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A Determination of the Modulus of Elasticity for Small Loads

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A DETERMINATION OF THE MODULUS OF ELASTICITY FOR SMALL LOADS.

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At the suggestion of Professor James S. Stevens, and under his supervision, I undertook to apply interference methods of measurement to an experimental determination of Young's modulus for copper, brass and steel. The metals were in the form of bars 66 cm. × 1 cm. × 0.5 cm. (approximate dimensions). The general method followed was the so-called "method of flexure" in which a bar, supported at each end on knife edges, is deflected by a weight applied at the center, and the deflection at the central point, together with the dimensions of the bar and the value of the deflecting force are substituted in the formula

$$M = \frac{\text{Force} \times \text{length}^3}{4 \times \text{breadth} \times \text{depth}^3 \times \text{deflection.}}$$

This formula is readily derived from a consideration of the relation between the stresses and their effects.

In my experiment the deflections were measured by the interferometer in terms of wave-lengths of sodium light and then reduced to centimeters, using 0.00005893 cm. as the value of the wave-length. The fineness of my unit of measurement allowed me to use deflecting forces very much smaller than is usual in this type of determinations.

The set up of the apparatus was as follows: The bar in test, supported by steel knife edges 60 cm. apart, was surrounded at the middle point by a loop of silk thread, A in the figure, to which the deflecting weights B could be attached. The movable mirror C and the plate to which it is attached were removed from the interferometer and the mirror alone was fastened by a pellet of soft wax to the upper side of the bar at the mid point. Then the interferometer, turned on its side, was adjusted until clear cut fringes were obtained, the source of light being a sodium flame at the right of the apparatus. At the beginning of the experiment an attempt was made to place the interferometer on a movable platform by means of which the adjustments could be made, but it was found advisable, on account of vibrations, to place it solidly on the slate table and make the adjustments by raising and lowering the knife edges or by moving the mirror on its pellet of wax for slight changes.

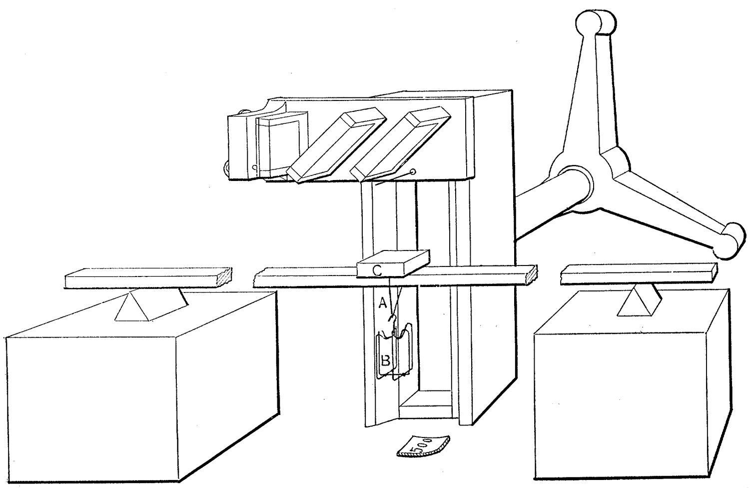

Fig. 1.

When the adjustment was satisfactory the deflecting weight was very carefully lowered until it hung suspended from the silk loop, and a count of the number of fringes passing the reference wire was made. The movable mirror was found to be so sensitive to external vibrations that no observations could be taken when any one was moving around in the building, and consequently the work had to be done in the evening or when the building was unoccupied.

For lowering the weights upon the thread slowly enough so that a good count could be made, a small platform which could be moved vertically by a tangent screw was devised. The weights used consisted of a wire basket of 0.5 g. mass and additional weights each of 0.5 g. mass, to be placed in the basket. The experiment was also tried using 0.2 g. mass and multiples, but these proved too small for reliable work.

The accompanying Tables I., II., III. show the results obtained with the three bars :

Table I.

STEEL BAR.

