Theoretical derivation of the depth average of remotely sensed optical parameters

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Abstract: The dependence of the reflectance at the surface on the vertical structure of optical parameters is derived from first principles. It is shown that the depth dependence is a function of the derivative of the round trip attenuation of the downwelling and backscattered light. Previously the depth dependence was usually modeled as being dependent on the round trip attenuation. Using the new relationship one can calculate the contribution of the mixed layer to the overall reflectance at the surface. This allows one to determine whether or not to ignore the vertical structure at greater depth. It is shown that the important parameter to average is the ratio of the backscattering and absorption coefficients. The surface reflectance is related to the weighted average of this parameter, not the ratio of the weighted average of the backscattering and the weighted average of the absorption. Only in the special case of "optical homogeneity" where the ratio of the backscattering and absorption coefficients does not vary with depth, can the vertical structure be ignored. Other special cases including constant backscattering and variable absorption are also investigated.

OCIS codes: (010.4450) Ocean optics; (120.0280) Remote sensing.

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#8803 - $15.00 USD Received 15 September 2005; revised 20 October 2005; accepted 24 October 2005

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1. Introduction

A major tool in the study of the distribution of optical parameters on global scales are inversions of such parameters obtained from color sensors such as SeaWiFS, MODIS, and MERIS. Such inversions do not take into account the vertical structure of the parameters. Especially in the coastal zones, there can be significant stratification, however. The remote sensing reflectance at the surface thus incorporates aspects of the vertical structure of the optical properties. The interpretation of optical parameters obtained from remote sensing in the presence of vertical structure is an important problem. The relationship between the reflectance at the surface and the vertical structure of the optical properties needs to be known in order to relate optical parameters derived from remote sensing to those measured *in situ* in the case of a stratified ocean. In this paper we derive an analytical expression for the dependence of the surface reflectance on the vertical structure of the reflectance. This in turn leads to relationships between optical properties derived from remote sensing and the vertical structure of optical properties in the ocean.

The irradiance reflectance R is often modeled as:

$$R = f\frac{b_b}{a} \quad , \tag{1a}$$

where b_b is the backscattering coefficient, a the absorption coefficient, and f is a parameter that depends on the shapes of the volume scattering function and the radiance distribution; it is usually set equal to 0.33. This relationship for irradiance reflectance was first derived by Gordon *et al.* [1] and Morel and Prieur [2], based on modeling of the results from radiative transfer calculations. Many authors (for example [3,4,5,6,7,8,9,10,11,12]) have used this equation as the starting point for inversions of reflectance to obtain backscattering and absorption properties of the ocean. It is thus important to examine the relationship of the parameters obtained from such inversions to the vertical structure of those parameters. The

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proportionality factor f , depends on how the backscattered light relates to the backscattering coefficient, and therefore to the details of the volume scattering function in the backward direction and the radiance distribution. Most of the directional effects of radiative transfer are thus contained in the factor f, and this factor has been studied in detail (for example [3,4,5]). Eq. (1a) is the starting point for many inversion algorithms, but it ignores the dependence of f on the shape of the volume scattering function, the radiance distribution, and the vertical structure of the optical properties.

The implicit assumption when using an inversion based on Eq. (1a) is therefore, that there is some remote sensing average (with a vertical weighting function that is to be determined) that can be applied to b_b and a (indicated by an over bar), such that the irradiance reflectance in a vertically inhomogeneous ocean can be modeled by:

$$R = f\ \overline{b_b}\ /\ \overline{a}\ . \qquad (1b)$$

Is this assumption correct? What kind of vertical average, if any, can be applied to b_b and a so that reflectance measurements made with spectral radiometers can be reconciled with scattering and absorption measurements, so that instrumental closure is achieved? If R is inverted to obtain $\overline{b_b}$ and $\overline{a}$, how do these averages relate to the actual vertical structure? Is it even correct to assume that the irradiance reflectance can be modeled as:

$$R = f\,(\overline{b_b / a}\,) \qquad (2)$$

These are questions that we address in this paper.

