Thermal Convection in Ice Sheets: We Look But Do Not See

Terence J. Hughes

University of Maine - Main, terry.hughes@maine.edu

Follow this and additional works at: http://digitalcommons.library.umaine.edu/ers_facpub

Part of the Earth Sciences Commons

Repository Citation
http://digitalcommons.library.umaine.edu/ers_facpub/110

This Article is brought to you for free and open access by DigitalCommons@UMaine. It has been accepted for inclusion in Earth Science Faculty Scholarship by an authorized administrator of DigitalCommons@UMaine.
THERMAL CONVECTION IN ICE SHEETS: WE LOOK BUT DO NOT SEE

By T. HUGHES

Abstract. Thermal convection in the Antarctic and Greenland ice sheets has been dismissed on the grounds that radio-echo stratigraphy is undisturbed for long distances. However, the undisturbed stratigraphy lies, for the most part, above the density inversion in polar ice sheets and therefore does not disprove convection. An echo-free zone is widespread below the density inversion, yet nobody has cited this as a strong indication that convection is indeed present at depth. A generalized Rayleigh criterion for thermal convection in elastic–viscoplastic polycrystalline solids heated from below is developed and applied to ice-sheet convection. An infinite Rayleigh number at the onset of primary creep decreases with time and becomes constant when secondary creep dominates, suggesting that any thermal buoyancy stress can initiate convection but convection cannot be sustained below a buoyancy stress of about 3 kPa. An analysis of the temperature profile down the Byrd Station core hole suggests that about 1000 m of ice below the density inversion will sustain convection. Creep along the Byrd Station strain network, radar sounding in East Antarctica, and seismic sounding in West Antarctica are examined for evidence of convective creep superimposed on advective creep. It is concluded that the evidence for convection is there, if we look for it with the intention of finding it.

Introduction. Glaciology is a small profession. Yet it is now in the process of handing the key to the riddle of climatic change, a major scientific concern, to the much larger profession of atmospheric sciences. This key provides the dynamic mechanism, a glaciological mechanism, which links Milankovitch variations in regional insolation to the global climatic cycles manifest by the waxing and waning of continental ice sheets. Glaciology also holds the key to the central scientific riddle in the earth sciences. That riddle is the mechanism whereby circulation of crystalline rock in the Earth’s mantle moves the great crustal plates on the Earth’s surface. The key is thermal convection in the much more accessible and chemically simple mantles of ice which cover Antarctica and Greenland. Yet, we do not realize that we hold this key. We look but do not see.

A RED HERRING

At the third International Symposium on Antarctic Glaciology in 1981, I stated that I see convection in the Antarctic ice sheet whenever I look at remote-sensing data from radio echo-sounding. In the second edition of The physics of glaciers, Paterson (1981, p. 170-71) discusses the possibility of thermal convection in the Antarctic ice sheet. He concludes his discussion with the statement, "The most convincing evidence against convection comes from radio echo-sounding records; these show layers within the ice that are continuous over distances of several hundred kilometres." In search for alternatives that would eliminate the need for convection, Paterson has blinded himself to the most exciting glaciological prospect of our time for a major scientific breakthrough. But Paterson is not alone. His position is the stance of our profession.

Modeling studies of ice-sheet flow along the Byrd Station strain network (Whillans, 1976) and pH lows in acidic ice layers deposited after volcanic eruptions (Hammer, 1980) have confirmed the view of Robin and others (1969) that internal radio-echo reflection horizons identify stratigraphic layers in ice sheets (although they ascribed these reflections to stratigraphic variations in density). The contention by Paterson (1981), presumably, is that circulatory flow caused by thermal convection would obliterate the internal reflections by scrambling the stratigraphy.

In response to this, I would point out that thermal convection transports heat vertically because a buoyancy stress causes vertical transport of mass. The buoyancy stress is unable to drive steady-state convection unless cold ice overlies warm ice and produces a superadiabatic temperature gradient in the vertical direction, as required by the classical theory of thermal convection (Strutt, 1916). These
conditions are never attained in the regions of the Antarctic ice sheet or the Greenland ice sheet where undisturbed radio-echo layers exist. In fact, advection of cold ice from higher elevations to lower elevations causes temperature to decrease with depth in the upper part of an ice sheet (Robin, 1955). The thermal buoyancy stress needed to drive convective flow cannot possibly exist in this upper layer, which is typically 1000 m thick, as measured directly down the Byrd Station core hole, for example, where the minimum temperature (Ueda and Garfield, 1970) corresponds precisely with the maximum density (Gow, 1970) at a depth of 1000 m. Consequently, radio-echo layers in the upper 1000 m of ice have no bearing whatever on whether or not thermal convection exists at greater depths. It is in this upper 1000 km where radio-echo layers extending for hundreds of kilometers are found. For Paterson to cite these layers as a reason to reject convection at deeper levels is a red herring, albeit unintentional.

Radio-echo layers can extend as deep as 3 km below the ice surface, but an echo-free zone typically occupies the lower 20 to 30% of ice over 2.5 km thick (Robin and Millar, 1982). As Robin and Millar (1982) point out, the radar pulse is strong enough to detect internal stratigraphic horizons in many regions at depths where the echo-free zone is observed. The conclusion is that the stratigraphy has been destroyed; they say by bed roughness on a local scale, I say by convective scrambling. I do not think that bed roughness of the dimensions of the echo-free zone, 200 m to 1000 m thick, can completely obliterate the stratigraphy. Stratigraphy is preserved in ice flowing over bedrock hills having these dimensions (see figure 1 of Robin and Millar, 1982), and should also be preserved in ice flowing around the hills.