No.	Mass in g.	Defl. in Sodium waves.	Defl. in cm.	Defl. in cm. for 0.5 g = l.
1	0.5	3.0	0.00008839	0.00008839
2	1.0	6.0	0.00017678	
3	1.5	9.0	0.00026517	
4	2.0	12.0	0.00035356	

Length of bar = 60 cm. = L.

Depth of bar = 0.537 cm. = d.

Breadth of bar = 1.008 cm. = b.

Force on bar = 0.5 g. × 980.5 = F.

$$M = \frac{FL^3}{4bd^3l} = \frac{0.5 \times 980.5 \times 216000}{4 \times 1.008 \times 0.154854 \times 0.00008839}.$$

$$M = 19.19 \times 10^{11}.$$

Table II.

COPPER BAR.

No.	Mass in g.	Waves.	Defl. in cm. for 0.5 g. = l.
1	0.5	2.6	0.00007857
2	1.3	5.3	
3	1.5	8.0	
4	2.0		

Length = 60 cm. = L.

Breadth = 1.093 cm. = b.

Depth = 0.638 cm. = d.

$$M = \frac{FL^3}{4bd^3l} = \frac{0.5 \times 980.5 \times 216000}{4 \times 1.093 \times 0.259694 \times 0.00007857}.$$

$$M = 11.87 \times 10^{11}.$$

TABLE III.

BRASS BAR.

No.	Mass in g.	Waves.	Defl. in cm.	Defl. in cm. for 0.5 g. = l.
1	0.5	3	0.00008839	0.00008839
2	1.0	6	0.00017678	
3	1.5	9	0.00026517	
4	2.0	12	0.00035356	

$$\text{Length} = 60 \text{ cm.} = L.$$
$$\text{Breadth} = 1.094 \text{ cm.} = b.$$
$$\text{Depth} = 0.636 \text{ cm.} = d.$$

$$M = \frac{FL^3}{4bd^3l} = \frac{0.5 \times 980.5 \times 216000}{4 \times 1.094 \times 0.257259 \times 0.00008839}.$$

$$M = 10.64 \times 10^{11}.$$

A word of explanation ought perhaps to be said in regard to the symmetry of the results in the second columns of Tables I. and III. (deflection in wave-lengths). The values were derived as follows: the deflection produced by 0.5 g. is noted several times. As nearly as can be read it is 3 waves. With the thought in mind that this is liable to be in error 0.1 either way, the deflections produced by 1.0 g., 1.5 g., etc., were noted. Any error in the first reading would become multiplied in the successive readings and be very readily detected. The fact that there is no detectable deviation from 12 complete waves for 2.0 g., argues that the first reading was a correct one. In the case of the copper bar the deflection for 1.5 g., came so exactly 8 waves that the third of that amount was used as the deflection corresponding to the excess of 0.5 g. rather than the estimated value of the deflection actually observed for 0.5 g.

After completing these determinations, using the interferometer and 0.5 g. excess, I made determinations for the same bars using excesses of 50 g. and measuring the deflections by means of a micrometer microscope reading directly to 1/28263 of a cm. The results of these determinations are embodied in the Tables IV., V., VI., while Table VII. gives a comparison of the mean values of M obtained by the different methods.

Table IV.

STEEL BAR.

Mass.	1st Rd.	2d Rd.	Diff.	Diff. for 50.	Mean.	l (cm.).
50	0.877	3.327	2.450	2.450		
100	0.794	5.690	4.896	2.448		
150	0.844	8.145	7.301	2.433		
200	0.828	10.633	9.805	2.451		
250	0.300	12.641	12.341	2.468	2.45	0.008668

Microscope constant = 282.63 turn to cm.
Length of bar = 60 cm. = L.
Breadth of bar = 1.008 cm. = b.
Depth of bar = 0.537 = d.

$$M = \frac{FL^2}{4bd^3l} = \frac{50 \times 980.5 \times 216000}{4 \times 1.008 \times 0.154854 \times 0.008668} = 19.56 \times 10^{11}.$$

Table V.

COPPER BAR.