Gordon and Clark [13] proposed that the influence of an optical component with a vertical structure, $C(z)$, on optical remote sensing would be given by:

$$\overline{C_S} = \left[\int_0^{z90} C(z)G(z)dz\right] \Big/ \left[\int_0^{z90} G(z)dz\right] , \qquad (3a)$$

where

$$G(z) = exp\left[-2\int_0^{z} K(z')\,dz'\right], \qquad (3b)$$

and where $K(z)$ is the downwelling plane irradiance attenuation coefficient, and $z90$ is the depth at which 90% of the surface downwelling plane irradiance at a given wavelength has been attenuated. (Actually, as Gordon and Clark [13] state in a footnote, $2K(z)$ should be the sum of the downwelling, $K_d(z)$, and upwelling, $K_u(z)$, plane irradiance attenuation coefficients, but this was not thought to lead to any serious errors). We shall call the average as suggested in Eq. (3) the Gordon and Clark average or GCA.

This suggests that the remotely sensed reflectance of a water mass with an optically active substance C which has a vertical structure of $C(z)$, would have the same reflectance as a homogeneous water mass with a concentration $\overline{C_S}$. This assumption appeared to work well in many cases, but by no means always. In a subsequent analysis Gordon [14] investigated the above hypothesis for varying vertical structures of chlorophyll, using Monte Carlo numerical modeling. The hypothesis was found to work well when the absorption coefficient, $a(z)$, and the backscattering coefficient, $b_b(z)$, covaried with the chlorophyll concentration $Chl(z)$. When only $a(z)$ covaried with $Chl(z)$, but $b_b(z)$ was kept constant with depth, the GCA did not work well, especially so in highly stratified cases. The GCA was used by Voss and Morel [15] to calculate the chlorophyll concentration that relates to remote sensing.

Stramska and Stramski [16] have provided a good review of the need for understanding the relationship between the vertical structure of Inherent Optical Properties (IOP, the scattering and absorption characteristics of the water and its constituents [24]) and the remote sensing reflection. Based on models of the vertical chlorophyll distribution they calculated the reflectance and compared it to homogeneous cases. Stramska and Stramski [16] found that the larger the background chlorophyll concentration was compared to the subsurface

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chlorophyll maximum, the closer the reflectance was to the uniform case. They also found that larger deviations from the constant case were found when $b_b(z)$was kept constant, but $a(z)$ covaried with $Chl(z)$.

Zaneveld [17] previously derived the theoretical dependence of the remotely sensed reflectance on the vertical structure of optical properties. This derivation, while correct, contains the full volume scattering function as well as the radiance distribution, and therefore is difficult to apply. What is needed is a simpler approach that can be used with parameters such as spectral $b_b(z)$and $a(z)$ that can now be readily measured *in situ* (see, for example, Twardowski *et al*. [18]).

In this paper we will address the above questions and observations using a two-flow model. We will demonstrate why the GCA works well for optically homogeneous situations when b_b/a does not vary with depth, even though both $b_b(z)$ and $a(z)$ covary with $Chl(z)$. We will show why averages such as those in Eq. (1b) are increasingly worse for more stratified cases where a varies with depth, but b_b is constant, as was observed both by Gordon and Clark [13] and Stramska and Stramski [16]. Such a situation can arise, for example, due to photo-acclimation of phytoplankton that increase their cellular pigment content with depth to adjust for a reduction of light with depth (e.g. Kitchen and Zaneveld, [19]).

Below we introduce averaging rules derived for various optical structures. We show that with a proper averaging rule, Eq. (2) is correct when f is depth independent. On the other hand no single averaging rule works when absorption and backscattering are separated, so that merely writing Eq. (1b) makes an assumption about the optical structure of the water column.

