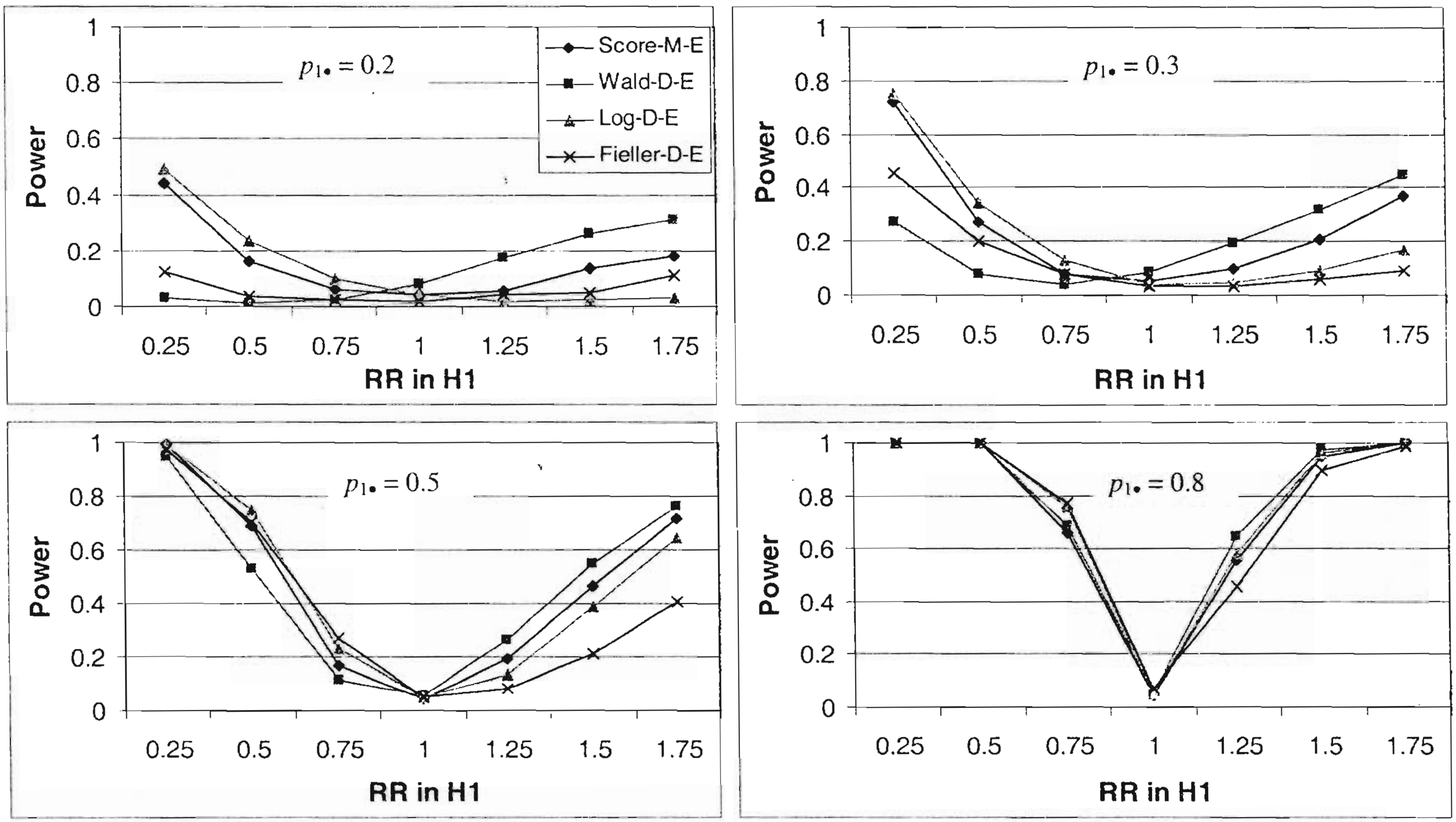

Figure 5.4. Empirical Power of Four Tests. Sample Size $n = 50$.