After studying the echo-free zone at many locations and along intersecting radio-echo flight lines, Robin and Millar (1982) concluded: "While two-dimensional analyses involve a gradual transition of ice velocity from flowing parallel to the bed to flowing approximately parallel to the surface, the field evidence presented above suggests, when considered as a whole, that the transition between shear controlled by bedrock surfaces and shear controlled internally due to mean surface slope, takes place within ice sheets at a relatively sharp boundary and the upper limit of the radio-echo non-reflecting zone". I contend that a better explanation for the "relatively sharp boundary" is that it represents the top of convecting ice that occupies the echo-free zone. A transition zone from 100 m to 200 m thick separates the echo-free basal zone from the overlying zone of continuous radio-echo layers. The transition zone consists of "broken-up layering with very short horizontal continuity, and near-vertical cusps and fingers" that bear no relationship to bedrock topography (see figure 4 and accompanying text of Robin and Millar, 1982). This transition zone is what would be expected to separate upper ice, flowing by advection only, from lower ice, flowing by both advection and convection.

Had Paterson (1981) contemplated the lack of radio-echo stratigraphy below the density inversion, where thermal convection may be possible, instead of contemplating the abundance of radio-echo stratigraphy above the density inversion, where thermal convection definitely is impossible, he might not have dropped his red herring across the glaciological trail. If thermal convection is responsible for the echo-free zone, a criterion for thermal convection in polar ice sheets must be satisfied in the zone. I shall now attempt to develop this criterion.

A CRITERION FOR ICE-SHEET CONVECTION

I believe that the classical Rayleigh criterion for initiating convection in viscous fluids (Strutt, 1916) does not have to be satisfied for initiating convection in polar ice sheets. In viscous fluids heated from below, convection begins when the classical Rayleigh number (Ra) exceeds a critical value. By definition:

$$\text{(Ra)} = \frac{g \rho a \Delta T}{\eta}$$  (1)

where $\rho$ is the fluid density, $g$ is gravity acceleration, $a$ is the thickness of the fluid layer, $\Delta T$ is the volume coefficient of thermal expansion in the fluid, $\eta$ is the temperature difference between the top and bottom surfaces of the fluid, $\kappa$ is the thermometric diffusivity of the fluid, and $\eta$ is the fluid viscosity. For ice, $\eta$ is the effective viscosity because it depends on the effective stress. Effective stress $\sigma$ and effective strain-rate $\dot{e}$ are defined by the expressions:

$$\sigma = \sigma_0 (1 + n \dot{e}^2) = [\frac{1}{2} \sigma_{xx} + \sigma_{yy} + \sigma_{zz} + \sigma_{xy}] + \sigma_{xy} + \sigma_{yz} + \sigma_{xz}$$

(2a)

$$\dot{e} = [\dot{e}_{x}^2 + \dot{e}_{y}^2 + \dot{e}_{z}^2]^{1/2}$$

(2b)

where indices $i$, $j$ represent rectilinear axes, $x$, $y$, $z$ in the usual tensor notation, $\sigma_{ij}$ is a given component of the deviator stress tensor, and $\dot{e}_{ij}$ is a given component of the strain-rate tensor. Effective stress and effective strain-rate equations are derived from without dilatation, and are therefore square roots of the second invariants of the deviator stress and strain-rate tensors (Glen, 1958; Deter, 1961, p. 50-51).

Newton defined fluid viscosity as component $\eta_{ij}$ of the deviator stress tensor divided by the gradient $\nabla_{ij}$ of velocity component $u_i$ in direction $j$:

$$n = \sigma_{ij}/\dot{e}_{ij} = \sigma_{ij}/2\dot{e}_{ij}$$  (3)

Fig. 1. The viscoplastic creep spectrum. Viscoplastic exponent $n$ controls the relative viscous ($n = 1$) and elastic ($n = 1$) behavior of strain-rate $\dot{e}$ due to effective stress $\sigma$. Strain-rate $\dot{e}$ exists at plastic yield stress $\sigma$ and is time-dependent during primary and tertiary creep, but not during secondary creep.
where, by definition:

\[ \dot{\epsilon}_{ij} = \dot{\gamma}_{ij} - \dot{\gamma}_{ij} = \dot{\epsilon}_{ij} + \alpha_{ij}/\alpha_{ij}. \]  

Effective viscosity is the derivative of \( \alpha_{ij} \) with respect to \( \dot{\gamma}_{ij} \):

\[ \eta = \dot{\alpha}_{ij}/\dot{\gamma}_{ij} = \dot{\alpha}_{ij}/\dot{\epsilon}_{ij}. \]

The reason for this is shown in Figure 1, which is a plot of the normalized creep equation

\[ \dot{\epsilon}_{ij} = (\alpha/\alpha_o)^n \]

where \( \dot{\epsilon}_{ij} \) is the strain-rate at plastic yield stress \( \alpha_o \), and \( n \) is a viscoplastic exponent such that \( n = 1 \) for viscous creep and \( n = \infty \) for plastic creep. It is clear that when \( 1 < n < \infty \), slope \( \alpha/\dot{\epsilon} \) varies with \( \alpha \). If \( \dot{\alpha}/\dot{\epsilon} \) varies with \( \alpha \), so does \( \dot{\alpha}/\dot{\epsilon} \), by Equations (2). Effective viscosity is proportional to the derivative \( \dot{\alpha}/\dot{\epsilon} \), not the ratio \( \alpha/\dot{\epsilon} \). However, \( \dot{\alpha}/\dot{\epsilon} = \dot{\epsilon}/\dot{\alpha} \) for Newtonian viscosity because \( n = 1 \).

For thermal convection to occur in a polar ice sheet, effective stress \( \sigma \) must include a thermal buoyancy stress in addition to the stresses associated with advective flow in the ice sheet. Thermal convection begins as primary creep, for which \( \dot{\epsilon}_o \) in Equation (6) is a time-dependent strain-rate. A complete theory of thermal convection in polar ice sheets must examine primary creep for initiating convection and secondary creep for sustaining convection. Primary creep is an exclusive feature of polycrystalline solids, and is therefore meaningless in analyzing convection in Newtonian viscous fluids.