Mass.	1st Rd.	2d Rd.	Diff.	Diff. for 50.	Mean.	l (cm.).
50 g	0.299	2.496	2.197	2.197		
100 g	0.337	4.733	4.396	2.198		
150 g	0.391	6.991	6.600	2.200		
200 g	0.435	9.200	8.765	2.191	2.1965	0.007772

$L = 60$ cm. $b = 1.093$ cm. $d = 0.638$.

$$M = \frac{FL^3}{bd^3l} = \frac{50 \times 980.5 \times 21600}{4 \times 1.093 \times 0.259694 \times 0.007772} = 12.00 \times 10.^{11}$$

Table VI.

BRASS BAR.

Mass	1st Rd.	2d Rd.	Diff.	Diff. for 50.	Mean.	l (cm.).
50 g	0.607	3.055	2.448	2.448		
100 g	0.040	5.070	5.030	2.515		
150 g	0.343	7.881	7.538	2.513		
200 g	0.349	10.374	10.025	2.506	2.4955	0.008868

$L = 60$ cm. $b = 1.094$ cm. $d = 0.636$ cm.

$$M = \frac{FL^3}{4bd^3l} = \frac{50 \times 980.5 \times 216000}{4 \times 1.094 \times 0.257259 \times 0.008868} = 10.616 \times 10^{11}.$$

Table VII.

COMPARISON.

	Interferometer.	Microm. Microp	Per Cent. of Deviation.
Steel	19.19×10^{11}	19.56×10^{11}	1.9
Copper	11.87×10^{11}	12.00×10^{11}	1.0
Brass	10.64×10^{11}	10.62×10^{11}	0.2

During the progress of the experiment an attempt was made to determine whether or not the bars returned to their original positions; that is, to determine if there was any "permanent set" for those small deflecting forces. The conclusion reached was that the return to initial position was absolute, within the limit of the fineness of observation possible with the instrument, which was to be expected, as the "set," if any, for so small a deflection would probably be beyond the range of observation even with this instrument.

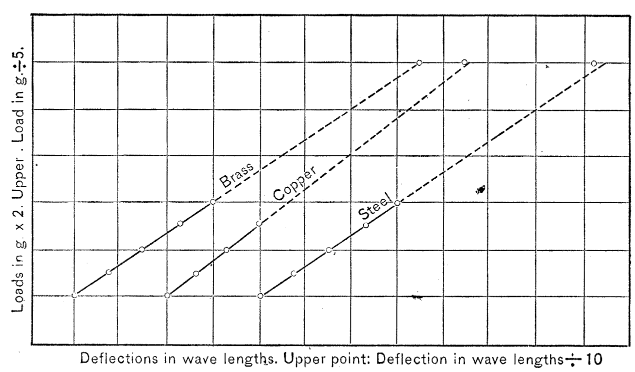

Fig. 2.

The accompanying curves show at one end the relation between loads differing by 0.5 g. and deflections measured in wave-lengths; and at the other a similar relation between loads differing by 50 g. and deflection reduced to wave-lengths, the latter reduced to a convenient scale.

Discussion of Results.

Since the unit was the wave-length of sodium light it is accurately known to at least four significant figures (cm.). All measurements therefore were carried out that far; but of course the precision reached was not of that order. A movement over a dark or yellow space is equivalent to one-quarter wave-length. These spaces could easily be estimated to tenths. The errors would, therefore, be likely to run about 1.5 to 0.4 per cent. It seems certain that in the case of copper and brass the same ratio which exists between large loads and the corresponding deflections holds for small loads and their corresponding deflections. In the case of steel the percentage of deviation between the two methods is 1.9. If we allow that an error of two per cent. might have been made in measuring the displacement of the fringes, or, what is equally probable, an error of that magnitude occurred in reading the micrometer microscope the two results will check within the limits of this error. The results in the above tables represent the mean of a series of measurements in each case. It may therefore be concluded that certainly for copper and brass and probably for steel the ratio between the deflection and a load is a constant quantity for loads ranging from 0.5 g. to those which would give the bar a permanent set.

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