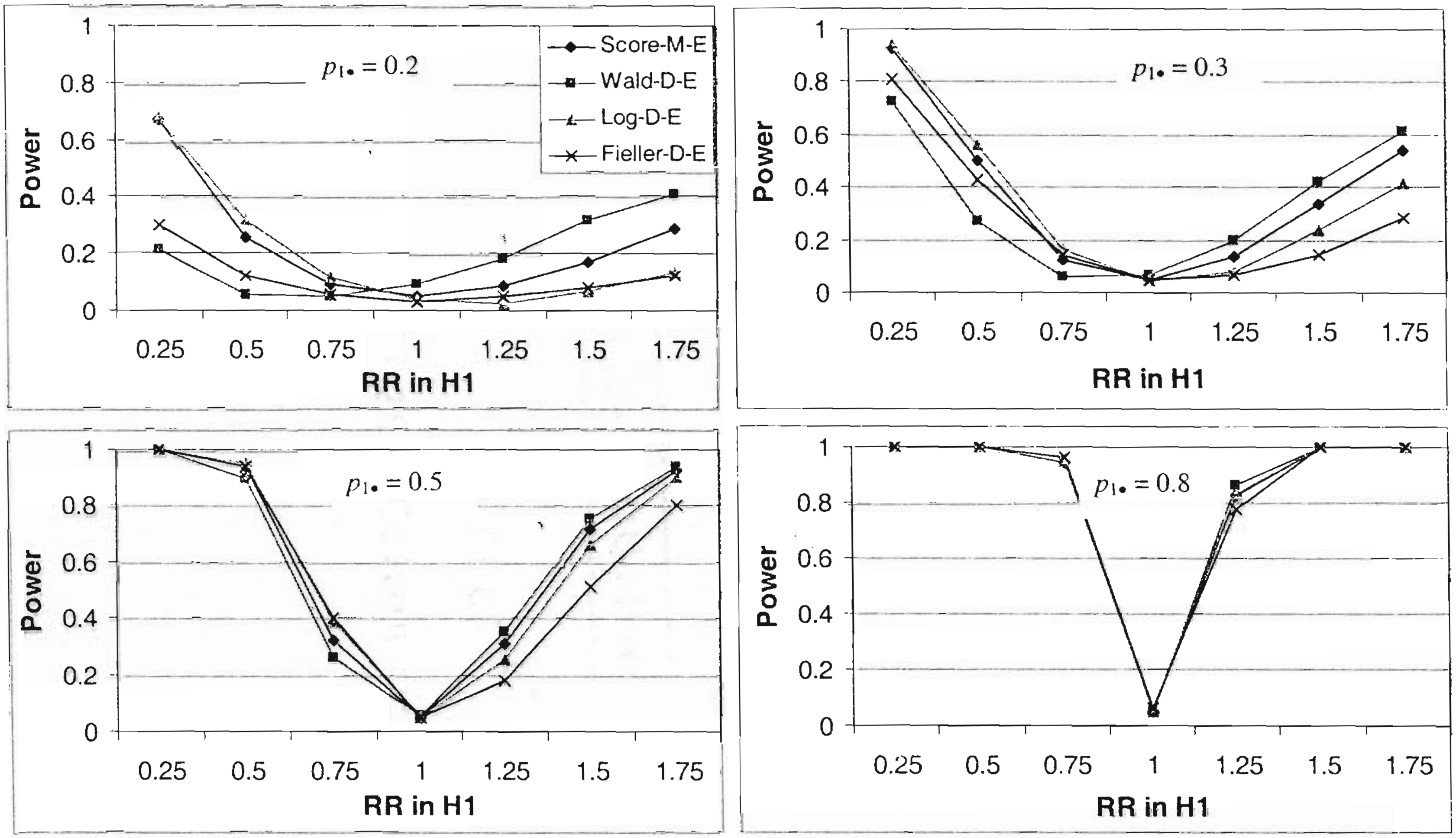

Figure 5.5. Empirical Power of Four Tests. Sample Size $n = 100$.