The first step in understanding thermal convection in polar ice sheets is examining its relationship to creep strain in polycrystalline ice. Figure 2 illustrates my conception of effective strain \( \epsilon \) in relation to lattice strain energy \( E \), effective stress \( \sigma \), and time \( t \) since \( \alpha \) was applied, provided that the initial grain fabric is not compatible with the stress field and recrystallizes to produce one that is compatible.

In terms of ice-sheet convection, \( \epsilon \) would be convection strain produced in time \( t \) by thermal buoyancy stress \( \sigma \) and causing lattice strain energy \( E \).

In Figure 2, small reversible displacements of atoms from their lattice sites are represented by elastic strain \( \epsilon_0 \). Primary strain \( \epsilon_1 \) begins when displacements allow chains of atoms to break free from their lattice sites and move as line imperfections having large irreversible displacements, and secondary strain \( \epsilon_2 \) exists when dislocations are generated and annihilated at the same rate. Secondary strain has stable and unstable states of equilibrium. Unstable equilibrium maximizes strain energy, and exists when the principal axes of the stress and strain tensors do not coincide. In violation of Neumann's Principle (Nye, 1957, p. 20-24, 104). Stable equilibrium minimizes strain energy, and exists after recrystallization to a new grain fabric which secures alignment of the principal axes, thereby satisfying Neumann's Principle. Being intermediate between these extremes of strain energy, elastic strain can be understood as a state of metastable equilibrium (Cottrell, 1957, p. 105-07).

As strain increases, strain energy increases during primary creep, causing strain hardening, reaches a maximum during unstable secondary creep, followed by recrystallization and strain softening that lead to minimum strain energy and stable secondary creep. In laboratory creep experiments at one atmosphere pressure, microcracks formed during recrystallization can cause fracture before stable secondary creep is attained. In this case, the creep curve during recrystallization is called tertiary creep, during which strain-rate accelerates to fracture. The creep curve shows how strain varies with time at constant stress, with slow unstable secondary creep at the end of strain hardening and fast stable secondary creep at the end of strain softening both satisfying the requirement that \( \dot{\epsilon}/\dot{t} = 0 \) for steady-state creep. Rates of strain hardening and strain softening are therefore balanced for the grain fabrics that exist for stable and unstable secondary creep.

The flow curve shows how strain varies with stress when strain-rate is kept constant as strain energy proceeds from metastable, to unstable, to stable equilibrium. A balance between strain hardening and strain softening for the "hard-glide" grain fabric before recrystallization and for the "easy-glide" grain fabric after recrystallization produces upper and lower yield stresses, respectively, for which \( \dot{\epsilon}/\dot{t} = 0 \). The upper yield stress records maximum strain hardening and the lower yield stress records maximum strain softening. These yield stresses differ from the upper and lower yield stresses which precede strain hardening in certain metals and alloys, notably low-carbon steel, and which are caused by dislocations being pinned and then released by an "atmosphere" of solute atoms (Oliker, 1961, p. 132-34).

Laboratory creep experiments generally show that recrystallization in polycrystalline ice with an initially random or hard-glide grain fabric does not occur until \( \epsilon = 0.5 \). This greatly exceeds the infinitesimal strain for which the Rayleigh criterion applies. Therefore, based on Figure 2, a useful expression for relating effective stress to the Ray-
leth number for thermal convection in polar ice sheets might be:

\[ \epsilon = \epsilon_0 + (\epsilon_1\tau^n + \epsilon_2\tau^2) \]  

where \( \epsilon_1 \) is the primary strain-rate for transient creep, \( \epsilon_2 \) is the secondary strain-rate for unstable steady-state creep, and \( \epsilon_0 \) is a primary creep exponent.

Laboratory creep experiments favor two ways for incorporating effective stress \( \sigma \) into Equation (7):

\[ \epsilon = (\epsilon_0\sigma)^m + (\epsilon_1\sigma^{n/3} + \epsilon_2\sigma^n) \tau, \]

or

\[ \epsilon = (\epsilon_0\sigma)^m + (\epsilon_1\sigma^{n/3} + \epsilon_2\sigma^n) \tau, \]

where \( \epsilon_0 \) is an elastic coefficient, \( \epsilon_1 \) is a softness coefficient for primary creep, \( \epsilon_2 \) is a softness coefficient for secondary creep, and \( n \) is the viscoplastic exponent in Equation (6). Time derivatives of Equations (8) give the following effective strain-rates:

\[ \dot{\epsilon} = m\epsilon_0\sigma^n + \dot{\epsilon}_2\sigma^n, \]

\[ \dot{\epsilon} = n\epsilon_0\sigma^n + m\epsilon_1\sigma^{n/3} + \epsilon_2\sigma^n, \]

Creep experiments (Glen, 1955; Butkovich and Landauer, 1983) support values \( m = (1/3) \) and \( n = 3 \), for which sheets might be:

Differentiating Equation (11) as follows

\[ \dot{\epsilon}_{ij}/\sigma_{ij} = (B_1\tau^{2/3} + B_2)\sigma_{ij} + \dot{\epsilon}_{ij}/\sigma_{ij} \]

where \( \sigma_{ij} \) is the thermal buoyancy stress, effective stress \( \sigma \) includes the thermal buoyancy stress and other deviator stresses associated with advective flow in ice sheets, \( \epsilon \) is time since convective flow began, \( d \) is ice thickness below the density inversion, especially where \( dT/dz \) is nearly constant, and softness coefficients \( B_1 \) and \( B_2 \) are evaluated for this ice layer.