2. Theory

In order to model the reflectance for various optical stratifications due to physical structure, it is useful to employ a simple two-flow model such as that used by Philpot and Ackleson [20], Philpot [21], Maritorena *et al*. [22] and others to study the effect of bottom albedo on the remotely sensed reflectance. These approaches all have in common that they start with the correct two flow assumptions [23,24]. It is then assumed that there is some backscattering parameter $B(z)$ that characterizes the redirection of light upward, and that there is some attenuation coefficient $g(z)$ that characterizes the round trip attenuation from the surface to a given depth z and back. The paper by Maritorena et al. [22] provides an excellent discussion of the errors resulting from these assumptions.

This relationship is:

$$R(0^-) = E_u(0^-) / E_d(0^-) = \int_0^\infty B(z)\, e^{-\tau_g(z)} dz, \tag{4}$$

where

$$\tau_g(z) = \int_0^z [K_u(z') + K_d(z')]dz' = \int_0^z [g(z')]dz' , \tag{5}$$

and where $E_u(0^-)$ and $E_d(0^-)$are the downwelling and upwelling plane irradiances just beneath the surface, respectively. Hence $e^{-\tau_g(z)}$ is the round trip attenuation of the signal. Substituting Eq. (5) into Eq. (4):

$$R(0^-) = \int_0^\infty B(z)\, exp\{-\int_0^z [g(z')]dz'\}dz. \tag{6}$$

Since:

$$\frac{d}{dz}\Big[exp\{-\int_0^z [g(z')]dz'\}\Big] = -\, g(z)\, exp\{-\int_0^z [g(z')]dz'\} \tag{7}$$

Equation (6) can be rewritten as:

$$R(0^-) = \int_0^\infty -\frac{B(z)}{g(z)} \frac{d}{dz}\Big[exp\{-\int_0^z [g(z')]dz'\}\Big]\ dz \quad (8)$$

If *B* and *g* are constant with depth, we get for the constant ocean

$$R_c(0^-) = \frac{B}{g}$$

We define $\frac{B}{g}(z) \equiv R_c(z)$. The physical interpretation of R_c is that it is the reflectance the ocean would have if the ocean were homogeneous and had vertically constant *B(z)* and *g(z)* values.

$$R(0^-) = \int_0^\infty R_c(z) \frac{d}{dz}\Big[-\,exp\{-\int_0^z [g(z')]dz'\}\Big]\ dz \quad (9)$$

Thus the proper depth average weighting function for reflectance is the *derivative* of the round trip attenuation.

It should be noted that the remote sensing reflectance (i.e., the ratio of upwelling nadir radiance and downwelling irradiance, rather than the irradiance reflectance, the ratio of upwelling irradiance and downwelling irradiance) can be modeled by the same mathematical formalism. Only the parameters would be different. In that case *g(z)* would represent the sum of the attenuation coefficients of downwelling irradiance and upwelling nadir radiance. Similarly *B(z)* would represent the function that transforms downwelling irradiance into upwelling nadir radiance (as opposed to irradiance). We have thus found an explicit relationship between the reflectance at the surface and the vertical structure of the reflectance. This has implications for the relationship of optical parameters determined from remote sensing and their vertical structure, as will be shown below.

Equation (3a) contained the normalization factor $\int_0^{z90} G(z)dz$. We will now derive the normalization factor for Eq. 9. The normalization factor is the integral of the weighting function over depth:

$$\int_0^\infty \frac{d}{dz}\Big[-\,exp\{-\int_0^z [g(z')]dz'\}\Big]\ dz = \int_0^\infty \frac{d}{dz}\Big[-\,exp\{-\tau_g(z)\ \}\Big]\ dz$$

$$= \Big[-\,exp\{-\tau_g(z)\ \}\Big]_0^\infty = -\,exp\{\tau_g(\infty)\ \} + exp\{-\tau_g(0)\ \} = 1, \quad (10)$$

We can set $-\,exp\{-\tau_g(\infty)\ \} = 0$, since $\tau_g(z)$ is a monotonically increasing function of z.