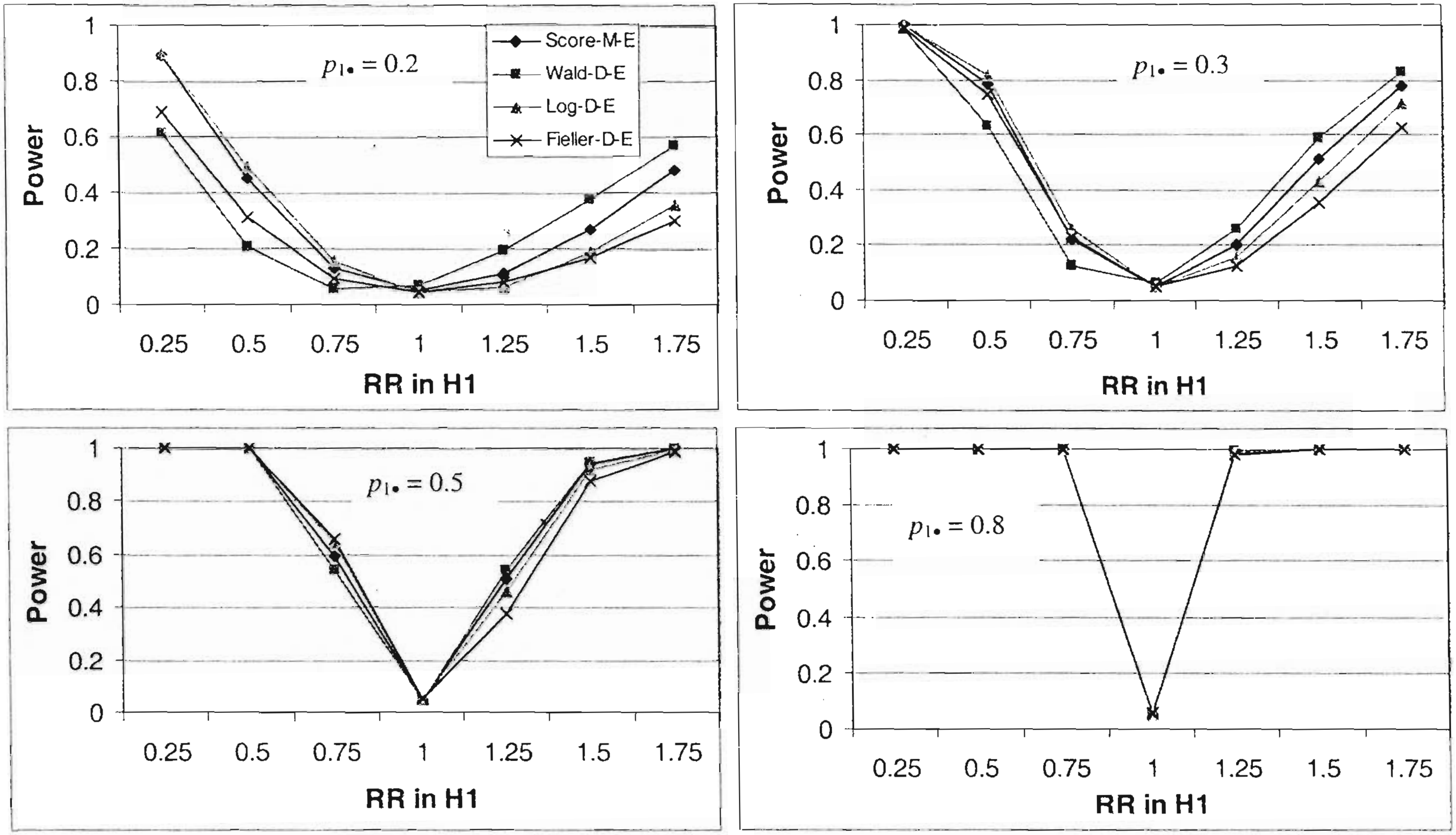

Figure 5.6. Empirical Power of Four Tests. Sample Size $n = 200$.