Primary convective creep begins anew whenever the vertical deviator stress \( \sigma_{zz} \) changes, where \( \sigma_{zz} \) has a relatively constant buoyancy component due to the ever-present density inversion and a relatively variable advective component due to the every-varying basal traction on a local scale. Since \( \sigma_{zz} = (\sigma_{xx} + \sigma_{yy}) \) and basaltic advective creep experiences continuous change in \( \sigma_{xx} \) and \( \sigma_{yy} \) that compare to the buoyancy stress, and these changes occur on a local scale within a time-span of days, this is the time-span of a transient convection episode of primary creep. It is advective creep that continually resets to zero time the clock for convective creep, and this is why there can be thousands of transient thermal convection episodes at any given site in a polar ice sheet where a density inversion persists.

Bed roughness and basal sliding variations on a local scale cause the variations in \( \sigma_{xx} \) and \( \sigma_{yy} \) for advective flow that make possible the recurring episodes of convective flow controlled by primary creep. Figure 3 shows the response of the Rayleigh number to the creep curve. At time since initiation of
thermal convection increases, \( \alpha_{zz} \) drops from infinity during primary creep, reaches a minimum in the inflection region of slow unstable secondary creep, increases during recrystallization and tertiary creep, and attains a constant value when fast stable secondary creep is established. Any change in \( \alpha_{zz} \) at any point along the time axis in Figure 3 resets the curves of \( \varepsilon_{zz} \) and \( \alpha_{zz} \) to \( t = 0 \), provided that the change is comparable to the buoyancy stress. As shown in Figure 4, a sudden increase in \( \alpha_{zz} \) causes a sudden increase in \( \varepsilon_{zz} \) and stabilizes convection by increasing \( \alpha_{zz} \), but a sudden decrease in \( \alpha_{zz} \) stops convection because \( \alpha_{zz} \) turns negative. In reality, \( \alpha_{zz} \) increases or decreases with time, causing rates of convection to increase or decrease, so long as \( \alpha_{zz} \) remains supercritical.

\[
\frac{1}{16} \alpha_{zz} \frac{d \varepsilon_{zz}}{d \varepsilon_{zz}} = \frac{1}{16} \alpha_{zz} \Delta T
\]

Where \( \Delta T \) is the temperature increase driving convection creep, \( \alpha_{zz} \) is the thermal stress driving convection, \( \varepsilon_{zz} \) is the total effective strain, and \( \Delta T \) is the temperature increase driving convection.

**INITIATING AND SUSTAINING ICE-SHEET CONVECTION**

Polar ice sheets are, of course, heated from below. Critical Rayleigh numbers for thermal convection in a Newtonian fluid depend on boundary conditions at the top and bottom surfaces of the fluid layer (Knopoff, 1964). This number is 657.5 when both surfaces are free, 1100.7 when one is free and the other is rigid, and 1707.8 when both are rigid. In polar ice sheets, boundary conditions at the top and bottom of an ice layer below the density inversion are probably intermediate between free and rigid, provided that the bed is thawed. Ice above this layer is deformable and basal sliding occurs; therefore surface traction at the top and bottom of this layer is greater than zero but less than surface traction at a rigid boundary. If the bed is frozen, however, a rigid basal boundary condition would exist. A reasonable assumption would be that the critical Rayleigh number for thermal convection in polar ice sheets is of the order of 10^9.

Thermal convection in a polar ice sheet must begin at the onset of primary creep. This condition is satisfied at \( t = 0 \) in Equations (13) and (14), giving \( n = 0 \) and \( \alpha_{zz} = \infty \). Provided that a thermal buoyancy stress is present, therefore, it seems to me that thermal convection must at least be initiated for the simple reason that \( \alpha_{zz} \) initially exceeds any reasonable critical value. This requirement is totally absent from initiation of thermal convection in Newtonian fluids. It forces me to completely re-think the meaning of thermal convection, since Newtonian creep is merely an end-member of the viscoplastic creep spectrum shown in Figure 1. The classical Rayleigh criterion for thermal convection in Newtonian fluids is therefore a limiting case of a more general Rayleigh criterion.

I believe the general Rayleigh criterion for thermal convection in elastic-viscoplastic materials heated from below ranges from the classical Rayleigh criterion at the viscous extreme to a criterion at the plastic extreme such that convection is absent when the thermal buoyancy stress lies below the plastic yield stress and is rampant at the yield stress (Brown, 1965). Between these extremes, convection may be initiated by any thermal buoyancy stress, but is sustained only if the thermal buoyancy stress persists over the time span needed for primary creep to be replaced by secondary creep. Equation (14) expresses this generalized Rayleigh criterion.

In laboratory creep experiments, \( t_0 \) is a few days when \( T = -10^9 \)°C and \( p = 1 \) bar. Although thermal convection is easily initiated, if it cannot be sustained it will be extremely transient, and capable of producing only a few per cent of strain at best.

The real meaning of Equation (14) is that it specifies conditions for sustaining, rather than initiating, thermal convection. As \( t \) increases from zero to infinity, \( \alpha_{zz} \) decreases from infinity at the onset of primary creep to a constant value for secondary creep. If \( \alpha_{zz} \) falls below its critical value at any time during this transition from primary to secondary creep, convection stops at that time. If not, convection is sustained heated from below depend on

To a first approximation, the thermal buoyancy stress driving convection creep is (Neeriman, 1967)

\[
\sigma_{zz} = \frac{1}{16} \alpha_{zz} \Delta T
\]

Where \( \Delta T \) is the temperature increase driving convection.
sheet drives advection creep, and to a first approxima-
tion it is (Budd, 1970):
\[ \sigma_{xz} = \rho g \theta \]  
where \( h \) is the ice-sheet thickness and \( \theta \) is its sur-
face slope in the horizontal direction \( x \) of advective
creep.