Having derived the dependence of the surface reflectance on its vertical structure, we now wish to explore how this relationship affects the dependence of the surface backscattering and absorption values on their depth dependence. In remote sensing studies $R(0^-)$ is often set equal to $f\frac{b_b}{a}$. We will indicate this remote sensing average value of $\frac{b_b}{a}$ by $<\frac{b_b}{a}>_{rs}$. If we set $R_c(z) = f(z)\frac{b_b}{a}(z)$, where $\frac{b_b}{a}$ is considered to be a single function of depth, we obtain:

$$<\frac{b_b}{a}>_{rs} = \int_0^\infty f(z)\frac{b_b}{a}(z)\frac{d}{dz}\Big[exp\{-\int_0^z [g(z')]dz'\}\Big]\ dz. \quad (11)$$

The parameter *f(z)* is relatively slowly varying (its total range is less than a factor of two [4]); its variability is likely to be much less than that of the weighting function. We therefore set $f = f(z) \approx constant$ (typically set = 0.33 in many algorithms). This results in:

$$< \frac{b_b}{a} >_{rs} \approx \int_0^{\infty} \frac{b_b}{a}(z) \frac{d}{dz} \Big[exp\{-\int_0^{z} [g(z')]dz'\} \Big] \; dz. \quad (12)$$

Thus the proper depth average weighting function for $\frac{b_b}{a}$ is also approximately equal to the derivative of the round trip attenuation. It is only an approximation in this case as we have assumed f to be a constant. With Eqs. (9) and (12) we have in hand fundamental relationships that will be used below to explain the results obtained by studies such as [14] and [16] that used non-uniform profiles of IOP to derive remote sensing parameters.

3. Numerical expressions

In many applications it is desirable to apply Eqs. (9) and (12) to numerical data. For that reason we derive the numerical equivalent to the weighting functions. We evaluate $\int_{z_1}^{z_2} R_c(z) \frac{d}{dz} \Big[exp\{-\int_0^{z} [g(z')]dz'\} \Big]$ for a small interval $(z_1,z_2) = \Delta z$ in which parameters are constant:

$$exp\{-\int_0^{z} [g(z')]dz'\} = exp\{-\int_0^{z} [K_u(z') + K_d(z')]dz' \} = \frac{E_u(z)}{E_u(0)} \frac{E_d(z)}{E_d(0)} \quad (13)$$

and
$$\frac{d}{dz} \Big[exp\{-\int_0^{z} [g(z')]dz'\} \Big] = \lim_{\Delta z \to 0} \frac{1}{\Delta z} \Big[\frac{E_u(z)}{E_u(0)} \frac{E_d(z)}{E_d(0)} - \frac{E_u(z+\Delta z)}{E_u(0)} \frac{E_d(z+\Delta z)}{E_d(0)} \Big] \quad (14)$$

Thus, if we have the vertical structure of the downwelling and upwelling irradiance we can determine the reflectance of an ocean made up out of N homogeneous layers.

$$(\frac{b_b}{a})_{rs} = <b_b/a> = \sum_{n=1}^{N} (\frac{b_b}{a})_n \, H_n = \sum_{n=1}^{N} (\frac{b_b}{a})_n \Big(\frac{E_{un-1} E_{dn-1} - E_{un} E_{dn}}{E_{u0} E_{d0}} \Big) \quad (15)$$

The H_n are the weighting parameters for the nth depth interval; and the triangular brackets < > indicate this kind of weighted average.