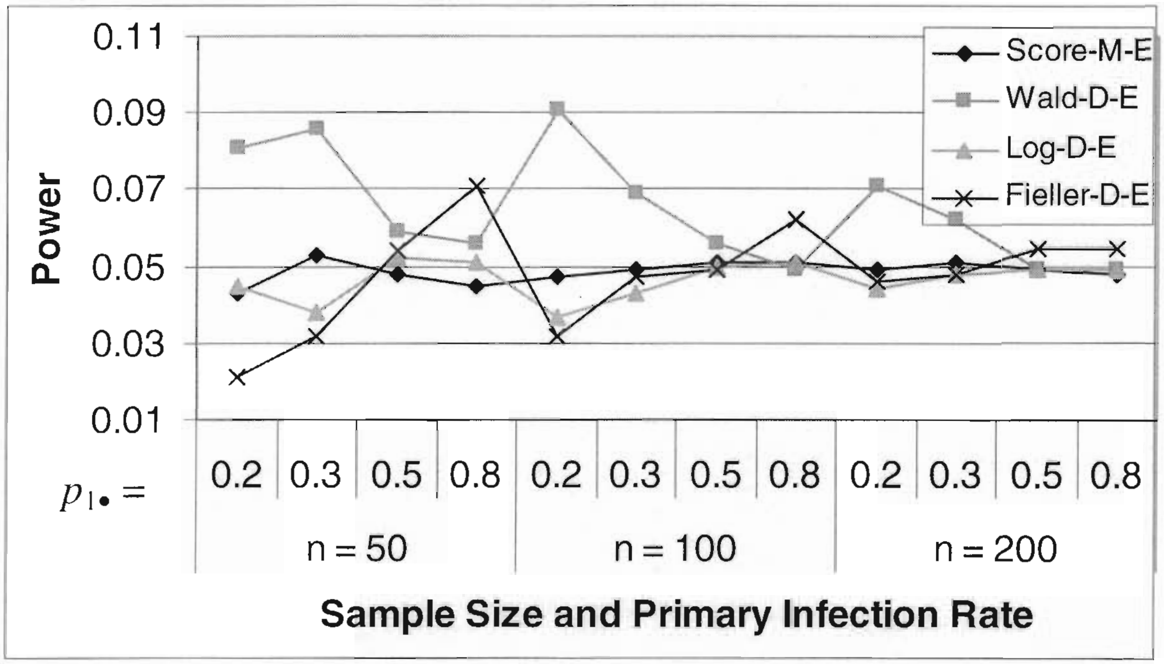

Figure 5.7. Empirical Power at the Points H_0: $RR = 1$ *vs.* H_1: $RR = 1$.

Table 5.1. The Power of the Tests---Simulation Results.

		n = 50					
$p_{1\bullet}$	RR in H1	Score-M-E	Wald-D-E	Log-D-E	Fieller-D-E	Wald-D-X	Log-D-X
0.2	0.25	0.438	0.031	0.492	0.126	0.223	0.198
0.2	0.5	0.162	0.014	0.237	0.040	0.125	0.115
0.2	0.75	0.062	0.027	0.101	0.023	0.079	0.066
0.2	1	0.043	0.081	0.045	0.021	0.094	0.050
0.2	1.25	0.055	0.174	0.020	0.044	0.157	0.066
0.2	1.5	0.135	0.260	0.022	0.048	0.243	0.115
0.2	1.75	0.178	0.309	0.032	0.113	0.337	0.198
0.3	0.25	0.721	0.273	0.752	0.454	0.389	0.381
0.3	0.5	0.274	0.079	0.347	0.199	0.177	0.201
0.3	0.75	0.079	0.038	0.131	0.077	0.084	0.087
0.3	1	0.053	0.086	0.038	0.032	0.089	0.050
0.3	1.25	0.097	0.193	0.047	0.034	0.180	0.087
0.3	1.5	0.210	0.316	0.091	0.056	0.311	0.201
0.3	1.75	0.373	0.448	0.168	0.092	0.445	0.381
0.5	0.25	0.991	0.950	0.992	0.974	0.922	0.838
0.5	0.5	0.691	0.531	0.750	0.701	0.544	0.531
0.5	0.75	0.169	0.113	0.235	0.272	0.136	0.179
0.5	1	0.048	0.059	0.052	0.054	0.069	0.050
0.5	1.25	0.191	0.266	0.134	0.081	0.247	0.179
0.5	1.5	0.467	0.549	0.389	0.210	0.518	0.529
0.5	1.75	0.715	0.763	0.644	0.408	0.727	0.837
0.8	0.25	1.000	1.000	1.000	1.000	1.000	1.000
0.8	0.5	1.000	1.000	1.000	1.000	1.000	0.991
0.8	0.75	0.661	0.684	0.756	0.772	0.701	0.660
0.8	1	0.045	0.056	0.051	0.071	0.054	0.050
0.8	1.25	0.556	0.648	0.579	0.456	0.637	0.660
0.8	1.5	0.947	0.971	0.959	0.893	0.972	0.991
0.8	1.75	0.998	0.999	0.999	0.988	0.999	1.000

Table 5.1. The Power of the Tests---Simulation Results (continued).