A first approximation for \( \sigma \) is obtained from Equations (2a), (16), and (17) as
\[ \sigma = [k (\beta_{zz} + \gamma_{zz})]^{j} + \left( p \frac{\rho g h}{16} \right)^{j}. \]

Performing the differentiation with respect to \( \gamma_{zz} \)
\[ \frac{d\sigma_{zz}}{d\gamma_{zz}} = 2 \left( \beta_{zz} + \gamma_{zz} \right)^{j} \]
where \( \beta_{zz} = 0 \) in my opinion. Substituting
Equations (16), (18), and (19) into Equation (14), for
which \( \sigma_{ij} = \sigma_{zz} \), gives a first approximation
for \( \sigma_{zz} \) in polar ice sheets. A second approximation
would include additional stresses in computing \( \sigma \),
particularly longitudinal stress \( \sigma_{xx} \), and more pre-
cise formulations of \( \sigma_{xz} \) and \( \sigma_{zz} \).

An estimate of the minimum thickness \( d \) of basal
ice that can sustain convection can be made from
Equation (14) based on the temperature profile down
the core hole to bedrock at Byrd Station in Antarc-
tica. Ueda and Garfield (1970) found that \( T_d = 3 \times 10^{-3}
\text{deg/m} \) in ice below the density inversion, a region
through which the temperature gradient is nearly con-
stant. For ice, \( p = 0.92 \text{ Mg/m}^{3}, \alpha = 1.53 \times 10^{-11}
\text{m}^{2}/\text{deg}, \) and \( \kappa = 10^{-6} \text{ m}^{2}/\text{s} \), Let \( (Ra) = 10^{3} \)
for boundary conditions intermediate between free and rigid
at the top and bottom of a basal ice layer having thickness
\( d \) below the density inversion. Set \( \tau = \) so that
secondary creep predominates. Values of \( d \) obtained in
most of the other laboratory experiments are greater than those quoted here. In some cases they
differ by a factor of ten (e.g. Butkovich and Landauer,
1958). More to the point, \( \tau_{yy} \) for fast, stable sec-
ondary creep may be most appropriate for solving Equation
(14) for \( d \) as \( \tau = \). A strong, single-maximum grain
fabric exists for most of the ice thickness below the
density Inversion at Byrd Station (Gow, 1970; Gow and
Williamson, 1976). This fabric approaches easy glide
in single crystals, which results in strain-rates
about 500 times faster than those in randomly
oriented polycrystalline ice or in hard-glide orienta-
tions in ice single crystals (Weertman, 1983). The
easy-glide orientation is produced by destrainer stress
\( \sigma_{yy} \) and adveactive flow dominated by simple shear.

However, thermal convection presumably causes upwar-
ning for ascending flow and downwarming for descending
flow, which also allows easy glide by bending creep
in the single-maximum ice fabric (Higashi and others,
1965). In this case, the ice softness coefficient for
fast, stable secondary creep is appropriate, and could
be as high as \( \tau_{yy} = 5 \times 10^{-11} \text{m}^{2}/\text{s} \) at \(-10^\circ\text{C}\) and
\( d = 1 \text{ bar}, \) where \( \sigma_{xx} = 1 \text{ bar} \) is the only important de-
viator stress component for a smooth bed.

For \( d = 1164 \text{ m}, \) the ice thickness below the den-
sity Inversion at Byrd Station, Equation (14) gives
\( \beta_{yy} = 4 \times 10^{-11} \text{bar}^{-3/2} \) in order to sustain thermal
convection. The single-maximum ice fabric vanishes
in the lowest 400 m of the Byrd Station core hole,
and large intergrown grains having a more random orienta-
tion were observed (Gow, 1970; Gow and Williamson,
1976). An abrupt change in ice fabric velocities coincided
with this fabric change, indicating that the fabric change is an \( \text{in situ} \) condition and not
produced by subsequent recrystallization in the ex-
tacted core (Gow and Kohlen, 1976). A fabric change
implies a complex stress field imposed by rugged bed topography on a scale of 400 m around
Byrd Station. I can then imagine that \( \sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 1 \text{ bar} \), giving
\( \tau_{yy} = 10 \text{ bars} \) and \( \beta_{yy} = \beta_{zz} = \beta_{xy} = \beta_{yx} = \beta_{xz} = \beta_{zx} = \beta_{xx} = \beta_{yy} = \beta_{zz} \)
in order to sustain convection for \( d = 1164 \text{ m}. \)

The above considerations convince me that reason-
able combinations of \( \beta_{yy} \) and \( d \) exist below the density
inversion at Byrd Station for sustained thermal con-
vection (the cushion between possible values of \( \beta_{yy} \)
and values needed to sustain convection may reach \( 10^{3} \)).
If convective flow exists, however, it is very weak
compared to gliding flow. Convective flow observed for
advection creep below the density inversion in the top
of the Byrd Station core hole is about triple the ice thickness are widespread on the
Antarctic ice sheet (Budd, 1970; Budd and Carter,
1971), and Whillans and Johnsen (1983) made a two-dimensional
analysis of the flow pattern that, when superimposed on the average flow pattern, produced the observed surface
undulations. The analysis was limited by employing a
constant Newtonian viscosity computed from strain-
rates along the BSSN and by disregarding transverse
and 6T as convection flow develops? Resulting flow field undulates gently upward and downward, as revealed by an undulating surface and undulating radio-echo layers. This is the situation that Whillans and Johnsen (1983) observed along the BSSN, where undulations in radio-echo stratigraphy occurred even when the bed was essentially flat. Downward undulations associated with descending convection flow will slope away from the bed, increase downward toward the bed, and upward undulations associated with ascending convection flow will lift slightly warmer ice upward from the bed. If the bed is uniformly at the pressure-melting point of ice, therefore, a tendency for basal freezing can be associated with downward undulations and basal melting with upward undulations. Regulation ice would then form below downward undulations, and warm ice above upward undulations. Strong basal radar reflections exist below downward undulations. Whillans and Johnsen (1983) attribute these reflections to ponded basal melt water, even though the bed is flat. This is consistent with convective flow, because basal melt water must flow from ascending to descending limbs of convection flow. Moreover, the strong basal reflections may be caused partly by the basal layer of regulation ice, with entrained debris, as was found in the bottom 4 to 5 m of the Byrd Station core hole (Gow, 1970). It should be emphasized that the 4 m elevation change from crest to trough of surface undulations on the 10 km scale are due to alternating basal compressive and extending flow caused by weak convective flow superimposed on strong advective flow, and not by differential thermal expansion between ascending the descending limbs of convective flow. A thermal buoyancy stress of 0.03 bars produces an elevation difference due to thermal expansion of 3 m, as is seen by solving Equation (16) with $\Delta T = 0.03$ bar and substituting for $a$, since the volume coefficient of thermal expansion is triple the linear coefficient. The temperature difference between ascending and descending limbs computed from Equation (16) for this stress is 1 deg for 2500 m of ice below the density inversion.