The weighting functions,

$$H_n = \frac{E_{un-1} E_{dn-1} - E_{un} E_{dn}}{E_{u0} E_{d0}} \quad (16a)$$

can be determined by direct measurement, or by modeling using measured IOP and a radiative transfer program such as Hydrolight. Profiles of irradiance can be obtained directly from measurements or from IOP using Gershun’s equation $K(z) = a(z)/\bar{\mu}(z)$ (assuming no internal sources or fluorescence) and determining $\bar{\mu}(z)$ from the algorithm in Berwald *et al.* [25]. In the case of the remote sensing reflectance (rather than irradiance reflectance), the attenuation of the upwelling radiance can be modeled as in Zaneveld et al. [26], and the definition of the coefficients would be:

$$H_n' = \frac{L_{un-1} E_{dn-1} - L_{un} E_{dn}}{L_{u0} E_{d0}} \; . \quad (16b)$$

4. Applications

Equation (9) describes the dependence of the reflectance at the surface on the vertical structure of the reflectance. This in turn leads to Eqs. (11) and (12) which provide the dependence of the b_b/a ratio as derived from remote sensing at the surface on the vertical structure of b_b/a. As mentioned in the introduction, Eq. (1) is usually used to derive values of b_b and a that are deemed to be representative of the pixel examined remotely. Since we have determined the dependence of the reflectance on the vertical structure of the IOP, we can

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determine whether certain common vertical optical structures will lead to reasonable results when comparing optical properties derived from remote sensing with those measured in stratified oceans. We pose the question whether the vertically averaged backscattering or absorption using the weighting function derived in this paper, or the GCA, will give an answer that is similar to that obtained from remote sensing inversion.

Equation (9) leads one to hypothesize that, similar to the Gordon and Clark average, the remote sensing average of a parameter C_{rs} (either b_b/a , b_b, a, chlorophyll or pigment concentration) might be given by:

$$<C>_{rs} = \sum_{n=1}^{N} (C_n)\ H_n = \int_0^{\infty} C(z) \frac{d}{dz} \left[- exp\{- \int_0^{z} [g(z')]dz'\} \right] dz \tag{17}$$

We will next test the applicability of Eq. (17) to common cases such as those studied by [2] and [4]. We have found that in general the following equation can be applied if f is considered to be constant:

$$R(0^-) = f <b_b/a> . \tag{18}$$

In this section we will start with $<b_b/a>$ and derive simplified expressions. We note that in general $<b_b/a> \neq <b_b>/<a>$.

4.1 The constant backscattering case

A simple situation is one where the backscattering coefficient is constant with depth, but the absorption coefficient is allowed to vary. This could be the case when the change in absorption is due to photoadaptation without change in biomass (e.g., [15]) or in extremely oligotrophic oceans where the backscattering is dominated by water. In that case:

$$<b_b/a> = \sum_{n=1}^{N} (b_{bn} / a_n)\ H_n = b_b \sum_{n=1}^{N} (1/a_n)\ H_n\ = b_b <1/a>. \tag{19}$$

Equation (19) shows that in this case $<b_b/a> \ \neq \frac{<b_b>}{<a>}$ because $1/<a> \ \neq <1/a>$. Thus an average for a using Eq. (17) will not give the correct average absorption coefficient. The GCA would also give the wrong reflectance. The assumption expressed in Eq. (1b) is incorrect for this vertical structure as already deduced by Gordon [14]. Stramska and Stramski's [16] calculations showing that larger deviations from the constant case were found when $b_b(z)$ was kept constant, but $a(z)$ covaried with $Chl(z)$, are thus also explained.

4.2 The constant absorption case

If a is constant with depth, an analysis such as in Eq.19 immediately shows that $<b_b/a> = <b_b> / a$, where a is constant. However if a is constant then $1/a = 1/<a> = <1/a>$. In this case we then have the result that:
$<b_b/a> = <b_b><1/a> = <b_b> / <a>$. This case can occur in nature in the red and near infrared when water absorption dominates a. Note that this kind of analysis cannot be applied near the chlorophyll fluorescence band at 681nm as fluorescence is not included in the preceding equations.