		n = 100					
$p_{1\bullet}$	**RR in H1**	**Score-M-E**	**Wald-D-E**	**Log-D-E**	**Fieller-D-E**	**Wald-D-X**	**Log-D-X**
0.2	0.25	0.673	0.211	0.675	0.296	0.344	0.331
0.2	0.5	0.258	0.054	0.315	0.122	0.168	0.177
0.2	0.75	0.091	0.046	0.116	0.056	0.082	0.081
0.2	1	0.047	0.091	0.037	0.032	0.077	0.050
0.2	1.25	0.084	0.181	0.026	0.048	0.157	0.081
0.2	1.5	0.172	0.316	0.070	0.077	0.277	0.176
0.2	1.75	0.288	0.407	0.132	0.122	0.410	0.331
0.3	0.25	0.929	0.724	0.940	0.807	0.700	0.636
0.3	0.5	0.497	0.269	0.560	0.429	0.320	0.348
0.3	0.75	0.128	0.057	0.165	0.143	0.101	0.124
0.3	1	0.049	0.069	0.043	0.047	0.071	0.050
0.3	1.25	0.141	0.200	0.080	0.066	0.195	0.124
0.3	1.5	0.335	0.419	0.234	0.145	0.401	0.349
0.3	1.75	0.537	0.614	0.416	0.281	0.589	0.636
0.5	0.25	1.000	1.000	1.000	1.000	0.999	0.983
0.5	0.5	0.939	0.894	0.953	0.941	0.895	0.809
0.5	0.75	0.319	0.257	0.390	0.404	0.260	0.305
0.5	1	0.051	0.056	0.050	0.049	0.060	0.050
0.5	1.25	0.311	0.349	0.252	0.176	0.344	0.305
0.5	1.5	0.716	0.754	0.663	0.514	0.721	0.810
0.5	1.75	0.924	0.939	0.902	0.805	0.904	0.983
0.8	0.25	1.000	1.000	1.000	1.000	1.000	1.000
0.8	0.5	1.000	1.000	1.000	1.000	1.000	1.000
0.8	0.75	0.942	0.944	0.960	0.962	0.960	0.908
0.8	1	0.051	0.049	0.051	0.062	0.052	0.050
0.8	1.25	0.827	0.861	0.833	0.775	0.871	0.908
0.8	1.5	0.999	1.000	0.999	0.997	0.999	1.000
0.8	1.75	1.000	1.000	1.000	1.000	1.000	1.000

Table 5.1. The Power of the Tests---Simulation Results (continued).

		n = 200					
$p_{1\bullet}$	RR in H1	Score-M-E	Wald-D-E	Log-D-E	Fieller-D-E	Wald-D-X	Log-D-X
0.2	0.25	0.892	0.614	0.897	0.688	0.612	0.565
0.2	0.5	0.452	0.205	0.495	0.312	0.283	0.302
0.2	0.75	0.129	0.054	0.155	0.093	0.097	0.112
0.2	1	0.049	0.071	0.044	0.046	0.065	0.050
0.2	1.25	0.113	0.196	0.063	0.080	0.171	0.112
0.2	1.5	0.267	0.372	0.188	0.170	0.363	0.302
0.2	1.75	0.480	0.566	0.357	0.300	0.549	0.563
0.3	0.25	0.997	0.982	0.998	0.990	0.967	0.896
0.3	0.5	0.784	0.632	0.816	0.744	0.634	0.596
0.3	0.75	0.219	0.129	0.262	0.229	0.167	0.200
0.3	1	0.051	0.062	0.048	0.048	0.060	0.050
0.3	1.25	0.200	0.260	0.157	0.128	0.254	0.200
0.3	1.5	0.514	0.589	0.430	0.353	0.568	0.597
0.3	1.75	0.779	0.826	0.718	0.629	0.780	0.895
0.5	0.25	1.000	1.000	1.000	1.000	1.000	1.000
0.5	0.5	0.999	0.998	1.000	1.000	0.999	0.978
0.5	0.75	0.590	0.538	0.642	0.655	0.537	0.532
0.5	1	0.049	0.049	0.049	0.055	0.056	0.050
0.5	1.25	0.507	0.538	0.458	0.376	0.524	0.532
0.5	1.5	0.938	0.947	0.921	0.874	0.914	0.978
0.5	1.75	0.998	0.998	0.996	0.990	0.989	1.000
0.8	0.25	1.000	1.000	1.000	1.000	1.000	1.000
0.8	0.5	1.000	1.000	1.000	1.000	1.000	1.000
0.8	0.75	0.999	0.999	0.999	1.000	1.000	0.996
0.8	1	0.048	0.049	0.049	0.055	0.051	0.050
0.8	1.25	0.987	0.991	0.987	0.979	0.987	0.996
0.8	1.5	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.75	1.000	1.000	1.000	1.000	1.000	1.000

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Suzhong Tian was born in Yutai, Shendong, China on June 5, 1963, and was raised in Genhe, Inner Mongolia, China. He graduated from Fourth Genhe High School in 1982. He studied Forestry at Beijing Forestry University from 1982 to 1989, and received a Bachelor of Science degree in Forest Protection in 1986 and a Master of Science degree in Forest Pathology in 1989. Then he was employed by the Beijing Forestry Bureau, Beijing until 1998 when he left for USA for further studies. He first studied Silviculture in the Department of Forest Ecosystem Science, the University of Maine from 1998 to 2002, and received a Doctor of Philosophy degree in Forest Resources in 2002.

Suzhong then entered the graduate program in the Department of Mathematics and Statistics in September 2001, and served as a teaching assistant and statistical consultant from September 2001 to August 2004. He is a member of Pi Mu Epsilon, mathematical honor society. Suzhong is a candidate for the Master of Arts degree in Mathematics from The University of Maine in December, 2004.