Advective flow moves the density inversion to greater depths with increasing distance from the ice divide (Robin, 1955). Given the strong dependence of Rayleigh number (Ra) on depth $d$ below the density inversion, convection that is marginally sustained near ice divides would probably be overwhelmed toward ice margins. Transient convection would, at most, produce only a few percent of strain; but it could occur virtually spontaneously because (Ra) would initially exceed any critical value. The density inversion exists, transient convection should exist, and density inversions are widespread in the Greenland and Antarctic ice sheets.

I think the echo-free zone that commonly exists away from ice divides, where the water depth in the lower 200 m to 1000 m of ice over 2000 m thick is a consequence of transient thermal convection below the density inversion. The pulse length for radar sounding is typically from 60 ns to 250 ns, which translates as 10 m to 40 m of ice, and therefore encompasses single annual precipitation layers or groups of these layers that may have high acidity from a single volcanic eruption or from a group of years marked by unusually frequent eruptions (Millar, 1982). These are the events recorded as radio-echo horizons. A vertical transient strain of only five percent, for example, would displace the local density inversion by 200 m to 1000 m thick thereby completely obliterating the radio-echo stratigraphy. This is particularly likely in view of the fact that transient convective strain can occur time and again, so long as the density inversion exists with each transient lasting perhaps only a few days, there could be literally thousands of these events in the very old ice at depths below 2000 m in the Greenland and Antarctic ice sheets. These events would presumably be rare and random, although they may cluster above deep troughs in bed topography because ice thickness below the density inversion would tend to be greater at these sites. I cannot imagine radio-echo stratigraphy surviving relentless repetitions of transient thermal convection.

Robin and Millar (1982) described conditions under which the echo-free zone is observed, and offered an explanation based on ice flowing around subglacial mountains as well as over the mountains. After noting that the energy of radar pulses was sufficient to detect reflecting horizons in deep ice where none existed, they stated that the echo-free zone typically began about 50 km from ice divides, and they linked the zone to increased velocity gradients of advective flow with distance from ice divides. In the lower 5 to 250 m of ice thickness, three-dimensional flow of warm ice over and around subglacial hills and mountains causes continuous recrystallization in the continuously changing stress field imposed by rugged basal topography. Above this topography, two-dimensional flow in cold ice is con-
trolled by the surface slope and ice accumulation, and causes predominantly eastward flow in simple shear on the middle 30 to 50% of the ice thickness and predominantly extending flow in perhaps the top 30% of ice. These stress fields tend to produce a random ice fabric in the upper zone, a single-maximum ice fabric in the middle zone, and a zone-maximum or circle fabric in the lower zone.

I agree that rugged bed topography can cause ice to flow around hills and mountains as well as over them. All other ice flowing over these features inhibits radio-echo stratigraphy that is warped to conform with the topography, this stratigraphy is retained. The radio-echo stratigraphy in ice flowing around pre-existing mountains may show or be warped, but retained. It could even be argued that these warps would straighten out as ice moved into the lee of hills and mountains. How could the stratigraphy be completely obliterated to create an echo-free zone?

The "near-vertical cusps or fingers" in the transition between the echo-free zone and the unwarped radio-echo layers along an ice flow line near Vostok Station in East Antarctica, shown in figure 4 of Robin and Millar (1982), bear no relationship to bed topography. The gently rolling bed topographic features have an amplitude of up to 500 m and a wavelength of up to 6 km. These features are on a much finer scale, are highly irregular, and lie in a horizontal band that parallels the flat radio-echo layers above, not the rolling and undulating bedrock landscape below. The echo-free zone first identified by Roy and others (1970) in the lower 1000 m of ice over the plain and the lower 500 m of ice over the massif. Radio-echo stratigraphy above the echo-free zone is severely buckled with respect to the ice surface, where the stratigraphy consists of broad upward bows 3 km to 10 km in width separated by sharp downward cusps, for an overall vertical relief of some 500 m in the deepest radio-echo layers. Only one upward bow exists over the three summits of the ice near the surface. In addition, the pattern of thermal convection transforms from parabolic to convective flow in the echo-free zone.

The "near-vertical cusps or fingers" in the transition between the echo-free zone and the unwarped radio-echo layers along an ice flow line near Vostok Station in East Antarctica, shown in figure 4 of Robin and Millar (1982), bear no relationship to bed topography. The gently rolling bed topographic features have an amplitude of up to 500 m and a wavelength of up to 6 km. These features are on a much finer scale, are highly irregular, and lie in a horizontal band that parallels the flat radio-echo layers above, not the rolling and undulating bedrock landscape below. The echo-free zone first identified by Roy and others (1970) in the lower 1000 m of ice over the plain and the lower 500 m of ice over the massif. Radio-echo stratigraphy above the echo-free zone is severely buckled with respect to the ice surface, where the stratigraphy consists of broad upward bows 3 km to 10 km in width separated by sharp downward cusps, for an overall vertical relief of some 500 m in the deepest radio-echo layers. Only one upward bow exists over the three summits of the ice near the surface. In addition, the pattern of thermal convection transforms from parabolic to convective flow in the echo-free zone.