4.3 The optically homogeneous case

In the optically homogeneous case it is assumed that b_b/a is constant with depth. Thus, b_b and a can vary with depth but must have the same depth structure. This would also be the case when both b_b and a covary with chlorophyll as studied by Gordon [14]. This might be the case in situations where the optical properties are dominated by phytoplankton, whose concentration determines both b_b and a at a given depth. At each depth we then have the

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same reflectance, so that $R/f = (b_{bn} / a_n)$ for all n. In that case the vertical structure of both $a(z)$ and $b_b(z)$ can be given by a depth function $h(z)$; $a(z)= h(z)a(0)$ and $b_b(z)= h(z)\, b_b(0)$.

Applying the < > average as defined in Eq. (12) gives:

$$<a(z)> = <h(z)> a(0) \text{ and } <b_b(z)> = <h(z)> b_b(0)$$
$$<b_b(z)/a(z)> = b_b(0)/a(0) = <b_b(z)>/<a(z)>. \tag{20}$$

For the optically homogeneous case we have thus shown that the ratio of the averages of b_b and a is the average of the ratio, where average is defined in the sense of Eq. (17). If, from remote sensing inversions, one thus derives a value for b_b/a, in the optically homogeneous case this means that we also have obtained a valid ratio of the averages. In this case the assumption expressed by Eq. (1b) is correct. The calculations of Stramska and Stramski [16] showed that the more closely both the scattering and the absorption followed the chlorophyll profile, the more similar the reflectance was to that of the homogeneous case.

4.4 Optical Homogeneity Index

In all cases $<b_b/a>$ can be determined from remote sensing. The separation into $<b_b>$ and $<a>$ depends on the vertical structure that is typically unknown when dealing with remotely sensed data alone. The closer the structure resembles the optically homogeneous case, the more accurate the determinations of $<b_b>$ and $<a>$. On this basis one can construct an optical homogeneity index, H,

$$H= <b_b/a> / [<b_b> /<a>], \tag{21}$$

which would be an indicator of the accuracy of the assumption that $<b_b/a> = <b_b> /<a>$.

4.5 Contribution of the mixed layer to the remotely sensed signal

We can use Eq. (9) to determine if the mixed layer (in which IOP are more or less constant) is sufficient by itself to generate the remotely sensed signal. In that case we determine the integral in Eq. (9) down to the Mixed Layer Depth (MLD):

$$Fr_{MLD} = \int_0^{MLD} R_c(z) \frac{d}{dz} \left[exp\{-\int_0^z [g(z')]dz'\} \right] dz / R(0^-) , \tag{22}$$

where Fr_{MLD} is the fraction of the reflectance at the surface determined by the mixed layer.

Similarly, the contribution to the signal by any layer between depths z_1 and z_2 can be determined by:

$$Fr_{z1,z2} = \int_{z_1}^{z_2} R_c(z) \frac{d}{dz} \left[exp\{-\int_0^z [g(z')]dz'\} \right] dz / R(0^-). \tag{23}$$

A similar expression can be obtained for b_b/a. For example (assuming again that f is constant, or alternatively instead of using $\frac{b_b}{a}(z)$, in what follows below, one could use $f\frac{b_b}{a}(z)$ as a single function with depth dependence) Eq. (13) can be expanded as follows:

$$Fr_{MLD} = \int_0^{MLD} \frac{b_b}{a}(z) \frac{d}{dz} \left[exp\{-\int_0^z [g(z')]dz'\} \right] dz / \int_0^{\infty} \frac{b_b}{a}(z) \frac{d}{dz} \left[exp\{-\int_0^z [g(z')]dz'\} \right] dz. \tag{24}$$

It is important to determine this for all wavelengths used in a particular algorithm. It is quite possible that, due to the spectral dependence of absorption and scattering by the several optical constituents, the fraction of the signal due to the mixed layer is quite different at different wavelengths. Expressions similar to Eq. (24) can be used to assess the relative contributions to the remotely sensed signal of other frequently used depth intervals, such as *z90* used in [13], the 1% light level, etc.