The "near-vertical cusps or fingers" in the transition between the echo-free zone and the unwarped radio-echo layers along an ice flow line near Vostok Station in East Antarctica, shown in figure 4 of Robin and Millar (1982), bear no relationship to bed topography. The gently rolling bed topographic features have an amplitude of up to 500 m and a wavelength of up to 6 km. These features are on a much finer scale, are highly irregular, and lie in a horizontal band that parallels the flat radio-echo layers above, not the rolling and undulating bedrock landscape below. The echo-free zone first identified by Roy and others (1970) in the lower 1000 m of ice over the plain and the lower 500 m of ice over the massif. Radio-echo stratigraphy above the echo-free zone is severely buckled with respect to the ice surface, where the stratigraphy consists of broad upward bows 3 km to 10 km in width separated by sharp downward cusps, for an overall vertical relief of some 500 m in the deepest radio-echo layers. Only one upward bow exists over the three summits of the ice near the surface. In addition, the pattern of thermal convection transforms from parabolic to convective flow in the echo-free zone.

Two important papers by Bentley (1971[a],[b]) bear on the possibility of thermal convection in West Antarctic ice sheet. A region in which deep ice is often so warm that absorption losses obscure radio-echo stratigraphy. Bentley (1971[a]) reported widespread grain control. An echo-free zone, a tendency for the optic axes of grains to change progressively from a near-vertical orientation near ice divides to a near-horizontal orientation in the downslope direction of advective flow, is characteristic of the upper 30% of ice. The "near-vertical cusps or fingers" in the transition between the echo-free zone and the unwarped radio-echo layers along an ice flow line near Vostok Station in East Antarctica, shown in figure 4 of Robin and Millar (1982), bear no relationship to bed topography. The gently rolling bed topographic features have an amplitude of up to 500 m and a wavelength of up to 6 km. These features are on a much finer scale, are highly irregular, and lie in a horizontal band that parallels the flat radio-echo layers above, not the rolling and undulating bedrock landscape below. The echo-free zone first identified by Roy and others (1970) in the lower 1000 m of ice over the plain and the lower 500 m of ice over the massif. Radio-echo stratigraphy above the echo-free zone is severely buckled with respect to the ice surface, where the stratigraphy consists of broad upward bows 3 km to 10 km in width separated by sharp downward cusps, for an overall vertical relief of some 500 m in the deepest radio-echo layers. Only one upward bow exists over the three summits of the ice near the surface. In addition, the pattern of thermal convection transforms from parabolic to convective flow in the echo-free zone.

Two important papers by Bentley (1971[a],[b]) bear on the possibility of thermal convection in West Antarctic ice sheet. A region in which deep ice is often so warm that absorption losses obscure radio-echo stratigraphy. Bentley (1971[a]) reported widespread grain control. An echo-free zone, a tendency for the optic axes of grains to change progressively from a near-vertical orientation near ice divides to a near-horizontal orientation in the downslope direction of advective flow, is characteristic of the upper 30% of ice. The "near-vertical cusps or fingers" in the transition between the echo-free zone and the unwarped radio-echo layers along an ice flow line near Vostok Station in East Antarctica, shown in figure 4 of Robin and Millar (1982), bear no relationship to bed topography. The gently rolling bed topographic features have an amplitude of up to 500 m and a wavelength of up to 6 km. These features are on a much finer scale, are highly irregular, and lie in a horizontal band that parallels the flat radio-echo layers above, not the rolling and undulating bedrock landscape below. The echo-free zone first identified by Roy and others (1970) in the lower 1000 m of ice over the plain and the lower 500 m of ice over the massif. Radio-echo stratigraphy above the echo-free zone is severely buckled with respect to the ice surface, where the stratigraphy consists of broad upward bows 3 km to 10 km in width separated by sharp downward cusps, for an overall vertical relief of some 500 m in the deepest radio-echo layers. Only one upward bow exists over the three summits of the ice near the surface. In addition, the pattern of thermal convection transforms from parabolic to convective flow in the echo-free zone.

Two important papers by Bentley (1971[a],[b]) bear on the possibility of thermal convection in West Antarctic ice sheet. A region in which deep ice is often so warm that absorption losses obscure radio-echo stratigraphy. Bentley (1971[a]) reported widespread grain control. An echo-free zone, a tendency for the optic axes of grains to change progressively from a near-vertical orientation near ice divides to a near-horizontal orientation in the downslope direction of advective flow, is characteristic of the upper 30% of ice. The "near-vertical cusps or fingers" in the transition between the echo-free zone and the unwarped radio-echo layers along an ice flow line near Vostok Station in East Antarctica, shown in figure 4 of Robin and Millar (1982), bear no relationship to bed topography. The gently rolling bed topographic features have an amplitude of up to 500 m and a wavelength of up to 6 km. These features are on a much finer scale, are highly irregular, and lie in a horizontal band that parallels the flat radio-echo layers above, not the rolling and undulating bedrock landscape below. The echo-free zone first identified by Roy and others (1970) in the lower 1000 m of ice over the plain and the lower 500 m of ice over the massif. Radio-echo stratigraphy above the echo-free zone is severely buckled with respect to the ice surface, where the stratigraphy consists of broad upward bows 3 km to 10 km in width separated by sharp downward cusps, for an overall vertical relief of some 500 m in the deepest radio-echo layers. Only one upward bow exists over the three summits of the ice near the surface. In addition, the pattern of thermal convection transforms from parabolic to convective flow in the echo-free zone.