#8803 - $15.00 USD Received 15 September 2005; revised 20 October 2005; accepted 24 October 2005

5. Discussion and conclusions

We derived from first principles a relationship between the vertical structure of the reflectance and the reflectance at the surface. This functional relationship is different from that of Gordon and Cark [13] in that they suggested a weighting function proportional to the round trip attenuation. We found from first principles that the weighting function should be proportional to the derivative of the round trip attenuation. In addition, it makes intuitive sense that when the attenuation is increasing rapidly in a layer, the influence of that layer on the remotely sensed parameters should be large.

The equations derived can be used to obtain insight into how aspects of the vertical structure affect the parameters obtained by remote sensing. When applying this model to the vertical structure cases investigated by [14] and [16], we are able to explain their results without numerical calculations. We found that only in the optically homogeneous case can the backscattering and absorption be separated, so that only in that case can the remote sensing parameters be related to vertically averaged parameters.

Often it is some weighted average of the vertical chlorophyll distribution that is compared to that inferred from remote sensing reflectance. The analysis presented here suggests that, in order to determine the value of *in situ* absorption, backscattering, or chlorophyll to be compared with that based on remote sensing reflectance, the vertical distribution of R or R_{rs} are needed. When band ratios are used to determine pigment concentrations, there is the further complication of the vertical structure of optical properties possibly not being the same at different wavelengths. The implication is that the simple vertical averaging rule derived here would not apply to vertical pigment averages if that is the case. Thus case 2 waters deserve special attention as in that case CDOM, sediments, and phytoplankton are likely to have very different vertical structures, making the comparison between derived and measured parameters uncertain. In such cases carrying out calculations with modeled vertical structures as in [14] and [16] is the way to gain insight into the errors.

We have found that, based on the two-flow approach (including its approximations) there always is a homogeneous ocean with a remote sensing reflectance that is the same as that of the stratified ocean. This conclusion is based on Eq. (9). With the simplifying assumption that f is constant we can then state that the vertical average of b_b/a that applies to the reflectance is also the derivative of the round trip attenuation. Thus, the homogeneous ocean with the same reflectance as the inhomogeneous one has a b_b/a that is equal to the $\langle b_b/a \rangle$ of the inhomogeneous ocean. We cannot say in general however, that therefore the b_b and a values of the homogeneous ocean are equal to the $\langle b_b \rangle$ and $\langle a \rangle$ values of the inhomogeneous ocean. We thus found here that there always is a meaningful vertical average of the backscattering to absorption ratio. The separation of these two components will in general not be possible however, without inducing errors. Only in special cases can the remotely sensed reflectance be expressed as being proportional to the ratio of some average $\langle b_b \rangle$ and an average $\langle a \rangle$. Yet almost all algorithms (for example [3,4,5,6,7,8,9,10,11,12]) assume that such a separation of $\langle b_b \rangle$ and $\langle a \rangle$ is possible. Such average values determined from remote sensing may be far from reality in the presence of vertical structure such as is often found in coastal waters. This explains the difficulty numerical studies such as [14] and [16] have in relating vertically weighted values with remotely sensed parameters, so that a consistent rule relating remotely sensed parameters to *in situ* parameters could not be deduced from such numerical studies. The present study provides such a relationship in Eqs. (9), (11), and (12).

A disturbing conclusion is that when measuring a depth dependent [chl] distribution in-situ, there is not a general expression for depth weighting that one could use to compare with the satellite [chl] estimate. If a direct link between depth dependent [chl] and depth dependent b_b/a could be found, then the equations derived in this paper could be used to find the relationship between the satellite [chl] estimate and the depth dependent [chl] distribution.

#8803 - $15.00 USD Received 15 September 2005; revised 20 October 2005; accepted 24 October 2005

Acknowledgments

This work was supported by the Ocean Biology/Biogeochemistry Program of NASA and the Environmental Optics Program of ONR.

#8803 - $15.00 USD Received 15 September 2005; revised 20 October 2005; accepted 24 October 2005