Two important papers by Bentley (1971[a],[b]) bear on the possibility of thermal convection in West Antarctic ice sheet. A region in which deep ice is often so warm that absorption losses obscure radio-echo stratigraphy. Bentley (1971[a]) reported widespread grain control. An echo-free zone, a tendency for the optic axes of grains to change progressively from a near-vertical orientation near ice divides to a near-horizontal orientation in the downslope direction of advective flow, is characteristic of the upper 30% of ice. The "near-vertical cusps or fingers" in the transition between the echo-free zone and the unwarped radio-echo layers along an ice flow line near Vostok Station in East Antarctica, shown in figure 4 of Robin and Millar (1982), bear no relationship to bed topography. The gently rolling bed topographic features have an amplitude of up to 500 m and a wavelength of up to 6 km. These features are on a much finer scale, are highly irregular, and lie in a horizontal band that parallels the flat radio-echo layers above, not the rolling and undulating bedrock landscape below. The echo-free zone first identified by Roy and others (1970) in the lower 1000 m of ice over the plain and the lower 500 m of ice over the massif. Radio-echo stratigraphy above the echo-free zone is severely buckled with respect to the ice surface, where the stratigraphy consists of broad upward bows 3 km to 10 km in width separated by sharp downward cusps, for an overall vertical relief of some 500 m in the deepest radio-echo layers. Only one upward bow exists over the three summits of the ice near the surface. In addition, the pattern of thermal convection transforms from parabolic to convective flow in the echo-free zone.
flow at depth. An anonymous referee stated, "I am not familiar enough with the echo-sounding data to have any grasp of the systematics of where echo-free zones are and how thick they are." He noted that this information is needed to properly assess the prospects for thermal convection. I also am not properly informed in this regard, and I invite those who are to address the referee's question.

In considering the possibility of an initially infinite Rayleigh number, the referee concluded that the initially infinite rate of transient creep may include "bound viscouselastic straining that is recovered on unloading" and if this is the case, "we would expect to see thermal convection in elastic bodies." In response, I would say that if thermal convection is defined as heat transport by mass transport in a circulatory motion, then thermal conduction can be regarded as a limiting case of thermal convection in which heat and mass transport are restricted and atoms vibrating (circulating) about their lattice sites and bumping into each other. When some atoms break free from their lattice sites and move as dislocations through the lattice, then the more general conditions for thermal convection become possible. During secondary steady-state creep, thermal convection exists as a limiting case in which atoms circulate within large, dense, and mobile blocks under a buoyancy stress. Between the extremes of microscopic and instantaneous "elastic convection" and macroscopic and steady-state "Rayleigh convection" lies "transient convection." If transient convection can be considered as an important indirect indication of weak but sustained thermal convection at depth, this migration must be distinguished from dune-migration of surface undulations exposed to katabatic winds and from down-slope migration of kinematic waves.

An important question is the relationship between the basal echo-free zone and buckled radio-echo stratigraphy above the echo-free zone on the one hand, and the density inversion and basal bedrock topography on the other hand. If the correlation is primarily with the height of the density inversion, thermal convection becomes an attractive possibility. However, if the correlation is primarily with bedrock topography, advective flow probably provides an adequate explanation. The same test should apply, I think, to the evidence for seismic anisotropy and the basal layer revealed by low-amplitude seismic reflections that Bentley (1971[a],b) reported to be widespread in West Antarctica. In applying this test, it is important to realize that neither transient nor steady-state convection produces a density inversion. Transient primary convection is probably too weak to influence advective flow at the height of the density inversion. Steady-state secondary convection may influence advective flow, but the density inversion because this represents a situation intermediate between free and rigid boundary conditions.

I am particularly intrigued by the possibility that lateral shear zones alongside ice streams may be related to the descending limb of thermal convection rolls aligned with stream flow. Simple shear from longitudinal advective flow and vertical convective flow combine in these zones to produce the easy-glide grain fabric and to create a warm vertical wall of ice. The ice stream lies at a lower elevation than its flanking shear zones, as if it constituted the descending limb of convective flow between ascending limbs in the shear zones. Longitudinal and graben structures on ice streams imply transverse tension, as if a lateral component of flow toward the shear zones was present. A link between thermal convection and ice streams is also implied by the observation that many ice streams move across a subglacial landscape that has no clear relationship to the development of stream-flow. Thermal convection may play an important role in ice-stream dynamics and, through ice-stream surges, in the stability of the West Antarctic ice sheet.

Evidence for thermal convection in basal ice of former ice sheets may exist in certain glacial landforms deposited on a wet bed. Basal melting should occur beneath ascending columns at the centers of convection cells, leaving drumlin-like accumulations of eroded material entrained in basal regulation ice. This association between drumlin fields and steady-state convection cells in former ice sheets has been suggested by Dand's (1961). Basal melting beneath ascending walls in convection rolls should leave esker-like accumulations of eroded material entrained in basal regulation ice. Convolution rolls should radiate from domes of former ice sheets, and long eskers radiate from the Keewatin and New Quebec (Labrador-Ungava) spreading centers of the late Wisconsin Laurentide ice sheet. Convective flow should be considered when assigning tags of glacial stratigraphy. Im mobile, but undulations may migrate with advective flow, provided the surface undulations lie transverse to advective flow (that is, the advective component does not spiral down-slope in aligned convection rolls). Down-slope migration of transverse undulations could be an important indirect explanation of stratigraphic lineations in bedrock pavements formed by former ice sheets.

My earlier work on ice-sheet convection was largely uninformed and I prefer not to reference it. For example, I should have realized that ice effectivity viscosities are far too high to allow the dike-still convection I once proposed. Moreover, sills of warm ice injected beneath ice streams could not account for the depth of the echo-free zone. For all that, I still would ask, by closing our minds to the possibility of ice-sheet convection, have we glaciologists fallen heir to the judgment of the apostle Paul: "You may listen carefully yet you will never understand; if the sea listens it will never see". (Acts XXIII 26)? That is the question.

REFERENCES

Hughes: Thermal convection in ice sheets


Budd, W.F., and others. 1971. Derived physical characteristics of the Antarctic ice sheet, by W.F. Budd, D. Jennisen, and U. Radok. ANARE Interim Reports. Ser. M[IV], Glaciology, Publication No. 120.
